

Question 1:

**A. Convert the following numbers to their decimal representation.
Show your work.**

1. $10011011_2 =$
 $1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 =$
 $128 + 0 + 0 + 16 + 8 + 0 + 2 + 1 =$
Answer: 155_{10}

2. $456_7 =$
 $4 \times 7^2 + 5 \times 7^1 + 6 \times 7^0 =$
 $196 + 35 + 6 =$
Answer: 237_{10}

3. $38A_{16} =$
 $3 \times 16^2 + 8 \times 16^1 + 10 \times 16^0 =$
 $768 + 128 + 10 =$
Answer: 906_{10}

4. $2214_5 =$
 $2 \times 5^3 + 2 \times 5^2 + 1 \times 5^1 + 4 \times 5^0 =$
 $125 + 50 + 5 + 4 =$
Answer: 184_{10}

B. Convert the following numbers to their binary representation:

1. $69_{10} =$
 $69/2 = 34 \text{ remainder } 1 \mid 34/2 = 17 \text{ remainder } 0 \mid 17/2 = 8 \text{ remainder } 1$
 $\mid 8/2 = 4 \text{ remainder } 0 \mid 4/2 = 2 \text{ remainder } 0 \mid 2/2 = 1 \text{ remainder } 0 \mid$
 $1/2 = 0 \text{ remainder } 1$
Answer: 1000101_2

2. $485_{10} =$
 $485/2 = 242 \text{ remainder } 1 \mid 242/2 = 121 \text{ remainder } 0 \mid 121/2 = 60 \text{ remainder } 1$
 $\mid 60/2 = 30 \text{ remainder } 0 \mid 30/2 = 15 \text{ remainder } 0 \mid 15/2 = 7 \text{ remainder } 1 \mid$
 $7/2 = 3 \text{ remainder } 1 \mid 3/2 = 1 \text{ remainder } 1 \mid 1/2 = 0 \text{ remainder } 1$
Answer: 111100101_2

3. $6D1A_{16} =$
 $6 = 0110 \mid D = 1101 \mid 1 = 0001 \mid A = 1010$
Answer: 0110110100011010_2

C. Convert the following numbers to their hexadecimal representation:

1. $1101011_2 =$
 $110 = 6 \mid 1011 = B$
Answer: $6B_{16}$

2. $895_{10} =$
 $895/16 = 55 \text{ remainder } 15 \mid 55/16 = 3 \text{ remainder } 7 \mid 3/16 = 0 \text{ remainder } 3$
Answer: $37F_{16}$

Question 2:

Solve the following, do all calculations in the given base. Show your work.

1. $7566_8 + 4515_8 =$

$$\begin{array}{r} 111 \\ 7566_8 \\ 4515_8 \\ + \text{----} \end{array}$$

Answer: 14303_8

2. $10110011_2 + 1101_2 =$

$$\begin{array}{r} 111111 \\ 10110011_2 \\ 00001101_2 \\ + \text{-----} \end{array}$$

Answer: 11000000_2

3. $7A66_{16} + 45C5_{16} =$

$$\begin{array}{r} 11 \\ 7A66_{16} \\ 45C5_{16} \\ + \text{-----} \end{array}$$

Answer: $C02B_{16}$

4. $3022_5 - 2433_5 =$

$$\begin{array}{r} 46 \\ 2517 \\ 3022_5 \\ 2433_5 \\ - \text{-----} \end{array}$$

Answer: $0034_5 \mid 34_5$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} =$

$$\begin{array}{l} 124/2 = 62 \text{ remainder } 0 \mid 62/2 = 31 \text{ remainder } 0 \mid 31/2 = 15 \text{ remainder } 1 \mid 15/2 = 7 \text{ remainder } 1 \mid 7/2 = 3 \text{ remainder } 1 \mid 3/2 = 1 \text{ remainder } 1 \mid 1/2 = 0 \text{ remainder } 1 \end{array}$$

Answer: 101111100
2'S COMPLEMENT

2. $-124_{10} =$

$$\begin{array}{r} 1111 \\ 01111100 \\ + 10000100 \\ \hline 100000000 \end{array}$$

Answer: 10000100
2'S COMPLEMENT

3. $109_{10} =$

$$\begin{array}{l} 109/2 = 54 \text{ remainder } 1 \mid 54/2 = 27 \text{ remainder } 0 \mid 27/2 = 13 \text{ remainder } 1 \mid 13/2 = 6 \text{ remainder } 1 \mid 6/2 = 3 \text{ remainder } 0 \mid 3/2 = 1 \text{ remainder } 1 \mid 1/2 = 0 \text{ remainder } 1 \end{array}$$

Answer: 01101101
2'S COMPLEMENT

4. $-79_{10} =$

$$\begin{array}{l} 79/2 = 39 \text{ remainder } 1 \mid 39/2 = 19 \text{ remainder } 1 \mid 19/2 = 9 \text{ remainder } 1 \mid 9/2 = 4 \text{ remainder } 1 \mid 4/2 = 2 \text{ remainder } 0 \mid 2/2 = 1 \text{ remainder } 0 \mid 1/2 = 0 \text{ remainder } 1 \end{array}$$

$$\begin{array}{r} 1001111_2 \\ 1111111 \\ 01001111 \\ + 10110001 \\ \hline 100000000 \end{array}$$

Answer: 10110001
2'S COMPLEMENT

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8 \text{ bit } 2's \text{ comp}} =$

$$\begin{array}{l} 00011110_2 = \\ 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = \\ 0 + 0 + 16 + 8 + 4 + 2 + 0 = \end{array}$$

Answer: 30
₁₀

2. $11100110_{8 \text{ bit } 2's \text{ comp}} =$

$$\begin{array}{r} 111111 \\ 11100110 \\ + 00011010 \end{array}$$

100000000

$$0011010_2 = 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 =$$

$$0 + 0 + 16 + 8 + 0 + 2 + 0 =$$

Answer: -26_{10}

3. $00101101_{8 \text{ bit } 2's \text{ comp}} =$

$$0101101_2 =$$

$$0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 =$$

$$0 + 32 + 0 + 8 + 4 + 0 + 1 =$$

Answer: 45_{10}

4. $10011110_{8 \text{ bit } 2's \text{ comp}} =$

111111

10011110

+01100010

100000000

$$11000100_2 =$$

$$1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 =$$

$$64 + 32 + 0 + 0 + 0 + 2 + 0 =$$

Answer: -98_{10}

Question 4: Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, sections b, c

b. $\neg(p \vee q)$ **Answer:**

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

c. $r \vee (p \vee \neg q)$ **Answer:**

p	q	r	$\neg q$	$p \vee \neg q$	$r \vee (p \vee \neg q)$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	T

2.Exercise 1.3.4,sections b, d

b. $(p \rightarrow q) \rightarrow (q \rightarrow p)$

Answer:

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

d. $(p \leftrightarrow q) \quad (p \leftrightarrow \neg q)$

Answer:

p	q	$\neg q$	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \quad (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5: Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7, sections b, c

b. **Answer:** $(b \rightarrow d) \vee (b \rightarrow m) \vee (m \rightarrow d)$

c. **Answer:** $b \vee (d \rightarrow m)$

2. Exercise 1.3.7, sections b – e

b. **Answer:** $(s \vee y) \rightarrow p$

c. **Answer:** $p \rightarrow y$

d. **Answer:** $p \leftrightarrow (s \rightarrow y)$

e. **Answer:** $p \rightarrow (s \vee y)$

3. Exercise 1.3.9, sections c, d

c. **Answer:** $c \rightarrow p$

d. **Answer:** $p \rightarrow c$

Question 6: Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6, sections b - d

b. **Answer:** **If** Joe maintains a B average, **then** Joe is eligible for the honors program.

c. **Answer:** **If** Rajiv can go on the roller coaster, **then** he is at least four feet tall.

d. **Answer:** **If** Rajiv is at least four feet tall, **then** he can go on the roller coaster.

2. Exercise 1.3.10, sections c - f

c. **Answer:** **False!** Both the hypothesis and the conclusion must be either true or false.

d. **Answer:** **Unknown!** IF r is false the expression is true, IF r is true the expression is false.

e. **Answer:** **Unknown!** IF r is false the expression is false, IF r is true the expression is true.

f. **Answer:** **True!** IF r is either true or false the expression is always true.

Question 7: Solve Exercise 1.4.5, sections b - d, from the Discrete Math zyBook

b. **Answer:**

j	l	r	$\neg j \rightarrow (l \vee \neg r)$	$(r \vee \neg l) \rightarrow j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

c. **Answer:**

j	l	$j \rightarrow \neg l$	$\neg j \rightarrow l$
T	T	T	F
T	F	T	T
F	T	T	T
F	F	F	T

d. **Answer:**

j	l	r	$(r \vee \neg l) \rightarrow j$	$j \rightarrow (r \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Question 8: Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2, sections c, f, i

c. **Answer:** $(p \rightarrow q) \rightarrow (p \rightarrow r) \equiv p \rightarrow (q \rightarrow r)$

$(p \rightarrow q) \rightarrow (p \rightarrow r)$	
$(\neg p \vee q) \rightarrow (\neg p \vee r)$	Conditional Identities Law
$\neg p \rightarrow (q \rightarrow r)$	Distributive Law
$p \rightarrow (q \rightarrow r)$	Conditional Identities Law

f. **Answer:** $\neg(p \vee (\neg p \rightarrow q)) \equiv \neg p \rightarrow \neg q$

$\neg(p \vee (\neg p \rightarrow q))$	
$\neg p \rightarrow \neg(\neg p \rightarrow q)$	DeMorgan's Law
$\neg p \rightarrow \neg\neg p \vee \neg q$	DeMorgan's Law
$\neg(p \rightarrow p) \vee \neg q$	Double Negation Law
$\neg p \rightarrow \neg q$	Idempotent Law

i. **Answer:** $(p \rightarrow q) \rightarrow r \equiv (p \rightarrow \neg r) \rightarrow \neg q$

$(p \rightarrow q) \rightarrow r$	
$\neg(p \rightarrow q) \vee r$	Conditional Identities
$\neg p \vee \neg q \vee r$	DeMorgan's Law
$\neg(p \vee r) \vee \neg q$	Associative Law
$\neg p \rightarrow \neg r \vee \neg q$	DeMorgan's Law
$\neg p \rightarrow \neg\neg r \vee \neg q$	Double Negation Law

$\neg q$	
$\neg(p \neg r) \vee \neg q$	DeMorgan's Law
$(p \neg r) \rightarrow \neg q$	Conditional Identities

2. Exercise 1.5.3, sections c, d

c. **Answer:** $\neg r \vee (\neg r \rightarrow p)$

$\neg r \vee (\neg r \rightarrow p)$	
$\neg r \vee (\neg \neg r \vee p)$	Conditional Identities
$\neg r \vee (r \vee p)$	Double Negation Law
$(\neg r \vee r) \vee p$	Associative Law
$T \vee p$	Complement Law
T	Domination Law

d. **Answer:** $\neg(p \rightarrow q) \rightarrow \neg q$

$\neg(p \rightarrow q) \rightarrow \neg q$	
$\neg(\neg p \vee q) \rightarrow \neg q$	Conditional Identities
$\neg \neg p \vee \neg q \rightarrow \neg q$	DeMorgan's Law
$p \vee \neg q \rightarrow \neg q$	Double Negation Law
$\neg(p \vee \neg q) \vee \neg q$	Conditional Identities
$\neg p \vee \neg \neg q \vee \neg q$	DeMorgan's Law
$\neg p \vee q \vee \neg q$	Double Negation Law

$\neg p \vee (q \vee \neg q)$	Associative Law
$\neg p \vee T$	Complement Law
$\neg T$	Domination Law
F	Complement Law

Question 9: Solve the following questions from the Discrete Math zyBook: 1. Exercise 1.6.3, sections c, d

c. There is a number that is equal to its square.

Answer: $\exists x (x^2=x)$

d. Every number is less than or equal to its square.

Answer: $\forall x (x \leq x^2)$

2. Exercise 1.7.4, sections b - d

b. **Answer:** $\forall x (\neg S(x) \rightarrow W(x))$

c. **Answer:** $\forall x (S(x) \rightarrow \neg W(x))$

d. **Answer:** $\exists x (S(x) \wedge W(x))$

Question 10: Solve the following questions from the Discrete Math zyBook: 1. Exercise 1.7.9, sections c - i

- c. **Answer:** True
 - d. **Answer:** When $x = e$, True
 - e. **Answer:** True
 - f. **Answer:** True
 - g. **Answer:** False
 - h. **Answer:** True
 - i. **Answer:** True
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2. Exercise 1.9.2, sections b - i

- b. **Answer:** True
 - c. **Answer:** True
 - d. **Answer:** False
 - e. **Answer:** False
 - f. **Answer:** True
 - g. **Answer:** False
 - h. **Answer:** True
 - i. **Answer:** True
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Question 11: Solve the following questions from the Discrete Math zyBook:

1.10.4, sections c - g

- c. **Answer:** $\exists x \exists y (x + y = xy)$
 - d. **Answer:** $\forall x \forall y ((x > 0 \wedge y > 0) \rightarrow x/y > 0)$
 - e. **Answer:** $\forall x ((x > 0 \wedge x < 1) \rightarrow 1/x > 1)$
 - f. **Answer:** $\neg \exists x \forall y (x \leq y)$
 - g. **Answer:** $\forall x \exists y (x \neq 0 \rightarrow xy = 1)$
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1.10.7, sections c - f

- c. **Answer:** $\exists x (N(x) \rightarrow D(x))$
 - d. **Answer:** $\exists x \forall y D(y) \rightarrow P(\text{Sam}, y)$
 - e. **Answer:** $\exists x \forall y N(x) \rightarrow P(x, y)$
 - f. **Answer:** $\exists x (N(x) \wedge \forall y ((x \neq y) \rightarrow \neg L(y)))$
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1.10.10, sections c - f

- c. **Answer:** $\forall x \exists y ((y \neq \text{Math } 101) \wedge T(x, y))$
 - d. **Answer:** $\exists x \forall y ((y \neq \text{Math } 101) \rightarrow T(x, y))$
 - e. **Answer:** $\forall x \exists y \exists z ((x \neq \text{Sam}) \rightarrow ((y \neq z) \wedge T(x, y) \wedge T(x, z)))$
 - f. **Answer:** $\exists y \exists z \forall w ((z \neq y) \wedge T(\text{Sam}, y) \wedge T(\text{Sam}, z) \wedge ((w \neq y \wedge w \neq z) \rightarrow \neg T(\text{Sam}, w)))$
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Question 12: Solve the following questions from the Discrete Math zyBook:

1.8.2, sections b – e

b. **Answer:** $\forall x (D(x) \vee P(x))$

Negation: $\neg \forall x (D(x) \vee P(x))$

Applying De Morgan's law: $\exists x (\neg D(x) \wedge \neg P(x))$

English: There is a patient who was not given the medication and not given the placebo.

c. **Answer:** $\exists x (D(x) \wedge M(x))$

Negation: $\neg \exists x (D(x) \wedge M(x))$

Applying De Morgan's law: $\forall x (\neg D(x) \vee \neg M(x))$

English: Every patient did not get the medication or did not have migraines or both.

d. **Answer:** $\forall x (P(x) \rightarrow M(x))$

Negation: $\neg \forall x (P(x) \rightarrow M(x))$

Applying De Morgan's law: $\exists x (P(x) \wedge \neg M(x))$

English: Some patient took the placebo and did not have migraines

e. **Answer:** $\exists x (M(x) \wedge P(x))$

Negation: $\neg \exists x (M(x) \wedge P(x))$

Applying De Morgan's law: $\forall x (\neg M(x) \vee \neg P(x))$

English: Every patient either did not have migraine or was not given the placebo or both.

1.9.4, sections c – e

c. **Answer:** $\exists x \forall y (P(x, y) \rightarrow Q(x, y))$

Negation: $\forall x \exists y (P(x, y) \wedge \neg Q(x, y))$

d. **Answer:** $\exists x \forall y (P(x, y) \leftrightarrow P(y, x))$

$\neg(\exists x \forall y ((P(x, y) \rightarrow P(y, x)) \wedge (P(y, x) \rightarrow P(x, y))))$

$\neg(\exists x \forall y ((\neg P(x, y) \vee P(y, x)) \wedge (\neg P(y, x) \vee P(x, y))))$

$\forall x \exists y (\neg (\neg P(x, y) \vee P(y, x)) \vee \neg (\neg P(y, x) \vee P(x, y)))$

Negation: $\forall x \exists y ((P(x, y) \wedge \neg P(y, x)) \vee (P(y, x) \wedge \neg P(x, y)))$

e. **Answer:** $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

$\neg(\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y))$

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Negation: $\forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y)$
