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Question 5:

1.12.2.B.
Answer:

1. Hypothesis : $\neg q$
2. Addition: $\neg q \lor \neg r$
3. Morgan's Law 2: $\neg(q \land r)$
4. Hypothesis: $p \to (q \land r)$
5. Modus tollens 3, 4: $\neg p$

1.12.2.E.

Answer:
1. Hypothesis : $p \lor q$
2. Hypothesis :$\neg q$
3. Disjunctive 1,2:p
4. Hypothesis: $\neg p \lor r$
5. Disjunctive 3,4: r

1.12.3.C.
Answer:
1. Hypothesis :$p \lor q$
2. Double negation 1: $\neg\neg p \lor q$
3. Conditional identity 2: $\neg p \to q$
4. Hypothesis: $\neg p$
5. Modus ponens 3, 4: q

1.12.5.C.

Answer:
$(c \land h) \to j$
$\neg j$

$\neg c$
1. Hypotheses: $(c \land h) \to j$
2. Hypotheses: $\neg j$
3. Modus Tollens 1,2: $\neg(c \land h)$
4. Morgan's law 3: $\neg c \lor \neg h$

1.12.5.D.

Answer:
$(c \land h) \to j$
$\neg j$
h

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¬c
1. Hypotheses: (c ∧ h) → j
2. Hypotheses: ¬ j
3. Modus Tollens 1,2: ¬(c ∧ h)
4. Morgan's Law 3 : ¬c ∨ ¬h
5. Hypothesis: h
6. Double negation 5: ¬¬h
7.Disjunctive Syllogism 4,6 : ¬c

1.13.5.D.

Answer:

-M(x): x misses the class
-D(x): x got detention

∀x(M(x) →D(x))
Penelope, is student in the class
¬M(Penelope)

¬D(Penelope)

	M	D	∀x(M(x) →D(x))	¬M(Penelope)	¬D(Penelope)
a	F	T	T	T	F

It is false if we give a value of M=F and D=T, both hypotheses will result true but the conclusions turn out to be false.

1.13.5.E.

Answer:
M(x): x misses the class
D(x): x got detention
A(x): x got an A
∀x((M(x) ∨ D(x)) → ¬A(x))
Penelope, is a student in the class
A(Penelope)

¬D(Penelope)

1. Hypothesis : ∀x((M(x) ∨ D(x)) → ¬A(x))
2. Hypothesis : Penelope, a student in the class
3. Universal Instantiation 1: ((M(Penelope) ∨ D(Penelope)) → ¬A(Penelope))
4. Hypothesis: A(Penelope)

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5. Double Negation: $\neg\neg A(Penelope)$
6. Modus Tollens 3,5: $\neg (M(Penelope) \vee D(Penelope))$
7. Morgans Law 6: $\neg M(Penelope) \wedge \neg D(Penelope)$
8. Simplification: $\neg D(Penelope)$

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Question 6:
2.4.1.D.
Answer:
The product of two odd integers is an odd integer.
Proof.

1. Let x and y be odd integers numbers. We will show that xy is also an odd integer.
2. Since x is an odd integer, there is an integer k such that x =2k +1. And, since y is an odd integer, there is an integer j such that y =2j +1
3. xy = (2k +1)(2j +1)
4. (2k +1)(2j +1) = 4kj + 2k +2j +1 = 2(2kj+k+j)+1
5. Since k and j are integers, then 2kj+k+j are integers.
6. Since xy is equal to 2m+1, where m=2kj+k+j is an integer, xy is odd.

2.4.1.B.
Answer:
If x is a real number and x≤3, then 12-7x+x2≥0.
Proof.

1. Let x be a real number and x≤3. We will prove that 12-7x+x2≥0.
2. Since x≤3, then 0≤3-x
3. Since 4 > 3, then 4-x > 3-x ≥ 0
4. Since (x-4) and (x-3) are both positive. The product of (x-4) and (x-3) is also positive.
5. (x-4)(x-3) ≥ 0. Multiplying out the expression on the left gives that 12-7x+x2≥0

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Question 7:
2.5.1.D.
Answer:
For every integer n, if n2-2n+7 is even, then n is odd
Proof.

1. Let n be an integer. If n is an even, then we will prove that n2-2n+7 is odd.
2. Since n is an even integer, there is an integer k that x =2k. Since n2-2n+7 is odd, there is an integer j that n2-2n+7 = 2j + 1.
3. n2 – 2n + 7 = (2k)^2-2(2k)+7 = 4k^2 -4k + 7 = 4k^2 -4k + 6 +1) = 2(2k-2k+6)+1
4. Since m is integer and m=2k-2k+6, then 2m + 1 is an odd integer.
5. So n2-2n+7 = 2m + 1 which makes n2-2n+7 an odd.

2.5.4.A.
Answer:
For every pair of real numbers x and y, if x3+xy2≤x2y+y3, then x≤y.
Proof.

1. Let x and y be a pair of real numbers.
   a. If x > y, then x-y>0
   b. Prove x3+xy2 > x2y +y3
2. x(x2+y2) > y(x2+y2) -> x(x2+y2) – y(x2+y2) > 0
3. Since x2 +y2 is a positive number, then x-y>0.

Statement is true

2.5.4.B.
Answer:
B. **Answer:**
For every pair of real numbers x and y, if x+y>20, then x>10 or y>10.
Proof.
1. Let x and y be a pair of real numbers.
2. If x <= 10 and y <= 10, we will prove that x + y <=20.
3. If x = 10 and y = 10, then x+y<=20.

Statement is true

2.5.5.C.
Answer:
For every non-zero real number x, if x is irrational, then 1/x is also irrational.
Proof.

1. Let x be a non-zero real number such that 1/x is not irrational. We will prove that x is not irrational
2. Since x != 0, and 1/x is not irrational then 1/x is rational.
3. Therefore, 1/x = a/b,for some two integers and b where b !=0.
4. x=b/a 5. Since a and b are integers and a!=0 and b!=0, then x>0.
6. x is equal to the inverse of its rational with non-zero denominator, so x is a rational number. Therefore r is not rational.

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Question 8:
2.6.6.C.
Answer:
The average of three real numbers is greater than or equal to at least one of the numbers.
Proof.

- There are real numbers a, b, and c such that all three numbers are less than the average of the three numbers.
- Let x,y,z be three real numbers which is less than any of the three numbers.
- Assume that A = (x+y+z)/3 is the average of three real numbers.
- If y is less than all three numbers as:

  $A < x$
  $A < y$
  $A < z$
- Then plugging in the average equation it will be

  $A < (A + A + A)/3$
  $A < 3A/3$
  $A < A$
- It is a contradiction proof, which proves the average of three real numbers is greater than or equal to at least one of the numbers

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2.6.6.D.
Answer:
There is no smallest integer
Proof.

- Assume that there is a smallest integer. Let it be r.
- If r is an integer, then r -1 is an integer. Therefore, r>r-1.
- This contradicts the fact that r is the smallest number

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Question 9:
2.7.2.B.
Answer:
If integers x and y have the same parity, then x+y is even.The parity of a number tells whether the number is odd or even. If x and y have the same parity, they are either both even or both odd.

Proof:

Let x and y be integer numbers, which x + y is even.

Case 1:
- Since x is odd which k is an integer such that x =2k +1. Since y is odd which j is an integer such that y= 2j + 1.
- (2j+1)+(2k+1) = 2j + 2k + 2= 2(j+k+1)
- Since m is an integer, an m = j + k +1. Then 2m is an even.
- Therefore x + y = 2m, which is an even number.

Case 2:
- Since x is even which k is an integer such that x =2k. Since y is even which j is an integer such that y= 2j .
- (2j)+(2k) = 2j + 2k = 2(j+k)
- Since m is an integer, an m = j + k. Then 2m is an even.
- Therefore x + y = 2m, which is an even number.
