

Question 7:

3.1.1.A

Answer:

True. 27 is multiple of 3, so 27 is an element of A.

3.1.1.B

Answer:

False. The square root of 27 is not an integer, so 27 is not a perfect square, therefore not an element of B.

3.1.1.C

Answer:

True. The square root of 100 is 10 an integer, so 100 is a perfect square, therefore an element of B.

3.1.1.D

Answer:

False. 3 is an element of E but not an element of C, so $E \not\subseteq C$. 4, 5, and 10 are elements of C but not elements of E, so $C \not\subseteq E$.

3.1.1.E

Answer:

True. every element of E is an integer multiple of 3, so every element of E is an element of A, therefore $E \subseteq A$. (f) $A \subset E$

3.1.1.F

Answer:

False. For example, 12 is an element of A but not an element of E, so A is not a subset of E.

3.1.1.G

Answer:

False. The symbol should be \subseteq , not \in , as \in is used to indicate that an element is in a set. $\in \in$

3.1.2.A

Answer:

F. $15 \in A$, but 15 is not a subset of A since 15 is an element of A

3.1.2.B

Answer:

True. $\{15\} \subseteq A$ and there is an element of A (for example, 27) that is not an element of $\{15\}$, so $\{15\}$ is proper set of A.

3.1.2.C

Answer:

True. $\emptyset \neq A$ and $\emptyset \subseteq A$, so \emptyset is a proper subset of A

3.1.2.D

Answer:

True. since every set is a subset of itself, the statement is true

3.1.2.E

Answer:

False. \emptyset is an empty set which cannot be an element of B

3.1.5.B

Answer:

{ $x \in \mathbb{Z} : x = 3n, n \in \mathbb{Z}$ }; the set is infinite

3.1.5.D

Answer:

{ $x \in \mathbb{Z} : x$ is an integer multiple of 10 and $0 \leq x \leq 1000$ }; the cardinality is 101

3.2.1.A

Answer:

True. 2 is an element of X

3.2.1.B

Answer:

True. Since 2 is an element of X, {2} is a subset of X.

3.2.1.C

Answer:

False. {2} is not an element of X.

3.2.1.D

Answer:

False. 3 is not an element of X.

3.2.1.E

Answer:

True. {1, 2} is an element of X.

3.2.1.F

Answer:

True. 1 and 2 are both elements of X, so {1, 2} is subset of X

3.2.1.G

Answer:

True. 2 and 4 are both element of X, so {2, 4} is a subset of X.

3.2.1.H

Answer:

False. {2, 4} is not an element of X.

3.2.1.I

Answer:

False. 3 is not an element of X , so $\{2, 3\}$ is not a subset of X .

3.2.1.J

Answer:

False. $\{2, 3\}$ is not an element of X .

3.2.1.K

Answer:

False. X contains 6 elements.

Question 8:

3.2.4.D.

Answer:

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Therefore, $X = \{\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$

Question 9:

3.3.1.C

Answer:

| |
|---|
| $A = \{-3, 0, 1, 4, 17\}$ |
| $C = \{x \in \mathbb{Z} : x \text{ is odd}\}$ |

$$A \cap C = \{-3, 1, 17\}$$

3.3.1.D

Answer:

| |
|---|
| $A = \{-3, 0, 1, 4, 17\}$ |
| $B = \{-12, -5, 1, 4, 6\}$ |
| $C = \{x \in \mathbb{Z} : x \text{ is odd}\}$ |
| $B \cap C = \{-5, 1\}$ |

$$A \cap (B \cap C) = \{-5, -3, 0, 1, 4, 17\} \cup$$

3.3.1.E

Answer:

| |
|---------------------------|
| $A = \{-3, 0, 1, 4, 17\}$ |
| $B \cap C = \{-5, 1\}$ |

$$A \cap B \cap C = \{1\}$$

3.3.3.A

Answer:

When $i = 2$, $i^2 = 4$, $A = \{1, 2, 4\}$

When $i = 3$, $i^2 = 9$, $A = \{1, 3, 9\}$

When $i = 4$, $i^2 = 16$, $A = \{1, 4, 16\}$

When $i = 5$, $i^2 = 25$, $A = \{1, 5, 25\}$

3.3.3.B

Answer:

When $i = 2$, $i^2 = 4$, $A = \{1, 2, 4\}$

When $i = 3$, $i^2 = 9$, $A = \{1, 3, 9\}$

When $i = 4$, $i^2 = 16$, $A = \{1, 4, 16\}$

When $i = 5$, $i^2 = 25$, $A = \{1, 5, 25\}$

Therefore, the union of all 5 sets is $\{1, 2, 3, 4, 5, 9, 16, 25\}$

3.3.3.E

Answer:

$B_1 = \{x \in \mathbb{R}, -1 \leq x \leq 1\}$

$B_2 = \{x \in \mathbb{R}, -1/2 \leq x \leq 1/2\}$

$B_3 = \{x \in \mathbb{R}, -1/3 \leq x \leq 1/3\}$

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$B_{100} = \{x \in \mathbb{R}, -1/100 \leq x \leq 1/100\}$

So, the range of values common to all 100 sets is $[-1/100, 1/100]$

Therefore, the intersection of all 100 sets is $\{x \in \mathbb{R}, -1/100 \leq x \leq 1/100\}$

3.3.3.F

Answer:

$B_1 = \{x \in \mathbb{R}, -1 \leq x \leq 1\}$

$B_2 = \{x \in \mathbb{R}, -1/2 \leq x \leq 1/2\}$

$B_3 = \{x \in \mathbb{R}, -1/3 \leq x \leq 1/3\}$

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$B_{100} = \{x \in \mathbb{R}, -1/100 \leq x \leq 1/100\}$

So, the union of all 100 sets is $\{x \in \mathbb{R}, -1 \leq x \leq 1\}$

3.3.4.B

Answer:

| |
|-----------------------------------|
| $A = \{a, b\}$ and $B = \{b, c\}$ |
| $A \cup B = \{a, b, c\}$ |

$$P(A \cup B) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$$

3.3.4.D

Answer:

| |
|--|
| $P(A) = \{ \emptyset, \{a\}, \{b\}, \{a, b\} \}$ |
| $P(B) = \{ \emptyset, \{b\}, \{c\}, \{b, c\} \}$ |

$$P(A) \cup P(B) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\} \}$$

Question 10:

3.5.1.B.

Answer:

(foam, tall, non-fat)

3.5.1.C.

Answer:

| |
|---------------------------------|
| $C = \{\text{non-fat, whole}\}$ |
| $B = \{\text{foam, no-foam}\}$ |

| | non-fat | whole |
|---------|--------------------|------------------|
| foam | (foam, non-fat) | (foam, whole) |
| No foam | (no foam, non-fat) | (no foam, whole) |

$B \times C = \{(\text{foam, non-fat}), (\text{foam, whole}), (\text{no-foam, non-fat}), (\text{no-foam, whole})\}$

3.5.3.B.

Answer:

True. $Z^2 = \{(a, b) : a, b \in Z\}$. $Z \subseteq \mathbb{R}$, so for every $a, b \in Z$, $a, b \in \mathbb{R}$, therefore $Z \times Z \subseteq \mathbb{R}^2$

3.5.3.C.

Answer:

True. The elements in Z^2 are pairs. The elements in Z^3 are triples. Therefore, the two sets have no elements in common.

3.5.3.E

Answer:

True. let any $(a, c) \in A \times C$, then $a \in A$ and $c \in C$. Since $A \subseteq B$, every a is an element of B , therefore $a \in B$ and $c \in C$, therefore $(a, c) \in B \times C$. Since (a, c) is an arbitrary element of $A \times C$, $\forall (a, c) \in A \times C \Rightarrow (a, c) \in B \times C$, therefore $A \times C \subseteq B \times C$

3.5.6.D.

Answer:

| |
|--|
| $\{0\}^2 = \{00\}, \{1\}^2 = \{11\}$ |
| $\{0\} \cup \{0\}^2 = \{0, 00\}, \{1\} \cup \{1\}^2 = \{1, 11\}$ |

$\{xy : \text{where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\} = \{01, 011, 001, 0011\}$

3.5.6.E.

Answer:

$$\{a\}^2 = \{aa\}$$

$$\{a\} \cup \{a\}^2 = \{a, aa\}$$

$$\{xy : x \in \{aa, ab\} \text{ and } y \in \{a\}^2\} = \{aaa, aaaa, aba, abaa\}$$

3.5.7.C

Answer:

$$A \times B = \{(a, b), (a, c)\}$$

$$A \times C = \{(a, a), (a, b), (a, d)\}$$

$$(A \times B) \cup (A \times C) = \{(a, a), (a, b), (a, c), (a, d)\}$$

3.5.7.F

Answer:

$$A \times B = \{(a, b), (a, c)\}$$

$$P(A \times B) = \{\emptyset, \{(a, b)\}, \{(a, c)\}, \{(a, b), (a, c)\}\}$$

3.5.7.G

Answer:

$$P(A) = \{\emptyset, \{a\}\}$$

$$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$$

$$P(A) \times P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{a\}, (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$$

Question 11:

3.6.2.B

Answer:

| | |
|-----------------------------------|-------------------|
| $(B \cap A) \cup (B' \cap A) = A$ | |
| $(B \cap B') \cup A$ | Distributive laws |
| $\emptyset \cup A$ | Complement laws |
| A | Identity laws |

3.6.2.C

Answer:

| | |
|--------------|-----------------------|
| $A \cap B'$ | |
| $A' \cup B'$ | De Morgan's Law |
| $A' \cup B$ | Double Complement Law |

3.6.3.B

Answer:

If $A = \{a, b\}$ and $B = \{b, c\}$, then $B \cap A = \{b\}$, so $A - (B \cap A) = \{a\}$, which is not equal to A .

3.6.3.D

Answer:

If $A = \{a, b\}$ and $B = \{b, c\}$, then $B - A = \{c\}$, so $(B - A) \cup A = \{a, b, c\}$, which is not equal to A .

3.6.4.B

Answer:

| | |
|----------------------|---------------------|
| $A \cap (B - A)$ | |
| $A \cap (B \cap A')$ | Set subtraction law |
| $A \cap (A' \cap B)$ | Commutative laws |
| $(A \cap A') \cap B$ | Associative laws |
| $\emptyset \cap B$ | Complement laws |
| \emptyset | Domination laws |

3.6.4.C

Answer:

| | |
|-------------------------------|---------------------|
| $A \cup (B - A)$ | |
| $A \cup (B \cap A')$ | Set subtraction law |
| $(A \cup B) \cap (A \cup A')$ | Distributive laws |
| $(A \cup B) \cap U$ | Domination laws |
| $A \cup B$ | Identity laws |