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HW8
Extended Bridge Spring ‘23

Question 7:

6.1.5.B
Answer:

First, choose the rank of the three matching cards, which can be done in 13 ways (one for each rank). Then, choose one card from the remaining three cards of that rank, which can be done in 4 ways (one for each suit).

Next, choose two cards from the remaining 48 cards that are not of the same rank as the three matching cards, which can be done in (48 choose 2) ways.

Finally, multiply all of these possibilities together to get the total number of three of a kind hands:

13 * 4 * (48 choose 2) = 54,912

To calculate the probability, divide the total number of three of a kind hands by the total number of possible five-card hands:

54,912 / (52 choose 5) = 0.02112845

Therefore, the probability of getting a three of a kind in a five-card hand from a standard deck of 52 cards is approximately 0.021 or 2.1%.

6.1.5.C
Answer:
The probability of being dealt a 5-card hand with all cards of the same suit is:

P(all cards same suit) = (4 choose 1) * (13 choose 5) / (52 choose 5)

where:

(4 choose 1) is the number of ways to choose one suit out of the four possible suits in a deck of playing cards.
(13 choose 5) is the number of ways to choose any five ranks out of the thirteen possible ranks in a suit.
(52 choose 5) is the number of ways to choose any 5 cards from a deck of 52 cards.
Using a calculator or software to perform the calculations, we get:

P(all cards same suit) = (4 * 1,287) / 2,598,960 ≈ 0.00198

Therefore, the probability of being dealt a 5-card hand with all cards of the same suit from a perfectly shuffled deck of playing cards is approximately 0.198%.

6.1.5.D
Answer:
The probability of being dealt a two of a kind hand (also called one pair) in

a 5-card hand is:

P(one pair) = (13 choose 1) * (4 choose 2) * (12 choose 3) * (4 choose 1)^3 / (52 choose 5)

where:

(13 choose 1) is the number of ways to choose one rank out of the thirteen possible ranks in a deck of playing cards.
(4 choose 2) is the number of ways to choose two suits out of the four possible suits for the two cards of the same rank.
(12 choose 3) is the number of ways to choose three different ranks out of the twelve possible ranks for the remaining three cards.
(4 choose 1)^3 is the number of ways to choose one suit out of the four possible suits for each of the three remaining cards.
(52 choose 5) is the number of ways to choose any 5 cards from a deck of 52 cards.
Using a calculator or software to perform the calculations, we get:

P(one pair) = (13 * 6 * 2,760 * 64) / 2,598,960 ≈ 0.42257

Therefore, the probability of being dealt a two of a kind hand (one pair) in a 5-card hand from a perfectly shuffled deck of playing cards is approximately 42.26%.

6.2.4.A
Answer:
To calculate the probability of the hand having at least one club, we need to calculate the probability of the complement event, i.e., the hand having no clubs, and subtract it from 1.

The probability of the first card being a club is 13/52. The probability of the second card not being a club is 39/51 (because there are 39 cards left in the deck that are not clubs out of a total of 51 cards remaining). Similarly, the probability of the third card not being a club is 38/50, the probability of the fourth card not being a club is 37/49, and the probability of the fifth card not being a club is 36/48. Therefore, the probability of the hand having no clubs is:

(39/51) x (38/50) x (37/49) x (36/48) x (39/52) ≈ 0.395

Therefore, the probability of the hand having at least one club is:

1 - 0.395 ≈ 0.605

So the probability of the hand having at least one club is approximately 0.605.

6.2.4.B

Answer:
To calculate the probability of the hand having at least two cards with the same rank, we need to calculate the probability of the complement event, i.e., the hand having no pairs, and subtract it from 1.

The first card can be any of the 52 cards. The second card must not have the same rank as the first card, so there are 3 cards of the same rank left in the deck and 48 other cards, giving a probability of 48/51. The third card must also have a different rank than the first two cards, so there are 2 cards of the same rank left in the deck and 47 other cards, giving a probability of 47/50. The fourth card must have a different rank than the first three cards, so there is only 1 card of the same rank left in the deck and 46 other cards, giving a probability of 46/49. Finally, the fifth card must also have a different rank than the first four cards, so there are no cards left with the same rank, giving a probability of 45/48. Therefore, the probability of the hand having no pairs is:

$(48/51) \times (47/50) \times (46/49) \times (45/48) \approx 0.724$

Therefore, the probability of the hand having at least two cards with the same rank is:

$1 - 0.724 \approx 0.276$

So the probability of the hand having at least two cards with the same rank is approximately 0.276.

6.2.4.C
Answer:
To calculate the probability of the hand having exactly one club or exactly one spade, we need to calculate the probability of each of the two events separately and add them.

The probability of the hand having exactly one club is equal to the probability of the first card being a club times the probability of the next four cards not being clubs, plus the probability of the first card not being a club times the probability of the next four cards having exactly one club. That is:

$(13/52) \times (39/51) \times (38/50) \times (37/49) \times (36/48) + (39/52) \times (13/51) \times (38/50) \times (37/49) \times (36/48) \times 5 \approx 0.374$

Similarly, the probability of the hand having exactly one spade is equal to:

$(13/52) \times (39/51) \times (38/50) \times (37/49) \times (36/48) + (39/52) \times (13/51) \times (38/50) \times (37/49) \times (36/48) \times 5 \approx 0.374$

Therefore, the probability of the hand having exactly one club or exactly one spade is:

$0.374 + 0.374 \approx 0.748$

So the probability of the hand having exactly one club or exactly one spade is approximately 0.748.

6.2.4.D

Answer:

To calculate the probability of the hand having at least one club or at least one spade, we can use the inclusion-exclusion principle. The probability of the hand having at least one club is 0.605 (as calculated in part (a)), the probability of the hand having at least one spade is the same, and the probability of the hand having both at least one club and at least one spade is the product of the probabilities of the two events, which is:

(13/52) x (12/51) x (39/50) x (38/49) x (37/48) ≈ 0.026

Therefore, the probability of the hand having at least one club or at least one spade is:

0.605 + 0.605 - 0.026 ≈ 1.184 - 0.026 ≈ 1.158

Note that this probability is greater than 1 because we have overcounted the cases where the hand has both at least one club and at least one spade. In reality, the probability of the hand having at least one club or at least one spade is 1.

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Question 8:

6.3.2.A
Answer:
To calculate the probabilities of each individual event, we need to count the number of permutations that satisfy the conditions of the events and divide that by the total number of possible permutations.

There are 3 letters before and 3 letters after the letter b, so there are 5 positions for the letter b to be in the middle. For each of those positions, there are 4 letters that can be placed in the first position, 3 letters that can be placed in the second position, 2 letters that can be placed in the third position, and the remaining 2 letters can be placed in the remaining positions. Therefore, the number of permutations that satisfy event A is:

5 x 4 x 3 x 2 x 2 x 1 x 1 = 240

The total number of permutations is 7! = 5040, so the probability of event A is:

p(A) = 240/5040 = 1/21

For event B, the letter b can be placed in any of the 7 positions, and once it is placed, the remaining 6 letters can be arranged in any order. If the letter c appears to the right of b, then it must be in one of the 3 positions to the right of b. Therefore, the number of permutations that satisfy event B is:

7 x 6! x 3! = 1512

The probability of event B is therefore:

p(B) = 1512/5040 = 7/24

For event C, the three letters d, e, and f must occur together in that order. There are 5 positions where the sequence can begin (the first 3 positions, the second 3 positions, etc.), and once the position is chosen, the remaining 4 letters can be arranged in any order. Therefore, the number of permutations that satisfy event C is:

5 x 4! = 120

The probability of event C is therefore:

p(C) = 120/5040 = 1/42

6.3.2.B
Answer:
To calculate p(A|C), we need to find the probability that event A occurs given that event C occurs. Event C requires that the letters d, e, and f occur together in that order, which means that the letter b cannot be in the middle. Therefore, p(A|C) = 0.

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6.3.2.C
Answer:
To calculate p(B|C), we need to find the probability that event B occurs given that event C occurs. Event C requires that the letters d, e, and f occur together in that order, so if the letter c appears to the right of b, it must appear to the right of the sequence d, e, and f as well. There are 3 positions where the letter c can appear to the right of the sequence, and once the position is chosen, the remaining 3 letters can be arranged in any order. Therefore, the number of permutations that satisfy both events B and C is:

3 x 3! x 3! = 54

The probability of event B given that event C occurs is therefore:

p(B|C) = 54/120 = 9/20

6.3.2.D
Answer:
To find p(A|B), we need to find the probability that event A occurs given that event B has occurred. We know that event B requires that the letter c appears somewhere to the right of b. Since there are seven letters in total and each permutation is equally likely, there are 6 possible positions for b to occupy, namely the second, third, fourth, fifth, sixth, or seventh position.

If b is in the second position, then c must be in one of the last four positions. There are 4! = 24 permutations of the remaining four letters, so there are 24 permutations in which b is in the second position and c appears to its right. Similarly, if b is in the third position, then c can appear in one of the last three positions, and there are 3! = 6 permutations of the remaining three letters. Thus, there are 6 * 24 = 144 permutations in which b is in the second or third position and c appears to its right.

If b is in the fourth position, then c can appear in one of the last two positions, and there are 2! = 2 permutations of the remaining two letters. If b is in the fifth position, then c must be in the last position, and there is only 1 permutation of the remaining one letter. Finally, if b is in the sixth or seventh position, then there are no positions for c to appear to its right. Thus, there are 2 + 1 = 3 permutations in which b is in the fourth or fifth position and c appears to its right.

Therefore, the total number of permutations in which event B occurs is 144 + 3 = 147. Of these, we know that there are 2 possible permutations in which event A and event B both occur, namely "bdcefga" and "bdcfega" (where the letters to the left and right of b can be any of the remaining four letters, and the remaining three letters can be arranged in any order). Thus, p(A|B) = 2/147.

6.3.2.E
Answer:
To determine which pairs of events are independent, we need to check if the occurrence of one event affects the probability of the other event occurring.

We can see that events A and C are independent, because the occurrence of one does not affect the probability of the other occurring. For example, the letter b could fall in any of the seven positions, regardless of whether the letters "def" occur together in a certain order.

On the other hand, events A and B are not independent, because the occurrence of event B affects the probability of event A occurring. If c appears immediately to the right of b (i.e., b is in the second or third position), then event A cannot occur.

Similarly, events B and C are not independent, because the occurrence of event B affects the probability of event C occurring. If c appears immediately to the right of b (i.e., b is in the first position), then it is impossible for the letters "def" to occur together in that order.

6.3.6.B

Answer:

P(HHHHHTTTTT) = P(H)^5 * P(T)^5 = (1/3)^5 * (2/3)^5 ≈ 0.0001286

6.3.6.C

Answer:

P(HTTTTTTTTT) = P(H) * P(T)^9 = (1/3) * (2/3)^9 ≈ 0.00136

6.4.2.A

Answer:

Let F be the event that the fair die was chosen, and B be the event that the biased die was chosen. Let E be the event that the rolls resulted in the sequence 4, 3, 6, 6, 5, 5. We want to find the probability that the fair die was chosen given the rolls, i.e., P(F|E).

By Bayes' theorem, we have:

P(F|E) = P(E|F) * P(F) / P(E)

where P(E|F) is the probability of observing the rolls given that the fair die was chosen, P(F) is the prior probability of choosing the fair die, and P(E) is the probability of observing the rolls regardless of which die was chosen.

We can calculate each of these probabilities as follows:

P(E|F) is the probability of observing the sequence of rolls given that the fair die was chosen. Since the rolls are mutually independent and the fair die has equal probability of landing on each face, we have:
P(E|F) = (1/6)^6

P(F) is the prior probability of choosing the fair die, which is 1/2 since there are two dice and we chose one at random.

To calculate P(E), we use the law of total probability:

P(E) = P(E|F) * P(F) + P(E|B) * P(B)

where P(E|B) is the probability of observing the rolls given that the biased die was chosen, and P(B) is the prior probability of choosing the biased die. Since the biased die has a 0.25 probability of landing on six and a 0.15 probability of landing on each other face, we have:

P(E|B) = (0.15)^2 * (0.25)^2 * (0.85)^2

And since P(B) = 1/2, we get:

P(E) = (1/6)^6 * (1/2) + (0.15)^2 * (0.25)^2 * (0.85)^2 * (1/2)

Now we can substitute these values into Bayes' theorem:

P(F|E) = (1/6)^6 * (1/2) / [(1/6)^6 * (1/2) + (0.15)^2 * (0.25)^2 * (0.85)^2 * (1/2)]

Evaluating this expression, we get:

P(F|E) ≈ 0.9655

So the probability that the fair die was chosen given the rolls is approximately 0.9655.

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Question 9:

6.5.2.A
Answer:
The range of A is {0, 1, 2, 3, 4}.

6.5.2.B
Answer:
We can compute the distribution over the random variable A by considering the probability of each possible value of A:

P(A=0) = (48 choose 5) / (52 choose 5) ≈ 0.656
P(A=1) = (4 choose 1) * (48 choose 4) / (52 choose 5) ≈ 0.290
P(A=2) = (4 choose 2) * (48 choose 3) / (52 choose 5) ≈ 0.047
P(A=3) = (4 choose 3) * (48 choose 2) / (52 choose 5) ≈ 0.003
P(A=4) = (4 choose 4) * (48 choose 1) / (52 choose 5) ≈ 0.00002

Therefore, the distribution over the random variable A is:

A	P(A)
0	0.656
1	0.290
2	0.047
3	0.003
4	0.00002

6.6.1.A
Answer:
We can use the formula for expected value:

E[G] = Σ g P(G=g)

where g is the possible values of G, and P(G=g) is the probability that G takes on the value g.

In this case, G can take on values of 0, 1, or 2, since we are choosing 2 students and there are 0, 1, or 2 girls in the group.

To calculate the probabilities, we can use combinations. The total number of ways to choose 2 students from a group of 10 is:

C(10,2) = 45

To choose 0 girls, we must choose 2 boys:

C(3,2) = 3

So P(G=0) = 3/45 = 1/15.

To choose 1 girl and 1 boy, we must choose 1 girl from 7 and 1 boy from 3:

C(7,1) * C(3,1) = 21

So P(G=1) = 21/45 = 7/15.

To choose 2 girls, we must choose 2 girls from 7:

C(7,2) = 21

So P(G=2) = 21/45 = 7/15.

Now we can calculate the expected value:

E[G] = 0 * P(G=0) + 1 * P(G=1) + 2 * P(G=2)
= 0 * (1/15) + 1 * (7/15) + 2 * (7/15)
= 1.4

Therefore, the expected number of girls chosen is 1.4.

6.6.4.A
Answer:
E[X] = (1^2)(1/6) + (2^2)(1/6) + (3^2)(1/6) + (4^2)(1/6) + (5^2)(1/6) + (6^2)(1/6)
= (1/6)(1 + 4 + 9 + 16 + 25 + 36)
= (1/6)(91)
= 15.1666666667

Therefore, the expected value of X is 15.1666666667.

6.6.4.B
Answer:
There are $2^3=8$ possible outcomes of three tosses of a fair coin. Let H denote a head and T denote a tail. The outcomes are:

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

For each outcome, we can find the value of the random variable Y by squaring the number of heads. The values of Y and their probabilities are:

$Y=0$, probability $1/8$ (no heads)
$Y=1$, probability $3/8$ (one head)
$Y=4$, probability $3/8$ (two heads)
$Y=9$, probability $1/8$ (three heads)

Therefore, the expected value of Y is:

$$E[Y] = 0\cdot\frac{1}{8} + 1\cdot\frac{3}{8} + 4\cdot\frac{3}{8} + 9\cdot\frac{1}{8} = \frac{3+12+9}{8} = \frac{24}{8} = 3$$

So, $E[Y]=3$.

6.7.4.A
Answer:
Let X be the number of children who get their own coat. Then we have X = 0, 1,

2, ..., 10.

To find the expected value of X, we can use the formula:

E[X] = Σ x * P(X = x)

where the summation is over all possible values of X.

Since the coats are distributed at random, each child has an equal probability of getting any of the 10 coats. Therefore, the probability that a particular child gets his or her own coat is 1/10. Let's denote this probability by p.

If we consider one particular child, there are 10 coats to choose from. If that child gets his or her own coat, then there are 9 coats left for the remaining 9 children. The probability of this happening is:

p * (9/9) * (8/8) * ... * (1/1) = p

If that child does not get his or her own coat, then there are 9 coats left, but one of them is the correct coat for that child. The probability of this happening is:

(1 - p) * (1/9) * (8/8) * ... * (1/1) = (1 - p)/9!

Therefore, the probability of x children getting their own coats is given by the binomial distribution:

P(X = x) = (10 choose x) * p^x * (1-p)^(10-x)

Substituting this into the formula for the expected value of X, we get:

E[X] = Σ x * (10 choose x) * p^x * (1-p)^(10-x)

We can simplify this expression by recognizing that it is the expected value of a binomial distribution with parameters n = 10 and p = 1/10. Therefore, we can use the formula for the expected value of a binomial distribution:

E[X] = n * p = 10 * 1/10 = 1

Therefore, the expected number of children who get their own coat is 1.

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<u>Question 10:</u>

6.8.1.A
Answer:
Using the binomial distribution, the probability of getting exactly 2 defective circuit boards out of 100 is:

${100\choose 2}(0.01)^2(0.99)^{98} \approx 0.1827$

6.8.1.B
Answer:
The probability of at least 2 defective circuit boards is the complement of getting 0 or 1 defective circuit boards:

$1 - [{100\choose 0}(0.01)^0(0.99)^{100} + {100\choose 1}(0.01)^1(0.99)^{99}] \approx 0.1391$

6.8.1.C
Answer:
The expected number of circuit boards with defects out of 100 is:

$E[X] = np = 100 \times 0.01 = 1$

6.8.1.D
Answer:
Let X be the number of batches that have at least one defective circuit board. Then X follows a binomial distribution with $n=50$ and $p=1\%$. The probability of at least 2 defective circuit boards out of 100 is the same as the probability of at least 2 batches having at least one defective circuit board, which is:

$1 - [{50\choose 0}(0.01)^0(0.99)^{50} + {50\choose 1}(0.01)^1(0.99)^{49}] \approx 0.3954$

The expected number of circuit boards with defects out of 100 is:

$E[X] \times 2 = np \times 2 = 50 \times 0.01 \times 2 = 1$

This is the same as the answer to part (c), since the probability of a circuit board having a defect is the same in both cases, regardless of whether they are made in batches or separately. However, the probability of at least 2 defective circuit boards is higher when they are made in batches, because batches of two are more likely to both be defective than individual circuit boards.

6.8.3.B
Answer:
If the coin is biased, the probability of getting a head in a single flip is 0.3 and the probability of getting a tail is 0.7.

To calculate the probability of reaching an incorrect conclusion, we need to

consider two cases:

Case 1: The coin is biased and we conclude it is fair.
Case 2: The coin is biased and we conclude it is biased.

Case 1:
The probability of getting at least 4 heads in 10 flips with a fair coin is given by the binomial distribution:

P(X ≥ 4) = 1 - P(X < 4) = 1 - (P(X=0) + P(X=1) + P(X=2) + P(X=3))
= 1 - [(0.5)^10 + 10*(0.5)^9 + 45*(0.5)^8 + 120*(0.5)^7]
≈ 0.1719

So the probability of mistakenly concluding that the coin is fair when it is actually biased is approximately 0.1719.

Case 2:
The probability of getting less than 4 heads in 10 flips with a biased coin is also given by the binomial distribution:

P(X < 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)
= (0.7)^10 + 10*(0.3)(0.7)^9 + 45(0.3)^2*(0.7)^8 + 120*(0.3)^3*(0.7)^7
≈ 0.9831

So the probability of mistakenly concluding that the coin is biased when it is actually fair is approximately 0.9831.

Therefore, the probability of reaching an incorrect conclusion if the coin is biased is the probability of Case 1, which is approximately 0.1719.
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