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HW11
Extended Bridge Spring ‘23

Question 5:

5.A
Answer:
Base Case - Step 1: Show true for n=1

For n=1, n^3**+2n=(1)^3+2(1)**
n^3**+2n=3**
3 is definitely divisible by 3 so the statement is true for n=1.

Inductive Step - Step 2: Assume true for n=k

We assume that for any integer k, n^3**+2n is divisible by 3. We can write this mathematically as:**
k^3**+2k=3m, where m is an integer**

Proof - Step 3: Show true for k+1

For n=k+1,

n^3**+2n=**$(k+1)^3$**+2(k+1)**

=(k^3**+**$3k^2$**+3k+1)+2k+2**

=(k^3**+2k)+3(**k^2**+k+1)**

Subbing in from part 2 for (k^3**+2k), we get:**

n^3**+2n=3m+3(**k^2**+k+1)**

=3(m+k^2+k+1)

which is divisible by 3.

This means that the statement being true for n=k implies the statement is true for n=k+1, and as we have shown it to be true for n=1 the proof of the statement follows by induction.

5.B
Answer:
Base Case - Step 1:
Let n = 2
2 is a prime number itself therefore it is true for 2

Inductive Step - Step 2:
Assume That the statement is true for 2 ≤ k ≤ n
Then k is a prime or product of primes.
Prove that n+1 is a product of prime numbers.

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If n+1 is prime there is nothing to prove.

Suppose n+1 is not prime.
Then there exist k_1, k_2 such that $k_1 * k_2$= n+1 and k_1, k_2<n

Hence by assumption k_1, k_2 can be expressed as a product of primes.
Therefore n+1 can be expressed as prime numbers. Hence, Proved.

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Question 6:

7.4.1.A
Answer:

P(n):
$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$
P(3):
$$\sum_{j=1}^{3} j^2 = \frac{(3)((3)+1)(2(3)+1)}{6}$$
$$1 + 2^2 + 3^2 = \frac{(3)(4)(7)}{6}$$
$$1 + 4 + 9 = \frac{84}{6}$$
$14 = 14$ **Hence P(3) is true**

7.4.1.B
Answer:
P(k):
$$\sum_{j=1}^{k} j^2 = \frac{(k)((k)+1)(2(k)+1)}{6}$$

7.4.1.C
Answer:
P(k+1):
$$\sum_{j=1}^{k+1} j^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$
$$\sum_{j=1}^{k+1} j^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

7.4.1.D
Answer:
We must prove P(1) is true in the base step:
$$\sum_{j=1}^{1} j^2 = \frac{(1)((1)+1)(2(1)+1)}{6}$$
$$1^2 = \frac{(1)(2)(3)}{6}$$
$$1 = \frac{(6)}{6}$$
$$1 = 1$$
Hence P(1) is true

7.4.1.E
Answer:
We must prove P(k+1) is true in the inductive step:
$$\sum_{j=1}^{k+1} j^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

7.4.1.F
Answer:
We must prove P(k) is true in the inductive step:

$\sum_{j=1}^{k} j^2 = \frac{(k)((k)+1)(2(k)+1)}{6}$

7.4.1.G
Answer:
Proof:

$\sum_{j=1}^{k+1} j^2 = \sum_{j=1}^{k} j^2 + (k+1)^2$

$= \frac{(k)((k)+1)(2(k)+1)}{6} + (k+1)^2$

$= \frac{(k)((k)+1)(2(k)+1)}{6} + (k+1)^2$

$= (k+1)(\frac{(k)(2(k)+1)}{6} + (k+1))$

$= (k+1)(\frac{(2k^2+k)}{6} + \frac{6(k+1)}{6})$

$= (k+1)(\frac{(2k^2+k)}{6} + \frac{6k+6}{6})$

$= (k+1)(\frac{(2k^2+7k+6}{6})$

$= (k+1)(\frac{(k+2)(2k+3)}{6})$

$= \frac{(k+1)(k+2)(2k+3)}{6}$

So $\sum_{j=1}^{k+1} j^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

P(k+1) Hence by induction on k

$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$

7.4.3.C
Answer:

$\sum_{j=1}^{n} \frac{1}{j^2} \leq 2 - \frac{1}{n}$

Base Case - Step 1:
For n = 1

$\sum_{j=1}^{n} \frac{1}{j^2} \leq 2 - \frac{1}{n}$

$\sum_{j=1}^{1} \frac{1}{j^2} = 2 - \frac{1}{(1)}$

$\frac{1}{1^2} = 2 - \frac{1}{(1)}$

$1 = 2 - 1$

$1 = 1$

Hence n = 1 is true

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Inductive step - Step 2:
Assume the statement is true for 1≤k≤n

$\sum_{j=1}^{k} \frac{1}{k^2} = 2 - \frac{1}{(k)}$

$\frac{1}{k^2} = 2 - \frac{1}{(k)}$

Proof - Step 3:
Proof for n = k+1

$\sum_{j=1}^{k+1} \frac{1}{(k+1)^2}$

$\leq \frac{1}{k^2}$

$= 2 - \frac{1}{k^2}$

$\leq 2 - \frac{1}{(k)}$

$=2 - \frac{1}{(k)}$

Therefore the given statement is true for k+1 Kence the given statement is true for all possible number n (1≤n)

7.5.1.A
Answer:

Base Case - Step 1:
For n = 1

$3^{2n} - 1$

$3^{2(1)} - 1$

$9 - 1$

8

8 is divisible by 4 Hence n = 1

Inductive step - Step 2:
Assume the statement is true for n=k

$3^{2k} - 1$

$3^{2(k)} - 1 = 4x$

$9^k - 1 = 4x$

8 is divisible by 4 Hence n = 1

Proof - Step 3:
Proof for n = k+1

$3^{2(k+1)} - 1 = (9^k * 9) - 1$

$= (9^k * 9) - 1 - 8 + 8$

$= (9^k * 9) - 9) + 8$

$= 9(9^k - 1) + 8$

$= 9(9^k - 1) + 8$
$= 9(4x) + 8$
$= 4(9x + 2)$
$= 4p$

Since $3^{2k+1} - 1$ can be represented as a multiple of 4 it is evenly divisible by 4.Therefore the given statement is true for k+1 Kence the given statement is true for all possible number n (n>0)
