NEW YORK UNIVERSITY TANDON SCHOOL OF ENGINEERING
Department of Mechanical and Aerospace Engineering

Foundations of Robotics (ROB-GY 6003)

Homework Assignment | Chapter 6

Homework Problems: 6.15, 6.16*, 6.20

Instructor's Note: For 6.16, you are free to use the Newton-Euler or Lagrangian method. As you work on this assignment, note when Coriolis terms appear (or don't appear),

6.15 [28] Derive the dynamic equations for the RP manipulator of Example 6.5, using the Newton–Euler procedure instead of the Lagrangian technique.

EXAMPLE 6.5

The links of an RP manipulator, shown in Fig. 6.7, have inertia tensors

$$ {}^{C_1}I_1 = \begin{bmatrix} I_{xx1} & 0 & 0 \\ 0 & I_{yy1} & 0 \\ 0 & 0 & I_{zz1} \end{bmatrix}, $$
$$ {}^{C_2}I_2 = \begin{bmatrix} I_{xx2} & 0 & 0 \\ 0 & I_{yy2} & 0 \\ 0 & 0 & I_{zz2} \end{bmatrix}, \tag{6.78} $$

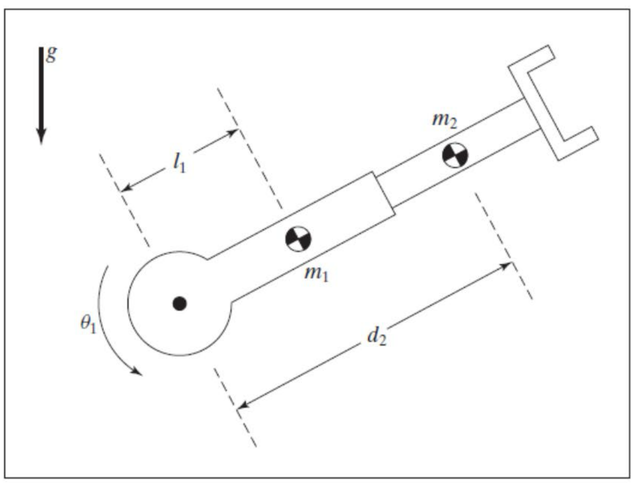

FIGURE 6.7: The RP manipulator of Example 6.5.

and total mass m_1 and m_2. As shown in Fig. 6.7, the center of mass of link 1 is located at a distance l_1 from the joint-1 axis, and the center of mass of link 2 is at the variable distance d_2 from the joint-1 axis. Use Lagrangian dynamics to determine the equation of motion for this manipulator.

6.16 [25] Derive the equations of motion for the PR manipulator shown in Fig. 6.10. Neglect friction, but include gravity. (Here, $\hat{X}_0$ is upward.) The inertia tensors of the links are diagonal, with moments I_{xx1}, I_{yy1}, I_{zz1} and I_{xx2}, I_{yy2}, I_{zz2}. The centers of mass for the links are given by

$$^{1}P_{C_1} = \begin{bmatrix} 0 \\ 0 \\ -l_1 \end{bmatrix},$$

$$^{2}P_{C_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

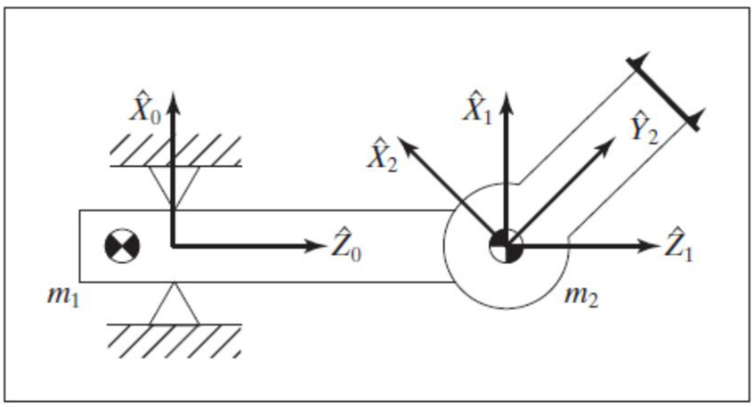

FIGURE 6.10: PR manipulator of Exercise 6.16.

6.20 [28] Derive the dynamic equations of the 2-DOF manipulator of Section 6.7, using a Lagrangian formulation.

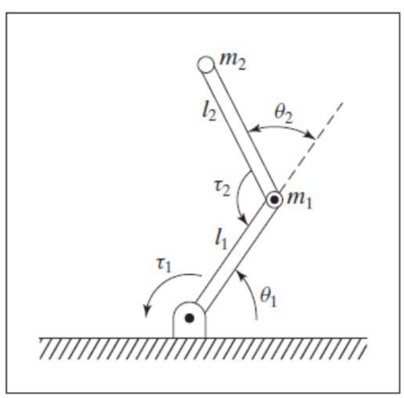

FIGURE 6.6: Two-link planar manipulator with point masses at distal ends of links.

HW CH2 Solution

Craig 4th ed. Prob.: 2.1, 2.3, 2.12, 2.14, 2.19, 2.20, 2.21, 2.22, 2.27, 2.37, 2.38

2.1) Fixed frame rotation: apply rotations "from right to left."

$$R = rot(\hat{x},\phi)rot(\hat{z},\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta & -S\theta & 0 \\ C\phi S\theta & C\phi C\theta & -S\phi \\ S\phi S\theta & S\phi C\theta & C\phi \end{bmatrix}$$

2.3) Since rotations are performed about axes of the frame being rotated, these are Euler-Angle rotations: apply rotations "from left to right."

$R = rot(\hat{z},\theta)rot(\hat{x},\phi)$

We might also use the following reasoning:

$${}^{A}R_{B}(\theta,\phi) = {}^{B}R_{A}^{-1}(\theta,\phi) = [rot(\hat{x},-\phi)rot(\hat{z},-\theta)]^{-1} = rot^{-1}(\hat{z},-\theta)rot^{-1}(\hat{x},-\phi) = rot(\hat{z},\theta)rot(\hat{x},\phi)$$

Another way of viewing the same operation:

1st rotate by $rot(\hat{z},\theta)$; 2nd rotate by $rot(\hat{z},\theta)rot(\hat{x},\phi)rot^{-1}(\hat{z},\theta)$

(See similarity transform in Problem 2.19.)

2.12) Velocity is a "free vector" and only will be affected by rotation, and not by translation:

$${}^{A}V = {}^{A}R_{B}\,{}^{B}V = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} = \begin{bmatrix} -1.34 \\ 22.32 \\ 30.0 \end{bmatrix}$$

2.14)

[Method 1] This rotation can be written as: ${}^{A}T_{B} = \text{trans}({}^{A}\hat{P},|{}^{A}P|)\cdot\text{rot}(\hat{K},\theta)\cdot\text{trans}(-{}^{A}\hat{P},|{}^{A}P|)$

where $\text{rot}\left(\hat{K},\theta\right)$ is written as in eq. (2.77),

$$\text{trans}\left({}^{A}\hat{P},\left|{}^{A}P\right|\right) = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } \text{trans}\left(-{}^{A}\hat{P},\left|{}^{A}P\right|\right) = \begin{bmatrix} 1 & 0 & 0 & -P_x \\ 0 & 1 & 0 & -P_y \\ 0 & 0 & 1 & -p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Multiplying out we get:

$${}^{A}T_{B} = \left[\begin{array}{ccc|c} R_{11} & R_{12} & R_{13} & Q_x \\ R_{21} & R_{22} & R_{23} & Q_y \\ R_{31} & R_{32} & R_{33} & Q_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

where the R_{ij} are given by eq. (2.77), and:

$$Q_x = P_x - P_x\left(K_x^2 V\theta + C\theta\right) - P_y\left(K_x K_y V\theta - K_z S\theta\right) - P_z\left(K_x K_z V\theta + K_y S\theta\right)$$

$$Q_y = P_y - P_x\left(K_x K_y V\theta + K_z S\theta\right) - P_y\left(K_y^2 V\theta + C\theta\right) - P_z\left(K_y K_z V\theta + K_x S\theta\right)$$
$$Q_z = P_z - P_x\left(K_x K_z V\theta - K_y S\theta\right) - P_y\left(K_y K_z V\theta + K_x S\theta\right) - P_z\left(K_z^2 V\theta + C\theta\right)$$

[Method 2] See Fig. 2.20. ${}^{A}T_B = {}^{A}T_{A'}\,{}^{A'}T_{B'}\,{}^{B'}T_B$

(i) Rotation

$${}^{A'}\hat{\mathbf{X}}_B = R_K(\theta)\,{}^{A'}\mathbf{P}_{A\hat{\mathbf{X}}} - R_K(\theta)\,{}^{A'}\mathbf{P}_{AORG} = R_K(\theta)({}^{A'}\mathbf{P}_{A\hat{\mathbf{X}}} - {}^{A'}\mathbf{P}_{AORG}) = R_K(\theta)\,{}^{A'}\hat{\mathbf{X}}_A = R_K(\theta)\,{}^{A}\hat{\mathbf{X}}_A$$

Similarly for **Y** and **Z**.

$${}^{A'}R_B = \left[{}^{A'}\hat{\mathbf{X}}_B \quad {}^{A'}\hat{\mathbf{Y}}_B \quad {}^{A'}\hat{\mathbf{Z}}_B\right] = \left[R_K(\theta)\,{}^{A}\hat{\mathbf{X}}_A \quad R_K(\theta)\,{}^{A}\hat{\mathbf{Y}}_A \quad R_K(\theta)\,{}^{A}\hat{\mathbf{Z}}_A\right] = R_K(\theta) I_d = R_K(\theta) = {}^{A'}R_{B'}$$

Therefore, the rotation portion of Frame $\{B\}$ is same as the rotation portion of Frame $\{B'\}$.

(ii) Translation

$${}^{A'}\mathbf{P}_{AORG} = -{}^{A}P_{A'ORG} = -{}^{A}\mathbf{P}$$
$${}^{A'}\mathbf{P}_{BORG} = R_K(\theta)\,{}^{A'}\mathbf{P}_{AORG} = -R_K(\theta)\,{}^{A}\mathbf{P}$$
$${}^{B'}\mathbf{P}_{BORG} = {}^{B'}R_{A'}\,{}^{A'}\mathbf{P}_{BORG} = {}^{B'}R_{A'}\left(-R_K(\theta)\,{}^{A}\mathbf{P}\right) = -{}^{A}\mathbf{P} \quad (\because\ {}^{B'}R_{A'} = R_K(\theta)^{-1})$$

$$\therefore\ {}^{A}T_B = {}^{A}T_{A'}\,{}^{A'}T_{B'}\,{}^{B'}T_B = \begin{bmatrix} I_d & {}^{A}\mathbf{P} \\ 000 & 1 \end{bmatrix}\begin{bmatrix} R_K(\theta) & 0 \\ 000 & 1 \end{bmatrix}\begin{bmatrix} I_d & -{}^{A}\mathbf{P} \\ 000 & 1 \end{bmatrix} = \begin{bmatrix} R_K(\theta) & {}^{A}\mathbf{P} - R_K(\theta)\,{}^{A}\mathbf{P} \\ 000 & 1 \end{bmatrix}$$

2.19) In the *Z-Y-Z* Euler Angle set, the first rotation is: $R_1 = rot(\hat{z},\alpha)$

The second rotation expressed in fixed coordinates is: $R_2 = rot(\hat{z},\alpha)rot(\hat{y},\beta)rot^{-1}(\hat{z},\alpha)$

The third is: $R_3 = (R_2R_1)rot(\hat{z},\gamma)(R_2R_1)^{-1}$

The result is: $R = R_3R_2R_1 = rot(\hat{z},\alpha)rot(\hat{y},\beta)rot(\hat{z},\gamma)$, which gives the result of (2.72).

<u>Additional explanation</u> about the description given in the problem statement: In order to perform the rotation about the fixed frame's *y* axis, the *y* axes of the fixed and moving frames need to be made coincident. Therefore, first, bring back the previous rotation about the fixed frame's *x* axis, perform the rotation about the fixed frame's *y* axis, and then re-perform the first rotation about the fixed frame's *x* axis.

2.20)

[Method 1] Transform matrix operations into vector operations.

$$R_K(\theta) = \begin{bmatrix} k_xk_xv\theta + c\theta & k_xk_yv\theta - k_zs\theta & k_xk_zv\theta + k_ys\theta \\ k_yk_xv\theta + k_zs\theta & k_yk_yv\theta + c\theta & k_yk_zv\theta - k_xs\theta \\ k_zk_xv\theta - k_ys\theta & k_zk_yv\theta + k_xs\theta & k_zk_zv\theta + c\theta \end{bmatrix}$$

$$= c\theta \cdot I_d + s\theta\begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{bmatrix} + v\theta\begin{bmatrix} k_xk_x & k_xk_y & k_xk_z \\ k_yk_x & k_yk_y & k_yk_z \\ k_zk_x & k_zk_y & k_zk_z \end{bmatrix}$$

$$Q' = R_K(\theta)Q = c\theta \cdot Q + s\theta \underbrace{\begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{bmatrix} Q}_{=\hat{K}\times Q} + v\theta \begin{bmatrix} k_x k_x & k_x k_y & k_x k_z \\ k_y k_x & k_y k_y & k_y k_z \\ k_z k_x & k_z k_y & k_z k_z \end{bmatrix} \begin{bmatrix} Q_x \\ Q_y \\ Q_z \end{bmatrix}$$

$$= Q \cdot c\theta + s\theta(\hat{K} \times Q) + v\theta \underbrace{\begin{bmatrix} k_x(k_x Q_x + k_y Q_y + k_z Q_z) \\ k_y(k_x Q_x + k_y Q_y + k_z Q_z) \\ k_z(k_x Q_x + k_y Q_y + k_z Q_z) \end{bmatrix}}_{=\hat{K}(\hat{K}\bullet Q)} = Q \cdot c\theta + s\theta(\hat{K} \times Q) + (1 - c\theta)(\hat{K} \bullet Q)\hat{K}$$

[Method 2] Derive backwards, i.e., expand the right-hand side of Rodriques' formula.

$$\begin{bmatrix} Q_x \cos\theta \\ Q_y \cos\theta \\ Q_z \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} k_y Q_z - k_z Q_y \\ k_z Q_x - k_x Q_z \\ k_x Q_y - k_y Q_x \end{bmatrix} + (1 - \cos\theta)(k_x Q_x + k_y Q_y + k_z Q_z) \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

$$= \begin{bmatrix} Q_x \cos\theta + \sin\theta(k_y Q_z - k_z Q_y) + (1 - \cos\theta)(k_x Q_x + k_y Q_y + k_z Q_z)k_x \\ Q_y \cos\theta + \sin\theta(k_z Q_x - k_x Q_z) + (1 - \cos\theta)(k_x Q_x + k_y Q_y + k_z Q_z)k_y \\ Q_z \cos\theta + \sin\theta(k_x Q_y - k_y Q_x) + (1 - \cos\theta)(k_x Q_x + k_y Q_y + k_z Q_z)k_z \end{bmatrix}$$

$$= \begin{bmatrix} [k_x k_x(1 - \cos\theta) + \cos\theta]Q_x + [k_x k_y(1 - \cos\theta) - k_z \sin\theta]Q_y + [k_x k_z(1 - \cos\theta) + k_y \sin\theta]Q_z \\ [k_y k_x(1 - \cos\theta) + k_z \sin\theta]Q_x + [k_y k_y(1 - \cos\theta) + \cos\theta]Q_y + [k_y k_z(1 - \cos\theta) - k_x \sin\theta]Q_z \\ [k_z k_x(1 - \cos\theta) - k_y \sin\theta]Q_x + [k_z k_y(1 - \cos\theta) + k_x \sin\theta]Q_y + [k_z k_z(1 - \cos\theta) + \cos\theta]Q_z \end{bmatrix}$$

$$= \begin{bmatrix} k_x k_x v\theta + c\theta & k_x k_y v\theta - k_z s\theta & k_x k_z v\theta + k_y s\theta \\ k_y k_x v\theta + k_z s\theta & k_y k_y v\theta + c\theta & k_y k_z v\theta - k_x s\theta \\ k_z k_x v\theta - k_y s\theta & k_z k_y v\theta + k_x s\theta & k_z k_z v\theta + c\theta \end{bmatrix} \begin{bmatrix} Q_x \\ Q_y \\ Q_z \end{bmatrix}$$

2.21) Just use the given approximations in (2.80) to obtain:

$$R_K(\delta\theta) = \begin{bmatrix} 1 & -K_Z\delta\theta & K_Y\delta\theta \\ K_Z\delta\theta & 1 & -K_X\delta\theta \\ -K_Y\delta\theta & K_X\delta\theta & 1 \end{bmatrix}$$

More on this is in Chapter 5.

2.22) So, given $R_1 = R_J(\alpha)$ and $R_2 = R_K(\beta)$ with $\alpha << 1$ and $\beta << 1$; show $R_1R_2 = R_2R_1$. If we form the product R_1R_2 and use $\alpha\beta \cong 0$ we have:

$$R_1R_2 = \begin{bmatrix} 1 & -J_Z\alpha - K_Z\beta & J_Y\alpha + K_Y\beta \\ J_Z\alpha + K_Z\beta & 1 & -J_X\alpha - K_X\beta \\ -J_Y\alpha - K_Y\beta & J_X\alpha + K_X\beta & 1 \end{bmatrix}$$

We see that j and k, as well as α and β, appear symmetrically, so $R_1R_2 = R_2R_1$.

2.27) For rotation part, use the definition of the rotation matrix in Equation (2.2). For translation part, write the position vector of the origin of Frame $\{B\}$ with respect to Frame $\{A\}$.

$$\therefore \quad {}^{A}T_{B} = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.37) Form (2, 4) element of $-{}^{A}R_{B}^{T}\,{}^{A}P_{BORG}$ → to get: –6.4
(See Equation (2.45).)

2.38) $v_1 \bullet v_2 = v_1^T v_2 = \cos\theta$, R preserves angles, so, $\left(Rv_1\right)^T\left(Rv_2\right) = v_1^T v_2$

$v_1^T R^T R v_2 = v_1^T v_2 \quad \therefore\ R^T R = Id \ \Rightarrow\ R^T = R^{-1}$

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Foundations of Robotics (ROB-GY 6003)

Homework Assignment | Chapter 2

There is a total of seven homework assignments for this course, corresponding to Chapters 2–8. Problems are taken from Craig 4th ed.

Homework Problems: 2.1, 2.3, 2.12, 2.14, 2.19, 2.20, 2.21, 2.22, 2.27, 2.37, 2.38

Instructor's Note: *2.19 is intended to be a head-scratcher. You will emerge from it with a much better sense of what frames are.*

2.1 [15] A vector ${}^{A}P$ is rotated about $\hat{Z}_A$ by θ degrees and is subsequently rotated about $\hat{X}_A$ by ϕ degrees. Give the rotation matrix that accomplishes these rotations in the given order.

2.3 [16] A frame $\{B\}$ is located initially coincident with a frame $\{A\}$. We rotate $\{B\}$ about $\hat{Z}_B$ by θ degrees, then we rotate the resulting frame about $\hat{X}_B$ by ϕ degrees. Give the rotation matrix that will change the descriptions of vectors from ${}^{B}P$ to ${}^{A}P$.

2.12 [14] A velocity vector is given by

$$ {}^{B}V = \begin{bmatrix} 10.0 \\ 20.0 \\ 30.0 \end{bmatrix}. $$

Given

$$ {}^{A}_{B}T = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 11.0 \\ 0.500 & 0.866 & 0.000 & -3.0 \\ 0.000 & 0.000 & 1.000 & 9.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

compute ${}^{A}V$.

2.14 [31] Develop a general formula to obtain ${}^{A}_{B}T$, where, starting from initial coincidence, $\{B\}$ is rotated by θ about $\hat{K}$ where $\hat{K}$ passes through the point ${}^{A}P$ (not through the origin of $\{A\}$ in general).

2.19 [24] An object is rotated about its $\hat{X}$ axis by an amount ϕ, then it is rotated about its new $\hat{Y}$ axis by an amount ψ. From our study of Euler angles, we know that the resulting orientation is given by

$$R_x(\phi)R_y(\psi),$$

whereas, if the two rotations had occurred about axes of the fixed reference frame, the result would have been

$$R_y(\psi)R_x(\phi).$$

It appears that the order of multiplication depends upon whether the rotations are described relative to fixed axes or those of the frame being moved. It is more appropriate, however, to realize that, in the case of specifying a rotation about an axis of the frame being moved, we are specifying a rotation in the fixed system given by (for this example)

$$R_x(\phi)R_y(\psi)R_x^{-1}(\phi).$$

This *similarity transform* [1], multiplying the original $R_x(\phi)$ on the left, reduces to the resulting expression in which *it looks as if* the order of matrix multiplication has been reversed. Taking this viewpoint, give a derivation for the form of the rotation matrix that is equivalent to the Z–Y–Z Euler angle set (α, β, γ). (The result is given by (2.72).)

2.20 [20] Imagine rotating a vector Q about a vector $\hat{K}$ by an amount θ to form a new vector, Q'—that is,

$$Q' = R_K(\theta)Q.$$

Use (2.80) to derive **Rodriques's formula,**

$$Q' = Q\cos\theta + \sin\theta(\hat{K} \times Q) + (1 - \cos\theta)(\hat{K} \cdot Q)\hat{K}.$$

2.21 [15] For rotations sufficiently small that the approximations $\sin\theta = \theta$, $\cos\theta = 1$, and $\theta^2 = 0$ hold, derive the rotation-matrix equivalent to a rotation of θ about a general axis, $\hat{K}$. Start with (2.80) for your derivation.

2.22 [20] Using the result from Exercise 2.21, show that two infinitesimal rotations commute (i.e., the order in which the rotations are performed is not important).

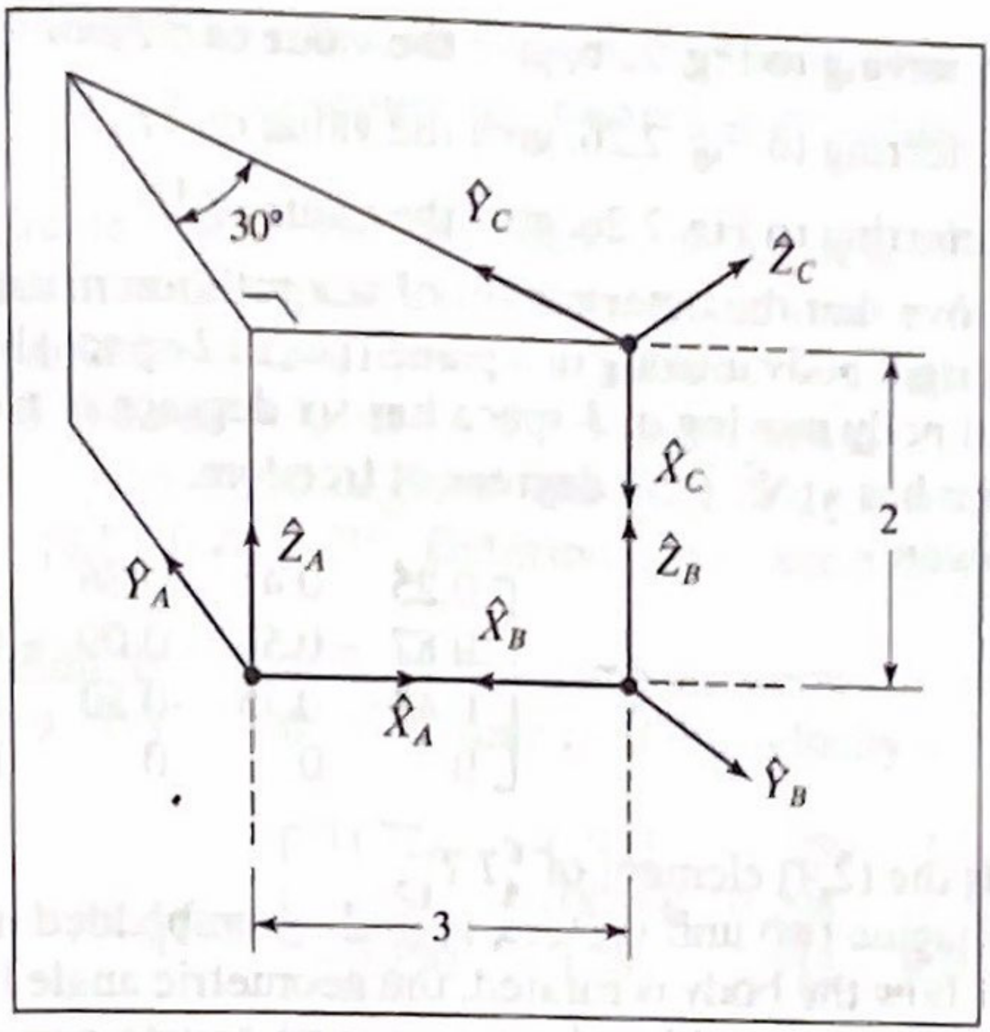


FIGURE 2.25: Frames at the corners of a wedge.

matrices.

2.27 [15] Referring to Fig. 2.25, give the value of ${}^{A}_{B}T$.

2.37 [15] Given

$$
{}^{A}_{B}T = \begin{bmatrix} 0.25 & 0.43 & 0.86 & 5.0 \\ 0.87 & -0.50 & 0.00 & -4.0 \\ 0.43 & 0.75 & -0.50 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

what is the (2,4) element of ${}^{B}_{A}T$?

what is the (2,4) element of ${}^{B}_{A}T$?

2.38 [25] Imagine two unit vectors, v_1 and v_2, embedded in a rigid body. Note that, no matter how the body is rotated, the geometric angle between these two vectors is preserved (i.e., rigid-body rotation is an "angle-preserving" operation). Use this fact to give a concise (four- or five-line) proof that the inverse of a rotation matrix must equal its transpose, and that a rotation matrix is orthonormal.

Foundations of Robotics (ROB-GY 6003)

Homework Assignment | Chapter 3

Homework Problems: 3.1, 3.4 (regard $\{S\}$ as $\{0\}$, and $\{T\}$ as $\{3\}$), 3.8, 3.12, 3.16, 3.17

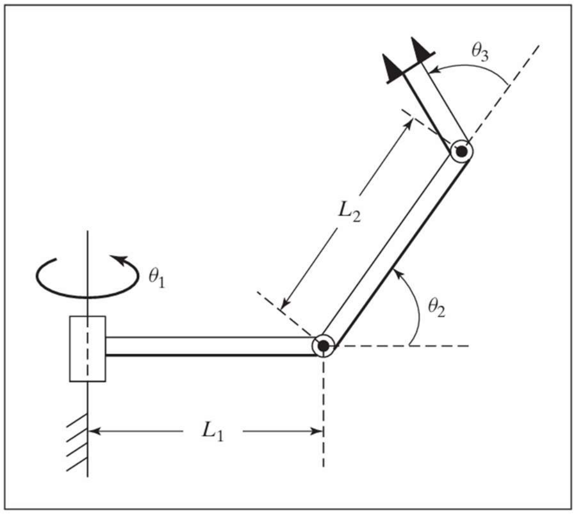


FIGURE 3.29: The 3R nonplanar arm (Exercise 3.3).

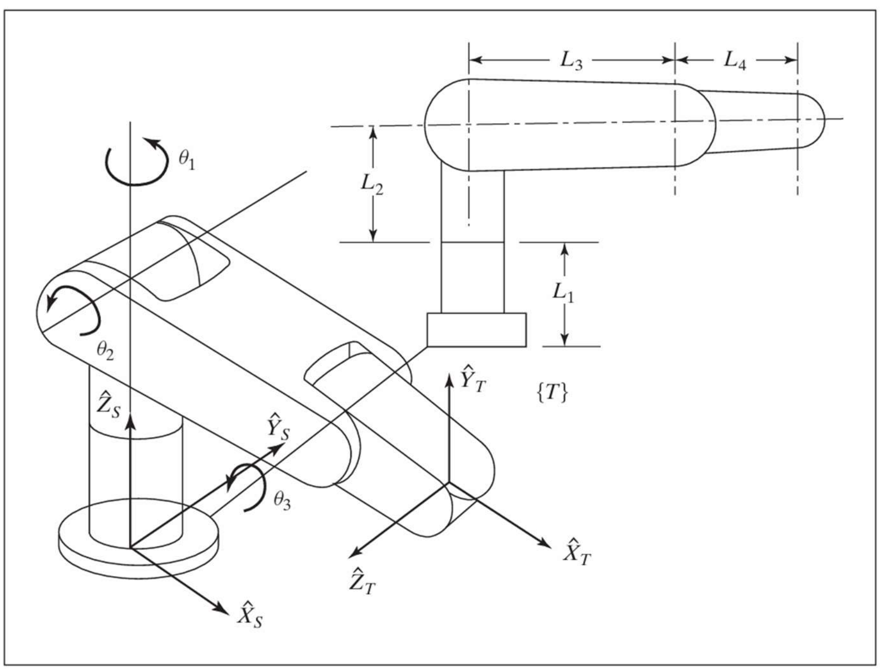
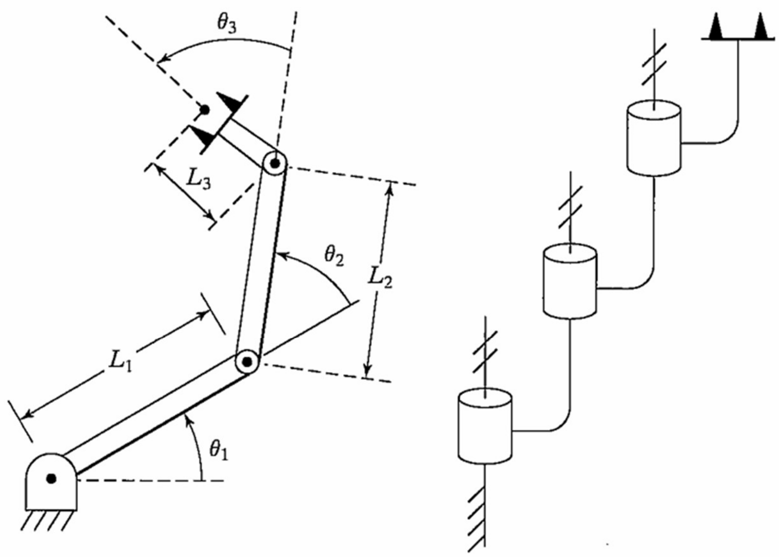


FIGURE 3.30: Two views of a 3R manipulator (Exercise 3.4).

EXAMPLE 3.3

Figure 3.6(a) shows a three-link planar arm. Because all three joints are revolute, this manipulator is sometimes called an ***RRR*** (or **3*R***) **mechanism**. Fig. 3.6(b) is a schematic representation of the same manipulator. Note the double hash marks



(a) (b)

3.1 [15] Compute the kinematics of the planar arm from Example 3.3.

3.4 [22] The arm with three degrees of freedom shown in Fig. 3.30 has joints 1 and 2 perpendicular, and joints 2 and 3 parallel. As pictured, all joints are at their zero location. Note that the positive sense of the joint angle is indicated. Assign link frames {0} through {3} for this arm—that is, sketch the arm, showing the attachment of the frames. Then derive the transformation matrices ${}^{0}_{1}T$, ${}^{1}_{2}T$, and ${}^{2}_{3}T$.

3.8 [13] In Fig. 3.31, the location of the tool, ${}^{W}_{T}T$, is not accurately known. Using force control, the robot feels around with the tool tip until it inserts it into the

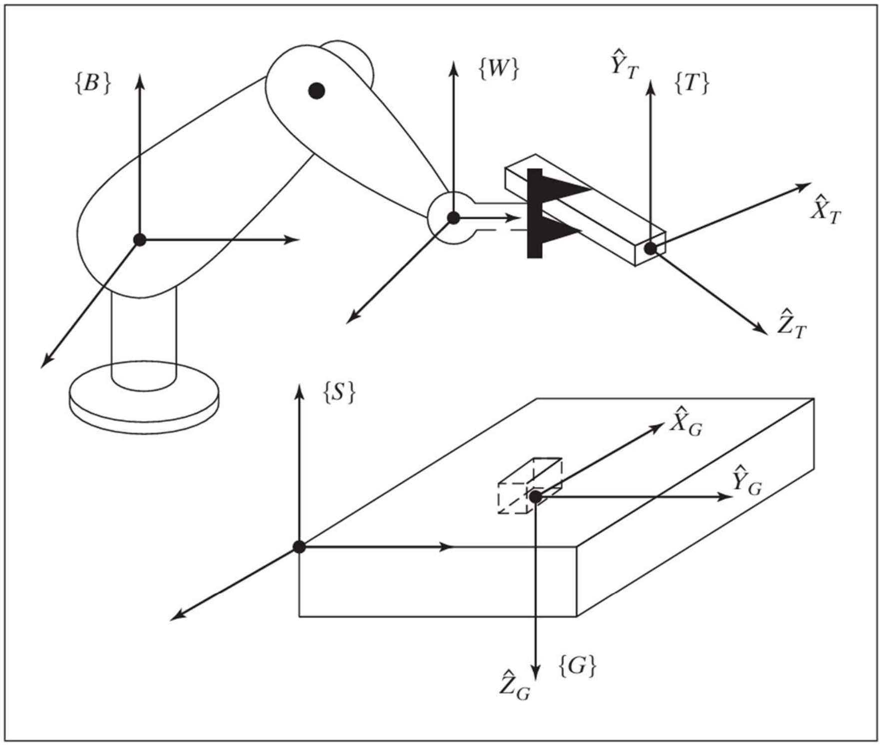


FIGURE 3.31: Determination of the tool frame (Exercise 3.8).

socket (or Goal) at location ${}^{S}_{G}T$. Once in this "calibration" configuration (in which $\{G\}$ and $\{T\}$ are coincident), the position of the robot, ${}^{B}_{W}T$, is figured out by reading the joint angle sensors and computing the kinematics. Assuming ${}^{B}_{S}T$ and ${}^{S}_{G}T$ are known, give the transform equation to compute the unknown tool frame, ${}^{W}_{T}T$.

3.12 [08] Can an arbitrary rigid-body transformation always be expressed with four parameters (a, α, d, θ) in the form of equation (3.6)?

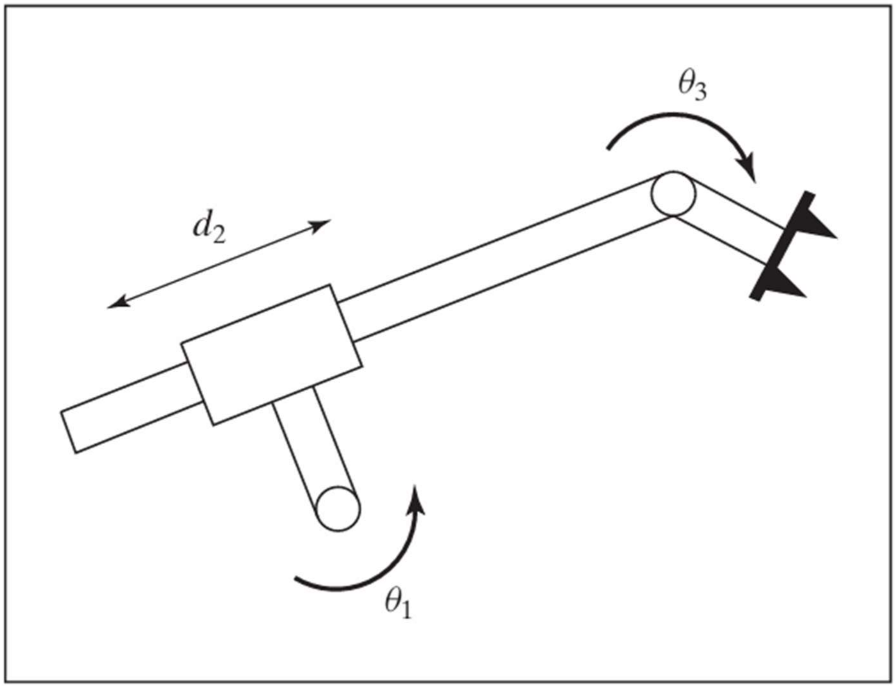


FIGURE 3.36: *RPR* planar robot (Exercise 3.16).

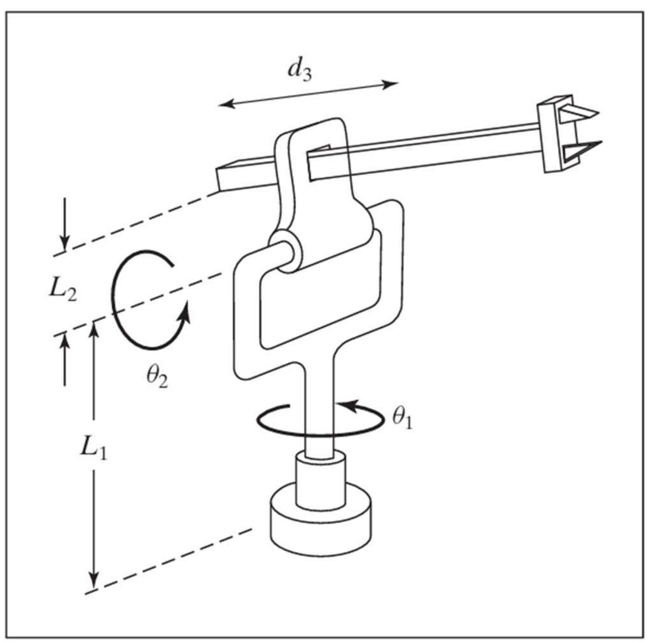


FIGURE 3.37: Three-link *RRP* manipulator (Exercise 3.17).

3.16 [15] Assign link frames to the RPR planar robot shown in Fig. 3.36, and give the linkage parameters.

3.17 [15] Show the attachment of link frames on the three-link robot shown in Fig. 3.37.

Foundations of Robotics (ROB-GY 6003)

Homework Assignment | Chapter 4

Homework Problems: 4.8, 4.9, 4.18, 4.19, 4.24

Instructor's Note: *For questions that ask for the number of solutions, you only need to indicate the number as your answer (e.g., none, one, two, infinite…). However, you may find it intellectually rewarding to try to "prove" your answer arrived at by physical intuition through either attempting to analytically or numerically solve for the inverse kinematics.*

4.8 [12] Given a desired position and orientation of the hand of a three-link planar rotary-jointed manipulator, there are two possible solutions. If we add one more rotational joint (in such a way that the arm is still planar), how many solutions are there?

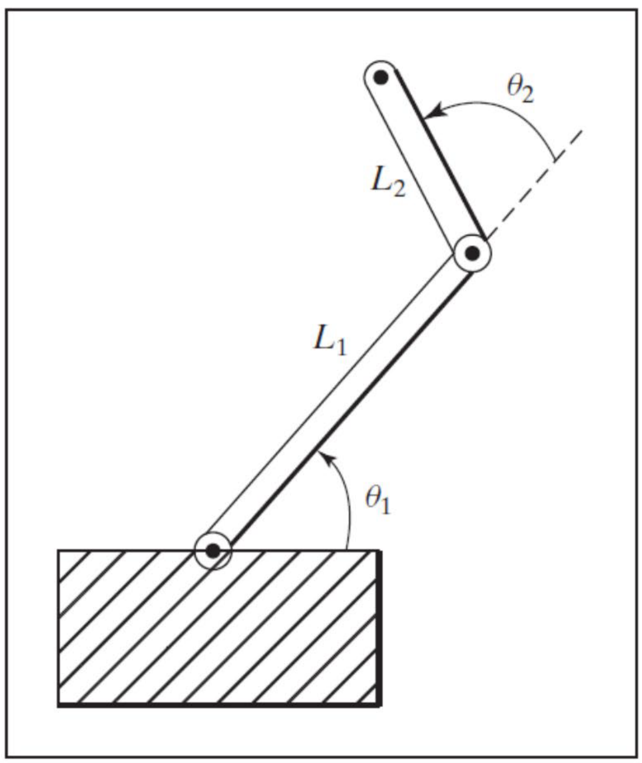


FIGURE 4.13: Two-link planar manipulator (Exercise 4.9).

4.9 [26] Figure 4.13 shows a two-link planar arm with rotary joints. For this arm, the second link is half as long as the first—that is, $l_1 = 2l_2$. The joint range limits in degrees are

$$0 < \theta_1 < 180,$$
$$-90 < \theta_2 < 180.$$

Sketch the approximate reachable workspace (an area) of the tip of link 2.

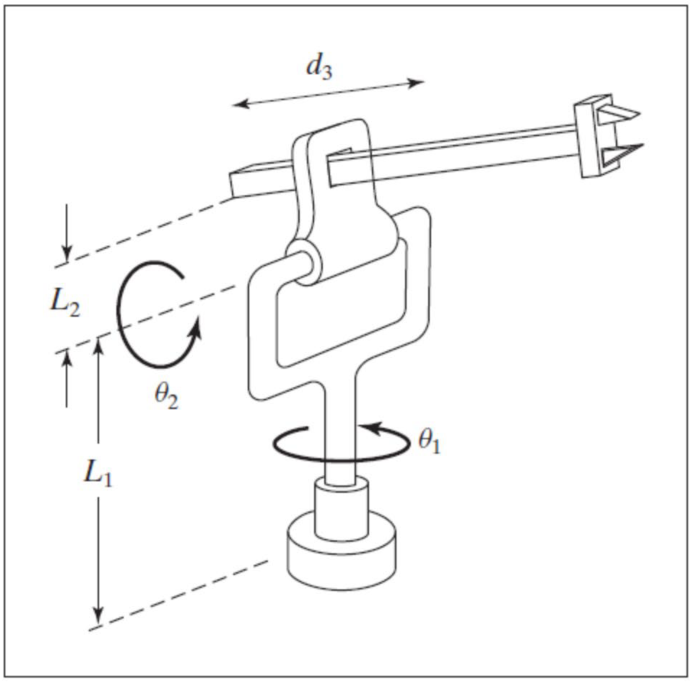


FIGURE 3.37: Three-link *RRP* manipulator (Exercise 3.17).

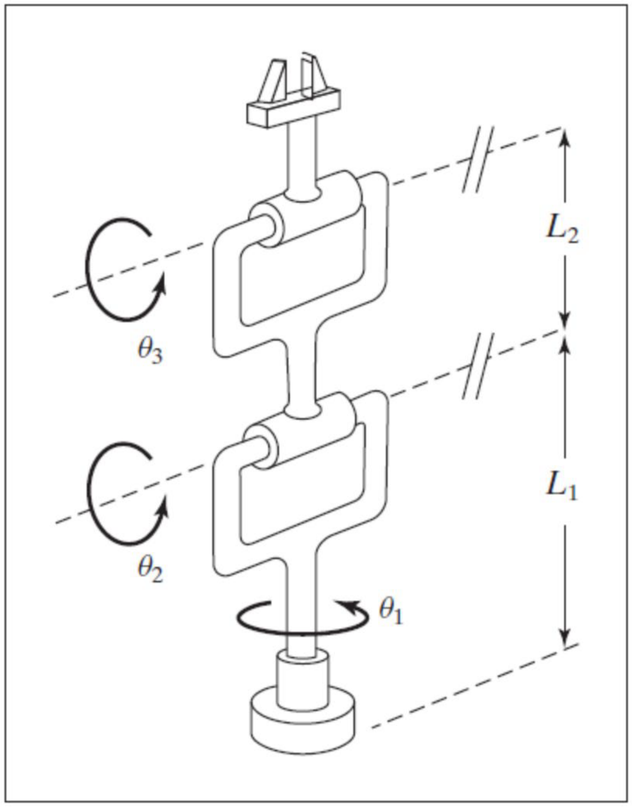


FIGURE 3.38: Three-link *RRR* manipulator (Exercise 3.18).

4.18 [15] Consider the *RRP* manipulator shown in Fig. 3.37. How many solutions do the (position) kinematic equations possess?

4.19 [15] Consider the *RRR* manipulator shown in Fig. 3.38. How many solutions do the (position) kinematic equations possess?

4.24 [20] Given the description of link frame $\{i\}$ in terms of link frame $\{i-1\}$, find the four Denavit–Hartenberg parameters as functions of the elements of ${}^{i-1}_{i}T$.

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Foundations of Robotics (ROB-GY 6003)

Homework Assignment | Chapter 5

Homework Problems: 5.4, 5.8, 5.11, 5.13, 5.16, 5.20

5.4 [8] Prove that singularities in the force domain exist at the same configurations as singularities in the position domain.

5.8 [18] General mechanisms sometimes have certain configurations, called "isotropic points," where the columns of the Jacobian become orthogonal and of equal magnitude [7]. For the two-link manipulator of Example 5.3, find out if any isotropic points exist. *Hint:* Is there a requirement on l_1 and l_2?

5.11 [14] Given

$$ {}^{A}_{B}T = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 10.0 \\ 0.500 & 0.866 & 0.000 & 0.0 \\ 0.000 & 0.000 & 1.000 & 5.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

if the velocity vector at the origin of $\{A\}$ is

$$ {}^{A}\nu = \begin{bmatrix} 0.0 \\ 2.0 \\ -3.0 \\ 1.414 \\ 1.414 \\ 0.0 \end{bmatrix}, $$

find the 6×1 velocity vector with reference point the origin of $\{B\}$.

5.13 [9] A certain two-link manipulator has the following Jacobian:

$$^{0}J(\Theta) = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}.$$

Ignoring gravity, what are the joint torques required in order that the manipulator will apply a static force vector $^{0}F = 10\hat{X}_0$?

5.16 [20] A $3R$ manipulator has kinematics that correspond exactly to the set of Z–Y–Z Euler angles (i.e., the forward kinematics are given by (2.72) with $\alpha = \theta_1$, $\beta = \theta_2$, and $\gamma = \theta_3$). Give the Jacobian relating joint velocities to the angular velocity of the final link.

5.20 [20] Explain what might be meant by the statement: "An n-DOF manipulator at a singularity can be treated as a redundant manipulator in a space of dimensionality $n - 1$."