NEW YORK UNIVERSITY TANDON SCHOOL OF ENGINEERING
Department of Mechanical and Aerospace Engineering

Foundations of Robotics (ROB-GY 6003)

Homework Assignment | Chapter 2

There is a total of seven homework assignments for this course, corresponding to Chapters 2–8. Problems are taken from Craig 4th ed.

Homework Problems: 2.1, 2.3, 2.12, 2.14, 2.19, 2.20, 2.21, 2.22, 2.27, 2.37, 2.38

Instructor's Note: *2.19 is intended to be a head-scratcher. You will emerge from it with a much better sense of what frames are.*

2.1 [15] A vector ${}^{A}P$ is rotated about $\hat{Z}_A$ by θ degrees and is subsequently rotated about $\hat{X}_A$ by ϕ degrees. Give the rotation matrix that accomplishes these rotations in the given order.

2.3 [16] A frame $\{B\}$ is located initially coincident with a frame $\{A\}$. We rotate $\{B\}$ about $\hat{Z}_B$ by θ degrees, then we rotate the resulting frame about $\hat{X}_B$ by ϕ degrees. Give the rotation matrix that will change the descriptions of vectors from ${}^{B}P$ to ${}^{A}P$.

2.12 [14] A velocity vector is given by

$$ {}^{B}V = \begin{bmatrix} 10.0 \\ 20.0 \\ 30.0 \end{bmatrix}. $$

Given

$$ {}^{A}_{B}T = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 11.0 \\ 0.500 & 0.866 & 0.000 & -3.0 \\ 0.000 & 0.000 & 1.000 & 9.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

compute ${}^{A}V$.

2.14 [31] Develop a general formula to obtain ${}^{A}_{B}T$, where, starting from initial coincidence, $\{B\}$ is rotated by θ about $\hat{K}$ where $\hat{K}$ passes through the point ${}^{A}P$ (not through the origin of $\{A\}$ in general).

2.19 [24] An object is rotated about its $\hat{X}$ axis by an amount ϕ, then it is rotated about its new $\hat{Y}$ axis by an amount ψ. From our study of Euler angles, we know that the resulting orientation is given by

$$R_x(\phi)R_y(\psi),$$

whereas, if the two rotations had occurred about axes of the fixed reference frame, the result would have been

$$R_y(\psi)R_x(\phi).$$

It appears that the order of multiplication depends upon whether the rotations are described relative to fixed axes or those of the frame being moved. It is more appropriate, however, to realize that, in the case of specifying a rotation about an axis of the frame being moved, we are specifying a rotation in the fixed system given by (for this example)

$$R_x(\phi)R_y(\psi)R_x^{-1}(\phi).$$

This *similarity transform* [1], multiplying the original $R_x(\phi)$ on the left, reduces to the resulting expression in which *it looks as if* the order of matrix multiplication has been reversed. Taking this viewpoint, give a derivation for the form of the rotation matrix that is equivalent to the Z–Y–Z Euler angle set (α, β, γ). (The result is given by (2.72).)

2.20 [20] Imagine rotating a vector Q about a vector $\hat{K}$ by an amount θ to form a new vector, Q'—that is,

$$Q' = R_K(\theta)Q.$$

Use (2.80) to derive **Rodriques's formula**,

$$Q' = Q\cos\theta + \sin\theta(\hat{K} \times Q) + (1 - \cos\theta)(\hat{K} \cdot Q)\hat{K}.$$

2.21 [15] For rotations sufficiently small that the approximations $\sin\theta = \theta$, $\cos\theta = 1$, and $\theta^2 = 0$ hold, derive the rotation-matrix equivalent to a rotation of θ about a general axis, $\hat{K}$. Start with (2.80) for your derivation.

2.22 [20] Using the result from Exercise 2.21, show that two infinitesimal rotations commute (i.e., the order in which the rotations are performed is not important).

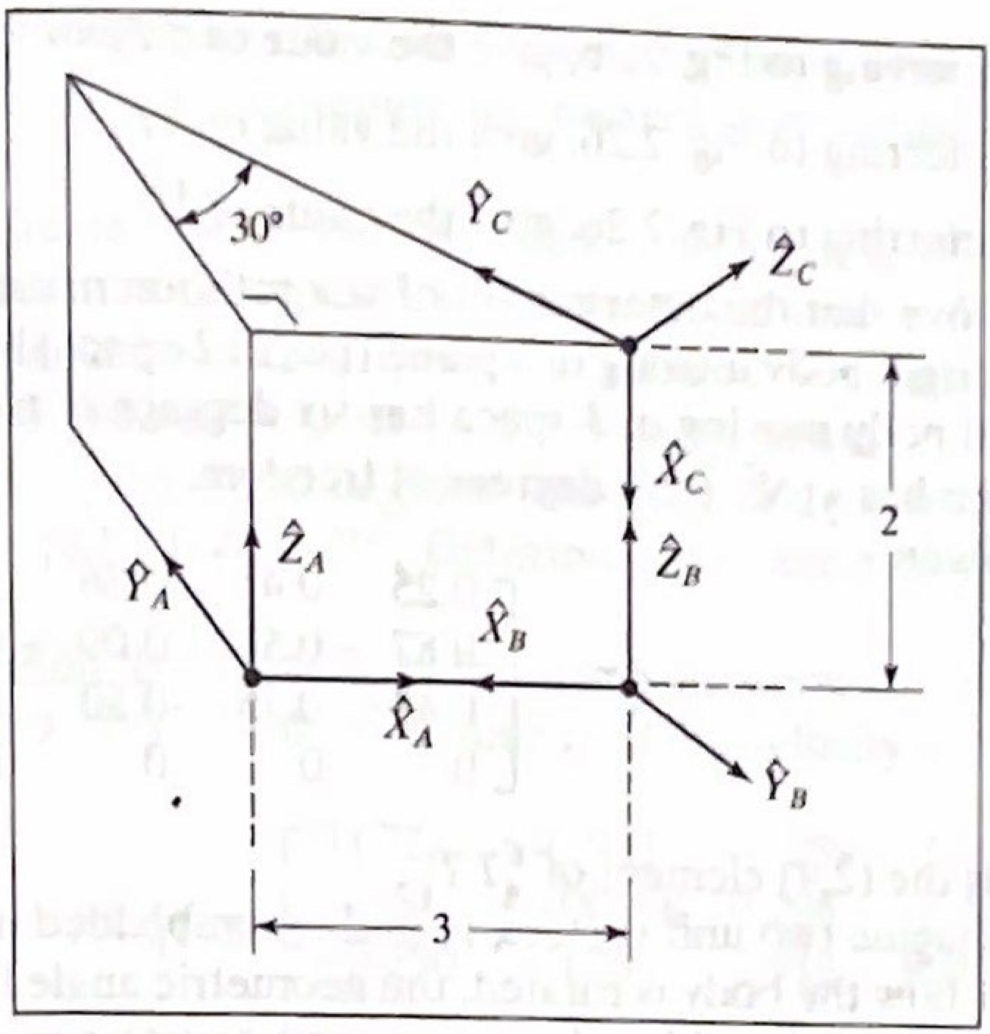

FIGURE 2.25: Frames at the corners of a wedge.

2.27 [15] Referring to Fig. 2.25, give the value of ${}^{A}_{B}T$.

2.37 [15] Given

$$
{}^{A}_{B}T = \begin{bmatrix} 0.25 & 0.43 & 0.86 & 5.0 \\ 0.87 & -0.50 & 0.00 & -4.0 \\ 0.43 & 0.75 & -0.50 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

what is the (2,4) element of ${}^{B}_{A}T$?

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2.38 [25] Imagine two unit vectors, v_1 and v_2, embedded in a rigid body. Note that, no matter how the body is rotated, the geometric angle between these two vectors is preserved (i.e., rigid-body rotation is an "angle-preserving" operation). Use this fact to give a concise (four- or five-line) proof that the inverse of a rotation matrix must equal its transpose, and that a rotation matrix is orthonormal.