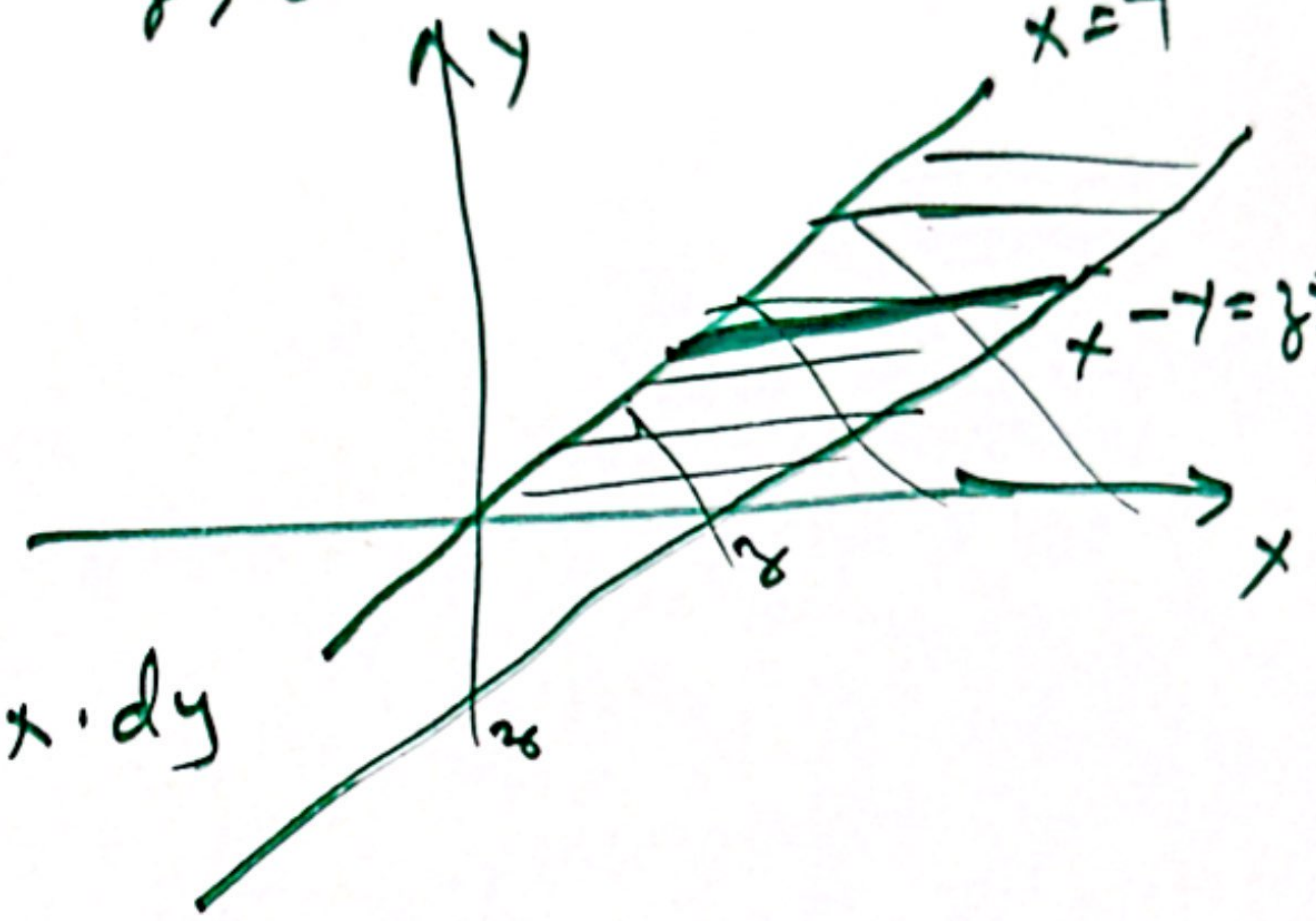
1) a) Solutions to Quiz #2

$$f_{XY}(x,y) = \begin{cases} e^{-x} & x > y > 0 \\ 0 & \text{otherwise} \end{cases}$$

$Z = X - Y \Rightarrow z > 0$



$z > 0$

$$F_Z(z) = P(Z \le z)$$

$$= P(X - Y \le z)$$

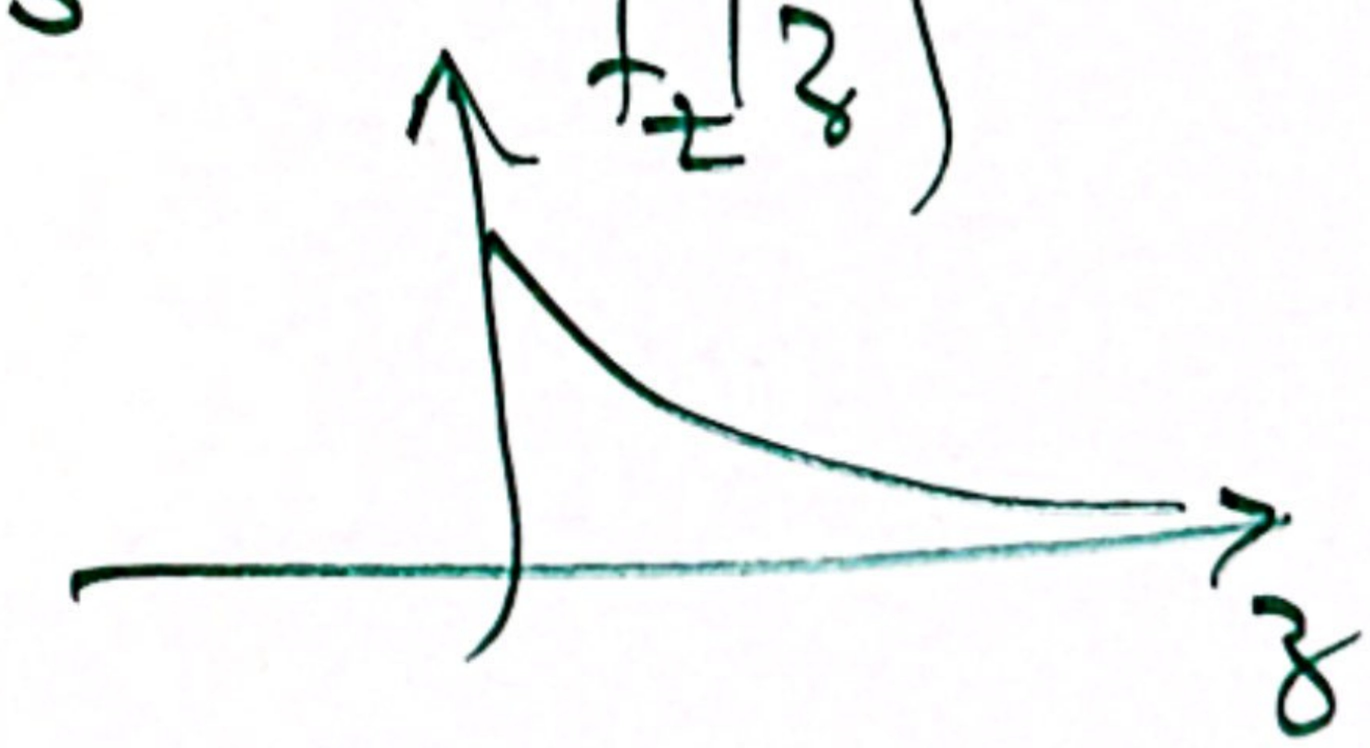
$$= \int_0^\infty \int_{x=y}^{y+z} f_{XY}(x,y)\,dx \cdot dy$$

$$f_Z(z) = \frac{d}{dz} F_Z(z) = \int_0^\infty 1 \cdot f_{XY}(y+z, y)\,dy$$

$$= \int_0^\infty e^{-(y+z)}\,dy = e^{-z} \underbrace{\int_0^\infty e^{-y}\,dy}_{1}$$

$$= e^{-z}, \quad z > 0$$

$\Rightarrow Z$ is exponential

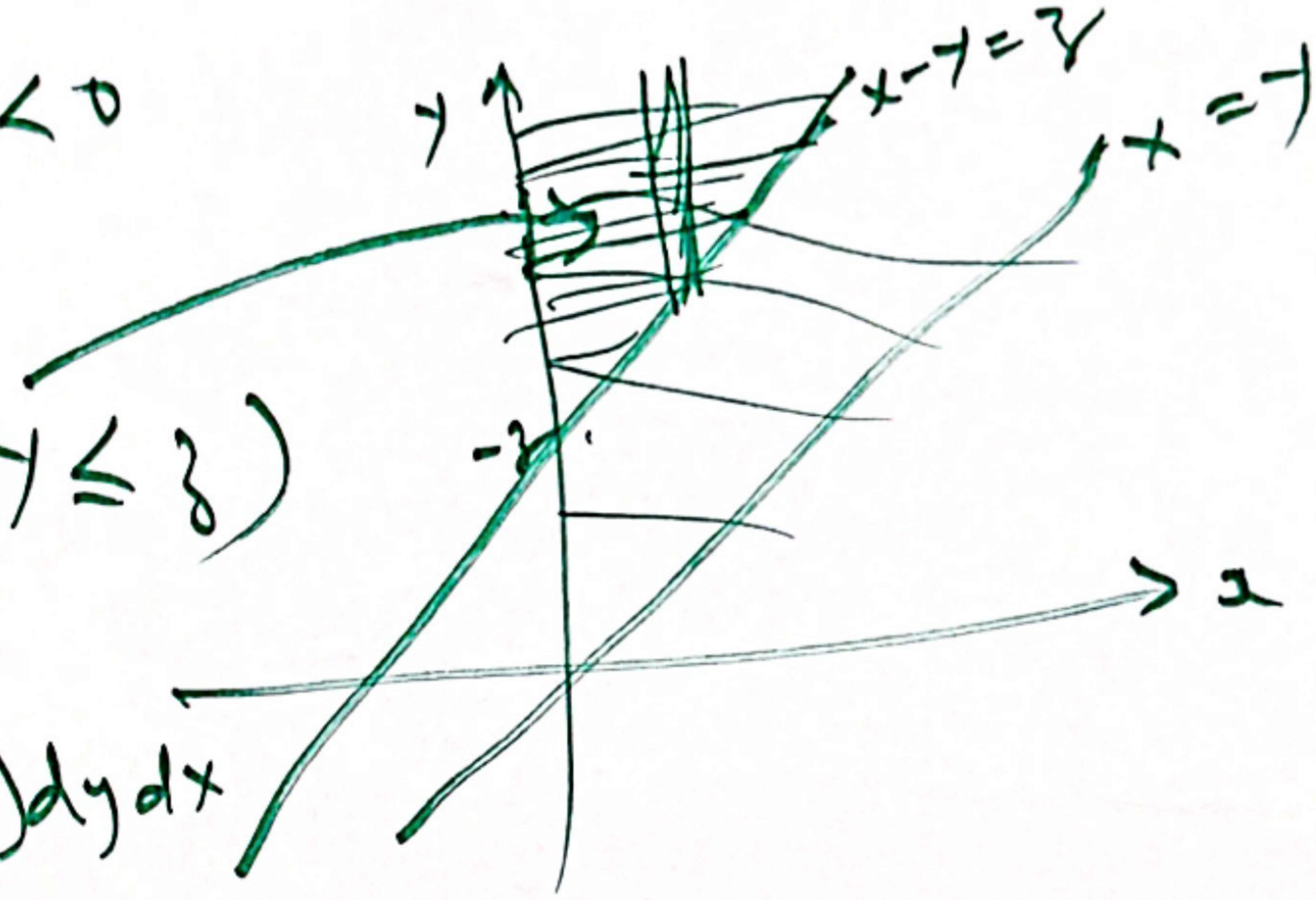


Solution to Quiz #2

2b) $f_{XY}(x,y) = \begin{cases} e^{-y}, & y > x > 0 \\ 0 & \text{otherw} \end{cases}$

$y > x \Rightarrow x - y \le z < 0$

For $z < 0$



$F_Z(z) = P(X - Y \le z)$

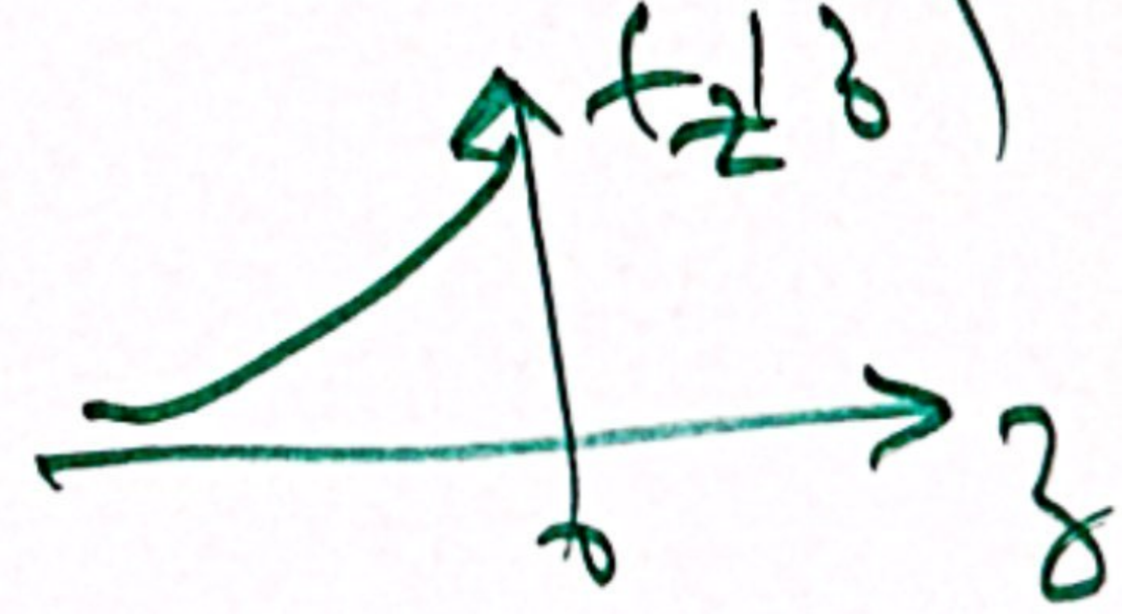
$= \int_0^\infty \int_{x-z}^\infty f_{XY}(x,y)\,dy\,dx$

$f_Z(z) = \int_0^\infty (-1)(-1)\, f_{XY}(x, x-z)\,dx$

$= \int_0^\infty f_{XY}(x, x-z)\,dx = \int_0^\infty e^{-(x-z)}\,dx$

$= e^{z} \underbrace{\int_0^\infty e^{-x}\,dx}_{1} = e^{+z}, \quad z < 0$

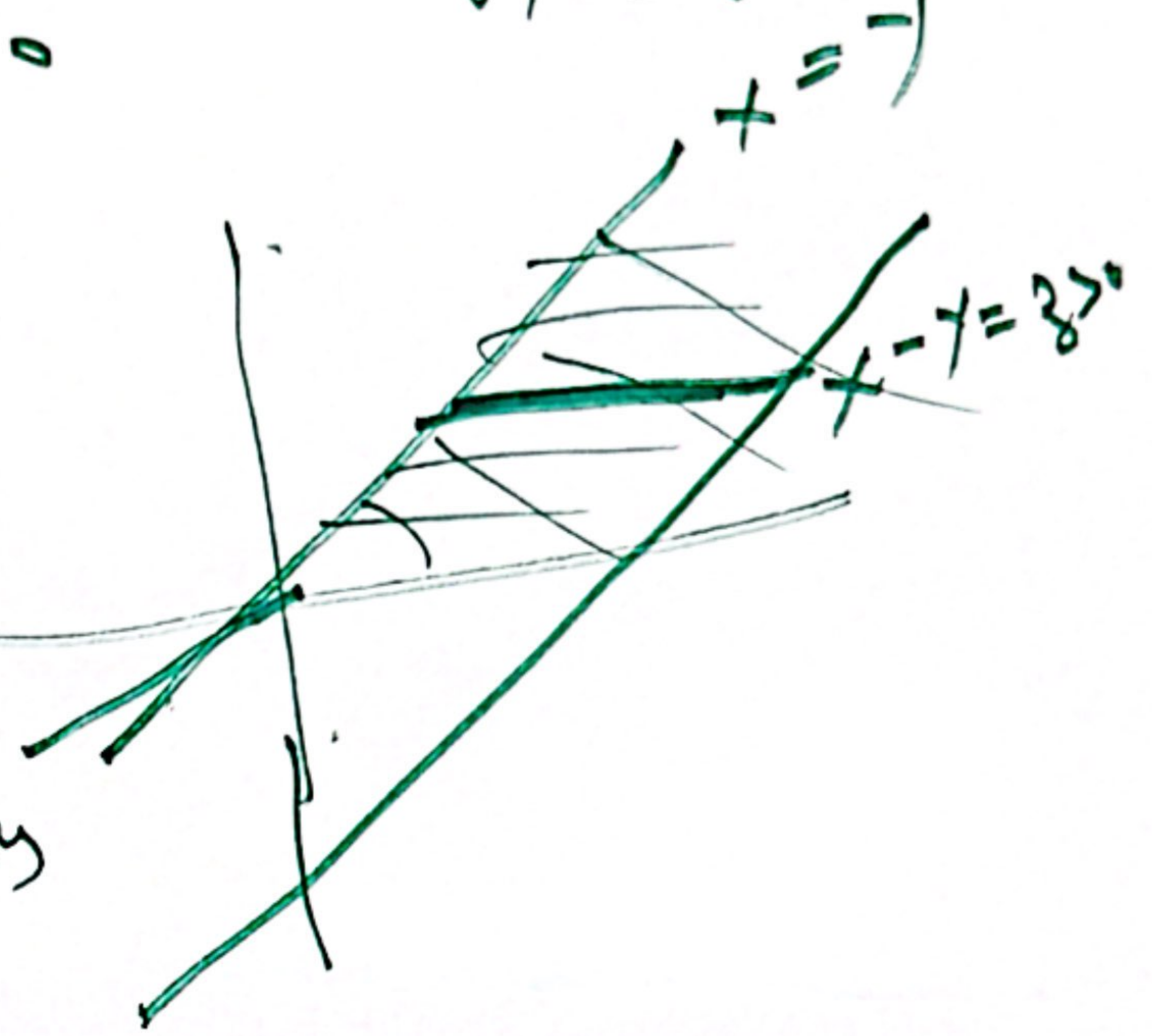
Z is -ve exponential.



Solution to Quiz 2

1c) $f_{XY}(x,y) = \begin{cases} \frac{x}{2}e^{-x}, & x>y>0 \\ 0 & \text{otherw} \end{cases}$

$x>y \Rightarrow x-y=z>0$



For $z>0$

$$F_Z(z) = P(\overbrace{X-Y}^{Z}\le z)$$

$$= \int_0^\infty \int_{x=y}^{y+z} f_{XY}(x,y)\,dx\,dy$$

$$f_Z(z) = \frac{d}{dz}F_Z(z) = \int_0^\infty 1\cdot f_{XY}(y+z,y)\,dy$$

$$= \frac{1}{2}\int_0^\infty (y+z)e^{-(y+z)}\,dy$$

$$= \frac{1}{2}e^{-z}\Big[\underbrace{\int_0^\infty y e^{-y}}_{1} + z\underbrace{\int_0^\infty e^{-y}\,dy}_{1}\Big]$$

$$= \frac{(1+z)}{2}e^{-z}, \quad z>0$$