Quiz #1a

Name: ____________________ NetID: ____________________

Problem 1

On holidays the probability that there are no accidents on the highway is twice the probability that there is at least one accident. If we model the accidents occurring on the highway as a Poisson random variable, what is the condition on the Poisson parameter λ?

$$X \sim P(\lambda) \Rightarrow P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}, \; k=0,1,\ldots$$

Prob.{"no accidents"} $= P(X=0) = e^{-\lambda}$ (1)

"At least one accident" $= (X \geq 1)$

$$P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0)$$

Given

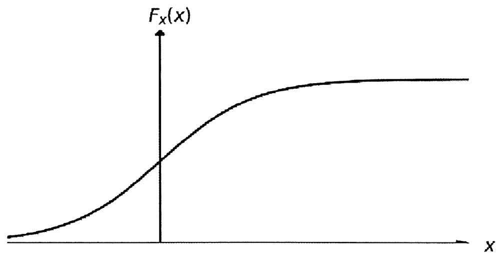
$$P(X=0) = 2P(X \geq 1)$$
$$= 2(1 - P(X=0))$$

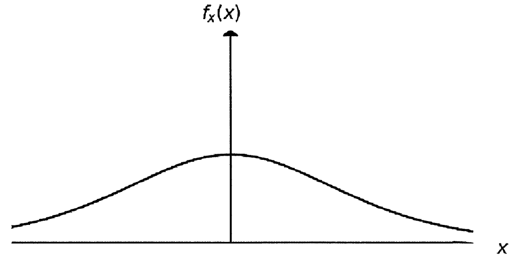
$$\Rightarrow 3P(X=0) = 2 \Rightarrow 3e^{-\lambda} = 2$$

$$e^{\lambda} = \frac{3}{2} \Rightarrow \lambda = \ln\frac{3}{2}$$

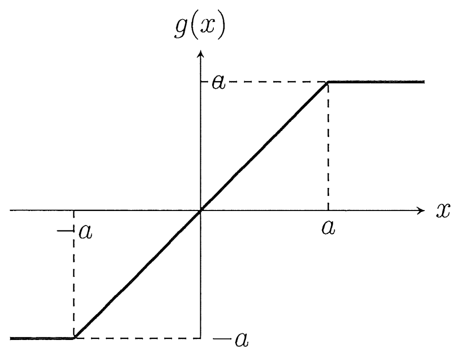
Problem 2

Given $F_x(x)$, $f_x(x)$ as shown





Sketch $F_y(y)$ and $f_y(y)$ for $y = g(x)$ when



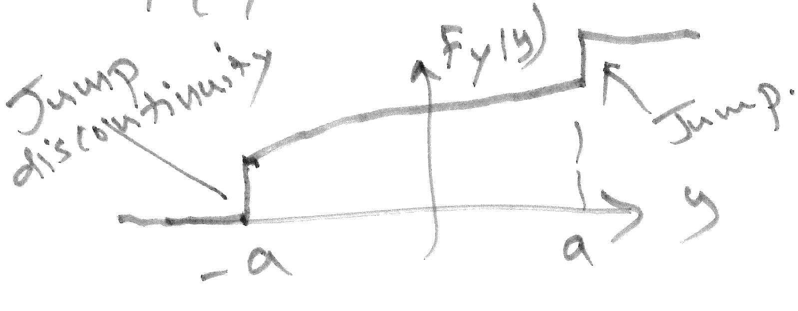
$$Y = \begin{cases} a, & x > a \\ x & -a < x \le a \quad (-a < y \le a) \\ -a & x \le -a \end{cases}$$

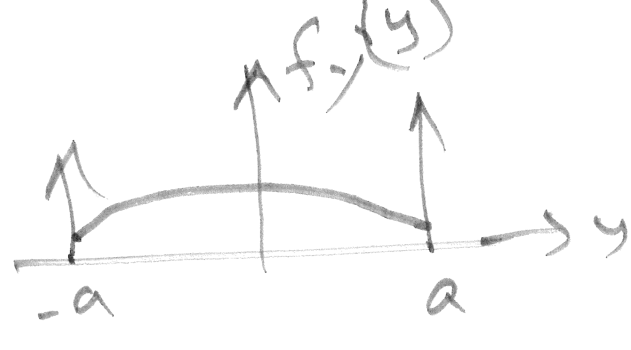
$$P(Y = -a) = P(X \le -a) = F_x(-a)$$

$$-a < y \le a \Rightarrow Y = X$$

$$F_y(y) = P(Y \le y) = P(X \le y) = F_X(y)$$

$$P(Y = a) = P(X > a) = 1 - P(X \le a) = 1 - F_X(a)$$





Quiz #1b

Name: ______________________ NetID: ______________________

Problem 1

On holidays the probability that there are no accidents on the highway is twice the probability that there is at most one accident. If we model the accidents occurring on the highway as a Poisson random variable, what is the condition on the Poisson parameter λ?

$X \sim P(\lambda) \Rightarrow P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}, \; k=0,1,2\ldots$

"No accident" $= (X=0)$

"At most one accident" $= \underbrace{\{X=0) \cup (X=1)}_{ME}$

Given

$P(X=0) = 2P(X \le 1)$

$= 2(P(X=0)+P(X=1))$

$\Rightarrow e^{-\lambda} = 2(e^{-\lambda} + \lambda e^{-\lambda})$

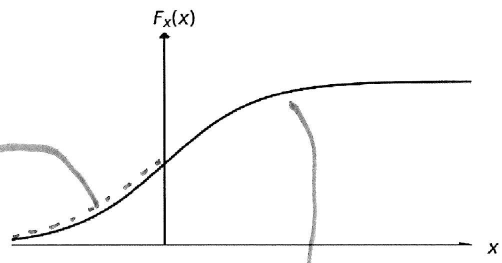
$\Rightarrow 1 = 2(1+\lambda) \Rightarrow \lambda + 1 = \frac{1}{2}$

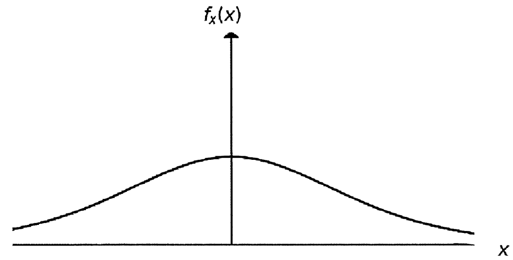
$\lambda = -\frac{1}{2}$

For $\lambda = -\frac{1}{2}$, this is unrealistic, since $\lambda > 0$ always.
There is no positive process for which the condition is $-\frac{1}{2}$.

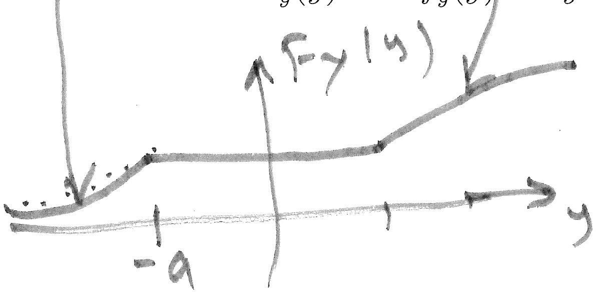
Problem 2

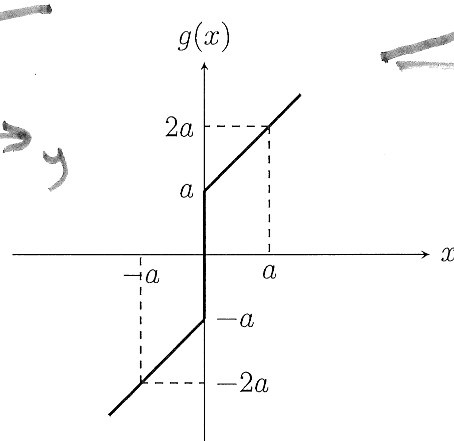
Given $F_x(x)$, $f_x(x)$ as shown





Sketch $F_y(y)$ and $f_y(y)$ for $y = g(x)$ when





$$Y = \begin{cases} X + a, & x > 0 \quad (Y > a) \\ X - a, & x \leq 0 \quad (Y \leq -a) \end{cases}$$

Notice Y doesn't take any values in $-a < y < a$.

$y > a$

$$F_Y(y) = P(Y \leq y) = P(X + a \leq y) = P(X \leq y - a)$$
$$= F_X(y - a), \quad y > a$$

$y < -a$

$$F_Y(y) = P(Y \leq y) = P(X - a \leq y) = P(X \leq y + a)$$
$$= F_X(y + a), \quad y < -a$$

See graph above.

Quiz #1c

Name: ______________________ NetID: ______________________

Problem 1

On holidays the probability that there are no accidents on the highway is twice the probability that there is exactly either one or two accidents. If we model the accidents occurring on the highway as a Poisson random variable, what is the condition on the Poisson parameter λ?

$$X \sim P(\lambda) \Rightarrow P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k=0,1,2$$

"No accidents" $= (X=0)$

"Exactly either one or two accidents"

$$= \underbrace{(X=1) \cup (X=2)}_{ME}$$

Given

$$P(X=0) = 2\left[P(X=1) + P(X=2)\right]$$

$$\Rightarrow e^{-\lambda} = 2\left[\lambda e^{-\lambda} + \frac{\lambda^2}{2}e^{-\lambda}\right]$$

$$\Rightarrow 1 = 2\lambda + \lambda^2$$

$$= \lambda^2 + 2\lambda - 1 = 0$$

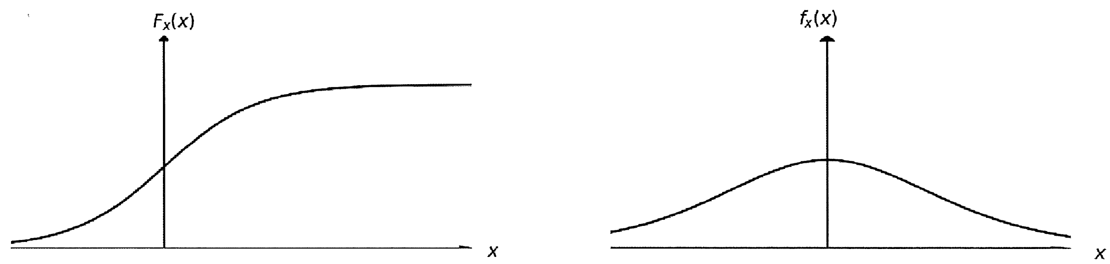
$$\lambda = \frac{-2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

$\lambda_1 = 1+\sqrt{2}$ or $\lambda_2 = 1-\sqrt{2}$.

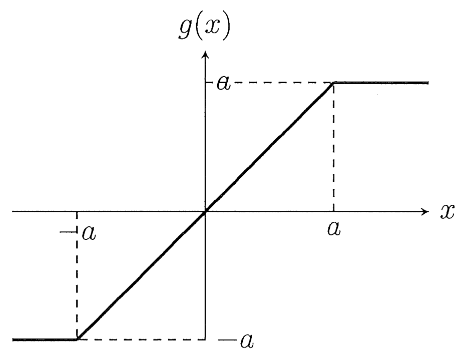
λ_2 is unrealistic since $\lambda_2 < 0$.
$\lambda_1 > 0$, so λ_1 is the only solution.

Problem 2

Given $F_x(x)$, $f_x(x)$ as shown



Sketch $F_y(y)$ and $f_y(y)$ for $y = g(x)$ when



$$y = \begin{cases} a, & x > a \\ x & -a < x \leq a \\ -a, & x \leq -a \end{cases}$$

$$P(y = -a) = P(x \leq -a) = F_x(-a)$$

$$-a < y \leq a \Rightarrow y = x$$

$$F_y(y) = P(Y \leq y) = P(X \leq y) = F_x(y)$$

$$P(y = a) = P(x > a) = 1 - P(x \leq a) = 1 - F_x(a)$$