

## Foundations of Robotics (ROB-GY 6003)

### Homework Assignment | Chapter 5

**Homework Problems:** 5.4, 5.8, 5.11, 5.13, 5.16, 5.20

**5.4** [8] Prove that singularities in the force domain exist at the same configurations as singularities in the position domain.

**5.8** [18] General mechanisms sometimes have certain configurations, called “isotropic points,” where the columns of the Jacobian become orthogonal and of equal magnitude [7]. For the two-link manipulator of Example 5.3, find out if any isotropic points exist. *Hint:* Is there a requirement on  $l_1$  and  $l_2$ ?

**5.11** [14] Given

$${}^A_B T = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 10.0 \\ 0.500 & 0.866 & 0.000 & 0.0 \\ 0.000 & 0.000 & 1.000 & 5.0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

if the velocity vector at the origin of  $\{A\}$  is

$${}^A v = \begin{bmatrix} 0.0 \\ 2.0 \\ -3.0 \\ 1.414 \\ 1.414 \\ 0.0 \end{bmatrix},$$

find the  $6 \times 1$  velocity vector with reference point the origin of  $\{B\}$ .

---

**5.13** [9] A certain two-link manipulator has the following Jacobian:

$${}^0J(\Theta) = \begin{bmatrix} -l_1s_1 - l_2s_{12} & -l_2s_{12} \\ l_1c_1 + l_2c_{12} & l_2c_{12} \end{bmatrix}.$$

Ignoring gravity, what are the joint torques required in order that the manipulator will apply a static force vector  ${}^0F = 10\hat{X}_0$ ?

**5.16** [20] A 3R manipulator has kinematics that correspond exactly to the set of Z–Y–Z Euler angles (i.e., the forward kinematics are given by (2.72) with  $\alpha = \theta_1$ ,  $\beta = \theta_2$ , and  $\gamma = \theta_3$ ). Give the Jacobian relating joint velocities to the angular velocity of the final link.

**5.20** [20] Explain what might be meant by the statement: “An  $n$ -DOF manipulator at a singularity can be treated as a redundant manipulator in a space of dimensionality  $n - 1$ .”

---