MATH 233H ATTENDANCE PROBLEMS

These are the quick problems given in class to (randomly) take attendance. Please let me know if you find any mistakes.

(1) Let $\mathbf{v} = \langle 1, 1, 1\rangle, \mathbf{w} = \langle 1, 3, -1\rangle$. Find a unit vector perpendicular to $\mathbf{v}$ and $\mathbf{w}$. **Answer:** Take the cross product to get $\mathbf{r} = \langle -4, 2, 2\rangle$. Using the dot product we see that $\mathbf{r}$ is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$. The length of $\mathbf{r}$ is $\sqrt{16+4+4} = \sqrt{24} = 2\sqrt{6}$. So the unit vector is $\langle -2, 1, 1\rangle/\sqrt{6}$.

(2) Find the distance between the lines $\langle t, -t, 1\rangle$ and the x-axis. **Answer:** The lines are skew. The direction vectors are $\mathbf{i}$ and $\langle 1, -1, 0\rangle = \mathbf{i} - \mathbf{j}$, so a common perpendicular is $\mathbf{k}$. A vector going in between is $\mathbf{w} = \langle 0, 0, 1\rangle$ (take $t = 0$ and the origin) so the distance is $|\mathbf{w} \cdot \mathbf{k}| = 1$.

(3) Let $\mathbf{r}(t) = \langle te^t, t^2 + t, \sin t^2\rangle$. Compute $\mathbf{r}'$ and $\mathbf{r}''$. **Answer:** $\mathbf{r}' = \langle te^t + e^t, 2t + 1, 2t\cos t^2\rangle$, $\mathbf{r}'' = \langle 2e^t + te^t, 2, -4t^2 \sin t^2 + 2\cos t^2\rangle$.

(4) Let $f(x, y) = \cos(\sqrt{x^2 + y^2})$. (a) Draw some contour lines for f. (b) Describe/sketch the graph. **Answer:** Let $r = \sqrt{x^2 + y^2}$. Then r measures the distance to the origin in the xy-plane, and the function is really $z = \cos r$ (Figure 1). This means the contour lines are circles centered at the origin (why?). The graph of the surface is what you get when you rotate the graph of cosine about the vertical axis (see Figure 2, although it's somewhat distorted). If you want to plot it yourself, you can try `plot cos sqrt (x x + y y)` at `wolframalpha.com`.

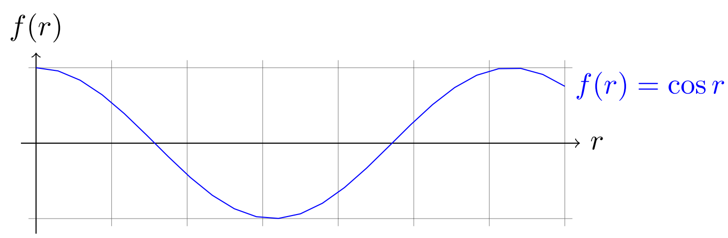

FIGURE 1.

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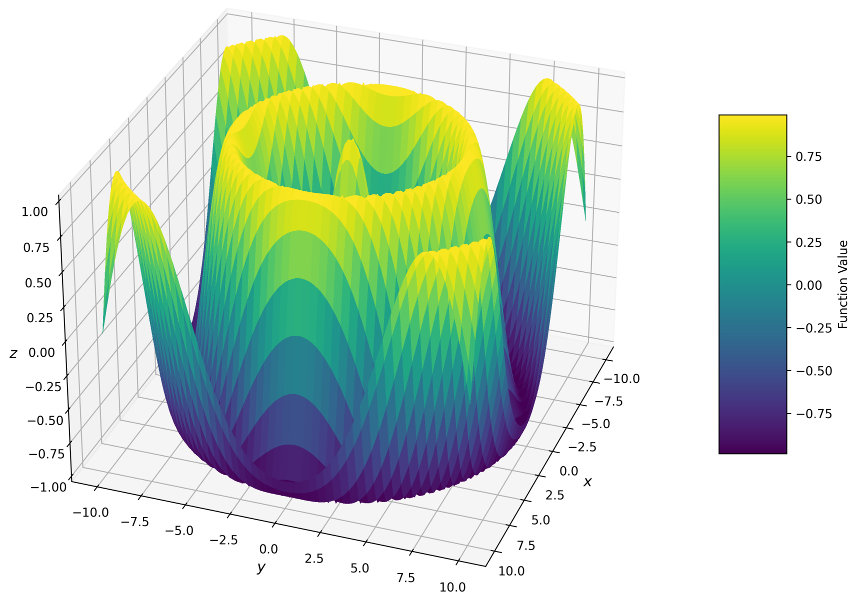

FIGURE 2. $z = \cos\sqrt{x^2 + y^2}$