MATH 233H ATTENDANCE PROBLEMS

These are the quick problems given in class to (randomly) take attendance. Please let me know if you find any mistakes.

(1) Let $\mathbf{v} = \langle 1, 1, 1\rangle, \mathbf{w} = \langle 1, 3, -1\rangle$. Find a unit vector perpendicular to $\mathbf{v}$ and $\mathbf{w}$. **Answer:** Take the cross product to get $\mathbf{r} = \langle -4, 2, 2\rangle$. Using the dot product we see that $\mathbf{r}$ is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$. The length of $\mathbf{r}$ is $\sqrt{16+4+4} = \sqrt{24} = 2\sqrt{6}$. So the unit vector is $\langle -2, 1, 1\rangle/\sqrt{6}$.

(2) Find the distance between the lines $\langle t, -t, 1\rangle$ and the x-axis. **Answer:** The lines are skew. The direction vectors are $\hat{\boldsymbol{\imath}}$ and $\langle 1, -1, 0\rangle$, so a common perpendicular is $\hat{\boldsymbol{k}}$. A vector going in between is $\mathbf{w} = \langle 0, 0, 1\rangle$ (take $t = 0$ and the origin) so the distance is $|\mathbf{w} \cdot \hat{\boldsymbol{k}}| = 1$.

(3) Let $\mathbf{r}(t) = \langle te^t, t^2 + t, \sin t^2\rangle$. Compute $\mathbf{r}'$ and $\mathbf{r}''$. **Answer:** $\mathbf{r}' = \langle te^t + e^t, 2t + 1, 2t\cos t^2\rangle$, $\mathbf{r}'' = \langle 2e^t + te^t, 2, -4t^2 \sin t^2 + 2\cos t^2\rangle$.

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