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# Lost Sales Inventory Models with Batch Ordering and Handling Costs

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**Abstract**

In this paper, we integrate inventory and handling into a single model for analysis and optimization of inventory replenishment decisions for a grocery retail store. We consider a retailer who periodically manages his inventory of a single item facing stochastic demand. The retailer may only order in multiples of a fixed batch size, the lead time is less than the review period length and all unmet demand is lost, which is a realistic situation for a large part of the assortment of grocery retailers. The replenishment cost includes both fixed and variable components, dependent on the number of batches and units in the order. This structure captures the shelf-stacking costs in retail stores. We investigate the optimal policy structure under the long-run average cost criterion. Our results show that it is worthwhile to explicitly take handling costs into account when making inventory decisions. We use parameter values typical for grocery retail environments. For an important subset of the retail assortment, we show that significant cost reductions exist by explicitly considering handling in the inventory policy.

*Keywords*: Retail inventory control, Handling, Lost sales, Periodic review, Fixed batch sizes

# 1 Introduction

The grocery retail inventory problem has several rich features (Broekmeulen et al., 2004). A store orders inventory on a periodic basis and receives replenishment according to a fixed schedule. Demand for products is stochastic and is typically lost if inventory is not available on the shelf. Orders are usually constrained to batches of fixed sizes (case packs). In this paper, we build upon the shelf-stacking insights of Curşeu et al. (2009) to investigate inventory replenishment decisions including merchandise handling. The handling of goods mainly represented the daily process of manually refilling the shelves in the store with products from new deliveries (i.e. shelf stacking), which is typically time consuming and costly.

The case we consider is a lost sales inventory system, with fixed ordering cost, quantized ordering quantity, the lead-time being less than review period, as well as additional handling cost. In the literature, to our knowledge, no study exists that analyses the optimal policy structure similar to the inventory problem discussed in this paper. Additional complexity caused by cost components related to handling costs, substantially complicates the analysis of the optimal solution structure.

We observed in the different in-company projects executed at various grocery retailers that in-store retail inventory management ignores handling costs. This is remarkable since Van Zelst et al. (2009) reported that the handling of goods in the store accounted for 75% of the total logistics store costs, while inventory accounted for the remaining 25% of total costs (see also Saghir and Jonson (2001) for similar insights on the importance of handling). Van Donselaar et al. (2010) discussed in their paper that retailers do not always follow order advices generated by an automated inventory replenishment system. These authors showed that store managers consistently modify automated order advices by advancing orders from peak to non-peak days. One important reason for doing so is to incorporate in-store handling costs, which were not considered in their standard inventory replenishment systems. Several other authors, not necessarily with a specific focus on inventory policy formulations, also emphasize the importance of handling and labor in retail operations (see e.g. Fisher et al. (2006), Fisher (2009), Randall et al. (2006), Roumiantsev and Netessine (2007)).

In this paper, we show that ignoring handling costs leads to serious cost increases. We show that depending upon the specific product (category), significant gains can be realized by considering handling costs in the objective function. These gains are especially pronounced for slow to medium moving products. Based on the dataset used in Curşeu et al. (2009), we find that around $25-30\%$ of the SKUs match these specifications, showing the potential for using an inventory policy including handling costs. We also compare the optimal policy with the best $(s, S, nq)$ and $(s, Q, nq)$ policies [1] both with handling and without handling in the

[1] The $(s, S, nq)$ policy's advice is as follows: whenever the inventory level at a review period is less than or equal to $s$, order the largest integer multiple $n$ of $q$ which results in an inventory position less than or equal

objective function. Interestingly, a large part of the potential gains of considering handling can also be used when utilizing the best $(s, S, nq)$ policy but with a complete objective function (including handling).

Our paper makes a number of contributions to the existing literature. First, we study a stochastic lost sales system that combines important features as seen in practice: fixed batch sizes and inventory handling costs. We formulate the problem as a Markov decision process and explore the structure of the optimal policies. We formulate a number of propositions based on the observed structures. Secondly, we base our analysis and inventory models on the real-life field study as presented in Curşeu et al. (2009). We draw upon this field study to demonstrate the importance of considering merchandize handling. The field study also provides us with realistic numbers to incorporate in our numerical analysis. Last, we numerically illustrate the impact of handling cost components on the optimal policy and the associated long-run average cost. We investigate the impact of inventory handling costs on the structure of the optimal policies and the corresponding long-run average cost. Finally, we analyze the added value of including the handling costs into decision making.

The remainder of this paper is organized as follows. The next section gives a brief review of related literature. Section 2 gives a detailed discussion for the handling costs function used in this paper. A literature review on the relevant references within inventory management is given in Section 3. Section 4 formally introduces the problem and formulates the model using a Markov decision process. The optimal policy is illustrated in Section 5.1. Specifically, we illustrate the complexity of the optimal policies and identify some structural properties in Section 5.2. In Section 5.3, we investigate the added value of handling costs and compare our optimal policy to two policies well-known in the literature (and often employed in practice). Section 6 discusses the results for the real-life data obtained in Curşeu et al. (2009). Finally, we present our conclusions.

to $S$. For the $(s, Q, nq)$ policy, one orders always $Q$ if the inventory level at a review period is less than or equal to $s$; $Q$ is constrained to be an integer multiple $n$ of $q$.

# 2 In-store handling costs

Curşeu et al. (2009) estimated the handling time per Stock Keeping Unit (SKU) required to execute the shelf-stacking operation depending on the number of batches and the number of consumer units stacked. The authors estimated the Total Stacking Time per SKU ($TST$), based on a reduced set of underlying factors, given a specific inventory replenishment rule, assortment, shelf space and package.

Two grocery retail chains (denoted as A and B) agreed to participate in this study. Empirical data on the stacking process was collected using a motion and time study approach. Data from chain A are used to test the hypotheses, and data from chain B are used to validate the results. In four stores, (two for each supermarket chain) employees familiar with the operations, were videotaped during the shelf stacking process. The product subgroups were selected such that they: (1) contain both fast- and slow movers; (2) contain different case pack sizes; (3) contain SKUs for which sufficient shelf space is available to accommodate more than one case pack in a delivery; (4) contain items that are comparable in terms of the handling process and productivity. Finally, we note that the data collection period did not include days with peak or dropping demand, and the stores were consistent in their operations. The stacking of items on the shelves is observed and recorded for each SKU. Afterwards, the execution time of each individual sub-activity and the Total Stacking Time per order line ($TST$) was registered using a computerized time registration tool, and results were entered into a database. Additional information necessary to identify the stacking process for each SKU was added as well, such as the SKU type, the number of case packs and case pack size per order line or the product category each SKU belongs to.

The final dataset contained 1048 observations, for chain A, across nine different product categories. The authors adopted two strategies for estimating the $TST$ per order line and evaluating the relative impact of each factor identified (sequential vs. overall regression). The two approaches served two different practical purposes. On one hand, the sequential approach, allows one for a better insight into the details of the shelf-stacking process, identifying those sub-activities that are mostly affected by the number of items being handled (case packs and number of consumer units), and those for which the variation in workload is potentially affected by other factors. At the same time, the approach indicates which sub-activities contribute mostly to the total variation in the stacking time of a new order line. Using the overall regression strategy, the authors found enough support to conclude that a simple prediction model, depending only on the number of case packs and the number of consumer units, offers already a reliable estimate of the $TST$. Results from testing and validation show that the model is stable and it explains the $TST$ to a large extent. Although the influence of case packs and number of consumer units on the shelf-stacking process may be implicitly recognized, it was demonstrated that both variables are relevant predictors for $TST$. For more details on this study, the reader is refered to Curşeu et al. (2009).

Based on the collected data, a predictive model is developed that allows estimating the total stacking time. The model is tested and validated using the collected real-life data and allows for evaluating the workload required for the shelf-stacking operations. The analysis lead to a general handling cost structure in which each replenishment is associated with costs of the following structure

$$c(nq) = \delta(nq)K + K_1 n + K_2 nq$$

where $q$ is the fixed batch size, order quantities $nq$ are nonnegative integers multiple of these $q$ consumer units, $K$ is the fixed cost incurred for each order, $K_1$ is the variable cost per batch, $K_2$ is the variable unit handling cost, and $\delta(a)$ is a function with a value equaling one if $a$ is strictly positive, and zero otherwise. Table 1 gives the observed sales characteristics (i.e. the average, minimum and maximum weekly demand) of the Chain A product categories used in Curşeu et al. (2009). The table also gives details for the rounded handling coefficients as estimated through the regression analysis. Ignoring the specific product category, i.e. for an arbitrary product, the parameters are estimated in Curşeu et al. (2009) to be $K = 10, K_1 = 20, K_2 = 1$. Note that since Curşeu et al. (2009) only were able to statistically significantly capture the effect of the product group on the fixed cost

$K$, the variable costs ($K_1$ and $K_2$) are independent of the product category. Table 1 clearly shows the large spread in characteristics of the different product categories. Note that this is observed sales and not necessarily equal to actual demand as lost sales might have occurred (see also Huh et al., 2009b).

Table 1: Characteristics of the product categories used in Curşeu et al. (2009)

| | # of | Weekly observed sales | | | Casepacksize | | | Handling | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Category | products | Avg. | Min. | Max. | Avg. | Min. | Max. | $K$ | $K_1$ | $K_2$ |
| Arbitrary product | 1048 | 17.11 | 0.17 | 491.46 | 12 | 1 | 36 | 10 | 20 | 1 |
| Baby food | 31 | 5.91 | 0.51 | 29.31 | 10 | 3 | 16 | 18 | 20 | 1 |
| Chocolate | 168 | 13.66 | 2.10 | 64.22 | 17 | 6 | 33 | 10 | 20 | 1 |
| Coffee | 163 | 18.81 | 0.77 | 491.46 | 12 | 1 | 30 | 21 | 20 | 1 |
| Coffee milk | 56 | 42.20 | 1.74 | 243.24 | 16 | 6 | 30 | 25 | 20 | 1 |
| Candy | 248 | 13.21 | 2.08 | 52.34 | 16 | 6 | 36 | 12 | 20 | 1 |
| Sugar | 18 | 23.86 | 1.31 | 226.84 | 10 | 5 | 20 | 14 | 20 | 1 |
| Canned Meat | 47 | 17.89 | 1.35 | 122.56 | 13 | 6 | 24 | 16 | 20 | 1 |
| Canned Fruit | 32 | 11.47 | 0.78 | 47.32 | 12 | 8 | 24 | 6 | 20 | 1 |
| Personal Care | 285 | 4.45 | 0.17 | 55.45 | 8 | 3 | 24 | 4 | 20 | 1 |

In this study, we adapt our cost function to include $c(nq)$. Additionally, we make use of the data summarised in Table 1 for benchmark computations.

# 3 Literature Review

We give a brief review of the literature on periodic review lost sales inventory models with positive lead times and possibly batch (quantized) ordering. The complexity of optimal policies for lost sales inventory models contrasts with that of classical backorder models, which are known to have solutions of the $(s, S)$ type, when only fixed ordering cost are considered. For a lost sales formulation, the optimal control policy will, in general, be neither of the $(s, S)$ nor of the $(s, Q)$ type, but will depend in a much more complex way on the physical stock at the time of placing an order.

The classical lost sales problem, originally formulated by Karlin and Scarf (1958), is known to be far less analytically tractable than the corresponding backorder problem. If the lead time is positive, the complete structure of the optimal ordering policy is unknown (Hadley and Within, 1963). In the presence of a positive setup cost, reorder-point polices of $(s, S)$ type (Wagner, 1962) or $(s, Q)$ type (Johansen and Hill, 2000) are considered in a lost sales setting. Hill and Johansen (2006) presents numerical examples for detecting optimal policy structure; concluding that neither $(s, S)$ nor $(s, Q)$ is optimal and the optimal structure may look different depending on the inventory level . Chiang (2006, 2007) proposed a dynamic programming model for periodic-review systems in which a replenishment cycle consists of a number of small periods (each of identical but arbitrary length) and holding and shortage costs are charged based on the ending inventory of small periods, rather than ending inventory of replenishment cycles. Note that the way costs are charged becomes especially important when the review cycle is considered as a decision variable. Chiang (2006) considers the case with no fixed ordering cost, whereas Chiang (2007) concentrates on the case with a fixed ordering cost. Both papers analyze backorder and lost sales inventory models (under the assumptions of lead times shorter than the replenishment cycle length, but a multiple of known length of time period). Chiang (2006) provides some properties and structure, however Chiang (2007) provides mostly computational results. Other interesting papers are considering a number of structural properties and have different extensions (see e.g. Huh et al. (2009), Huh et al. (2011), Huh and Janakiraman (2010), Levi et al. (2008), Zipkin (2008a), Zipkin (2008b)) but are different than our consider environment, as they do not consider fixed ordering costs and have integer lead times. Li and Yu (2012) describe a unified lost sales inventory model that takes a number of issues into consideration: capacity constraints, two-ordering modes and supply capacity constraint. They show that, under quite general conditions, with few restrictions, the expected profit function is quasi-concave. The difference is, again, our lead-time structure, as well as we have discrete demand. Further details and additional issues regarding lost sales systems can be found in the recent review by Bijvank and Vis (2011).

The incorporation of a batch size that is a multiple of fixed amount is hardly taken into account in lost sales inventory control models. The above mentioned policies can be extended to the batch ordering policy variants: $(s, S, nq)$ and $(s, Q, nq)$. Veinott (1965) points out that $(R, nQ)$ policies are not optimal in general if a fixed cost is taken into consideration and he proposes a two parameter policy instead. Zheng and Chen (1992) provide an efficient heuristic to compute the best $R$ and $Q$ parameters and later on, Hill (2006) uses this heuristic in the analysis and optimization of an $(s, S, q)$ policy, where $s$ and $S$ are assumed to be multiples of $q$. The structure of the optimal policy for lost sales systems with batch ordering remains an open question (Veinott, 1965; Hill and Johansen, 2006). A recent result of Huh et al. (2009) shows that, when there are no setup costs, the order-up-to policies are asymptotically optimal as the penalty cost becomes large compared to the holding cost and the lead time is an integral multiple of the review period length.

Given a fixed batch size $q$, our specific handling cost structure described in the previous section can be rewritten as $c(nq) = \delta(nq)K + k_q n$, with $k_q = K_1 + K_2 q$. This model can be motivated as a special case in an inventory control setting in which the order quantities are not necessarily restricted to multiples of the batch size, and we interpret $K$ as the fixed administrative cost incurred for each order, and $k_q$ as the fixed cost charged per batch (irrespective of being fully or partially filled). In a recent study, Caliskan-Demirag et al. (2012) describe and study several variants of stochastic periodic-review inventory systems with quantity-dependent fixed costs, including batch-dependent fixed costs.

# 4 The Mathematical Model

We use Markov Decision Processes to model the inventory system and to explore the structure of the optimal policy. The main notation used throughout this paper is given in Table 2. We assume the period demand to be a known, nonnegative discrete random variable and the demand process is i.i.d. stationary. The lead time is fixed and shorter than the review period length. The review cycle length $R$ is exogenous to the model and set as the time unit.

Figure 1 depicts the sequence of events in each period.

More specifically, at the beginning of each review period, the inventory on-hand $X_t$ is observed and an order $a_t$ is placed, which will arrive $L$ time units later (but within the same

Table 2: Notation used in this paper

| | |
|---|---|
| $R$ | Review period |
| $L$ | Lead time ($0 \leq L \leq R$) |
| $t$ | Period index, $t = 0, 1, 2, \ldots$ |
| $X_t$ | Inventory on hand at the beginning of period $t = 0, 1, 2, \ldots$ |
| $a_t$ | Quantity ordered in period $t = 0, 1, 2, \ldots$ |
| $D_L$ | Demand during lead time (i.e. the demand occurring between the start of a period and the time the order $a_t$ is received) |
| $D_{R-L}$ | Demand after the receipt of the orders (and until the end of the period) |
| $D_R$ | Total demand during a review period, $D_R = D_L + D_{R-L}$ |
| $q$ | Fixed (exogenously determined) batch size, $q = 1, 2, \ldots$ |
| $K$ | Fixed cost per order |
| $K_1$ | Handling cost per batch |
| $K_2$ | Handling cost per unit |
| $h$ | Holding cost per unit of inventory (charged at the end of the period) |
| $p$ | Penalty cost for each unit of sales lost during a period (charged at the end of the period) |
| $s$ | Reorder level |
| $I_{Max}$ | Maximum stock level |

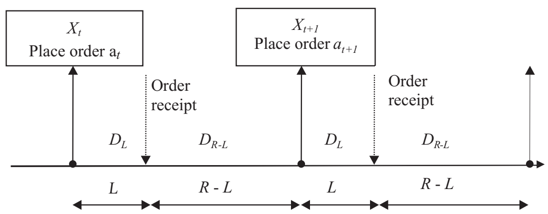


Figure 1: Sequence of events

review cycle). The order $a_t$ is restricted to be a nonnegative integer, multiple of the fixed batch size $q$, i.e. $a_t \in \{0, q, 2q, \ldots\}$. Next, the stochastic demand $D_L$ is realized and satisfied with on-hand inventory (if possible); unsatisfied demand is lost. Then, the order placed at the beginning of the period arrives and afterwards stochastic demand $D_{R-L}$ continues to occur, up until the beginning of the next ordering moment. All demand occurring in this time period not directly satisfied is again assumed to be lost. The random variables $D_L$ and $D_{R-L}$ are stochastically independent. In practice, independency is not necessarily guaranteed, but this assumption is needed for obtaining a tractable model that can be analyzed.

The decision epoch is the beginning of each review period and the inventory on-hand at the beginning of a review moment characterizes the system state, with state space $SS = \{0, 1, 2, \ldots\}$. At each review moment a decision is made regarding the ordering quantity, which is limited to the set $A(i) = \{0, q, 2q, \ldots\}$, for every $i \in SS$. Due to assumption $L \leq R$, the expected transition times from one decision epoch to the next are deterministic and equal $R$. The evolution of on-hand inventory from one decision epoch to the next is given by the following recursive relation:

$$X_{t+1} = ((X_t - D_L)^+ + a_t - D_{R-L})^+, \quad t = 0, 1, 2, \ldots,$$

where $(x)^+ = \max\{0, x\}$ for any $x \in \mathbb{R}$.

The probability $p_{ij}(a_i)$ of a transition from state $i$ at one decision epoch to state $j$ at the next epoch, given decision $a_i$ at the first decision epoch is defined as:

$$p_{ij}(a_i) = \mathsf{P}\left(j = ((i - D_L)^+ + a_i - D_{R-L})^+\right), \quad i, j = 0, 1, \ldots, \quad a_i = 0, q, 2q, \ldots,$$

and is given by:

$$p_{i0}(0) = \mathsf{P}(D_R \geq i), \quad i = 1, 2, \ldots,$$

$$p_{ij}(0) = \mathsf{P}(D_R = i - j), \quad i = 0, 1, \ldots, \; j = 1, 2, \ldots, i,$$

$$p_{00}(0) = 1,$$

when there is no order, and when the order amounts to $a_i = n_i q > 0$ by:

$$\begin{aligned}
p_{i0}(a_i) &= \sum_{k=0}^{i-1} \mathsf{P}(D_L = k)\mathsf{P}(D_{R-L} \geq i + a_i - k) + \mathsf{P}(D_L \geq i)\mathsf{P}(D_{R-L} \geq a_i), \quad i = 1, 2, \ldots, \\
p_{ij}(a_i) &= \sum_{k=0}^{i-1} \mathsf{P}(D_L = k)\mathsf{P}(D_{R-L} = i + a_i - j - k) + \mathsf{P}(D_L \geq i)\mathsf{P}(D_{R-L} = a_i - j), \quad i = 1, 2, \ldots, \\
&\phantom{=} j = 1, 2, \ldots, a_i, \\
p_{ij}(a_i) &= \sum_{k=0}^{i+a_i-j} \mathsf{P}(D_L = k)\mathsf{P}(D_{R-L} = i + a_i - j - k), \quad i = 1, 2, \ldots, \; j = a_i + 1, \ldots, a_i + i, \\
p_{00}(a_0) &= \mathsf{P}(D_{R-L} \geq a_0), \\
p_{0j}(a_0) &= \mathsf{P}(D_{R-L} = a_0 - j), \quad j = 1, 2, \ldots, a_0, \\
p_{ij}(a_i) &= 0, \quad \text{otherwise.}
\end{aligned}$$

The total expected cost from one decision epoch to the next (i.e., the one-period transition cost), given that we are in state $i$ and order $a_i = n_i q$, is defined as:

$$c_i(a_i) = c_i^r(a_i) + c_i^h(a_i) + c_i^p(a_i), \quad a_i \in A(i),\ i \in SS, \tag{1}$$

where the expected replenishment cost $c_i^r$ and the expected holding $c_i^h$ and penalty costs $c_i^p$ are given by:

$$\begin{aligned} c_i^r(0) &= 0, \\ c_i^r(a_i) &= K + K_1 a_i/q + K_2 a_i \quad a_i = n_i q > 0 \end{aligned} \tag{2}$$

and

$$\begin{aligned} c_i^h(0) + c_i^p(0) &= h\mathsf{E}\left[(i - D_R)^+\right] + p\mathsf{E}\left[(i - D_R)^-\right], \\ c_i^h(a_i) + c_i^p(a_i) &= h\mathsf{E}\left[((i - D_L)^+ + a_i - D_{R-L})^+\right] \\ &\quad + p\left\{\mathsf{E}\left[(D_L - i)^+\right] + \mathsf{E}\left[(D_{R-L} - a_i - (i - D_L)^+)^+\right]\right\} \\ &= h\left\{\sum_{k=0}^{i-1} \mathsf{P}(D_L = k)\mathsf{E}\left[(i - k + a_i - D_{R-L})^+\right] + \mathsf{P}(D_L \geq i)\mathsf{E}\left[(a_i - D_{R-L})^+\right]\right\} \\ &\quad + p\left\{\mathsf{E}\left[(D_L - i)^+\right] + \sum_{k=0}^{i-1} \mathsf{P}(D_L = k)\mathsf{E}\left[(D_{R-L} - a_i - i + k)^+\right]\right\} \\ &\quad + p\mathsf{P}(D_L \geq i)\mathsf{E}\left[(D_{R-L} - a_i)^+\right], \end{aligned}$$

for $a_i > 0$, respectively, where $(x)^- = \max\{0, -x\} = -\min\{0, x\}$ for any $x \in \mathbb{R}$.

From (1) and (2), we derive the total one-period transition cost as follows:

$$\begin{aligned} c_i(0) &= h\mathsf{E}\left[(i - D_R)^+\right] + p\mathsf{E}\left[(i - D_R)^-\right], \\ c_i(a_i) &= K + K_1 a_i/q + K_2 a_i \\ &\quad + h\mathsf{E}\left[((i - D_L)^+ + a_i - D_{R-L})^+\right] \\ &\quad + p\left\{\mathsf{E}\left[(D_L - i)^+\right] + \mathsf{E}\left[(D_{R-L} - a_i - (i - D_L)^+)^+\right]\right\}, \quad a_i = n_i q > 0. \end{aligned} \tag{3}$$

We aim to determine the inventory policy $U^*$, solving the following optimization problem:

$$\min_U C(U) = \frac{1}{R}\sum_{i\in SS}\pi_i c_i(a_i) \overset{R=1}{=} \sum_{i\in SS}\pi_i c_i(a_i),$$

where $C(U)$ denotes the long-run expected average cost under policy $U$, and $(\pi_i)_{i\in SS}$ represents the steady-state distribution of the inventory on hand (provided it exists).

Note that $C(U)$ is generally a very complex function of $U$ and the steady-state distribution may not be determined in closed form. For the exact conditions that guarantee the existence of an average-cost optimal policy see, e.g., Puterman (1994), or Cavazos-Catena and Senott (1992). If an optimal policy exists, then there exist relative values $(v_i)_{i\in SS}$ and the long-run average cost $g$, such that they satisfy the average-cost optimality equations:

$$v_i = \min_{a\in A(i)}\{c_i(a) - g + \sum_{j\in SS} p_{ij}(a)v_j\}, \quad i \in SS. \tag{4}$$

Unique relative values $(v_i)_{i\in SS}$ are obtained if we provide an initial condition, such as $v_0 = 0$. In order to solve the optimality equations and determine an optimal policy, it is common to use an algorithm such as policy iteration or value iteration (Puterman, 1994).

# 5 Structure of the optimal policy

For the lost-sales problem introduced in Section 4, there is no apparent analytic solution of a simple form. To calculate the optimal stationary policy under the long-run average cost criterion, we numerically solved the average cost optimality equations of the inventory system using the standard relative value iteration algorithm with $\epsilon = 10^{-12}$ (see, e.g., Puterman, 1994 or Bertsekas, 1995). The state space was truncated to a size sufficiently large to ensure finding a global optimum. The computational time increases with the state space size due to the generation of all needed transition probabilities. In our numerical computations, the state space is truncated to a size sufficiently large to ensure we find a global optimum.

The state space size increases with the mean demand per period. The truncated state space is determined by testing larger and larger sizes until the results are insensitive to the increments. We observed that the computational time increases with the state space, which indicates the size of the system of equations to be solved in the policy evaluation step. The most time consuming part is the generation of transition probabilities in each step. For any given policy, we evaluate numerically the long run average cost by solving the average-cost optimality equations. The best policy within a given policy class is determined by exhaustive search over its defining parameters.

We first illustrate the structure of the optimal policy, then we develop some structural insights based on the optimal policy.

## 5.1 Illustrations of the optimal policy

As an illustration, Figure 2 depicts the optimal policies for three different lead times $L$ (right side) and the effect of the batch sizes, $q = 1$ vs. $q = 6$ vs. $q = 12$(left side). The top row in Figure 2 assumes $\lambda = 20, h = 1, p = 50, K = 10, K_1 = 20, K_2 = 1, q = 1, L = 0.5$ in the base case. The bottom row in Figure 2 assumes $\lambda = 10, h = 1, p = 50, K = 10, K_1 = 20, K_2 = 1, q = 1, L = 0.5$ in the base case.

Generally, when $q > 1$, the optimal order quantity is a stepwise decreasing function of on-hand inventory, and the step size equals $q$. Note that the structure of the optimal solution differs from an $(s, S, nq)$ policy, as for this policy, for low levels of inventory on-hand, the advice is to still order the same quantity, multiple of $q$. The optimal order quantity as a function of on-hand inventory exhibits no clear structure. It is observed that there exists a reorder point ($s = \max\{i \in SS | a_i > 0\}$) below which it is always optimal to place an order and beyond which it is never optimal to order. Thus, $s$ plays the role of a reorder level in a general inventory policy. A similar observation was made by Hill and Johansen (2006), both for a continuous review setting, as well as periodic review setting for the lost sales system without handling costs. Moreover, as illustrated in Figure 2 bottom row, we

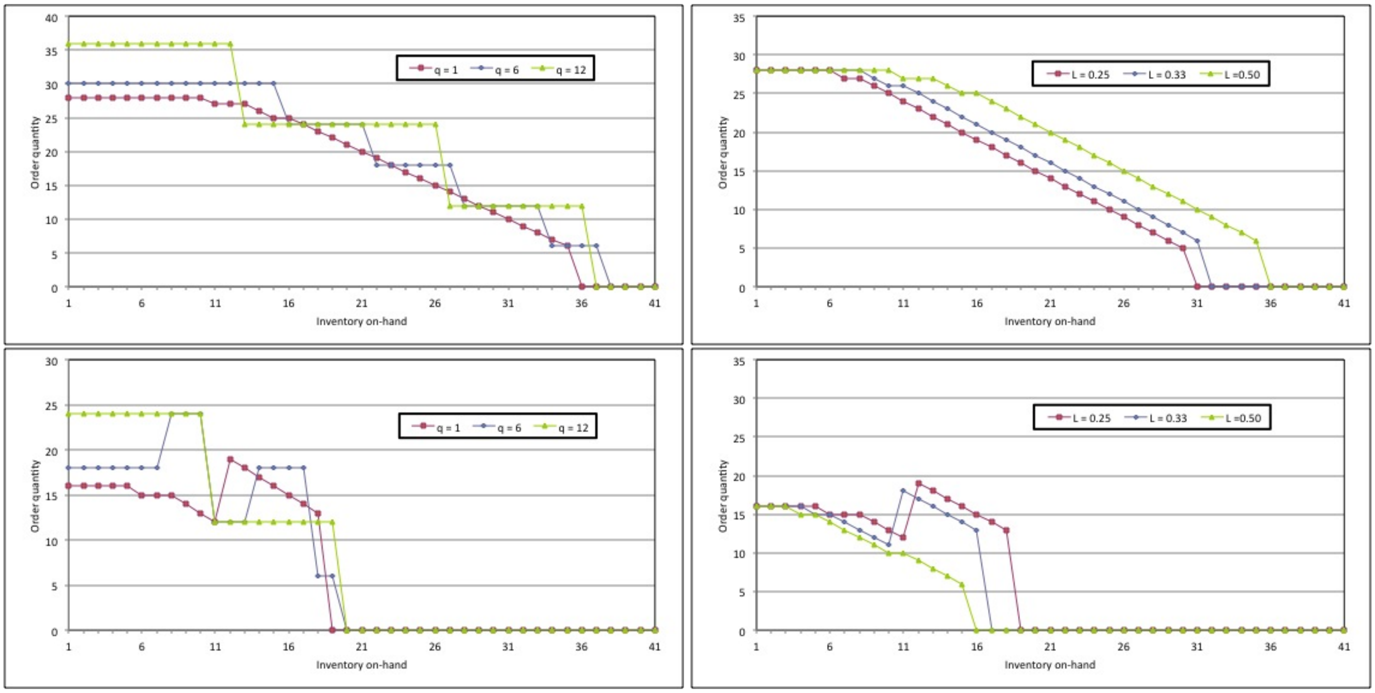


Figure 2: Optimal order quantity as a function of inventory on-hand

find similar peculiar non-monotonic behavior of our policy as observed earlier by Hill and Johansen (2006) and Jansen (1998). A complete and detailed sensitivity analysis for a broad set of parameter values can be found in Appendix A.

## 5.2 Structural insights from the optimal policy

Observe from Figure 2 that the advice is either to order a fixed amount $Q$ (and thus act like an $(s, Q)$ policy), for low levels of stock on-hand; or, for high levels of inventory, order enough to reach a target stock level $S$ (and act like an $(s, S)$ policy). Outside these regions however, the optimal policy structure remains unclear. This observation also suggests that a policy combining the logic of both $(s, Q)$ and $(s, S)$ policies might perform close to optimal. The proposed policy is a structured way of taking into consideration the observations made earlier in this paper, as well as Hill and Johansen (2006) and Chiang (2007). Both Hill and Johansen (2006) and Chiang (2007) computationally came up with similar structures. However the environment we consider has handling cost compared to both papers.

Note that there is sufficient complexity to the periodic cost function for making a proof of the optimal policy structure for the entire state space analytically challenging. However, based on the numerical insights for the optimal policy, we present next three propositions. Proposition C.1 discusses an upper threshold for the state variable, i.e. an inventory level above which we will never order. Proposition C.2 discusses the instances where we never order at all. Lastly, Proposition C.3 presents a sufficient condition that gives the existence of a lower threshold for the state variable, i.e. an inventory level below which we will always order. The proofs for the different propositions are given in Appendix C. Note that these results can be utilised to have a more efficient search procedure for the optimal solution of an instance.

**Proposition 5.1.** *There exists a state $i_u$ such that for $i \geq i_u$ the optimal decision is not to order.*

**Proposition 5.2.** *There exists a set of parameter values for which we would not operate the lost sales inventory system under consideration, i.e. we will never order.*

Next, we identify a condition that guarantees a threshold value for the initial inventory level below which it is always optimal to order.

**Proposition 5.3.** *If*

$$
\begin{aligned}
(h+p)E[D_R] \quad \geq \quad & K + K_1 a^{NV}_{R-L}/q + K_2 a^{NV}_{R-L} + h\mathsf{E}\left[(a^{NV}_{R-L} - D_{R-L})^+\right] \\
+ \quad & p\mathsf{E}\left[(D_{R-L} - a^{NV}_{R-L})^+\right] + p\mathsf{E}\left[D_L\right] + h\mathsf{E}\left[D_R\right].
\end{aligned}
$$

*is satisfied, then there exists a state $i$ for which it is optimal to order. Here $a^{NV}_{R-L}$ is the optimal order-up-to-level for a news vendor problem with Poisson demand over $R-L$ time units.*

## 5.3 Ignoring retail handling costs in the optimal policy

Retail handling costs, although acknowledged in practice, are not taken into account when making inventory replenishment decisions. We study the extra costs of using a suboptimal policy (i.e. ignoring the handling costs). We first compute the optimal policy and its optimal average cost under the assumption of no handling costs. Then, we evaluate this policy with the lost sales model considering handling costs to obtain the corresponding average cost. Finally, we compare the results against the true optimal cost, associated to the optimal policy determined while taking the handling costs into account. The percentage difference in costs for not explicitly considering handling, $\xi$, is calculated as follows.

$$\xi = 100 \times \frac{C_{(K,K_1,K_2)}(U_{(0,0,0)}) - C^*_{(K,K_1,K_2)}}{C^*_{(K,K_1,K_2)} - (K_1/q + K_2) \cdot \lambda} \tag{5}$$

where $U_{(0,0,0)}$ denotes the optimal policy assuming no handling costs ($K = K_1 = K_2 = 0$), $C_{(K,K_1,K_2)}(U)$ is the average cost of the original model with cost components $K, K_1$ and $K_2$ corresponding to policy $U$, and $C^*_{(K,K_1,K_2)}$ represents the true minimum average cost, which is re-scaled to better reflect policy-related costs. Table 3 gives the average error averaged over different $q$ values ($q \in \{1, 3, 6, 9, 12\}$).

Table 3: Cost of ignoring handling $\xi$

| $K$ | $\lambda = 0.1$ | $\lambda = 1$ | $\lambda = 5$ | $\lambda = 10$ | $\lambda = 15$ | $\lambda = 20$ | Average |
|---|---|---|---|---|---|---|---|
| $L = 0.25$ | | | | | | | |
| 5 | 1.15% | 4.80% | 1.81% | 0.37% | 0.28% | 0.36% | 1.46% |
| 10 | 1.72% | 10.42% | 8.22% | 1.14% | 0.42% | 0.36% | 3.71% |
| 15 | 3.62% | 16.70% | 15.45% | 7.66% | 1.26% | 0.41% | 7.51% |
| 20 | 5.21% | 22.12% | 22.50% | 14.11% | 6.51% | 1.07% | 11.92% |
| 25 | 6.57% | 27.18% | 29.54% | 20.28% | 12.40% | 5.35% | 16.89% |
| $L = 0.33$ | | | | | | | |
| $K$ | $\lambda = 0.1$ | $\lambda = 1$ | $\lambda = 5$ | $\lambda = 10$ | $\lambda = 15$ | $\lambda = 20$ | Average |
| 5 | 1.19% | 4.56% | 1.77% | 0.36% | 0.47% | 0.42% | 1.46% |
| 10 | 1.77% | 10.30% | 7.89% | 1.16% | 0.56% | 0.41% | 3.68% |
| 15 | 3.64% | 16.17% | 14.81% | 7.46% | 1.35% | 0.45% | 7.31% |
| 20 | 5.21% | 21.58% | 21.91% | 13.69% | 6.54% | 1.11% | 11.67% |
| 25 | 6.56% | 26.53% | 29.05% | 19.76% | 12.19% | 5.28% | 16.56% |
| $L = 0.50$ | | | | | | | |
| $K$ | $\lambda = 0.1$ | $\lambda = 1$ | $\lambda = 5$ | $\lambda = 10$ | $\lambda = 15$ | $\lambda = 20$ | Average |
| 5 | 1.26% | 4.01% | 1.90% | 0.45% | 0.39% | 0.38% | 1.40% |
| 10 | 1.88% | 10.26% | 7.69% | 1.31% | 0.50% | 0.38% | 3.67% |
| 15 | 3.69% | 15.91% | 14.21% | 7.23% | 1.28% | 0.43% | 7.12% |
| 20 | 5.22% | 20.98% | 21.01% | 13.12% | 6.20% | 1.07% | 11.27% |
| 25 | 6.53% | 25.84% | 27.90% | 18.93% | 11.49% | 5.05% | 15.95% |
| Average | 3.68% | 15.82% | 15.04% | 8.47% | 4.12% | 1.50% | 8.11% |

Note: $h = 1, K_1 = 20, K_2 = 1, p = 50$

The results in Table 3 show that the cost impact of ignoring the handling costs may be significant. Two effects are pronounced. First, for slow to medium moving products (i.e. $\lambda \in \{1, 5, 10\}$), the error made is high. Fast movers and extreme slow movers are less affected by ignoring the handling aspects in the inventory policy. Secondly, increasing $K$ values lead to a larger error. Intuitively, the fixed ordering cost, $K$, affects the total costs more substantially when ignored. From a broad retail assortment point of view, many products in the categories referred to in Table 1 are affected by this significant cost of ignoring the handling operations in the inventory ordering process.

# 6 Detailed results for the data from Curşeu et al. (2009)

In this section, we return to the dataset available from Curşeu et al. (2009). For the different product categories presented in Table 1, we generated the optimal policy for the average demand $\lambda$ and the average casepack size $q$. The results are reported in Table 4. Next to characterizing the optimal policy, we give the percentage error made if a retailer would use the best $(s, S, nq)$ or $(s, Q, nq)$ policy including handling in the cost function. Additionally, we also show in the last two columns the same two policies but excluding the handling component in the cost function when identifying the best policy. The policy obtained as such (i.e. excluding handling costs) is then evaluated in the full objective function (including handling). Finally, note that this table does not represent a complete parametric design (which is available in Appendix D).

Based on the results presented in Table 4, we make the following observations:

- In a number of cases, extending the objective function with a handling cost component for the $(s, S, nq)$ or $(s, Q, nq)$ policy leads to errors less than 1%. This is an important observation, since using an extended standard policy is more straightforward from a practice point of view. In this case, retailers can use the same inventory logic but only need to adapt their cost functions used, which is a small effort compared to implementing a completely new policy.

- There are product categories where ignoring handling has a large effect on the costs.

Table 4: Optimal policy for the product categories as reported in Curşeu et al. (2009)

| | | | | | Optimal Policy | | | Handling | | No handling | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\lambda$ | $q$ | $K$ | $L$ | $s$ | $I_{max}$ | $C^*$ | $(s, S, nq)$ | $(s, Q, nq)$ | $(s, S, nq)$ | $(s, Q, nq)$ |
| Arbitrary product | 17.11 | 12 | 10 | 0.5 | 30 | 44 | 78.4119 | 0.20% | 12.53% | 0.64% | 12.53% |
| | | | | 0.33 | 27 | 41 | 74.9735 | 0.19% | 5.45% | 0.39% | 5.45% |
| | | | | 0.25 | 25 | 39 | 73.2901 | 0.20% | 6.12% | 0.69% | 6.12% |
| Baby food | 5.91 | 10 | 18 | 0.5 | 11 | 29 | 39.7233 | 0.00% | 0.81% | 6.15% | 6.46% |
| | | | | 0.33 | 10 | 28 | 38.4331 | 0.00% | 0.64% | 6.67% | 7.01% |
| | | | | 0.25 | 9 | 27 | 37.7752 | 0.00% | 0.84% | 6.56% | 6.92% |
| Chocolate | 13.66 | 17 | 10 | 0.5 | 25 | 43 | 59.8172 | 0.00% | 0.39% | 0.04% | 0.39% |
| | | | | 0.33 | 22 | 41 | 57.0048 | 0.00% | 0.67% | 0.06% | 0.67% |
| | | | | 0.25 | 21 | 40 | 55.7162 | 0.00% | 0.77% | 0.06% | 0.77% |
| Coffee | 18.81 | 12 | 21 | 0.5 | 31 | 59 | 94.5068 | 0.00% | 1.96% | 3.11% | 2.04% |
| | | | | 0.33 | 27 | 56 | 90.5513 | 0.00% | 2.76% | 3.72% | 2.76% |
| | | | | 0.25 | 26 | 54 | 88.7614 | 0.00% | 2.99% | 3.87% | 2.99% |
| Coffee milk | 42.20 | 16 | 25 | 0.5 | 69 | 89 | 163.4191 | 0.00% | 11.67% | 0.01% | 11.67% |
| | | | | 0.33 | 62 | 81 | 155.3126 | 0.00% | 27.88% | 0.00% | 27.88% |
| | | | | 0.25 | 58 | 78 | 151.4613 | 0.00% | 19.53% | 0.00% | 19.53% |
| Candy | 13.21 | 16 | 12 | 0.5 | 23 | 42 | 53.3841 | 0.00% | 0.70% | 0.23% | 0.70% |
| | | | | 0.33 | 21 | 39 | 54.5734 | 0.00% | 0.55% | 0.14% | 0.55% |
| | | | | 0.25 | 20 | 38 | 53.3841 | 0.00% | 0.70% | 0.23% | 0.70% |
| Sugar | 23.86 | 10 | 14 | 0.5 | 40 | 52 | 113.7879 | 0.13% | 19.73% | 0.14% | 19.73% |
| | | | | 0.33 | 35 | 48 | 109.068 | 0.03% | 26.65% | 0.03% | 26.65% |
| | | | | 0.25 | 33 | 46 | 106.8054 | 0.00% | 17.39% | 0.01% | 17.39% |
| Canned Meat | 17.89 | 13 | 16 | 0.5 | 31 | 58 | 85.2725 | 0.12% | 0.62% | 0.83% | 0.62% |
| | | | | 0.33 | 27 | 55 | 81.636 | 0.11% | 0.91% | 0.88% | 1.03% |
| | | | | 0.25 | 26 | 53 | 79.9394 | 0.10% | 0.95% | 0.84% | 0.95% |
| Canned Fruit | 11.47 | 12 | 6 | 0.5 | 21 | 34 | 54.5348 | 0.00% | 9.77% | 0.22% | 18.74% |
| | | | | 0.33 | 18 | 31 | 52.0773 | 0.00% | 11.14% | 0.69% | 22.76% |
| | | | | 0.25 | 17 | 30 | 50.9219 | 0.00% | 11.91% | 0.46% | 11.91% |
| Personal Care | 4.45 | 8 | 4 | 0.5 | 8 | 16 | 28.3628 | 0.08% | 0.00% | 0.08% | 0.00% |
| | | | | 0.33 | 7 | 15 | 27.3204 | 0.08% | 0.00% | 0.08% | 0.00% |
| | | | | 0.25 | 7 | 15 | 26.8417 | 0.08% | 0.00% | 0.08% | 0.00% |

Note: $h = 1, K_1 = 20, K_2 = 1, p = 50$. Percentage errors computed based on the normalized costs

A detailed analysis of the different scenarios (the full table is given in Appendix D) reveals that settings where $\lambda \in \{10, 15\}$, $L \in \{0.33, 0.50\}$, $K \in \{5, 10\}$, $p \in \{10, 25\}$ and $q \in \{1, 6, 12\}$ gives improvements of over 1% when comparing the optimal policy with the best $(s, S, nq)$ policy.

- It is clear that the $(s, Q, nq)$ policy may lead to a significant optimality gap, even when explicitly considering the handling costs in the policy. Interestingly, the $(s, S, nq)$ policy performs remarkably well with a maximum optimality gap of less than 1% (considering handling in the cost function).

- The results show that for a significant part of the retail assortment, especially the slow to medium moving products, significant costs reductions can be expected. Note that a 1% cost reduction directly translates into important higher margins, which are already very slim.

We observed in our experiments (Appendix D) that the cost $C(s, Q, S)$ is relatively flat around the optimal $s$ and $Q$ values, respectively and more sensitive to changes in $S$. Similar to Hill and Johansen (2006), we observe the occurrence of local minima in the cost function. In view of these numerical results, we may expect that the $(s, S, nq)$ policies are also asymptotically optimal (see also Huh et al., 2009), as $p$ gets large and $K > 0$. Regarding the solution space, we observed that, in nearly all cases, the order-up-to levels and the reorder points are nearly identical for both the optimal policy and the $(s, S, nq)$ policies (the occasional differences are small). Apparently, the average cost $C(s, Q, S)$ is less sensitive to changes in the values of parameter $Q$. These insights could be exploited in the derivation of easily computable policy parameters, which is left for future research.

# 7 Conclusions and Future Research

We studied a single-location, single-item periodic-review lost-sales inventory control problem with the following features: there are stochastic customer demands, lead time is less than the review period length, there is a fixed (predetermined) batch size for ordering and orders are restricted to integers multiples of the batch size. Furthermore, we assume a replenishment cost structure that includes a fixed cost, as well as linear components depending on the number of batches, and the number of units in a replenishment order. Our motivation and data comes from the retail environment, but the analysis is appropriate for systems in which there is a fixed unit-size and there are economies of scale in the replenishment component. Using Markov decision processes, we explore the structure of the optimal policies and investigate, in particular, the impact of handling costs on the optimal policy and the long-run average cost.

Our results show that it is worthwhile to explicitly take handling costs into account when making inventory decisions. We have used parameter values that are typical for grocery retail environments, where decision models typically abstain from including these costs. Using an $(s, S, nq)$ policy rather than the often used in practice $(s, Q, nq)$ policy seems from a cost perspective more plausible. A detailed ABC analysis of the product portfolio is important in using the obtained results. The results showed that for a significant part of the retail assortment, especially the slow to medium moving products, higher reductions are expected. Moreover, a 1% cost reduction directly translates into important higher margins.

Future research could involve a number of directions. In many contexts there is also consideration of a minimum stock presentation, i.e., retailers do not want the stock on the shelf to go below some positive number of units, e.g. because of its marketing impact on the visual impression given to customers. The incorporation of this minimum stock level is an interesting path for future research.

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# A Sensitivity Analysis

We discuss the sensitivity of the optimal policy and its associated average cost to the relevant system parameters. In the absence of simple-structured policies, it is difficult to evaluate the impact of changes in the problem parameters on the optimal solution. Therefore, we use the reorder point ($s = \max\{i \in SS | a_i > 0\}$) and the maximum stock level ($I_{max} = \max\{i + a_i | 0 \leq i \leq s\}$) as main operational indicators.

## Sensitivity on $\lambda, L, K$ and $q$

Table 5 provides a representative set of our results, where $p = 50, L \in \{0.25, 0.33, 0.5\}, \lambda \in \{0.1, 1, 5, 15, 25, 50\}, K \in \{5, 10, 15, 20, 25\}, q \in \{1, 3, 6, 12, 18, 36\}$.

Clearly, the results in Table 5 are in agreement with expectations. We observe that the batch size $q$ mostly affects $I_{max}$, which increases, in general, as $q$ increases (other parameters being equal). Numerical results suggest that, in general, $s$ increases with $L$, and $I_{max}$ also increase with $L$ and $q$ (and there is little interaction between $L$ and $q$). Additionally, $s$ decreases with $K$, while $I_{max}$ increases with $K$ and $q$. Furthermore, regarding the sensitivity of the average cost to changes in problem parameters, our numerical studies suggest that the optimal long-run average cost is increasing in $L$ and $K$(all other things being equal). Note that similar monotonic results (w.r.t. $L$) of the average cost (as well as the infinite horizon discounted costs) are claimed by Zipkin (2008) for the lost sales model with lead times which are integers multiple of the review period length. Note that previous research (Janakiraman and Muckstadt, 2004) shows that linear purchase costs ($K_2$) can be assumed to be zero without loss of generality, for general distribution and assembly systems with lost sales and/or backorders, when lead times are integers. Janakiraman and Muckstadt (2001) extend the result to a lost sales model with lead times which are a fraction of the review period length (see Lemma B.1 in Appendix B). The result states that the finite horizon, discounted cost problem with no setup cost and a positive unit purchasing cost can be

Table 5: Sensitivity analysis on $L, \lambda, K, q$

$L = 0.25$

| | | $\lambda = 0.1$ | | | $\lambda = 1$ | | | $\lambda = 5$ | | | $\lambda = 10$ | | | $\lambda = 15$ | | | $\lambda = 20$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $K$ | $q$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ |
| 5 | 1 | 0 | 1 | 3.68 | 2 | 5 | 26.12 | 8 | 13 | 116.8 | 15 | 19 | 225.86 | 23 | 27 | 333.87 | 30 | 34 | 441.62 |
| | 6 | 0 | 6 | 4.02 | 2 | 8 | 10.15 | 9 | 17 | 34.2 | 16 | 24 | 60.61 | 24 | 31 | 85.42 | 31 | 39 | 109.91 |
| | 12 | - | - | 4.98 | 1 | 13 | 10.62 | 8 | 20 | 26.4 | 16 | 29 | 44.90 | 23 | 36 | 61.95 | 30 | 43 | 78.17 |
| | 18 | - | - | 4.98 | 1 | 19 | 12.63 | 8 | 26 | 25.5 | 15 | 33 | 40.48 | 23 | 41 | 55.03 | 30 | 48 | 68.95 |
| | 36 | - | - | 4.98 | 0 | 36 | 20.56 | 6 | 42 | 30.0 | 14 | 50 | 41.25 | 21 | 57 | 52.20 | 28 | 64 | 63.15 |
| 10 | 1 | 0 | 2 | 4.11 | 1 | 6 | 27.30 | 7 | 16 | 119.3 | 14 | 20 | 230.79 | 22 | 27 | 338.87 | 29 | 34 | 446.62 |
| | 6 | 0 | 6 | 4.10 | 2 | 8 | 10.98 | 8 | 20 | 36.8 | 15 | 30 | 65.09 | 23 | 31 | 90.40 | 30 | 39 | 114.91 |
| | 12 | - | - | 4.98 | 1 | 13 | 11.03 | 8 | 20 | 28.5 | 15 | 30 | 48.69 | 22 | 36 | 66.65 | 29 | 43 | 83.14 |
| | 18 | - | - | 4.98 | 1 | 19 | 12.91 | 8 | 26 | 26.9 | 15 | 33 | 43.25 | 22 | 42 | 58.97 | 29 | 49 | 73.58 |
| | 36 | - | - | 4.98 | 0 | 36 | 20.69 | 6 | 42 | 30.7 | 14 | 50 | 42.63 | 21 | 57 | 54.27 | 28 | 64 | 65.92 |
| 15 | 1 | 0 | 2 | 4.35 | 1 | 7 | 28.10 | 7 | 18 | 121.3 | 14 | 29 | 233.43 | 21 | 27 | 343.85 | 28 | 34 | 451.62 |
| | 6 | 0 | 6 | 4.18 | 2 | 8 | 11.80 | 8 | 22 | 38.7 | 15 | 32 | 67.83 | 22 | 43 | 95.26 | 29 | 39 | 119.91 |
| | 12 | - | - | 4.98 | 1 | 13 | 11.44 | 8 | 24 | 30.6 | 15 | 36 | 51.66 | 22 | 46 | 70.91 | 29 | 55 | 88.08 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.18 | 8 | 26 | 28.2 | 15 | 33 | 46.01 | 22 | 44 | 62.80 | 29 | 49 | 78.18 |
| | 36 | - | - | 4.98 | 0 | 36 | 20.83 | 6 | 42 | 31.4 | 14 | 50 | 44.02 | 21 | 57 | 56.35 | 28 | 64 | 68.69 |
| 20 | 1 | 0 | 2 | 4.58 | 1 | 8 | 28.83 | 7 | 20 | 123.0 | 14 | 30 | 235.88 | 20 | 41 | 346.89 | 27 | 34 | 456.62 |
| | 6 | 0 | 6 | 4.26 | 2 | 8 | 12.62 | 8 | 23 | 40.5 | 15 | 34 | 70.23 | 22 | 45 | 98.18 | 28 | 39 | 124.90 |
| | 12 | - | - | 4.98 | 1 | 13 | 11.85 | 8 | 27 | 32.5 | 15 | 38 | 54.00 | 22 | 48 | 73.65 | 28 | 57 | 92.02 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.46 | 8 | 26 | 29.6 | 15 | 41 | 48.69 | 22 | 51 | 66.11 | 28 | 61 | 82.39 |
| | 36 | - | - | 4.98 | 0 | 36 | 20.97 | 6 | 42 | 32.1 | 14 | 50 | 45.40 | 21 | 57 | 58.43 | 28 | 64 | 71.46 |
| 25 | 1 | 0 | 2 | 4.82 | 1 | 8 | 29.48 | 7 | 22 | 124.5 | 13 | 31 | 238.18 | 20 | 42 | 349.43 | 27 | 54 | 459.97 |
| | 6 | 0 | 6 | 4.34 | 2 | 8 | 13.45 | 7 | 25 | 42.0 | 15 | 35 | 72.52 | 21 | 45 | 100.73 | 28 | 57 | 128.07 |
| | 12 | - | - | 4.98 | 1 | 13 | 12.25 | 8 | 28 | 34.2 | 15 | 39 | 56.09 | 21 | 49 | 76.20 | 28 | 59 | 95.29 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.73 | 8 | 26 | 31.0 | 15 | 43 | 50.99 | 21 | 54 | 68.59 | 28 | 64 | 85.12 |
| | 36 | - | - | 4.98 | 0 | 36 | 21.10 | 6 | 42 | 32.8 | 14 | 50 | 46.79 | 21 | 57 | 60.50 | 28 | 64 | 74.23 |

$L = 0.33$

| | | $\lambda = 0.1$ | | | $\lambda = 1$ | | | $\lambda = 5$ | | | $\lambda = 10$ | | | $\lambda = 15$ | | | $\lambda = 20$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $K$ | $q$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ |
| 5 | 1 | 0 | 1 | 3.70 | 2 | 5 | 26.24 | 8 | 14 | 117.36 | 16 | 20 | 226.87 | 24 | 28 | 335.35 | 32 | 36 | 443.52 |
| | 6 | 0 | 6 | 4.02 | 2 | 8 | 10.27 | 9 | 17 | 34.69 | 17 | 25 | 61.64 | 25 | 32 | 86.95 | 33 | 40 | 111.84 |
| | 12 | - | - | 4.98 | 1 | 13 | 10.80 | 9 | 21 | 27.01 | 17 | 30 | 45.91 | 24 | 37 | 63.45 | 32 | 45 | 80.07 |
| | 18 | - | - | 4.98 | 1 | 19 | 12.76 | 8 | 26 | 25.95 | 16 | 34 | 41.48 | 24 | 42 | 56.42 | 31 | 50 | 70.84 |
| | 36 | - | - | 4.98 | 0 | 36 | 20.70 | 7 | 43 | 30.49 | 15 | 51 | 42.23 | 22 | 58 | 53.62 | 30 | 66 | 65.02 |
| 10 | 1 | 0 | 2 | 4.12 | 2 | 6 | 27.41 | 8 | 17 | 119.81 | 15 | 28 | 231.79 | 23 | 28 | 340.35 | 30 | 36 | 448.52 |
| | 6 | 0 | 6 | 4.11 | 2 | 8 | 11.09 | 9 | 20 | 37.34 | 16 | 31 | 66.11 | 24 | 33 | 91.93 | 32 | 40 | 116.84 |
| | 12 | - | - | 4.98 | 1 | 13 | 11.20 | 9 | 21 | 29.08 | 16 | 31 | 49.69 | 24 | 38 | 68.12 | 31 | 45 | 85.03 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.03 | 8 | 26 | 27.33 | 16 | 34 | 44.25 | 24 | 43 | 60.41 | 31 | 51 | 75.49 |
| | 36 | - | - | 4.98 | 0 | 36 | 20.83 | 7 | 43 | 31.18 | 15 | 51 | 43.62 | 22 | 58 | 55.70 | 30 | 66 | 67.79 |
| 15 | 1 | 0 | 2 | 4.36 | 1 | 7 | 28.28 | 8 | 19 | 121.84 | 15 | 30 | 234.41 | 22 | 28 | 345.34 | 29 | 36 | 453.52 |
| | 6 | 0 | 6 | 4.19 | 2 | 8 | 11.91 | 8 | 22 | 39.27 | 16 | 33 | 68.83 | 23 | 45 | 96.73 | 31 | 40 | 121.83 |
| | 12 | - | - | 4.98 | 1 | 13 | 11.61 | 9 | 25 | 31.15 | 16 | 37 | 52.66 | 23 | 47 | 72.35 | 31 | 57 | 89.97 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.31 | 8 | 26 | 28.71 | 16 | 34 | 47.01 | 23 | 45 | 64.28 | 30 | 51 | 80.08 |
| | 36 | - | - | 4.98 | 0 | 36 | 20.97 | 7 | 43 | 31.87 | 15 | 51 | 45.00 | 22 | 58 | 57.77 | 30 | 66 | 70.56 |
| 20 | 1 | 0 | 2 | 4.59 | 1 | 8 | 28.98 | 7 | 21 | 123.56 | 15 | 31 | 236.86 | 22 | 43 | 348.31 | 29 | 36 | 458.51 |
| | 6 | 0 | 6 | 4.27 | 2 | 8 | 12.73 | 8 | 24 | 40.98 | 16 | 35 | 71.24 | 23 | 46 | 99.61 | 30 | 40 | 126.83 |
| | 12 | - | - | 4.98 | 1 | 13 | 12.02 | 8 | 27 | 33.08 | 16 | 39 | 55.00 | 23 | 49 | 75.07 | 30 | 59 | 93.91 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.58 | 8 | 26 | 30.09 | 16 | 42 | 49.69 | 23 | 53 | 67.53 | 30 | 63 | 84.26 |
| | 36 | - | - | 4.98 | 0 | 36 | 21.11 | 7 | 43 | 32.56 | 15 | 51 | 46.39 | 22 | 58 | 59.85 | 30 | 66 | 73.33 |
| 25 | 1 | 0 | 2 | 4.82 | 1 | 9 | 29.63 | 7 | 22 | 125.04 | 14 | 32 | 239.13 | 21 | 43 | 350.84 | 28 | 55 | 461.80 |
| | 6 | 0 | 6 | 4.35 | 2 | 8 | 13.56 | 8 | 25 | 42.50 | 15 | 36 | 73.50 | 23 | 47 | 102.17 | 30 | 59 | 129.94 |
| | 12 | - | - | 4.98 | 1 | 13 | 12.42 | 8 | 29 | 34.67 | 15 | 40 | 57.06 | 23 | 51 | 77.62 | 30 | 61 | 97.17 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.85 | 8 | 26 | 31.47 | 16 | 44 | 52.00 | 23 | 55 | 70.02 | 30 | 66 | 87.00 |
| | 36 | - | - | 4.98 | 0 | 36 | 21.24 | 7 | 43 | 33.25 | 15 | 51 | 47.77 | 22 | 58 | 61.92 | 30 | 66 | 76.10 |

$L = 0.50$

| | | $\lambda = 0.1$ | | | $\lambda = 1$ | | | $\lambda = 5$ | | | $\lambda = 10$ | | | $\lambda = 15$ | | | $\lambda = 20$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $K$ | $q$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ | $s$ | $I_{max}$ | $C^*$ |
| 5 | 1 | 0 | 1 | 3.73 | 2 | 6 | 26.55 | 10 | 15 | 118.50 | 18 | 22 | 229.01 | 27 | 31 | 338.44 | 35 | 40 | 447.53 |
| | 6 | 0 | 6 | 4.04 | 2 | 8 | 10.56 | 10 | 18 | 35.89 | 19 | 27 | 63.79 | 28 | 36 | 90.08 | 37 | 44 | 115.91 |
| | 12 | - | - | 4.98 | 2 | 14 | 11.09 | 10 | 22 | 28.16 | 19 | 31 | 48.03 | 27 | 40 | 66.57 | 36 | 49 | 84.08 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.06 | 9 | 27 | 27.08 | 18 | 36 | 43.60 | 27 | 45 | 59.43 | 35 | 54 | 74.85 |
| | 36 | - | - | 4.98 | 1 | 37 | 20.89 | 8 | 44 | 31.57 | 17 | 53 | 44.30 | 25 | 61 | 56.62 | 34 | 70 | 68.99 |
| 10 | 1 | 0 | 2 | 4.14 | 2 | 7 | 27.63 | 9 | 18 | 120.93 | 17 | 30 | 233.86 | 26 | 31 | 343.43 | 34 | 40 | 452.53 |
| | 6 | 0 | 6 | 4.12 | 2 | 8 | 11.38 | 10 | 21 | 38.48 | 18 | 34 | 68.26 | 27 | 36 | 95.05 | 36 | 44 | 120.91 |
| | 12 | - | - | 4.98 | 2 | 14 | 11.50 | 10 | 22 | 30.23 | 18 | 33 | 51.84 | 27 | 41 | 71.23 | 35 | 49 | 89.04 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.33 | 9 | 27 | 28.46 | 18 | 36 | 46.36 | 27 | 46 | 63.46 | 35 | 55 | 79.52 |
| | 36 | - | - | 4.98 | 1 | 37 | 21.02 | 8 | 44 | 32.26 | 17 | 53 | 45.68 | 25 | 61 | 58.70 | 34 | 70 | 71.76 |
| 15 | 1 | 0 | 2 | 4.38 | 2 | 8 | 28.53 | 9 | 20 | 122.94 | 17 | 32 | 236.48 | 25 | 31 | 348.42 | 33 | 40 | 457.53 |
| | 6 | 0 | 6 | 4.20 | 2 | 8 | 12.20 | 10 | 23 | 40.44 | 18 | 35 | 70.95 | 26 | 48 | 99.82 | 35 | 44 | 125.91 |
| | 12 | - | - | 4.98 | 2 | 14 | 11.91 | 10 | 26 | 32.29 | 18 | 39 | 54.78 | 26 | 50 | 75.42 | 34 | 61 | 93.97 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.60 | 9 | 27 | 29.84 | 18 | 36 | 49.13 | 26 | 48 | 67.32 | 34 | 55 | 84.08 |
| | 36 | - | - | 4.98 | 1 | 37 | 21.16 | 8 | 44 | 32.95 | 17 | 53 | 47.07 | 25 | 61 | 60.77 | 34 | 70 | 74.53 |
| 20 | 1 | 0 | 2 | 4.61 | 2 | 8 | 29.30 | 8 | 22 | 124.66 | 17 | 33 | 238.92 | 25 | 46 | 351.31 | 32 | 40 | 462.53 |
| | 6 | 0 | 6 | 4.29 | 2 | 8 | 13.02 | 9 | 25 | 42.12 | 18 | 37 | 73.34 | 26 | 49 | 102.66 | 34 | 44 | 130.90 |
| | 12 | - | - | 4.98 | 2 | 14 | 12.32 | 10 | 28 | 34.23 | 18 | 41 | 57.10 | 26 | 52 | 78.12 | 34 | 63 | 97.91 |
| | 18 | - | - | 4.98 | 1 | 19 | 13.87 | 9 | 27 | 31.22 | 18 | 44 | 51.80 | 26 | 56 | 70.56 | 34 | 67 | 88.24 |
| | 36 | - | - | 4.98 | 1 | 37 | 21.30 | 8 | 44 | 33.64 | 17 | 53 | 48.45 | 25 | 61 | 62.85 | 34 | 70 | 77.30 |
| 25 | 1 | 0 | 2 | 4.84 | 1 | 9 | 29.97 | 8 | 23 | 126.14 | 16 | 34 | 241.17 | 24 | 46 | 353.81 | 32 | 59 | 465.68 |
| | 6 | 0 | 6 | 4.37 | 2 | 8 | 13.83 | 9 | 26 | 43.63 | 17 | 38 | 75.59 | 26 | 50 | 105.21 | 34 | 63 | 133.92 |
| | 12 | - | - | 4.98 | 2 | 14 | 12.74 | 9 | 30 | 35.81 | 17 | 42 | 59.15 | 26 | 54 | 80.66 | 34 | 65 | 101.15 |
| | 18 | - | - | 4.98 | 1 | 19 | 14.15 | 9 | 27 | 32.60 | 18 | 46 | 54.10 | 26 | 58 | 73.05 | 33 | 70 | 90.95 |
| | 36 | - | - | 4.98 | 1 | 37 | 21.44 | 8 | 44 | 34.33 | 17 | 53 | 49.83 | 25 | 61 | 64.92 | 34 | 70 | 80.07 |

Note: $h = 1, K_1 = 20, K_2 = 1, p = 50$; $s = I_{max} = -$ refers to a do-not-order-policy with an average cost equal to $p \cdot \lambda$.

transformed into a finite horizon problem with zero unit purchase cost. We deduce that for any value of $q$, the optimal policy for a problem with parameters $(K_1, K_2, h, p, q)$ and $K = 0$ is the same as the optimal policy for a problem with parameters $(0, 0, h, p - K_1/q - K_2, q)$, and on the long-run, the average cost difference between the former and the latter model is given by $(K_1/q + K_2) * \lambda$, a term independent of the policy (see Proposition B.2 in Appendix B). Therefore, when $q$ is exogenous, the batch ($K_1$) and unit ($K_2$) handling costs can be assumed to be zero without loss of generality, for determining the optimal policy, provided the penalty cost transformation $p(q) := p - K_1/q - K_2$.

### Sensitivity on $p, \lambda$ and $L$

In the above scenarios, the penalty cost is quite high ($p = 50$) compared to the holding costs, resulting in high service levels. With a very high service level, shortages are rare events, and the lost sales model could be approximated by a backorder one. Table 6 shows the performance with regards to different values of the lost sales penalty cost $p$, together with $\lambda$ and $L$. Clearly, if the penalty is too low (i.e. $p = 5$) the policy is to not order anything and take the lost sales costs completely. Higher penalties for very low demand ($\lambda = 0.1$) also do not lead to changes in this policy behavior. On the other hand, for higher levels of demand ($\lambda \geq 1$) it becomes more efficient from a cost point of view to order positive amounts of inventory. Remember, that in the previous Table 5, we set $p = 50$ which resulted in policies where also positive orders are seen for low values of $\lambda$, but only if $q$ is small enough ($q \leq 6$).

## B On Unit vs. Batch Costs

**Lemma B.1.** *(Janakiraman and Muckstadt 2001, Lemma 1) For all valid sets of cost parameters* $(c'; h'; p')$ *(i.e.* $(\alpha(p' - h') \geq c'$ *)) there exists another set of cost parameters* $(0; h; p)$

Table 6: Sensitivity analysis on $p$

| $p$ | $\lambda = 0.1$ $s$ | $I_{max}$ | $C^*$ | $\lambda = 1$ $s$ | $I_{max}$ | $C^*$ | $\lambda = 5$ $s$ | $I_{max}$ | $C^*$ | $\lambda = 10$ $s$ | $I_{max}$ | $C^*$ | $\lambda = 15$ $s$ | $I_{max}$ | $C^*$ | $\lambda = 20$ $s$ | $I_{max}$ | $C^*$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $L = 0.25$ | | | | | | | | | | | | | | | | | | |
| 5 | - | - | 0.56 | - | - | 5.00 | - | - | 25.00 | - | - | 50.00 | - | - | 75.00 | - | - | 100.00 |
| 15 | - | - | 1.54 | 0 | 6 | 9.89 | 5 | 17 | 34.54 | 12 | 27 | 62.14 | 18 | 28 | 87.22 | 25 | 34 | 111.27 |
| 25 | - | - | 2.52 | 1 | 7 | 10.33 | 7 | 19 | 35.63 | 14 | 29 | 63.59 | 21 | 29 | 88.78 | 27 | 36 | 113.06 |
| 40 | - | - | 4.00 | 2 | 8 | 10.87 | 8 | 19 | 36.43 | 15 | 30 | 64.63 | 22 | 31 | 89.94 | 29 | 38 | 114.32 |
| $L = 0.33$ | | | | | | | | | | | | | | | | | | |
| 5 | - | - | 0.56 | - | - | 5.00 | - | - | 25.00 | - | - | 50.00 | - | - | 75.00 | - | - | 100.00 |
| 15 | - | - | 1.54 | 0 | 6 | 10.02 | 6 | 18 | 34.98 | 13 | 28 | 63.05 | 20 | 29 | 88.55 | 27 | 36 | 113.05 |
| 25 | - | - | 2.52 | 1 | 7 | 10.47 | 7 | 19 | 36.14 | 14 | 30 | 64.57 | 22 | 31 | 90.21 | 29 | 38 | 114.90 |
| 40 | - | - | 4.00 | 2 | 8 | 10.96 | 8 | 20 | 36.96 | 16 | 31 | 65.64 | 23 | 32 | 91.39 | 31 | 40 | 116.24 |
| $L = 0.50$ | | | | | | | | | | | | | | | | | | |
| 5 | - | - | 0.56 | - | - | 5.00 | - | - | 25.00 | - | - | 50.00 | - | - | 75.00 | - | - | 100.00 |
| 15 | - | - | 1.54 | 1 | 7 | 10.19 | 7 | 19 | 36.01 | 14 | 30 | 64.99 | 22 | 32 | 91.45 | 30 | 40 | 116.87 |
| 25 | - | - | 2.52 | 1 | 7 | 10.79 | 8 | 20 | 37.23 | 16 | 32 | 66.60 | 25 | 34 | 93.22 | 33 | 42 | 118.83 |
| 40 | - | - | 4.00 | 2 | 8 | 11.19 | 9 | 21 | 38.10 | 18 | 33 | 67.75 | 26 | 35 | 94.48 | 35 | 44 | 120.29 |

Note: $h = 1, q = 6, K = 10, K_1 = 20, K_2 = 1$ and $s = I_{max} = -$ refers to a do-not-order-policy with an average cost equal to $p \cdot \lambda$.

*with* $h = h' + c'(1-\alpha)/\alpha$ *and* $p = p' - c$ *such that*

$$f_n^{(c';h';p')}(x_n, q_n) = f_n^{(0;h;p)}(x_n, q_n) + \zeta,$$

*where* $f_n^{(c;h;p)}(x_n, q_n)$ *denotes the minimum expected sum of all discounted future costs (with discount factor* $\alpha$ *and cost parameters* $c$*,* $h$ *and* $p$*) and* $\zeta$ *is a term independent of the policy, if we start period* $n$ *with* $x_n$ *units of inventory on hand and we order* $q_n$ *units.*

For the proof, we refer the reader to Janakiraman and Muckstadt (2001). In view of this result, we derive the following result for the infinite horizon, average cost model with batch ordering and no setup cost.

**Proposition B.2.** *Consider the inventory system introduced in Section 4 and assume there is no setup cost. For any given batch size* $q$*, and all sets of cost parameters* $(K_1, K_2, h, p)$ *such that* $p - h \geq K_1/q + K_2$*, the parameter transformation*

$$(K_1, K_2, h, p, q) \mapsto (0, 0, h, p - K_1/q - K_2, q)$$

*leads to the following cost transformation:*

$$C^*_{(K_1,K_2,h,p,q)} = C^*_{(0,0,h,p-K_1/q-K_2,q)} + (K_1/q + K_2) \cdot \lambda,$$

*where $C^*_{(K_1,K_2,h,p,q)}$ denotes the minimum long-run average cost corresponding to parameters $(K_1, K_2, h, p, q)$ and $\lambda$ denotes the average demand per review period.*

*Proof.* For every fixed value of the batch size $q$, we can rewrite the replenishment cost as follows

$$c^r(nq) = \delta(nq)K + K_1 n + K_2 nq = \delta(nq)K + (K_1/q + K_2)nq = \delta(nq)K + c(q)nq,$$

where $c(q) = K_1/q + K_2$ is the per unit purchasing cost (given $q$). Then, since $K = 0$, we apply Lemma B.1 and a limiting argument and deduce:

$$C^*_{(K_1,K_2,h,p,q)} = C^*_{(0,0,h,p-c(q),q)} + \zeta. \tag{6}$$

Next, assume that the problem parameters are such that the optimal policy corresponding to $(0, 0, h, p - c(q); q)$ is to never order, in which case the minimum average cost equals $C^*_{(0,0,h,p-c(q),q)} = (p - c(q)) \cdot \lambda = (p - K_1/q - K_2) \cdot \lambda$. It follows that the optimal policy for the problem with parameters $(K_1, K_2, h, p, q)$ is also the policy of never ordering and the corresponding minimum average cost equals $C^*_{(K_1,K_2,h,p,q)} = p \cdot \lambda$. Replacing the average costs in (6) it follows that:

$$p \cdot \lambda = (p - c(q)) \cdot \lambda + \zeta,$$

and thus

$$\zeta = c(q) \cdot \lambda = (K_1/q + K_2) \cdot \lambda$$

# C Proofs for the different propositions

**Proposition C.1.** *There exists a state $i_u$ such that for $i \geq i_u$ the optimal decision is not to order.*

*Proof.* Consider the one-period transition cost $c_i(a_i)$ defined in Equation (3), and let $\underline{c}_i(a_i)$

be a lower bound for $c_i(a_i)$ when $a_i$ units are ordered. A possible $\underline{c}_i(a_i)$ can be obtained when we eliminate certain terms in Equation (3).

To simplify notation, let

$$\begin{aligned}\theta(i) &= ((i-D_L)^+ + a_i - D_{R-L})^+ \\ \theta_1(i) &= (D_L - i)^+ + (D_{R-L} - a_i - (i-D_L)^+)^+.\end{aligned}$$

Then, using the identify $(x-y)^+ = x - y + (x-y)^+$, we obtain

$$\begin{aligned}\theta(i) &= (i - D_L + (D_L - i)^+ + a_i - D_{R-L})^+ = (i + a_i - D_R + (D_L - i)^+)^+, \\ \theta_1(i) &= (D_L - i)^+ + D_{R-L} - a_i - (i - D_L)^+ + ((i-D_L)^+ - D_{R-L} + a_i)^+ \\ &= D_L - i + D_{R-L} - a_i + ((i-D_L)^+ + a_i - D_{R-L})^+ \\ &= D_R - i - a_i + ((i-D_L)^+ + a_i - D_{R-L})^+ \\ &= D_R - i - a_i + \theta(i).\end{aligned}$$

Next, observe that

$$\begin{aligned}\theta(i) &\geq (i + a_i - D_R)^+, \\ \theta_1(i) &\geq D_R - i - a_i + (i + a_i - D_R)^+ = (D_R - i - a_i)^+.\end{aligned}$$

Therefore, we obtain the following lower bounds for the expected one-period holding and penalty costs:

$$\begin{aligned}c_i^h(a_i) &= h\mathsf{E}\left[((i-D_L)^+ + a_i - D_{R-L})^+\right] \geq h\mathsf{E}\left[(i + a_i - D_R)^+\right], \\ c_i^p(a_i) &= p\left\{\mathsf{E}\left[(D_L - i)^+\right] + \mathsf{E}\left[(D_{R-L} - a_i - (i - D_L)^+)^+\right]\right\} \geq p\mathsf{E}\left[(D_R - i - a_i)^+\right].\end{aligned}$$

Now, we define $\underline{c}_i(a_i)$ as

$$\underline{c}_i(a_i) = \delta(a_i)K + h\mathsf{E}\left[(i + a_i - D_R)^+\right] + p\mathsf{E}\left[(D_R - i - a_i)^+\right]$$

to obtain a lower bound for the one-period transition cost $c_i(a_i)$. Note that $\underline{c}_i(a_i)$ has a newsvendor structure, except for the fixed cost $K$.

Let $a_R^{NV}$ be the optimal order quantity when $i$ is not considered. Hence, the optimal solution for $\underline{c}_i(a_i)$ would be

$$a_i^* = \begin{cases} a_R^{NV} - i, & \text{if } i \leq a_R^{NV} \\ 0, & \text{otherwise.} \end{cases}$$

Note that with this ordering policy $\underline{c}_i(a_i^*) \geq c_i(0)$, for $i \geq a_R^{NV}$. Hence, $i_u = a_R^{NV}$.

□

**Proposition C.2.** *There exists a set of parameter values for which we would not operate the lost sales inventory system under consideration, i.e. we will never order.*

*Proof.* No formal proof is necessary as one can set $K$ arbitrarily large in Equation (3) to satisfy this proposition. □

Next, we identify a condition that guarantees a threshold value for the initial inventory level below which it is always optimal to order.

**Proposition C.3.** *If*

$$\begin{aligned} (h+p)E[D_R] \quad &\geq \quad K + K_1 a_{R-L}^{NV}/q + K_2 a_{R-L}^{NV} + h\mathsf{E}\left[(a_{R-L}^{NV} - D_{R-L})^+\right] \\ &+ \quad p\mathsf{E}\left[(D_{R-L} - a_{R-L}^{NV})^+\right] + p\mathsf{E}\left[D_L\right] + h\mathsf{E}\left[D_R\right]. \end{aligned}$$

*is satisfied, then there exists a state $i$ for which it is optimal to order. Here $a_{R-L}^{NV}$ is the optimal order-up-to-level for a news vendor problem with Poisson demand over $R-L$ time units.*

*Proof.* Consider Equation (3) for $a_i = 0$ and observe that

$$\begin{aligned} c_i(0) &= h\mathsf{E}\left[(i - D_R)^+\right] + p\mathsf{E}\left[(D_R - i)^+\right] \\ &= (h+p)\mathsf{E}\left[(D_R - i)^+\right] + hi - h\mathsf{E}\left[D_R\right], \end{aligned}$$

using the identity $(x-y)^+ = x - y + (y-x)^+$. Using Equation (3) for $a_i > 0$ and defining $\overline{c}'_i(a_i)$ to be an upper bound for $c_i(a_i)$, we obtain the following:

$$\begin{aligned} \overline{c}'_i(a_i) &= K + K_1 a_i/q + K_2 a_i + h\mathsf{E}\left[(i + a_i - D_{R-L})^+\right] \\ &+ p\mathsf{E}\left[(D_{R-L} - a_i - i)^+\right] + p\mathsf{E}\left[D_L\right]. \end{aligned}$$

Assuming that we always carry the initial $i$ units and satisfy the demand from the incoming order only, we obtain another upper bound for $c_i(a_i)$, denoted as $\overline{c}_i(a_i)$:

$$\begin{aligned} \overline{c}_i(a_i) &= K + K_1 a_i/q + K_2 a_i + hi + h\mathsf{E}\left[(a_i - D_{R-L})^+\right] \\ &+ p\mathsf{E}\left[(D_{R-L} - a_i)^+\right] + p\mathsf{E}\left[D_L\right]. \end{aligned}$$

Hence, any non-optimal solution for the upper bound will still be an upper bound for the original problem. Similarly to Proposition C.1, define $a^{NV}_{R-L}$. Hence,

$$\overline{c}_i(a^{NV}_{R-L}) \geq \overline{c}_i(a^*_i).$$

As a result,

$$\begin{aligned} \overline{c}_i(a^{NV}_{R-L}) &\leq K + K_1 a^{NV}_{R-L}/q + K_2 a^{NV}_{R-L} + hi + h\mathsf{E}\left[(a^{NV}_{R-L} - D_{R-L})^+\right] \\ &+ p\mathsf{E}\left[(D_{R-L} - a^{NV}_{R-L})^+\right] + p\mathsf{E}\left[D_L\right]. \end{aligned}$$

Now, writing $c_i(0)$ and $\bar{c}_i(a^{NV}_{R-L})$ together, we want $c_i(0) \geq \bar{c}_i(a^{NV}_{R-L})$ or

$$\begin{aligned}(h+p)\mathsf{E}\left[(D_R-i)^+\right]+hi-h\mathsf{E}\left[D_R\right] \;\; &\geq \;\; K+K_1a^{NV}_{R-L}/q+K_2a^{NV}_{R-L} \\ &+ \;\; hi+h\mathsf{E}\left[(a^{NV}_{R-L}-D_{R-L})^+\right] \\ &+ \;\; p\mathsf{E}\left[(D_{R-L}-a^{NV}_{R-L})^+\right]+p\mathsf{E}\left[D_L\right].\end{aligned}$$

Eliminating $hi$ terms,

$$\begin{aligned}(h+p)\mathsf{E}\left[(D_R-i)^+\right] \;\; &\geq \;\; K+K_1a^{NV}_{R-L}/q+K_2a^{NV}_{R-L}+h\mathsf{E}\left[(a^{NV}_{R-L}-D_{R-L})^+\right] \\ &+ \;\; p\mathsf{E}\left[(D_{R-L}-a^{NV}_{R-L})^+\right]+p\mathsf{E}\left[D_L\right]+h\mathsf{E}\left[D_R\right].\end{aligned}$$

Note that the righthand side is constant in $i$ and the lefthand side is decreasing with $i$. The existence condition given in the statement of the proposition is the case when $i = 0$. Hence, if the condition is satisfied, the largest $i$ satisfying this condition is $i_L$, a lower bound for inventory level $i$ where it is optimal to order. □

# D Comparing with $(s, S, nq)$ and $(s, Q, nq)$ policies

We now report on the computational results on the performance of the best $(s, S, nq)$ and the best $(s, Q, nq)$ policies, compared to the optimal policy. This is an important analysis, since many retailers employ these policies. The key question to be answered is then whether by extending the cost function to include handling, these standard policies get closer in performance compared to the optimal policy. For any policy, we use a dynamic programming formulation (similar to (4) in Section 4) in order to determine the long-run average cost of the policy. In this case, for any state $i \in SS$, instead of minimizing over all possible order quantities, the order quantity is determined by the logic of the specific policy (either the $(s, S, nq)$ or the $(s, Q, nq)$ policy). We then solve the resulting system of equations to determine the long-run average cost $C(s, Q, S)$. To obtain the best policy, we exhaustively search over a sufficiently large feasible region to ensure finding a global optimum. Note that all policies have a cost function including handling (which is not done in retail practice).

Table 7: Performance of the optimal policy compared to the best $(s, S, nq)$ and the best $(s, Q, nq)$ policies

| | $(s,S,nq)$ | | | $(s,Q,nq)$ | | |
|---|---|---|---|---|---|---|
| $\lambda$ | Avg | Min | Max | Avg | Min | Max |
| 0.1 | 0.00% | 0.00% | 0.00% | 37.63% | 24.94% | 50.32% |
| 1 | 0.00% | 0.00% | 0.00% | 1.33% | 0.00% | 10.88% |
| 5 | 0.02% | 0.00% | 0.28% | 0.29% | 0.00% | 4.75% |
| 10 | 0.09% | 0.00% | 1.43% | 0.91% | 0.00% | 8.91% |
| 15 | 0.13% | 0.00% | 1.44% | 2.81% | 0.00% | 15.37% |
| 20 | 0.30% | 0.00% | 1.70% | 6.31% | 0.00% | 20.52% |
| $q$ | Avg | Min | Max | Avg | Min | Max |
| 1 | 0.30% | 0.00% | 1.39% | 4.84% | 0.01% | 50.32% |
| 6 | 0.18% | 0.00% | 1.44% | 4.14% | 0.00% | 24.94% |
| 12 | 0.15% | 0.00% | 1.70% | 2.73% | 0.00% | 13.27% |
| 18 | 0.01% | 0.00% | 0.39% | 2.06% | 0.00% | 20.52% |
| 36 | 0.00% | 0.00% | 0.00% | 0.23% | 0.00% | 2.27% |
| $K$ | Avg | Min | Max | Avg | Min | Max |
| 5 | 0.12% | 0.00% | 1.44% | 6.04% | 0.00% | 50.32% |
| 10 | 0.09% | 0.00% | 1.17% | 2.23% | 0.00% | 14.12% |
| 15 | 0.12% | 0.00% | 0.83% | 1.15% | 0.00% | 7.66% |
| 20 | 0.17% | 0.00% | 1.70% | 0.88% | 0.00% | 4.08% |
| 25 | 0.06% | 0.00% | 0.80% | 0.80% | 0.00% | 3.99% |
| $p$ | Avg | Min | Max | Avg | Min | Max |
| 10 | 0.17% | 0.00% | 1.70% | 1.85% | 0.00% | 18.83% |
| 25 | 0.11% | 0.00% | 1.70% | 2.03% | 0.00% | 20.52% |
| 50 | 0.03% | 0.00% | 0.54% | 5.48% | 0.00% | 50.32% |
| $L$ | Avg | Min | Max | Avg | Min | Max |
| 0.25 | 0.10% | 0.00% | 1.70% | 2.08% | 0.00% | 20.52% |
| 0.33 | 0.11% | 0.00% | 1.70% | 3.11% | 0.00% | 50.32% |
| 0.5 | 0.13% | 0.00% | 1.04% | 1.68% | 0.00% | 9.45% |
| Overall | 0.11% | 0.00% | 1.70% | 2.59% | 0.00% | 50.32% |

Note: $h = 1, K_1 = 20, K_2 = 1$. Percentage errors computed based on the normalized costs, i.e. total average costs $-(K_1/q + K_2) \cdot \lambda$

Table 7 summarizes the results of our experiments. Results for the best $(s, S, nq)$ and $(s, Q, nq)$ policy are stated as percentage increase in average costs from the cost of the optimal policy. In computing the percentage errors, all average costs are first normalized by subtracting from the total costs the constant $(K_1/q + K_2) \cdot \lambda$. The main observation from Table 7 is that, on average, the best $(s, S, nq)$ policy is performing relatively close to optimal, while the $(s, Q, nq)$ heuristic is remarkably worse. The best $(s, S, nq)$ policy, with an average error of 0.09%, deteriorates slightly. An important reason for the small optimality gap between these heuristics is the flatness of the cost curves $C(s, Q, S)$ around the optimal $Q$ value.

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