Correspondence Note: Periodic Review Multi-Period Inventory Control under a Mean-Variance Optimization Objective[1]

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September 4, 2010

Abstract: We study in this paper a solution scheme which solves a periodic review multi-period inventory problem under a mean-variance framework. We first investigate a primal inventory problem with a mean-variance objective function. Owing to the non-separable nature of variance, we construct an auxiliary problem which is separable. By solving the auxiliary problem, we identify the conditions under which the solutions of the primal and auxiliary problems converge. We hence propose the algorithm and show that a base-stock policy is optimal.

Keywords: Inventory control, base stock policy, mean-variance analysis.

[1] We sincerely thank Professor Witold Pedrycz (the editor-in-chief), the associate editor, and the three anonymous referees for their kind advice and comments. Their suggestions have led to a major improvement of the paper. This research is partially supported by the Hong Kong RGC Competitive Earmarked Research Grants under the grant numbers of PolyU 5143/07E and PolyU 5146/05E.

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I. INTRODUCTION

Inventory control is one of the most important topics in supply chain systems [1, 4, 22]. Despite having some computationally challenging problems [8], inventory control is still an ever-green topic in the literature. Actually, since the publications of various pioneering papers [2, 6], the single-item periodic review stochastic inventory control problems have been well-studied and extended in the literature. The (s, S) type of inventory policies, for both finite and infinite horizons, have been proven to be optimal and extended in many different ways. Some of remarkable results include the optimal inventory policy with non-stationary demands [16], the development of the algorithms for computing the (s, S) policies under different settings [8], and the optimal policy under a service level constraint [13]. Interested readers could find out more details from [18]. Despite the fact that the above extensions have captured nearly all important aspects of the problem formulations for the single-item periodic review stochastic inventory system, to the best of our knowledge, there is no mention of the control of the profit uncertainty (or risk) that is associated with the optimal inventory policy in the existing literature. However, the concern of risk is a common practice in business. For example, suppose that a company needs to decide the details of an optimal multi-period periodic review inventory control policy. Owing to high demand volatility in the market, in addition to achieving a certain reasonable profit target, the company would also like to minimize the associated risk. This practical decision making problem motivates us to explore a new optimal inventory control problem under an optimization objective which includes a trade-off of expected profit and risk.

Pioneered by the Nobel laureate Harry Markowitz in the 1950s, the use of the mean-variance (MV) model in the analysis of portfolio [14] has been demonstrated to be highly successful and the MV framework has been applied for many different types of models [1, 15]. In particular, risk analyses via MV models in supply chain management have received much attention and various publications have appeared. For instance, after the pioneering work done by [11] with the use of a mean-standard-deviation objective function for studying the newsboy problem, some papers which have made use of the mean-variance approach for analysis in supply chain inventory management problems include the study on risk intermediation challenge in [5], the study of newsvendor problem via an MV approach in [1], the risk analysis of the commitment-option contracts in [3], and the mean-variance analysis of supply chain contracts in [17]. Undoubtedly, we all know that variance control has many important applications but it is also notorious for some shortcomings. One of them is its non-separability nature in the sense of dynamic programming. As a result, optimization of the multi-period variance control problem becomes difficult, if not impossible, to be achieved. Fortunately, the approach proposed in [12] has provided a general solution mechanism for solving variance minimization problems in a multi-period setting and it forms the basis for the analysis of this paper.

Based on the above literature, we apply the MV model for studying the multi-period periodic review inventory system. We first investigate the optimal inventory policy under a control on the profit uncertainty via a variance of profit penalty term in the objective function. To be specific, we derive an optimal inventory policy for the risk averse decision maker under which an objective function with expected profit and variance of profit consideration is maximized. Since the variance of profit is non-separable in the sense of dynamic programming, we solve the original problem under the variance control by constructing and solving an auxiliary problem and finding the condition under which the solutions between them converge. A dynamic control system is proposed and an optimal policy is derived by using dynamic programming. The sufficient conditions for the existence of the optimal threshold policy have been proposed. An interesting finding is that: The existence of the optimal threshold policy is related to the number of periods and the degree of risk aversion of the decision maker. Our results also indicate that a base stock policy is optimal. To the best of our knowledge, different from all the above reviewed papers in inventory control, *this paper is the first piece of work which attempts to solve the non-separable multi-period inventory control problem under an MV optimization objective*. As a remark, there are some other works which explore the inventory control problem with a risk averse decision maker by using the exponential utility function. Despite being analytically more tractable, the use of exponential utility is being criticized by being impractical as it is nearly impossible to find the utility function of the decision maker. On the contrary, the MV model is being known to more applicable, implementable and intuitive because it simply relies on exploring two performance measures, namely the mean and the variance where the mean captures the payoff and the variance captures the risk. A tradeoff between the two measures would thus yield the optimal decision. Thus, we employ the MV model for our analysis in this paper.

II. BASIC INVENTORY MODEL

Consider a retailer who sells a product with a finite shelf-life of N replenishment periods. The retailer takes a periodic review stocking policy in which he places the order at the end of each selling period. The delivery time for the order is very short (i.e. zero delivery time) and the order placed will be able to arrive on time to meet the demand in the coming period. For each period, the product's demand is uncertain and follows a stationary distribution. Any unmet demand during the selling season is lost and any unsold inventory can be carried on to the next period. The product's cost structure is as follows: The per-period unit holding cost is h, and at the end of the last selling period $N-1$, any unsold product can be salvaged with a unit price of v. Moreover, each product incurs a unit ordering cost of c, and the unit revenue is r. Following the traditional inventory system's assumption which can avoid trivial cases, we have $r > c > v$ and $c > h$. Owing to the complexity of the problem, we consider the situation where the fixed setup cost is negligible. The order quantity is represented by q_k. We use the following notation:

$(A)^+ = \max(A, 0)$, $(i, j)^- = \min(i, j)$, x_k = inventory level at period k,

q_k = order quantity at period k,

w_k = demand at period k and it is a continuous *iid* random variable, $w_k \in [0, W]$ and $0 < W < \infty$.

$f(\cdot)$ = the probability density function of w_k,

y_k = inventory position, which is defined as $x_k + q_k$.

End of season salvage value $v_k = \begin{cases} 0, & k \neq N-1, \\ v, & k = N-1. \end{cases}$

With the above details, the profit at each period $k = 0, 1, ..., N-1$, $z_k(x_k)$, can be expressed as follows,

$$\begin{aligned} z_k(x_k) &= r(x_k + q_k, w_k)^- - cq_k + (v_k - h)(x_k + q_k - w_k)^+ \\ &= r[x_k + q_k - (x_k + q_k - w_k)^+] - cq_k + (v_k - h)(x_k + q_k - w_k)^+ \\ &= r(x_k + q_k) - cq_k - (r + h - v_k)(x_k + q_k - w_k)^+. \end{aligned} \tag{2.1}$$

Putting $y_k = x_k + q_k$ into (2.1) gives (2.2),

$$z_k(x_k) = (r - c)y_k + cx_k - (r + h - v_k)(y_k - w_k)^+. \tag{2.2}$$

Notice that y_k represents the inventory position at Period k. For the product with a finite selling season of N periods (Periods 0, 1, ..., $N-1$), we have the following expected profit maximization problem,

$$\max_{q_k} E\left(\sum_{k=0}^{N-1} z_k(x_k)\right). \tag{2.3}$$

In order to derive the optimal policy which solves (2.3), dynamic programming is applied. Each period represents a stage and x_k represents the state. Moreover, we have the following system equation,

$$x_{k+1} = (x_k + q_k - w_k)^+. \tag{2.4}$$

We define the benefit-to-go by $\hat{J}$ and we have the following dynamic programming formulation:

$$\hat{J}_N(x_N) = 0,$$

$$\hat{J}_k(x_k) = \max_{q_k \geq 0} E[z_k(x_k) + \hat{J}_{k+1}(x_{k+1})], \text{ for all } k = 0, 1, \ldots, N-1.$$

It is a classic result that $\hat{J}_k(x_k)$ is concave and an optimal threshold policy exists:

$$q_k^* = \begin{cases} \hat{y}_k^* - x_k, & \text{if } x_k < \hat{y}_k^*, \\ 0, & \text{if } x_k \geq \hat{y}_k^*, \end{cases}$$

where $\hat{y}_k^* = \arg\{\max_{y_k} E_{w_k}[\hat{J}_{k+1}(x_{k+1})]\}$, and q_k^* is the optimal order quantity at Period k. Thus, at each period, the order quantity is either equal to 0 or the difference between the order-up-to level $\hat{y}_k^*$ and the on-hand inventory (inventory level) x_k.

III. INVENTORY POLICY UNDER MV OBJECTIVE

We now consider the inventory policy under an MV objective. To be specific, for an inventory replenishment policy with Periods 0, 1, ..., $N-1$, the optimization problem under the MV formulation is defined as follows:

$$(O(\delta)) \quad \max_{q_k} E(Z_{N-1}) - \delta Var(Z_{N-1}), \tag{3.1}$$

where $Z_{N-1} = \sum_{k=0}^{N-1} z_k(x_k)$, $\delta \geq 0$.

Notice that in $O(\delta)$, the objective function of (3.1) carries very good economic meaning and hence it is applicable in practice. First of all, it is an MV model in which δ is called the risk sensitivity coefficient [11, 19, 20] and it captures the degree of risk aversion of the decision maker. A larger δ implies a more risk averse decision maker because the objective function gives more weight to the variance part and hence penalizes "risk" more. When δ is zero, the decision maker is risk neutral. Thus, we can take (3.1) as an objective function in which a tradeoff between the mean and the variance is present and it is related to the penalty function method as discussed in the literature [4]. Furthermore, the unconstrained problem $O(\delta)$ is actually also related to the constrained optimization in which the objective is on minimizing the variance of profit subject to the constraint on achieving a certain (minimum) amount of expected profit. By standard optimization methods such as the Lagrangian multiplier method and KKT conditions [21], we can convert the constrained MV model(s) into $O(\delta)$. Thus, finding the optimal solution for $O(\delta)$ also means solving the equivalent constrained MV problems.

As we have mentioned in Section I, the variance of profit is non-separable. This makes Problem ($O(\delta)$) non-separable for all $\delta > 0$. In order to use dynamic programming to derive an optimal policy, we would like to derive a solution scheme under which the primal problem, Problem ($O(\delta)$), is embedded into a tractable and separable auxiliary problem. After that, by solving the auxiliary problem and then investigating the condition under which the solutions of the auxiliary problem and Problem ($O(\delta)$) converge, an optimal dynamic control policy can be developed. Following this concept, we construct the auxiliary problem as follows.

First, by using the relationship of $Var(Z_{N-1}) = E[{Z_{N-1}}^2] - (E[Z_{N-1}])^2$, we rewrite Problem ($O(\delta)$) in (3.1) as (3.2),

$$\max_{q_k} \; E(Z_{N-1}) - \delta[E({Z_{N-1}}^2) - (E(Z_{N-1}))^2]. \tag{3.2}$$

Define: $U[E(Z_{N-1}), E({Z_{N-1}}^2)] = E(Z_{N-1}) - \delta[E({Z_{N-1}}^2) - (E(Z_{N-1}))^2]$.

Second, we construct a problem ($A(\mu,\delta)$) below:

($A(\mu,\delta)$) $\quad \max_{q_k} \; \mu E(Z_{N-1}) - \delta E[{Z_{N-1}}^2]$.

Define:

$Q_O(\delta) = \{Q \mid Q \text{ is a maximizer of } (O(\delta))\}$,

$Q_A(\mu,\delta) = \{Q \mid Q \text{ is a maximizer of } (A(\mu,\delta))\}$,

$$D(Q,\delta) = \left.\frac{\partial U[E(Z_{N-1}), E({Z_{N-1}}^2]}{\partial E(Z_{N-1})}\right|_Q$$

$$= 1 + 2\delta E(Z_{N-1})\big|_Q.$$

Corollary 3.1. *For any* $Q^* \in Q_O(\delta)$*:* $Q^* \in Q_A(D(Q^*,\delta),\delta)$.

Proof of Corollary 3.1: We prove by contradiction. Assuming that $Q^* \in Q_O(\delta)$ but $Q^* \notin Q_A(D(Q^*,\delta),\delta)$, then there exists a Q such that the following is true,

$$\{D(Q,\delta)E(Z_{N-1}) - \delta E[{Z_{N-1}}^2]\}\big|_Q > \{D(Q,\delta)E(Z_{N-1}) - \delta E[{Z_{N-1}}^2]\}\big|_{Q^*}. \tag{3.3}$$

Observe that $D(Q,\delta) = 1 + 2\delta E(Z_{N-1})\big|_Q$ and $\dfrac{\partial U[E(Z_{N-1}), E({Z_{N-1}}^2)]}{\partial E({Z_{N-1}}^2)} = -\delta$. Further notice that $U[E(Z_{N-1}), E({Z_{N-1}}^2)]$ is a convex function of $E(Z_{N-1})$ and $E({Z_{N-1}}^2)$. As a result, we have the following,

$$U[E(Z_{N-1}), E(Z_{N-1}{}^2)]|_Q$$

$$\geq U[E(Z_{N-1}), E(Z_{N-1}{}^2)]|_{Q^*} + \{D(Q,\delta)E(Z_{N-1}) - \delta E[Z_{N-1}{}^2]\}|_Q$$

$$-\{D(Q,\delta)E(Z_{N-1}) - \delta E[Z_{N-1}{}^2]\}|_{Q^*}. \tag{3.4}$$

(3.3) and (3.4) together implies the following,

$$U[E(Z_{N-1}), E(Z_{N-1}{}^2)]|_Q > U[E(Z_{N-1}), E(Z_{N-1}{}^2)]|_{Q^*},$$

which contradicts with the definition of Q^*. (Q.E.D.)

Notice that the above proof is based on the idea from [12]. From Corollary 3.1, we know that the solution set of $Q_O(\delta)$ is a subset of the solution set of $Q_A(\mu,\delta)$. As a result, we can embed Problem ($O(\delta)$) into Problem ($A(\mu,\delta)$). Proposition 3.2 gives the necessary condition under which the solution of ($A(\mu,\delta)$) converges to the solution of the primal problem ($O(\delta)$).

Proposition 3.2. *If* $Q^* \in Q_A(\mu,\delta)$, a *necessary condition for* $Q^* \in Q_O(\delta)$ *is* $\mu = 1 + 2\delta E(Z_{N-1})|_{Q^*}$.

Proof of Proposition 3.2: From the above discussion, observe that for a given δ, $A(\mu,\delta)$ is parameterized by μ. For a fixed μ, we have the maximizer for $A(\mu,\delta)$ given by $Q_A(\mu,\delta)$. From Corollary 3.1, we have the relationship that if $Q^* \in Q_O(\delta)$, then $Q^* \in Q_A(D(Q^*,\delta),\delta)$. As a consequence, we can rewrite Problem ($O(\delta)$) in the following equivalent form,

$$\max_{\mu}\ E(Z_{N-1}(\mu,\delta)) - \delta[E(Z_{N-1}{}^2(\mu,\delta)) - (E(Z_{N-1}(\mu,\delta)))^2].$$

A first-order necessary condition for the optimal μ^* is,

$$[(1+2\delta E(Z_{N-1}))|_{Q^*}]\frac{dE(Z_{N-1}(\mu^*,\delta))}{d\mu} - \delta\frac{dE(Z_{N-1}{}^2(\mu^*,\delta))}{d\mu} = 0. \tag{3.5}$$

Since $Q^* \in Q_A(D(Q^*,\delta),\delta)$, we have,

$$\mu^* \frac{dE(Z_{N-1}(\mu^*,\delta))}{d\mu} - \delta \frac{dE(Z_{N-1}{}^2(\mu^*,\delta))}{d\mu} = 0 . \tag{3.6}$$

From (3.5) and (3.6), we have $\mu^* = 1 + 2\delta E(Z_{N-1})|_{Q^*}$. (Q.E.D.)

As a notation, we define:

$\mu^* = 1 + 2\delta E(Z_{N-1})|_{Q^*}$.

The following optimization formulation gives the complete auxiliary problem,

$$(AP(\varepsilon)) \quad \max_{q_k} \; E(Z_{N-1}) - \varepsilon E(Z_{N-1}{}^2)$$

$$s.t. \quad Z_k = Z_{k-1} + z_k(x_k) \; ,$$

$$x_{k+1} = (x_k + q_k - w_k)^+ ,$$

$$k = 0, 1, ..., N-1.$$

The system states at Period k in ($AP(\varepsilon)$) include both x_k and Z_{k-1}. One of the prominent feature in the objective function of ($AP(\varepsilon)$) is that it is separable in the sense of dynamic programming and we can derive the optimal policy by using dynamic programming. Define:

$$\varepsilon^* = \delta / (1 + 2\delta E[Z_{N-1}]|_{Q^*}) . \tag{3.7}$$

Denote $K_* = E[Z_{N-1}]|_{Q^*}$, checking the first order partial derivative of ε^* with respect to δ, we have:

$$\frac{\partial \varepsilon^*}{\partial \delta} = \frac{1}{(1 + 2K_*\delta)^2} > 0 .$$

Thus, ε^* is an increasing function of δ.

$$Q_{AP}(\varepsilon) = \{Q \mid Q \text{ is a maximizer of } (AP(\varepsilon))\}, \tag{3.8}$$

With (3.7), (3.8) and Proposition 3.2, we can derive Lemma 3.3.

Lemma 3.3. *If* $Q^* \in Q_{AP}(\varepsilon)$, then a *necessary condition for* $Q^* \in Q_O(\delta)$ *is* $\varepsilon = \varepsilon^*$.

With this condition on ε, we can then develop an algorithm to solve the original problem by solving the auxiliary problem ($AP(\varepsilon)$). However, before that, we need to develop an optimal policy for ($AP(\varepsilon)$) first.

We represent the expected benefit-to-go for ($AP(\varepsilon)$) by *J*. At Stage $N-1$, the expected benefit-to-go is:

$$J_{N-1}(x_{N-1}, Z_{N-2}) = \max_{q_{N-1}} \underset{w_{N-1}}{E}[Z_{N-1} - \varepsilon Z_{N-1}^2].$$

At Stage k, $\forall k = 0, 1, ..., N-1$, the expected benefit-to-go is given by:

$$J_k(x_k, Z_{k-1}) = \max_{q_k} \underset{w_k}{E}[J_{k+1}(x_{k+1}, Z_k)].$$

Define:

$$\hat{\varepsilon} = 1/(2N(r-c)W),$$

where $N(r-c)W$ gives the maximum amount of profit that can be generated from the N selling periods under the N−period inventory problem. Lemma 3.4 gives us the sufficient conditions for the existence of the optimal threshold policy for ($AP(\varepsilon)$) and ($O(\delta)$).

Lemma 3.4. a) *For a finite N−period problem, a sufficient condition for the existence of an optimal threshold policy for Problem ($AP(\varepsilon)$) is $0 \le \varepsilon \le \hat{\varepsilon}$. b) If $0 \le \delta \le \hat{\varepsilon}$, Problem ($O(\delta)$) can be solved by the optimal threshold policy which solves Problem ($AP(\varepsilon)$).*

Proof of Lemma 3.4: a) This part can be proven by checking the second order condition that the expected benefit-to-go is concave if the sufficient condition $0 \le \varepsilon \le \hat{\varepsilon}$ holds. (b) From Lemma 3.3, we know that the necessary condition for the convergence of the solutions between ($O(\delta)$) and ($AP(\varepsilon)$) is: $\varepsilon = \varepsilon^*$, where $\varepsilon^* = \delta/(1 + 2\delta E[Z_{N-1}]|_{Q^*})$. Thus, if $0 \le \delta \le \hat{\varepsilon}$, then $\varepsilon^* = \delta/(1 + 2\delta E[Z_{N-1}]|_{Q^*}) \le \hat{\varepsilon}$

and $(AP(\varepsilon))$ (and also $(O(\delta))$) can be solved when $\varepsilon = \varepsilon^*$. (Q.E.D.)

Lemma 3.4 tells us that when ε is bounded between 0 and $\hat{\varepsilon}$, an optimal threshold policy exists. Moreover, from the definition of $\hat{\varepsilon}$, we can see that its value depends on the number of periods (*N*) and the maximum per period profit $(r-c)W$. So, for the problem with a relatively small number of stages and small profit margin, there is a higher chance of guaranteeing the existence of an optimal threshold policy for the problem. We summarize the threshold policy for ($AP(\varepsilon)$) as follows.

Optimal Threshold Policy for ($AP(\varepsilon)$): For any given ε which satisfies the sufficient condition as stated in Lemma 3.4a, the following optimal threshold policy for ($AP(\varepsilon)$) exists:

$$q_k^* = \begin{cases} y_k^* - x_k, & \text{if } x_k < y_k^*, \\ 0, & \text{if } x_k \geq y_k^*, \end{cases}$$

where $y_k^* = \arg\{\max_{y_k} \underset{w_k}{E}[J_{k+1}(x_{k+1}, Z_k)]\}$ and q_k^* is the optimal quantity at Period *k*.

We now discuss the algorithm for solving $O(\delta)$. First of all, we define: $Q^*(\varepsilon)$ = the optimal policy for $AP(\varepsilon)$ with a given ε, $e[Q^*(\varepsilon)] = \delta/(1+2\delta E[Z_{N-1}(Q^*(\varepsilon))])$. Notice that $e[Q^*(\varepsilon)]$ is the value of ε^* evaluated at $Q^*(\varepsilon)$. Further notice that when ε = 0, $AP(\varepsilon)$ becomes the classical expected profit maximizing problem and the resulting optimal policy will yield the largest possible expected profit. When ε gets larger and larger, the term $\varepsilon E({Z_{N-1}}^2)$ becomes more and more significant.

Proposition 3.5. *$\forall \varepsilon_1, \varepsilon_2 \in \varepsilon$, if $\varepsilon_1 > \varepsilon_2$, then $E\{Z_{N-1}[Q^*(\varepsilon_1)]\} \leq E\{Z_{N-1}[Q^*(\varepsilon_2)]\}$ and $e[Q^*(\varepsilon_1)] \geq e[Q^*(\varepsilon_2)]$.*

Proposition 3.5 tells us that a larger (smaller) ε implies a smaller (larger) $E\{Z_{N-1}[Q^*(\varepsilon)]\}$ and a larger (smaller) $e[Q^*(\varepsilon)]$. Since $\varepsilon^* \equiv e[Q^*(\varepsilon)]$, a larger (or smaller) value of ε thus implies a larger (or smaller) ε^*. Since ε^* is an increasing function of δ, we thus have the scenario that a larger δ (cf: for a more risk averse decision maker) corresponds to a smaller $E\{Z_{N-1}[Q^*(\varepsilon)]\}$ which follows the classic trade-off of return and risk under the MV model.

Suggested Algorithm for Solving (*O*(δ)): For any given δ in the primal problem ($O(\delta)$) which satisfies the sufficient condition as stated in Lemma 3.4b, the optimal threshold policy can be found by the following algorithm:

1. Define a "precision width" Δ and then divide the interval of $[0,\bar{\varepsilon}]$ into N equally spaced intervals with a width of Δ per interval. Define ε_i as the middle point in each interval, where $i = 1,2,...,N$.
2. Solve the auxiliary problem ($AP(\varepsilon)$) with a fixed $\varepsilon = \varepsilon_i$, for all $i = 1,2,...,N$.
3. Compute ε_i^* and hence $|\varepsilon_i - \varepsilon_i^*|$ for all $i = 1,2,...,N$. Select the ε_i which gives the smallest $|\varepsilon_i - \varepsilon_i^*|$, for all $i = 1,2,...,N$.

Explanation for the above algorithm: First of all, since we know that the optimal threshold ε must reside somewhere between 0 and $\hat{\varepsilon}$, our algorithm only searches in this region. For a fixed $\varepsilon = \varepsilon_i$, we can solve Problem ($AP(\varepsilon)$). However, when we compare the values between ε_i and ε_i^* and find that they are too far away, we are not able to know whether we should increase or reduce the value of ε_i. As an illustration, for instance, when $\varepsilon_i > \varepsilon_i^*$, we have,

$$\varepsilon_i > \varepsilon_i^* \Leftrightarrow \varepsilon_i > \frac{\delta}{1+2\delta E[Z_{N-1}[Q^*(\varepsilon_i)]]} \Leftrightarrow \varepsilon_i\{1+2\delta E(Z_{N-1}[Q^*(\varepsilon_i)])\} > \delta .$$

From Proposition 3.5, we know that when we reduce the value of ε_i, $E(Z_{N-1}[Q^*(\varepsilon_i)])$ is increased; when we increase the value of ε_i, $E(Z_{N-1}[Q^*(\varepsilon_i)])$ is decreased. Thus, unless we have some clue of the specific rate of decrease of $E(Z_{N-1}[Q^*(\varepsilon_i)])$ when ε_i increases, we are not able to know whether it is optimal to increase or reduce the value of ε_i. As a result, in the proposed algorithm, the best we can do is to conduct an exhaustive search and then get the best result among all the ε_i under investigation.

IV. CONCLUSION

In this paper, we have explored the periodic review multi-period inventory problem under an MV objective. Owing to the non-separable nature of variance of profit, we first develop a primal-dual problems solution scheme for finding the optimal inventory threshold policy in a multi-period setting with an MV objective function. We find that when the degree of risk aversion of the decision maker is within a certain range, the existence of the optimal policy, similar to the form reported in the literature, exists. The algorithm which solves the problem is developed.

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