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Theory and Methodology

EOQ models for perishable items under stock dependent selling rate

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Abstract

This paper presents inventory models for perishable items with stock dependent selling rate. The selling rate is assumed to be a function of current inventory level and rate of deterioration is taken to be constant. Under instantaneous replenishment with zero lead time, the model incorporates aspects such as complete, partial, and no backlogging. EOQ is determined for maximizing the total profit in each of the situations. The models developed are illustrated through numerical examples and sensitivity analysis is reported.

Keywords: Stock dependent selling rate; Perishable; Backlogging; Inventory; Optimization

1. Introduction

Classical inventory models developed for constant demand rate [5] can be applied to both manufacturing and sales environment. In the case of certain consumer products, the consumption rate may be influenced by the stock levels. This phenomenon is termed as 'stock dependent consumption rate' by Gupta and Vrat [2,3]. However, they assumed that the consumption rate was a function of order quantity. Padmanabhan and Vrat [10] developed models for sales environment to maximize the profit with the assumption that stock dependent selling rate is a function of initial stock level.

In a subsequent paper Padmanabhan and Vrat [11] defined stock dependent consumption rate more realistically by assuming it as a function of inventory level at any instant of time and developed models for non-sales environment.

For the perishable inventory, significant work has been done for determining the optimal ordering polices for fixed life items by Nahimas [6–8], Pierskalla [13,14], and Prastacos [15,16]. Detailed survey of fixed life models and on blood inventory management were presented by Nahimas [9] and Prastacos [17] respectively. Ghare and Schrader [1] have studied the effect of constant rate of deterioration on inventory and generalized the Wilson's EOQ Model without shortages. Many researchers have developed models for perishable items with exponential decay.

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The present work attempts to model the situations where selling rate depends on the current stock level and items have constant rate of deterioration with complete/partial backlogging and without backlogging.

2. Assumptions and notation

1. The selling rate $D(t)$ at time t is assumed to be $\alpha + \beta I(t)$, where α, β are positive constants and $I(t)$ is inventory level at time t.
2. Replenishment rate is infinite and lead time is zero.
3. Backlogging is not permitted in Model I, complete backlogging is permitted in Model II at a finite shortage cost C_2 per unit per unit time. In Model III, the partial backlogging is permitted at a finite shortage cost per unit per unit time.
4. The distribution of time to deterioration of the items follows exponential distribution with parameter θ (constant rate of deterioration).
5. The unit cost C and the inventory carrying cost as a fraction i, per unit per unit time, are known and constant.
6. S, the selling price per unit, R, the fixed opportunity cost of lost sales and A, the ordering cost per order, are known and constant.
7. T is the cycle time, t_1 is the time upto which inventory is positive in a cycle, B is the maximum inventory level and Q is the order quantity.
8. The inventory policy is a continuous review policy of EOQ type.

3. Model I

The objective of the first model is to determine the optimum order quantity for items having stock dependent selling rate, and exponential decay with no shortages permitted.

The inventory level depletes as the time passes due to selling and deterioration. The differential equation representing the inventory level at time t can be written as

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta I(t) = -\{\alpha + \beta I(t)\}, \quad 0 \le t \le T. \tag{1}$$

The solution of Eq. (1), for the boundary condition $I(T) = 0$, is

$$I(t) = \frac{\alpha}{\beta + \theta}\{\mathrm{e}^{(\beta+\theta)(T-t)} - 1\}. \tag{2}$$

$$\begin{pmatrix}\text{Sales revenue}\\ \text{per cycle}\end{pmatrix} = S\int_0^T \{\alpha + \beta I(t)\}\,\mathrm{d}t$$

$$= S\left\{\alpha T + \frac{\alpha\beta}{(\beta+\theta)^2}\{\mathrm{e}^{(\beta+\theta)T} - 1 - T(\beta+\theta)\}\right\}. \tag{3}$$

$$\begin{pmatrix}\text{Material cost}\\ \text{per cycle}\end{pmatrix} = \frac{\alpha}{\beta+\theta}\{\mathrm{e}^{(\beta+\theta)T} - 1\}C. \tag{4}$$

$$\begin{pmatrix}\text{Carrying cost}\\ \text{per cycle}\end{pmatrix} = \int_0^T I(t) Ci \, \mathrm{d}t$$

$$= \frac{Ci\alpha}{(\beta+\theta)^2}\{\mathrm{e}^{(\beta+\theta)T} - 1 - T(\beta+\theta)\}. \tag{5}$$

$$\begin{pmatrix}\text{Profit}\\ \text{per cycle}\end{pmatrix} = \begin{pmatrix}\text{sales revenue}\\ \text{per cycle}\end{pmatrix} - \begin{pmatrix}\text{total cost}\\ \text{per cycle}\end{pmatrix}.$$

$$\begin{pmatrix}\text{Total cost}\\ \text{per cycle}\end{pmatrix} = \begin{pmatrix}\text{material cost}\\ \text{(incl. deterioration loss)}\end{pmatrix} + \begin{pmatrix}\text{ordering}\\ \text{cost}\end{pmatrix} + \begin{pmatrix}\text{carrying}\\ \text{cost}\end{pmatrix}.$$

The profit per unit time is

$$P(T) = \frac{1}{T}\left[S\left[\alpha T + \frac{\alpha\beta}{(\beta+\theta)^2}\{\mathrm{e}^{(\beta+\theta)T} - 1 - T(\beta+\theta)\}\right] - A - \frac{\alpha}{\beta+\theta}\{\mathrm{e}^{(\beta+\theta)T} - 1\}C - \frac{Ci\alpha}{(\beta+\theta)^2}\{\mathrm{e}^{(\beta+\theta)T} - 1 - T(\beta+\theta)\}\right]. \tag{6}$$

The necessary condition for maximum profit per unit time is

$$(\mathrm{d}P(T)/\mathrm{d}T) = 0,$$

$$\frac{A}{T^2} + \frac{\alpha\{\beta S - C(i+\beta+\theta)\}[T(\beta+\theta)\,\mathrm{e}^{(\beta+\theta)T} - \{\mathrm{e}^{(\beta+\theta)T} - 1\}]}{(\beta+\theta)^2 T^2} = 0. \tag{7}$$

The optimum value of T can be obtained from expression (7) using the Newton–Raphson method. From this the optimum order quantity is

$$Q = \frac{\alpha}{\beta+\theta}\{\mathrm{e}^{(\beta+\theta)T} - 1\}. \tag{8}$$

The sufficient condition for optimum value of profit is

$$(\mathrm{d}^2P(T)/\mathrm{d}T^2) < 0,$$

$$\frac{\mathrm{d}^2P(T)}{\mathrm{d}T^2} = -\frac{A}{T^3} + \frac{\alpha[\beta S - C(i+\beta+\theta)]}{(\beta+\theta)^2 T^3}\left[\mathrm{e}^{(\beta+\theta)T}\{(T(\beta+\theta)-1)^2 + 1\} - 2\right]. \tag{9}$$

For $T > 0$, expression (9) is always negative.

3.1. Numerical example

Data considered to illustrate model I are as follows:
$a = 600$ units, $A =$ Rs. 250.00, $i = 0.35$, $C =$ Rs. 5.00, $S =$ Rs. 7.00.

Optimum cycle time (T), order quantity (Q) and profit (P) are determined using expressions (7), (8) and (6) respectively. To analyse the effect of stock dependent selling rate parameter (β) and rate of deterioration (θ), these are varied from 0 to 0.35 and the results are given in Table 1.

3.2. Analysis of results

As θ increases, optimum order quantity (Q) and profit (P) decrease, whereas if the stock-dependent selling rate parameter (β) increases, optimum order quantity (Q) and profit (P) also increase.

Table 1
Effect of β and θ on order quantity (Q) and profit (P) [a]

Selling rate parameter β		Rate of deterioration (θ)							
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35
0.00	Q	414.04	307.52	367.90	350.71	335.79	322.68	311.07	300.66
	P	475.43	421.27	370.15	321.59	275.21	230.73	187.91	146.56
0.05	Q	427.16	400.83	378.70	359.91	343.02	329.70	317.43	386.38
	P	492.16	436.73	384.38	334.96	287.77	242.58	199.14	157.24
0.10	Q	444.42	414.56	390.16	369.73	352.31	337.27	324.09	312.44
	P	509.69	452.76	399.28	348.76	300.70	254.76	210.66	160.18
0.15	Q	462.79	429.34	402.40	380.13	361.38	345.22	331.16	318.75
	P	527.95	469.38	414.59	362.96	313.98	267.25	222.46	179.37
0.20	Q	483.17	445.56	415.70	391.36	370.95	353.58	338.57	325.35
	P	547.02	486.65	430.42	377.61	327.65	280.07	234.55	190.98
0.25	Q	505.91	463.36	430.16	403.43	381.26	362.49	346.38	332.32
	P	567.00	504.62	446.82	392.74	341.71	293.24	246.94	202.53
0.30	Q	531.76	483.13	445.98	416.44	392.28	372.01	354.69	339.66
	P	588.00	523.37	463.84	408.38	356.21	306.78	259.66	214.54
0.35	Q	561.35	505.25	463.39	430.66	404.17	382.16	363.50	347.40
	P	610.14	542.99	481.54	424.57	371.18	320.71	272.72	226.04

[a] Q in units, P in Rs.

The effect of θ is more pronounced on order quantity and profit for items having higher value of selling rate parameter (β). For $\beta = 0$ and $\theta = 0.35$ the order quantity (Q) is 0.726 times the order quantity when $\beta = 0$ and $\theta = 0$. Similarly, profit for $\beta = 0$ and $\theta = 0.35$ is 0.308 times the profit when $\beta = 0$ and $\theta = 0$. For $\beta = 0.35$ and $\theta = 0.35$, the Q and P are respectively 0.617 and 0.372 times the Q and P when $\beta = 0.35$ and $\theta = 0$.

The effect of stock dependent selling rate parameter (β) is more significant on order quantity and profit for items having lower values of deterioration (θ). The order quantity when $\theta = 0$ and $\beta = 0.35$ is 1.36 times the order quantity when $\theta = 0$ and $\beta = 0$. Similarly profit is 1.28 times the respective profit when $\theta = 0$ and $\beta = 0$. For $\theta = 0.35$ and $\beta = 0.35$, the Q and P are respectively 1.58 and 1.54 times the corresponding values when $\theta = 0.35$ and $\beta = 0$.

4. Model II

The model developed in Section 3 has been enriched by permitting shortages at a finite shortage cost (C_2) per unit per unit time.

When the inventory is positive, selling rate is stock dependent, whereas for negative inventory the demand (backlogging) rate is constant. Therefore, the inventory level decreases due to stock dependent selling as well as deterioration during the period $(0, t_1)$. During the period (t_1, T) demand is backlogged. The differential equations governing the inventory status are given by

$$(\mathrm{d}I(t)/\mathrm{d}t) + \theta I(t) = -\{\alpha + \beta I(t)\}, \quad 0 \le t \le t_1, \tag{10}$$

$$(\mathrm{d}I(t)/\mathrm{d}t) = -\alpha, \quad t_1 \le t \le T. \tag{11}$$

The solutions of the above differential equations after applying the boundary conditions are

$$I(t) = \frac{\alpha}{\beta+\theta}(e^{((\beta+\theta)t_1 - t)} - 1), \quad 0 \le t \le t_1, \tag{12}$$

$$I(t) = \alpha(t_1 - t), \quad t_1 \le t \le T. \tag{13}$$

Therefore, the inventory level at the beginning of the cycle (maximum inventory level) is

$$B = \frac{\alpha}{\beta+\theta}\{e^{(\beta+\theta)t_1} - 1\}. \tag{14}$$

$$\begin{aligned}\begin{pmatrix}\text{Sales revenue}\\ \text{per cycle}\end{pmatrix} &= S\left\{\int_0^{t_1} D(t)\,dt + \alpha(T-t_1)\right\}\\ &= S\left[\alpha t_1 + \frac{\alpha\beta}{(\beta+\theta)^2}\{e^{(\beta+\theta)t_1} - 1 - t_1(\beta+\theta)\} + \alpha(T-t_1)\right].\end{aligned} \tag{15}$$

$$\begin{pmatrix}\text{Carrying cost}\\ \text{per cycle}\end{pmatrix} = Ci\int_0^{t_1} \text{I}(t)\,dt = \frac{\alpha Ci}{(\beta+\theta)^2}\{e^{(\beta+\theta)t_1} - 1 - t_1(\beta+\theta)\}. \tag{16}$$

$$\begin{pmatrix}\text{Shortage cost}\\ \text{per cycle}\end{pmatrix} = \tfrac{1}{2}\alpha(T-t_1)^2 C_2. \tag{17}$$

$$\begin{pmatrix}\text{Material cost}\\ \text{per cycle}\end{pmatrix} = C\left[\frac{\alpha}{\beta+\theta}\{e^{(\beta+\theta)t_1} - 1\} + \alpha(T-t_1)\right]. \tag{18}$$

Profit per unit time is

$$\begin{aligned}P(T, t_1) = \frac{1}{T}\Bigg[&\frac{\alpha\{\beta S - C(i+\beta+\theta)\}\{e^{(\beta+\theta)t_1} - 1\}}{(\beta+\theta)^2} - A\\ &- \frac{\alpha C_2(T-t_1)^2}{2} + \alpha(S-C)T + \alpha t_1 C - \frac{\alpha t_1}{\beta+\theta}(\beta S - Ci)\Bigg].\end{aligned} \tag{19}$$

The necessary criteria for maximum value of $P(T, t_1)$ are

$$(dP(T, t_1)/dT) = 0, \qquad (dP(T, t_1)/dt_1) = 0,$$

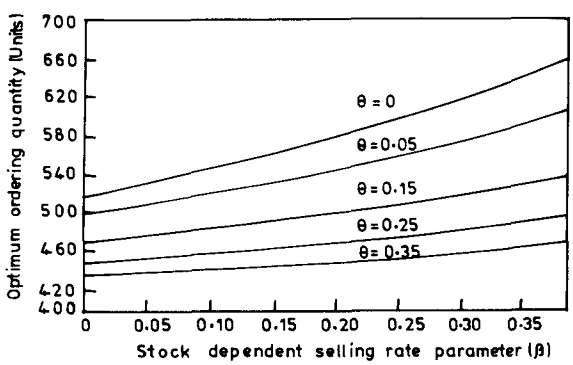

Fig. 1. Effect of β and θ on ordering quantity (Q).

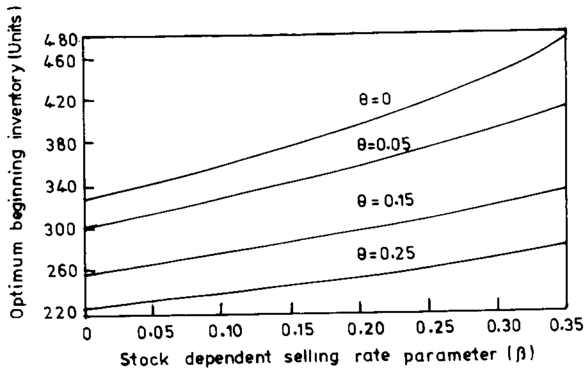

Fig. 2. Effect of β and θ on beginning inventory level (B).

giving

$$2\alpha\{\beta S - C(i+\beta+\theta)\}\{e^{(\beta+\theta)t_1} - 1\} - 2A(\beta+\theta)^2 - 2\alpha(\beta+\theta)(\beta S - Ci)t_1 + \alpha C_2(T^2 - t_2^1)(\beta+\theta)^2 + 2\alpha C t_1(\beta+\theta)^2 = 0, \tag{20}$$

$$\{\beta S - C(i+\beta+\theta)\}\ e^{(\beta+\theta)t_1} + C_2(T-t_1)(\beta+\theta) + C(\beta+\theta) - \beta S + Ci = 0. \tag{21}$$

The optimum values of T and t_1 can be obtained by solving the above non-linear expressions using numerical methods. From this the optimum order quantity is

$$Q = \frac{\alpha}{\beta+\theta}\{e^{(\beta+\theta)t_1} - 1\} + \alpha(T - t_1). \tag{22}$$

4.1. Numerical example

The numerical example solved in the previous section may again be considered with finite C_2 equal to Rs. 3 per unit per unit time. The effects of parameters θ and β on order quantity, maximum inventory level and profit are studied by varying each parameter from 0 to 0.35 and are portrayed in Fig. 1, Fig. 2 and Fig. 3 respectively. The following inferences may be drawn from the figures:

(i) The effect of β on order quantity, maximum inventory level and profit is more significant on items having lower rate of deterioration.

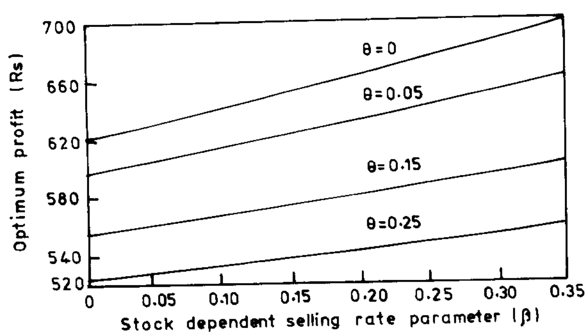

Fig. 3. Effect of β and θ on profit (P).

(ii) The effect of β on order quantity, maximum inventory level and profit is linear for items having higher rate of deterioration.

5. Model III

Park [8] considered a partial backlogging case when $I(t)$ is negative in which the demand backlogged is a fixed fraction (k) of constant demand rate (R). Under certain situations, the amount of demand backlogged is variable and may depend on the amount of orders already backlogged, i.e. the customers will not wait for goods if already many customers are waiting. To take care of this situation we have defined the demand (backlogging) rate $D_1(t)$ when inventory is negative as

$$D_1(t) = \alpha + \delta I(t), \quad t_1 \le t \le T, \tag{23}$$

where δ is a positive constant.

The differential equations representing the system are:

$$(\mathrm{d}I(t)/\mathrm{d}t) + \theta I(t) = -\{\alpha + \beta I(t)\}, \quad 0 \le t \le t_1, \tag{24}$$

$$(\mathrm{d}I(t)/\mathrm{d}t) + \delta I(t) = -\alpha, \quad t_1 \le t \le T. \tag{25}$$

Solutions of Eqs. (24), (25) after applying the boundary condition $I(t_1) = 0$ are as follows:

$$I(t) = \frac{\alpha}{\beta+\theta}\{\mathrm{e}^{(\beta+\theta)t_1} - 1\}\,\mathrm{e}^{-(\beta+\theta)t} + \frac{\alpha}{\beta+\theta}\{\mathrm{e}^{-(\beta+\theta)t} - 1\}, \quad 0 \le t \le t_1, \tag{26}$$

$$I(t) = (\alpha/\delta)\{\mathrm{e}^{\delta(t_1-t)} - 1\}, \quad t_1 \le t \le T. \tag{27}$$

$$\begin{pmatrix}\text{Sales revenue}\\ \text{per cycle}\end{pmatrix} = S\left[\alpha t_1 + \frac{\alpha\beta}{(\beta+\theta)^2}\{\mathrm{e}^{(\beta+\theta)t_1} - 1 - t_1(\beta+\theta)\} + \frac{\alpha}{\delta}\{1 - \mathrm{e}^{\delta(t_1-T)}\}\right]. \tag{28}$$

$$\begin{pmatrix}\text{material cost}\\ \text{per cycle}\end{pmatrix} = \frac{C\alpha}{\beta+\theta}\{\mathrm{e}^{(\beta+\theta)t_1} - 1\} + \frac{C\alpha}{\delta}\{1 - \mathrm{e}^{\delta(t_1-T)}\}. \tag{29}$$

$$\begin{pmatrix}\text{Shortage cost}\\ \text{per cycle}\end{pmatrix} = \frac{C_2\alpha}{\delta}\{\mathrm{e}^{\delta(t_1-T)} + \delta(T-t_1) - 1\}. \tag{30}$$

$$\begin{pmatrix}\text{Carrying cost}\\ \text{per cycle}\end{pmatrix} = \frac{\alpha Ci}{(\beta+\theta)^2}\{\mathrm{e}^{(\beta+\theta)t_1} - 1 - t_1(\beta+\theta)\}. \tag{31}$$

$$\begin{pmatrix}\text{Opportunity cost per cycle}\\ \text{due to lost sales}\end{pmatrix} = R\left[\alpha(T-t_1) + \frac{\alpha}{\delta}\{\mathrm{e}^{\delta(t_1-T)} - 1\}\right]. \tag{32}$$

The profit per unit time is

$$\begin{aligned} P(T, t_1) = \frac{S}{T}&\left[\alpha t_1 + \frac{\alpha\beta}{(\beta+\theta)^2}\{\mathrm{e}^{(\beta+\theta)t_1} - 1 - t_1(\beta+\theta)\} + \frac{\alpha}{\delta}\{1 - \mathrm{e}^{\delta(t_1-T)}\}\right] \\ &- \left[A + \frac{C\alpha}{\beta+\theta}\{\mathrm{e}^{(\beta+\theta)t_1} - 1\} + \frac{C\alpha}{\delta}\{1 - \mathrm{e}^{\delta(t_1-T)}\}\right. \\ &\left. + \frac{\alpha Ci}{(\beta+\theta)^2}\{e^{(\beta+\theta)t_1} - 1 - t_1(\beta+\theta)\} + \frac{C_2\alpha}{\delta}\{e^{\delta(t_1-T)} + \delta(T-t_1) - 1\}\right]. \end{aligned} \tag{33}$$

Table 2
Effect of i and δ on Q, B and P

Carrying charge (i)		Backlogging parameter (δ)							
		0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50
0.20	Q	536.27	520.89	509.37	519.47	493.38	482.88	475.49	470.04
	B	342.31	350.59	357.45	363.24	368.20	376.27	382.58	387.67
	P	618.07	603.99	592.33	582.49	574.06	560.34	549.61	540.97
0.25	Q	512.51	496.27	484.08	474.63	467.10	455.90	448.01	442.18
	B	310.62	318.71	325.43	331.14	336.04	344.08	350.40	355.52
	P	583.57	560.06	540.76	524.58	510.76	488.29	470.73	456.56
0.30	Q	493.50	476.52	463.73	453.79	445.86	434.04	425.69	419.51
	B	284.72	292.60	299.18	304.79	309.63	317.59	323.90	329.03
	P	573.61	556.29	541.83	529.47	518.82	501.30	487.43	476.14
0.35	Q	477.93	460.28	446.97	436.59	428.30	415.91	407.14	400.63
	B	263.09	270.75	277.19	282.68	287.44	295.32	301.58	306.70
	P	555.43	536.65	520.89	507.42	495.75	476.47	461.12	448.57
0.40	Q	464.93	446.68	432.89	422.12	413.50	400.58	391.43	384.62
	B	244.71	252.16	258.44	263.82	268.50	276.27	282.47	287.57
	P	539.29	519.17	502.21	487.67	475.04	454.08	437.33	423.57

The necessary conditions for maximum profit are

$$(\mathrm{d}P(T,t_1)/\mathrm{d}T)=0, \qquad (\mathrm{d}P(T,t_1)/\mathrm{d}t_1)=0,$$

giving

$$-\alpha t_1 S+\frac{\alpha}{(\beta+\theta)^2}\{1-\mathrm{e}^{(\beta+\theta)t_1}\}\{\beta S-C(i+\beta+\theta)\}+\frac{\alpha t_1}{\beta+\theta}(\beta S-Ci)$$
$$+A+\alpha\left(S-C+\frac{C_2}{\delta}\right)\left[T\,\mathrm{e}^{-\delta(T-t_1)}-\frac{1}{\theta}\{1-\mathrm{e}^{\delta(t_1-T)}\}\right]-\frac{C_2\alpha t_1}{\delta}=0, \tag{34}$$

$$\delta\,\mathrm{e}^{(\beta+\theta)t_1}\{\beta S-C(i+\beta+\theta)\}+\delta(S\theta+Ci)-C_2(\beta+\theta)-(\beta+\theta)\,\mathrm{e}^{\delta(t_1-T)}\{\delta(S-C)+C_2\}=0. \tag{35}$$

The expressions (34), (35) can be solved using numerical methods for optimum values of T and t_1.

5.1. Numerical example

To study the effect of δ on optimum order quantity, profit and maximum inventory level, different values of carrying charge (0.2, 0.25, 0.3, 0.35 and 0.4), shortage cost (Rs. 2.00, 2.50, 3.00, 3.50 and 4.00), ordering cost (Rs. 50.00, 100.00, 150.00, 200.00 and 250.00) and selling price (Rs. 6.50, 7.00, 7.50 and 8.00) are considered. The results are tabulated in Tables 2–5.

5.2. Analysis of results

The following inferences can be made from the results given in Tables 2–5:

(i) Optimum order quantity and profit are more sensitive to δ when its value is small.

(ii) The effect of δ on Q and P is greater for lower values of ordering cost.

Table 3
Effect of C_2 and δ on Q, B and P

Shortage cost (C_2)		Backlogging parameter (δ) 0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50
2.00	Q	527.90	493.66	470.89	454.41	441.98	424.52	412.91	404.69
	B	236.90	249.32	259.20	267.30	274.10	284.91	293.19	299.77
	P	619.58	589.16	564.95	545.11	528.47	501.97	481.68	465.57
2.50	Q	498.16	474.52	457.43	444.54	434.49	419.89	409.85	402.57
	B	251.60	261.20	269.08	275.68	281.33	290.49	297.66	303.44
	P	583.57	560.06	540.76	524.58	510.76	488.29	470.73	456.56
3.00	Q	477.93	460.28	446.97	436.59	428.30	415.91	407.14	400.63
	B	263.09	270.75	277.19	282.68	287.44	295.32	301.58	306.70
	P	555.43	536.65	520.89	507.42	495.75	476.47	461.12	448.57
3.50	Q	462.98	449.27	438.59	430.05	423.08	412.43	404.70	398.86
	B	272.35	278.62	283.98	288.62	292.70	299.53	305.05	309.62
	P	532.75	517.39	504.26	492.87	482.89	466.15	452.63	441.43
4.00	Q	451.46	440.49	431.72	424.57	418.63	408.38	402.51	397.25
	B	279.98	285.21	289.75	293.73	297.25	303.24	308.14	312.25
	P	514.05	501.23	490.12	480.37	471.72	457.06	445.05	435.00

(iii) The impact of selling price on maximum inventory level and profit is significant whereas on ordering quantity, it is negligible except when δ is very small.

(iv) As the inventory carrying charge (i) increases, order quantity and profit decrease. Effect of δ is relatively greater on Q and P for higher values of i.

(v) Effect of shortage cost on Q and B at lower values of δ is more pronounced than at higher values of δ.

Table 4
Effect of A and δ on Q, B and P

Ordering cost (A)		Backlogging parameter (δ) 0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50
150.00	Q	369.14	355.90	345.87	338.03	331.75	322.31	315.59	310.57
	B	203.20	209.35	214.49	218.87	222.65	228.86	233.77	237.75
	P	702.15	687.09	674.49	663.76	654.50	639.29	627.27	617.50
200.00	Q	426.90	411.35	399.60	390.43	383.09	372.10	364.30	358.50
	B	235.00	241.97	247.81	252.80	257.11	264.66	269.85	274.45
	P	624.25	607.17	592.85	580.64	570.08	552.67	538.86	527.60
250.00	Q	477.93	460.28	446.97	436.59	428.30	415.91	407.14	400.63
	B	263.09	270.75	277.19	282.68	287.44	295.32	301.58	306.70
	P	555.43	536.65	520.89	507.42	495.75	476.47	461.12	448.57
300.00	Q	499.26	478.45	462.70	450.41	440.56	425.82	415.38	407.63
	B	251.74	259.62	266.31	272.06	277.08	285.49	292.23	297.81
	P	457.38	434.11	414.40	397.42	382.60	357.86	337.93	321.44

Table 5
Effect of S and δ on Q, B and P

selling price (S)		Backlogging parameter (δ)							
		0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50
6.50	Q	473.80	459.20	447.47	437.86	429.86	417.32	407.99	400.80
	B	257.27	262.67	267.41	271.61	275.35	281.79	287.15	291.69
	P	250.39	236.75	224.79	214.20	204.73	188.47	174.95	163.48
7.00	Q	477.93	460.28	446.97	436.59	428.30	415.91	407.14	400.63
	B	263.09	270.75	277.19	282.68	287.44	295.32	301.58	306.70
	P	555.43	536.65	520.89	507.42	495.75	476.47	461.12	448.57
7.50	Q	482.30	461.86	447.32	436.47	428.09	416.02	407.80	401.86
	B	269.20	279.04	286.96	293.49	298.98	307.74	314.45	319.76
	P	860.65	837.27	818.49	802.97	789.92	769.11	753.19	740.58
8.00	Q	486.92	463.94	448.46	437.35	429.02	417.38	409.67	404.21
	B	275.63	287.57	296.78	304.12	310.13	319.42	326.28	331.58
	P	1166.06	1138.59	1117.42	1100.53	1086.70	1065.33	1049.55	1037.36

6. Sensitivity analysis

The effect of errors in the estimation of various parameters on the optimality of solution is studied through sensitivity analysis. Let the estimated values of order quantity and profit be Q' and P' respectively, while the true value of these are Q and P. In the example considered earlier for $\beta = 0.5$, $\theta = 0.10$ and $\delta = 0.50$, the effect of 20% over or under-estimation of the parameters A, i, C_2, β, θ, δ and α on Q and P has been examined. The results are tabulated in Table 6.

The following inferences can be made from the results obtained:

(i) Q and P are more sensitive to ordering cost as compared to other parameters.

Table 6
Sensitivity analysis of stock dependent selling rate model with partial backlogging

Parameter		Percentage of under- or over-estimation of parameter								
		−20	−15	−10	−5	0	5	10	15	20
Ordering cost	Q'/Q	0.8939	0.9216	0.9484	0.9746	1.0000	1.0248	1.0491	1.0728	1.0960
	P'/P	1.1177	1.0870	1.0572	1.0282	1.0000	0.9725	0.9455	0.9192	0.8935
Carrying charge	Q'/Q	1.0757	1.0544	1.0347	1.0617	1.0000	0.9845	0.9701	0.9567	0.9441
	P'/P	1.0690	1.0505	1.0328	1.0160	1.0000	0.9847	0.9701	0.9560	0.9426
Shortage cost	Q'/Q	1.0251	1.0181	1.0116	1.0056	1.0000	0.9948	0.9898	0.9852	0.9809
	P'/P	1.0344	1.0250	1.0162	1.0078	1.0000	0.9926	0.9856	0.9789	0.9726
Selling rate parameter β	Q'/Q	0.9877	0.9907	0.9938	0.9969	1.0000	1.0032	1.0064	1.0097	1.0130
	P'/P	0.9912	0.9934	0.9956	0.9978	1.0000	1.0022	1.0045	1.0068	1.0091
Deterioration rate	Q'/Q	1.0180	1.0134	1.0088	1.0044	1.0000	0.9957	0.9915	0.9874	0.9834
	P'/P	1.0196	1.0146	1.0097	1.0048	1.0000	0.9953	0.9906	0.9860	0.9814
Backlogging parameter δ	Q'/Q	1.0094	1.0069	1.0046	1.0023	1.0000	0.9978	0.9957	0.9936	0.9915
	P'/P	1.0086	1.0064	1.0042	1.0021	1.0000	0.9979	0.9959	0.9939	0.9920
Selling rate parameter α	Q'/Q	0.8950	0.9223	0.9489	0.9748	1.0000	1.0246	1.0486	1.0720	1.0949
	P'/P	0.6946	0.7697	0.8457	0.925	1.0000	1.0782	1.1571	1.2366	1.4166
All parameters	Q'/Q	0.9768	0.9829	0.9888	0.9945	1.0000	1.0053	1.0105	1.0155	1.0204
	P'/P	0.6186	0.7048	0.7971	0.8955	1.0000	1.1107	1.2277	1.3509	1.4804

Table 7
Comparison of Models I, II and II

		full back-logging (Model II)	δ					Without backlogging (Model I)
			0.25	0.50	1.00	2.50	5.00	
$\beta=0,\ \theta=0.1$	Q	485.15	467.82	454.75	436.42	409.15	392.28	367.90
	P	576.27	559.32	545.19	522.84	481.54	447.65	370.15
	B	277.24	284.77	291.05	300.99	319.35	334.41	367.90
$\beta=0.1,\ \theta=0$	Q	544.74	538.13	518.02	503.06	470.89	451.26	444.42
	P	642.10	630.37	620.28	604.62	576.73	527.71	509.69
	B	358.78	367.38	373.89	383.99	401.98	421.27	444.42
$\beta=0.15,\ \theta=0.20$	Q	477.94	460.28	446.97	428.30	400.63	383.71	361.38
	P	555.44	536.65	520.89	495.75	448.57	409.80	313.98
	B	263.09	270.75	277.19	287.45	386.70	322.86	361.38

Q in units, P in Rs., B in units

(ii) The under-estimation of parameters results in more profits than over estimation of parameters except in respect of selling rate parameter.
(iii) Under-estimation of parameters A, β, α results in lower ordering quantity, whereas in the case of parameters i, C_2, θ and δ it results in higher optimum order quantity.
(iv) The effect of α on Q and P is quite significant for 20% under or over estimation of α. Q'/Q varies from 0.8950 to 1.0949 and P'/P varies from 0.6946 to 1.4166.
(v) The effect of over or under estimation of α and A on order quantity is similar whereas on profit it is notably different.

7. Comparison of all the three models

In order to compare all the models developed in this paper, results of the illustrative example considered are tabulated in Table 7.

The following inferences can be made from the results in Table 7.

(i) As the value of backlogging parameter (δ) increases, optimum order quantity, optimum profit decrease, whereas the maximum inventory level increases implying that shortage period reduces.
(ii) As the value of δ increases, the optimum order quantity and maximum profit become close to the values of optimum order quantity and profit respectively when backlogging is not permitted.
(iii) Optimum order quantity and profit are more sensitive to δ when its value is small.

8. Conclusions

In this paper, more realistic parameters such as stock dependent selling rate and perishability are considered in developing the inventory models. Also a different type of partial backlogging is considered by defining it as a function of amount of orders already backlogged. Three models have been presented for items having stock dependent selling rate and constant rate of deterioration without backlogging, with full backlogging and partial backlogging. The impact of stock dependent selling rate, perishability and partial backlogging parameters on order quantity, profit and maximum inventory level are reported. The results indicate that the effects of stock dependent selling rate, perishability and partial backlogging on the system behaviour are significant, and hence should not be ignored in developing the inventory models. These models are of immense use to determine optimum inventory policies for the above situations. The models can further be enriched by incorporating quantity discounts, inflation, change in selling price with changes in demand rate etc. The models can be applied to determine optimal inventory policy in situations such as super market bakeries, stationery stores, and fancy items which may exhibit the characteristics modelled here.

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