The Newsvendor Model

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0.1 The Newsvendor Model

The host of problems, called **Newsvendor Problems**, is enormous. Consider the scenario faced by a retailer who must make a *one time* stocking decision about a particular item. If the retailer stocks too much, she is stuck with excess inventory holding cost and acquisition cost but may recover some salvage value for the left overs. If the retailer stocks too little, she loses opportunity profit and suffers loss of goodwill or lost future profit from customers. She also may need to satisfy customers with a more expensive alternative source. Since demand is uncertain, the question then is how much to stock to best match supply with demand?

These are called Newsvendor Problems because they mimic the dilemma faced by the manager of a newsstand stocking newspapers each morning. They are quite prevalent in the stocking of perishable goods, such as milk and baked goods, which must be discarded after their expiration dates. They also apply to seasonal products, such as Christmas lights and Easter eggs, which tend to be salvaged at the end of the season. Fashion goods, such as designer clothes, also exhibit Newsvendor economics.

0.1.1 Marginal analysis

We now develop the Newsvendor Model using an intuitive marginal analysis motivated by the fashion industry.

EXAMPLE 0.1. The Fashion Store

The Fashion Store sells fashion items. The store has to order these items many months in advance of the fashion season in order to get a good price on the items. Each unit costs Fashion $100. These units are sold to customers at a price of $250 per unit. Items not sold during the season can be sold to the outlet store at $80 per unit. If the store runs out of an item during the season it has to obtain the item from alternative sources and the cost including air freight to Fashion is $190 per unit.

Fashion wants help in choosing the initial order quantity to maximize net contribution from running the store. **What should we do?**

The first thing we should do is to analyze historical demand data, given in Table 1, from comparable items over the last few years. Using this data, we can construct a frequency table, shown in Table 2, with corresponding cumulative probabilities for each demand value. We can also determine that the sample mean is 85 units and the standard deviation is 4.43 units.

86	94	90	86	82	84	91	76	85	83	92	82	89	88	79	83	83	85	89	90
73	84	86	90	90	92	83	91	85	85	82	81	81	76	81	81	78	85	84	82
88	86	85	88	86	89	87	84	83	79	90	87	83	87	82	81	85	84	87	89
82	80	92	85	88	85	83	87	84	84	86	80	87	80	89	79	83	80	86	85
81	93	91	89	80	86	87	86	88	84	81	84	84	82	77	93	94	97	87	75

Table 1: 100 demand data points for the Fashion Store.

Suppose we are trying to decide whether to order an inventory of 85 units for the start of the season or whether we should increase the inventory ordered to 86 units. If the extra item is not sold we incur a marginal cost of $100 - 80 = 20$. On the other hand, if the extra item is sold we earn the difference between the contribution we make by having the item on hand and the contribution we would have made if we would not have had the item on hand. Thus we gain a marginal contribution of $(250 - 100) - (250 - 190) = 190 - 100 = 90$.

In order to compute the marginal net contribution, we need the probabilities

$$
\begin{aligned}
\text{P(The 86}^{\text{th}}\text{ item is not sold)} &= \text{P(Demand} \leq 85) \\
\text{P(The 86}^{\text{th}}\text{ item is sold)} &= \text{P(Demand} > 85),
\end{aligned}
$$

which we can look up in Table 2. With these probabilities, we can compute

$$
\begin{aligned}
\text{Expected Marginal Cost} &= \text{P(Demand} \leq 85) \times 20 \\
\text{Expected Marginal Profit} &= \text{P(Demand} > 85) \times 90
\end{aligned}
$$

Thus we would increase the order from 85 to 86 if

$$
\begin{aligned}
\text{Expected Marginal Cost} &< \text{Expected Marginal Profit} \\
\text{P(Demand} \leq 85) \times 20 &< \text{P(Demand} > 85) \times 90 \\
0.56(20) &< 0.44(90) \\
11.2 &< 39.6.
\end{aligned}
$$

Letting C_e be the cost of excess and C_s be the cost of shortage, in general we increase our inventory B while

$$
\text{P(Demand} \leq B)C_e < \text{P(Demand} > B)C_s.
$$

Demand (D)	Frequency	P(Demand) P(D)	Cum. Prob. P(Demand ≤ D)
73	1	0.01	0.01
75	1	0.01	0.02
76	2	0.02	0.04
77	1	0.01	0.05
78	1	0.01	0.06
79	3	0.03	0.09
80	5	0.05	0.14
81	7	0.07	0.21
82	7	0.07	0.28
83	8	0.08	0.36
84	10	0.10	0.46
85	10	0.10	0.56
86	9	0.09	0.65
87	8	0.08	0.73
88	5	0.05	0.78
89	6	0.06	0.84
90	5	0.05	0.89
91	3	0.03	0.92
92	3	0.03	0.95
93	2	0.02	0.97
94	2	0.02	0.99
97	1	0.01	1.00
	100	1.0	

Table 2: Frequency table from demand data for the Fashion Store.

We would stop increasing B at $B^\star$, the optimal stocking level, when

$$P(\text{Demand} \leq B^\star)C_e = P(\text{Demand} > B^\star)C_s.$$

Rewriting this last equation we get

$$
\begin{aligned}
P(\text{Demand} \leq B^\star)C_e &= (1 - P(\text{Demand} \leq B^\star))C_s \\
0 &= C_s - P(\text{Demand} \leq B^\star)(C_s + C_e),
\end{aligned}
$$

which becomes

$$P(\text{Demand} \leq B^\star) = \frac{C_s}{C_s + C_e}. \tag{1}$$

Since the left-hand side is by definition the service level, (1) is equivalent to (**??**).

From our Fashion Store example we have $C_e = 20$ and $C_s = 90$. So,

$$\text{SL}^\star = \frac{C_s}{C_s + C_e} = \frac{90}{90 + 20} = .8182.$$

From the frequency table, for a service level of .8182, we get $B^* = 89$. **Note that we always move up to cover the service level.**

0.1.2 Two Newsvendor Examples

As the above marginal analysis illustrates, we are in fact finding $B^\star$ so as to maximize the *expected net contribution*.[1]

EXAMPLE 0.2. Pat's Papers — Lost Sales

Pat sells Sunday papers outside a retirement home on Sunday mornings. Pat collects $1.75 per paper and pays $.98 for a paper. The retirement home residents think fondly of Pat and aren't too upset when papers run out. On the other hand, any papers Pat is left with are thrown into the recycling bin. Pat is trying to figure out a systematic ordering policy. When Pat runs out of papers, the sale is lost.

Many Newsvendor problems are similar to this *lost sales* example, so we will define some notation that will help to generalize such examples.

r = revenue received for the sale of each newspaper = $1.75

c = cost (wholesale) paid for each newspaper = $.98

h = cost to hold any extra newspapers after the day is over = $0

s = salvage value of a newspaper after the day is over or cost to dispose of a paper = $0
   (Note that s is positive if a salvage value and negative if a disposal cost.)

p = penalty for being unable to sell a customer a newspaper when one is desired = $0:
   includes loss-of-good will costs, backorder costs, emergency supply costs, etc.

We can write C_e and C_s in terms of the cost and revenue parameters we defined above.

$$C_s = r - c + p \qquad \text{and} \qquad C_e = h + c - s.$$

Looking a little more closely at C_s and C_e, we see that these costs intuitively make sense. The cost of being short one newspaper ought to include lost contribution $(r - c)$ as well as any additional loss-of-good-will costs (p). The cost of having one too many newspapers should include any holding costs (h) plus the unrecoverable investment in inventory $(c - s)$.

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[1]We could mathematically derive what follows by explicitly performing this optimization, but we'll spare you the details!

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Thus $C_s = r - c + p = 1.75 - .98 + 0 = .77$, and $C_e = h + c - s = 0 + .98 - 0$. Pat, then, should provide a service level of $\frac{.77}{.77+.98} = .44$

These choices of C_s and C_e maximize the firm's expected net contribution, which depends on the relationship of demand D and stocking level B. Pat has a shortage if $D > B$ and an excess if $D < B$. In this *particular* case, the total net contribution can be written as

$$\text{Net Contribution}(B) = \begin{cases} rD - cB + s(B - D) - h(B - D), & \text{if } D \leq B \\ rB - cB - p(D - B). & \text{if } D > B \end{cases}$$

This expression could be put into a spreadsheet to confirm that a service level of .44 indeed maximizes its expected value.

EXAMPLE 0.3. Pat's Papers — Alternative Source

Pat sells Sunday papers outside a retirement home on Sunday mornings. Pat collects $1.75 per paper and pays $.98 for a paper. The retirement home residents think fondly of Pat and aren't too upset when papers run out. On the other hand, any papers Pat is left with are thrown into the recycling bin. Pat is trying to figure out a systematic ordering policy. In this case, when Pat runs out of a paper he procures it from town at $1.5 per paper.

So, we have the following:

$$
\begin{aligned}
r &= \$1.75 \\
c &= \$.98 \\
h &= \$0 \\
s &= \$0 \\
p &= 1.5 - .98
\end{aligned}
$$

Here, since p is the penalty for being short a paper from *the original primary stock*, it is the difference in the cost of a paper from the alternative source and the primary source, $\$1.5 - \$.98 = \$.52$. In the case where all excess demand is satisfied from an alternative source, intuitively, the contribution terms $(r - c)$ should not appear from C_s, as this contribution is not lost. Indeed this is the case:

$$C_s = p \quad \text{and} \quad C_e = h + c - s.$$

Thus the optimal service level is $\frac{.52}{.52+.98} = .3467$. The Fashion Store example is another example of an alternative source case.

In this *particular* case, we get a slightly different expression for net contribution:

$$\text{Net Contribution}(B) = \begin{cases} rD - cB + s(B - D) - h(B - D), & \text{if } D \leq B \\ rD - cD - p(D - B). & \text{if } D > B \end{cases}$$

We must stress that all situations are different and don't necessarily fit into these expressions.

0.1.3 Service level versus fill rate

We end this section with a more detailed discussion of service level, and another often used measurement of service, **fill rate**.

Service level

> *Service Level is the probability that demand is satisfied immediately from on-hand inventory.*

Suppose a newsstand stocks B newspapers daily. They collected data which showed that they stocked out 73 out of 365 days. Thus the newsstand provided an average service level of (365-73)/365 = 80%. That is, they were able to satisfy all the demand for a day, 80% of the days. On a given day, the service level is either 0% when a stockout occurs, or 100%, when no stockout occurs. We use service level as a probability or proportion of periods in which no stockout occurs.

For the Fashion Store example, the average service level associated with an inventory of 89 units is 84%. This implies that if we used an inventory level of 89 units for 100 different time periods each of which has the same historical demand information as provided, on average, we would stock out for 16 of these periods, i.e., a 16% stockout rate.

Note that the stock out rate does *not* keep track of the number of units of demand not filled. It merely records if there was a stockout of an item or not, not differentiating whether we failed to satisfy the demand of one customer or 100 customers.

Fill rate

> *Fill Rate is the fraction of demand that is satisfied from on-hand inventory.*

Fill rate does keep track of the fraction of customer demand units that are filled. If demand (D) is less than the inventory level (B) then the fill rate is 100%. Otherwise the fill rate is the ratio of

inventory level to demand (D). So we can write:

$$\text{Fill rate} = \text{Expected} \left[\frac{\min(B, D)}{D} \right]$$

Quite often, when people refer to a measurement of service, they are referring to fill rate. The information needed to measure fill rate, however, is often very difficult to acquire. Quite simply, not every customer that walks into a store asks to see a particular product if they cannot find it on the shelves. Many customers leave without ever making their wishes known. Hence getting an exact count of the number of customers which get turned away is often very difficult, if not impossible. In contrast, service level is easy to measure – "What proportion of periods did I have enough stock?" Nonetheless, fill rate information, when available, is a very intuitive gauge of service.

Interestingly enough, we do not need to be able to calculate fill rate in order to find an inventory stocking policy which results in the maximum expected net contribution. We have shown that all we need to do is set an optimal service level. This service level will correspond to a particular fill rate, but calculating this fill rate is difficult because it depends on the demand probability distribution. In contrast, the optimal service level does not depend on the demand distribution, which makes it much easier to work with.

For the Fashion store example, if we had maintained an inventory of 89 units and we had 100 observations as shown previously, we would have service levels and fill rates shown in Table 3. Note that we would have recorded an average service level of 84% and an average fill rate of 99.56%.

EXAMPLE 0.4. Measuring customer service in practice

Suppose you are setting up an information system to track customer service, and wish to compute both fill rate and service level. How would you compute them?

The following table shows customer demand for each of 12 months for a particular item. Suppose, at the start of each month the inventory stock is replenished to exactly 100 units to satisfy demand over the month. So, in the first month the demand of 73 is satisfied, the stock is replenished back to 100 and so the next month the demand of 80 is satisfied. However in the third month the demand exceeds 100 units, 22 units are left unsatisfied.

Demand (D)	Frequency	P(Demand) P(D)	Cum. Prob. P(Demand ≤ D)	Service Level (%)	Fill Rate (%)
73	1	0.01	0.01	100	100.0
75	1	0.01	0.02	100	100.0
76	2	0.02	0.04	100	100.0
77	1	0.01	0.05	100	100.0
78	1	0.01	0.06	100	100.0
79	3	0.03	0.09	100	100.0
80	5	0.05	0.14	100	100.0
81	7	0.07	0.21	100	100.0
82	7	0.07	0.28	100	100.0
83	8	0.08	0.36	100	100.0
84	10	0.10	0.46	100	100.0
85	10	0.10	0.56	100	100.0
86	9	0.09	0.65	100	100.0
87	8	0.08	0.73	100	100.0
88	5	0.05	0.78	100	100.0
89	6	0.06	0.84	100	100.0
90	5	0.05	0.89	0	98.9
91	3	0.03	0.92	0	97.8
92	3	0.03	0.95	0	96.7
93	2	0.02	0.97	0	95.7
94	2	0.02	0.99	0	94.7
97	1	0.01	1.00	0	91.7
	100	1.0		84 %	99.56 %

Table 3: Daily and average fill rates and service levels when stocking 89 units.

Month	Demand	Service Level	Fill Rate
1	73	100	100.0
2	80	100	100.0
3	122	0	82.0
4	103	0	97.1
5	90	100	100.0
6	99	100	100.0
7	109	0	91.7
8	88	100	100.0
9	83	100	100.0
10	93	100	100.0
11	104	0	96.2
12	120	0	83.3
		58.3%	95.86%

The third column is used to compute the service level over the year. The service level is the proportion of the periods in which all the demand was satisfied from stock. Here, in seven out of 12 months all of the demand was satisfied, and thus we have a service level for the year of 7/12 =

58.3%. In a given monthly period the service level is either 100% or 0%.

The last column computes the fill rate. The fill rate is a very intuitive measure. What proportion of the demand did I satisfy from stock? So, for instance in Month 3, only $100/122 = 82\%$ units of demand were satisfied. At the bottom of the last column is the average *fill rate over the year.*

Question: Can you convince yourself that the fill rate will always be larger than the service level?