Homework set 12

Due by 15:00 on Monday, November 13, 2023.

Please select **three** problems to solve and hand in written solutions either in person or to `gunnar@magnusson.io`.

Problem 1. Let $V = \mathrm{Hom}(\mathbf{R}^2, \mathbf{R}^2)$ be the Banach space of endomorphisms of $\mathbf{R}^2$. Consider

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

in V. Show that $\sigma(A) = \varnothing$.

Recall that an *eigenspace* E_λ is the subspace generated by the eigenvectors corresponding to the eigenvalue λ of an operator.

Problem 2. A subspace $Y \subset X$ is *invariant* under an operator $T : X \to X$ if $T(Y) \subset Y$. Show that an eigenspace of T is invariant under T. Give an example.

Problem 3. Consider the right-shift operator $R : \ell^2 \to \ell^2$. Find its point, continuous, and residual spectra.

Problem 4. Consider $C^0([0,1])$ with the sup norm.

1. Fix $g \in C^0([0,1])$ and let $T(f) = fg$ be multiplication by g. Find $\sigma(T)$.
2. Fix real numbers a, b such that $[a, b] \subset [0, 1]$. Find a linear operator $T : C^0([0,1]) \to C^0([0,1])$ such that $\sigma(T) = [a, b]$.

Problem 5. Let $T : X \to X$ be an operator on a normed space. Let $\lambda_1, \dots, \lambda_n$ be pairwise different eigenvalues of T and $x_1, \dots, x_n$ eigenvectors such that $T(x_j) = \lambda_j x_j$. Show that $\{x_1, \dots, x_n\}$ is linearly independent.