

Given flow field:

$$X_{n \times m} = \begin{bmatrix} x_{11} & \dots & x_{1m} \\ x_{21} & & \vdots \\ \vdots & & \vdots \\ x_{n1} & \dots & x_{nm} \end{bmatrix}$$

$$\text{SVD} = X_{n \times m} = U_{n \times n} \Sigma_{n \times m} V_{m \times m}^T$$

$$X_{n \times m} = \hat{U}_{n \times m} \hat{\Sigma}_{m \times m} \hat{V}_{m \times m}^T$$

$$\begin{bmatrix} x_{11} & \dots & x_{1m} \\ x_{21} & & \vdots \\ \vdots & & \vdots \\ x_{n1} & \dots & x_{nm} \end{bmatrix} = \begin{bmatrix} | & | \\ u_1 & u_2 & \dots & u_n \\ | & | \end{bmatrix} \begin{array}{c} \hline \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} -v_1^T \\ -v_2^T \\ \vdots \end{bmatrix}^T \\ \hline \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} \end{array} \begin{array}{c} \hline \text{up to } m \\ \hline \end{array}$$

be such that:

Now γ be such that:

$$\frac{\sum_{j=1}^{\gamma} \lambda_j}{\sum_{j=1}^m \lambda_m} \approx 1$$

$$X = U \Sigma V^T$$

\downarrow \downarrow

Spatial Temporal
matrix matrix

Covariance

The diagram illustrates the SVD decomposition of a matrix X . The equation $X = U \Sigma V^T$ is written in red. Below the matrix U , a downward arrow points to the text "Spatial matrix". Below the matrix V^T , a downward arrow points to the text "Temporal matrix". A curved arrow originates from the Σ matrix and points to the word "Covariance" on the right side of the image.

$$\# \quad x - \bar{x} = \sum a_j(t) \phi_j(\epsilon_r)$$

$$\Rightarrow x - \bar{x} = a_1(t) \phi_1 + a_2(t) \phi_2 + \dots$$

Modes are static spatial flow fields
and their linear combi. will give
flow field at different times.

Original $\Rightarrow X(:, t)$

Using just K modes: [first K modes]

$$X' = \bar{X} + \sum_{i=1}^K U(:, i) * \Sigma(i, i) * (V^T(:, i))^T$$

Reduced SVD:

$$X - \bar{X} = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \dots & \\ | & | & & | \end{bmatrix} \begin{bmatrix} \downarrow \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_m \\ & & & & \downarrow \sigma \end{bmatrix} \begin{bmatrix} \leftarrow v_1^T \leftarrow \\ & \leftarrow v_2^T \leftarrow \\ & & \ddots \\ & & & \leftarrow v_m^T \leftarrow \\ & & & & \downarrow \sigma \end{bmatrix}^T$$

$\xrightarrow{\sigma}$

New $\Rightarrow \hat{U} \Rightarrow n \times r$

$\hat{\Sigma} \Rightarrow r \times r$

$\hat{V}^T \Rightarrow r \times m$