Given flow field :

$$X_{n\times m} : \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ x_{21} & & \vdots \\ \vdots & & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix}$$

SVD :

$$X_{n\times m} = U_{n\times n}\,\Sigma_{n\times m}\,V^T_{m\times m}$$

$$X_{n\times m} = \hat{U}_{n\times m}\,\hat{\Sigma}_{m\times m}\,\hat{V}^T_{m\times m}$$

$$\begin{bmatrix} x_{11} & \cdots & x_{1m} \\ x_{21} & & \\ \vdots & & \\ x_{n1} & & x_{nm} \end{bmatrix} = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \sigma_3 & \\ & & & \ddots \\ \hline & 0 & & \end{bmatrix} \begin{bmatrix} - & v_1^T & - \\ - & v_2^T & - \\ & \vdots & \end{bmatrix}^T$$

upto m upto m

be such that :

Now γ be such that:

$$\frac{\sum_{j=1}^{\gamma} \lambda_j}{\sum_{j=1}^{m} \lambda_m} \approx 1$$

$$X = U \Sigma V^T$$

Covariance

Spatial matrix

Temporal matrix

$X - \bar{X} = \sum a_j(t)\, \phi_j(\xi_f)$

$\Rightarrow X - \bar{X} = a_1(t)\, \phi_1 + a_2(t)\, \phi_2 + \text{----}$

Modes are static spatial flow feilds and their linear combi. ~~te~~ will give flow feild at different times.

Original $\Rightarrow$ $X(:, t)$

Using just K modes : [first K modes]

$$X' = \bar{X} + \sum_{i=1}^{K} U(:, i) * \Sigma(i, i) \times (V(:, i))^T$$

Reduced SVD :

$$X - \bar{X} = \underbrace{\begin{bmatrix} | & | & & \\ u_1 & u_2 & \cdots & \\ | & | & & \end{bmatrix}}_{r} \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_m \end{bmatrix} \begin{bmatrix} — v_1^T — \\ — v_2^T — \\ \vdots \end{bmatrix}^T$$

New $\Rightarrow$ $\hat{U} \Rightarrow n \times r$

$\hat{\Sigma} \Rightarrow r \times r$

$\hat{V}^T \Rightarrow r \times m$