

# gradient-descent

December 16, 2023

## 1 Gradient Descent

For a multivariable function, in  $N$  dimensions let's say,  $F(\mathbf{v})$  which is differentiable at a point  $\mathbf{v}$ , we say that  $F(\mathbf{v})$  decreases fastest in the direction of negative of the gradient at that point  $\mathbf{v}$  denoted by  $-\nabla F(\mathbf{v})$ .

### 1.1 Evaluating the descent

$$\mathbf{v}_{i+1} = \mathbf{v}_i - L \nabla F(\mathbf{v}_i) \quad (1)$$

where,  $L$  is the learning rate and  $L \in \mathbb{R}_+$  and  $\mathbf{v} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$  and  $\nabla F(\mathbf{v}) = \begin{bmatrix} \frac{\partial F}{\partial x_1} \\ \frac{\partial F}{\partial x_2} \\ \vdots \\ \frac{\partial F}{\partial x_n} \end{bmatrix}$ . Hence the equation becomes:

$$\begin{bmatrix} x_1^{i+1} \\ x_2^{i+1} \\ \vdots \\ x_N^{i+1} \end{bmatrix} = \begin{bmatrix} x_1^i \\ x_2^i \\ \vdots \\ x_N^i \end{bmatrix} - L \begin{bmatrix} \frac{\partial F}{\partial x_1} \\ \frac{\partial F}{\partial x_2} \\ \vdots \\ \frac{\partial F}{\partial x_n} \end{bmatrix} \quad (2)$$

### 1.2 Descent to a *minima*

Notice that the decrease in  $F(\mathbf{v})$  is guaranteed only to the nearest well, which may or may not be the global minima. We may run for a specified number of epochs or terminate at a set tolerance too.

### 1.3 2-dimensional example

Let  $F(\mathbf{v}) = F \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \sin(x) \cos(y)$ , then the gradient  $\nabla F(\mathbf{v}) = \begin{bmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \end{bmatrix} = \begin{bmatrix} \cos(x) \cos(y) \\ -\sin(x) \sin(y) \end{bmatrix}$  and starting from an initial point  $\mathbf{v}_0$ , we may reach the nearest local minima as:

$$\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} - L \begin{bmatrix} \cos(x) \cos(y) \\ -\sin(x) \sin(y) \end{bmatrix} \quad (3)$$

```
[23]: import numpy as np
import matplotlib.pyplot as plt
```

```
from mpl_toolkits.mplot3d import Axes3D  
# !pip3 install ipympl  
%matplotlib widget
```

Enable the support

```
[28]: from google.colab import output  
output.enable_custom_widget_manager()
```

To disable the support

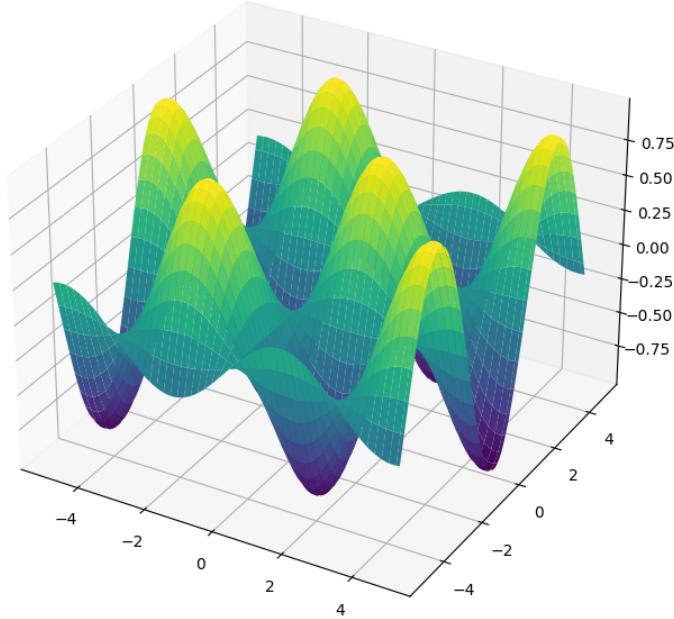
```
[ ]: from google.colab import output  
output.disable_custom_widget_manager()
```

```
[27]: def gradient_func2d(x, y):  
    return np.cos(x) * np.cos(y), - np.sin(x) * np.sin(y)
```

```
[26]: def func2d(x, y):  
    return np.sin(x) * np.cos(y)
```

### 1.3.1 The 2D Function $F(x, y)$

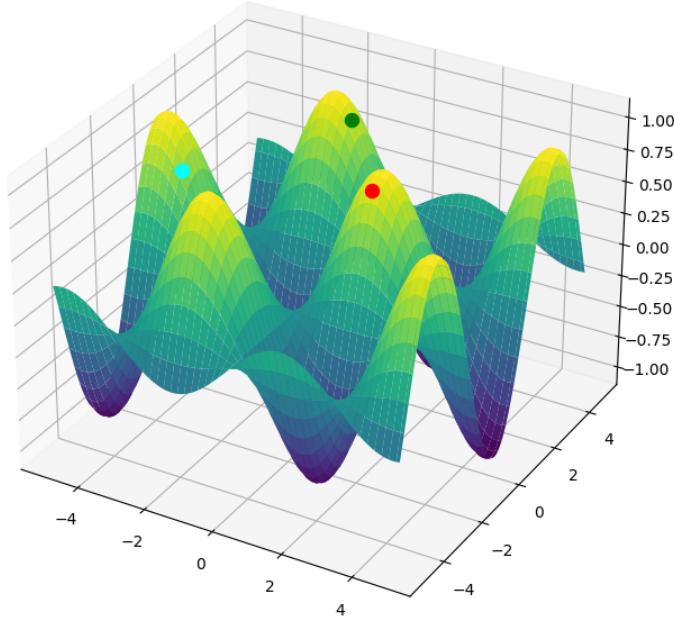
```
[29]: x = np.arange(-5, 5, 0.1)  
y = np.arange(-5, 5, 0.1)  
X, Y = np.meshgrid(x, y)  
  
fig = plt.figure(figsize = (10, 8))  
ax = fig.add_subplot(projection="3d", computed_zorder=False)  
ax.plot_surface(X, Y, func2d(X, Y), cmap="viridis")  
  
plt.show()
```



### 1.3.2 3 initial points on $F(x, y)$

```
[30]: p1 = (1, 0.5)
p2 = (-1, 3)
p3 = (-4, -0.5)
fig = plt.figure(figsize = (10, 8))
ax = fig.add_subplot(projection="3d", computed_zorder = False)
ax.plot_surface(X, Y, func2d(X, Y), cmap="viridis", zorder = 0)

ax.scatter(p1[0], p1[1], func2d(p1[0], p1[1]), c = "red", s = 75, zorder = 1)
ax.scatter(p2[0], p2[1], func2d(p2[0], p2[1]), c = "green", s = 75, zorder = 1)
ax.scatter(p3[0], p3[1], func2d(p3[0], p3[1]), c = "cyan", s = 75, zorder = 1)
plt.show()
```



### 1.3.3 Descent of the points

```
[31]: def iterate_gradient(point, L):
    Xgrad, Ygrad = gradient_func2d(point[0], point[1])
    Xnew, Ynew = point[0] - L * Xgrad, point[1] - L * Ygrad
    return (Xnew, Ynew, func2d(Xnew, Ynew))
```

```
[22]: L, epochs = 0.1, 100

ax = plt.subplot(projection="3d", computed_zorder = False)

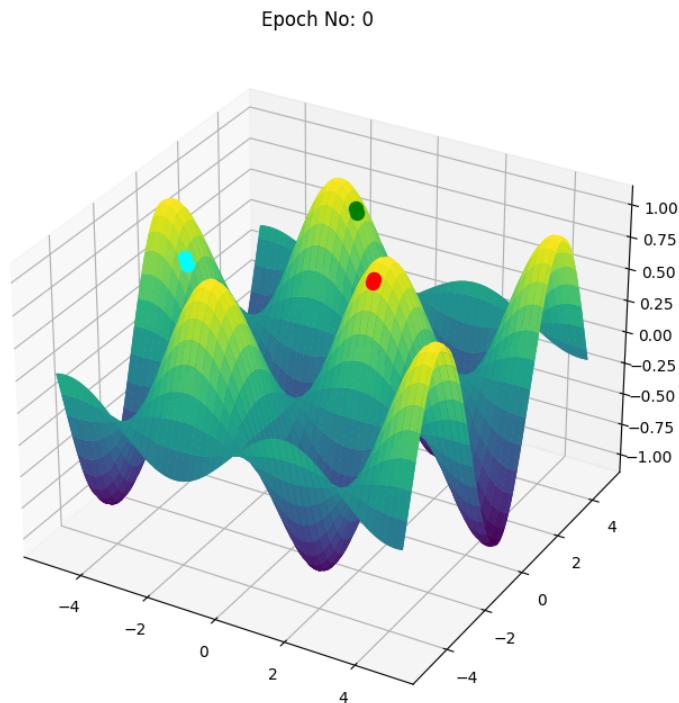
for i in range(epochs):
    p1 = iterate_gradient(p1, L)
    p2 = iterate_gradient(p2, L)
    p3 = iterate_gradient(p3, L)

ax.plot_surface(X, Y, func2d(X, Y), cmap="viridis", zorder = 0)
```

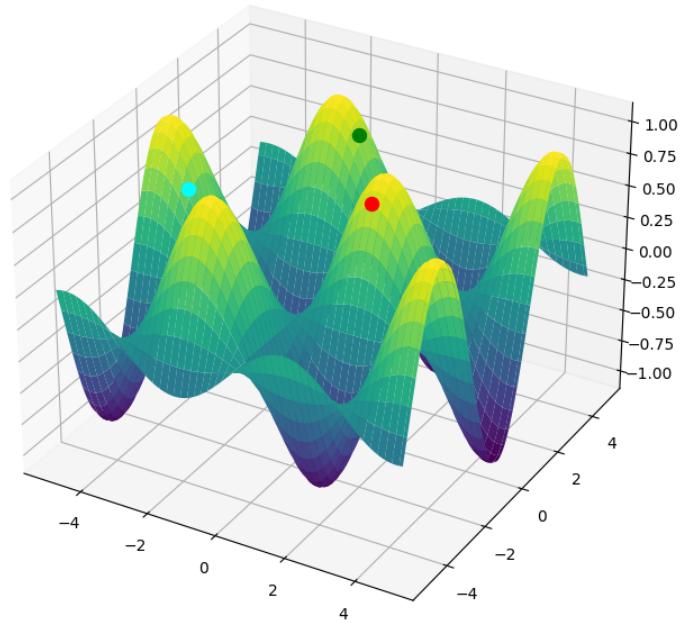
```
ax.scatter(p1[0], p1[1], func2d(p1[0], p1[1]), c = "red", s = 75, zorder = 1)
ax.scatter(p2[0], p2[1], func2d(p2[0], p2[1]), c = "green", s = 75, zorder = 1)
ax.scatter(p3[0], p3[1], func2d(p3[0], p3[1]), c = "cyan", s = 75, zorder = 1)

ax.set_title(f"Epoch No: {i}")

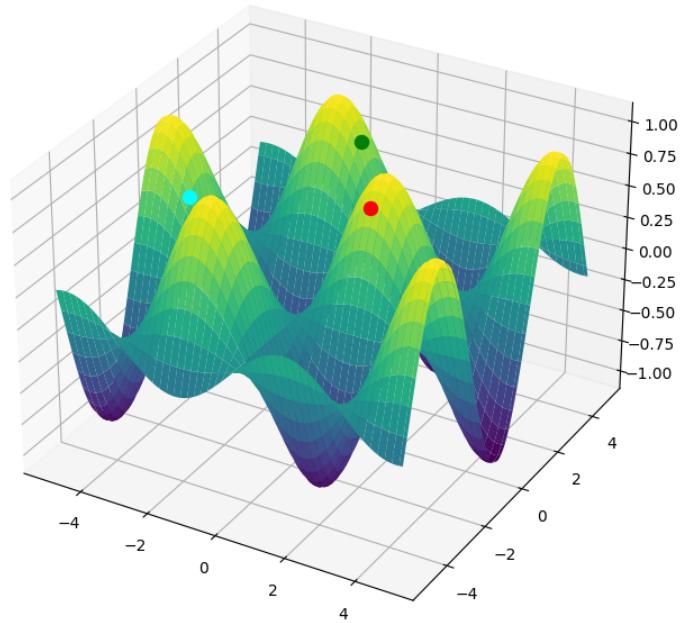
plt.pause(0.001)
ax.clear()
```



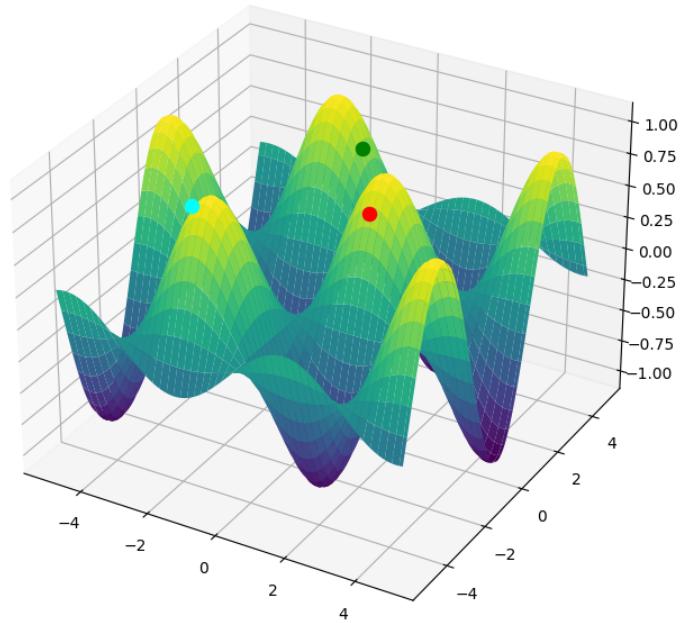
Epoch No: 1



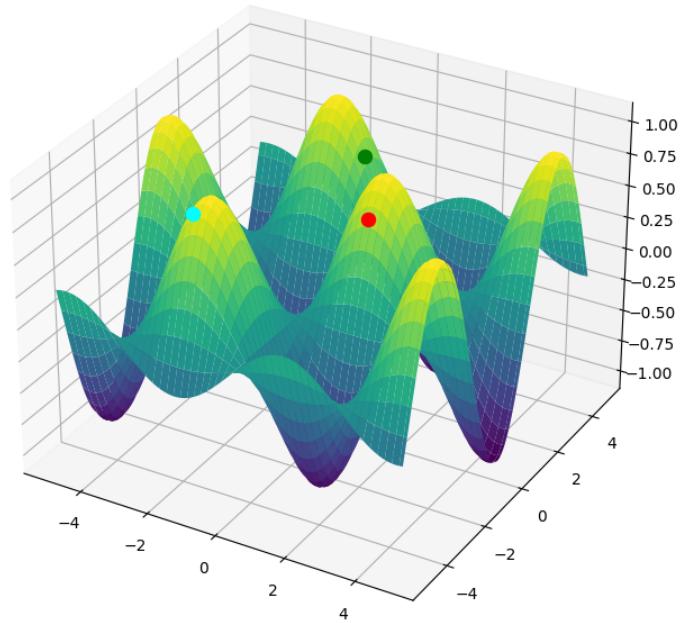
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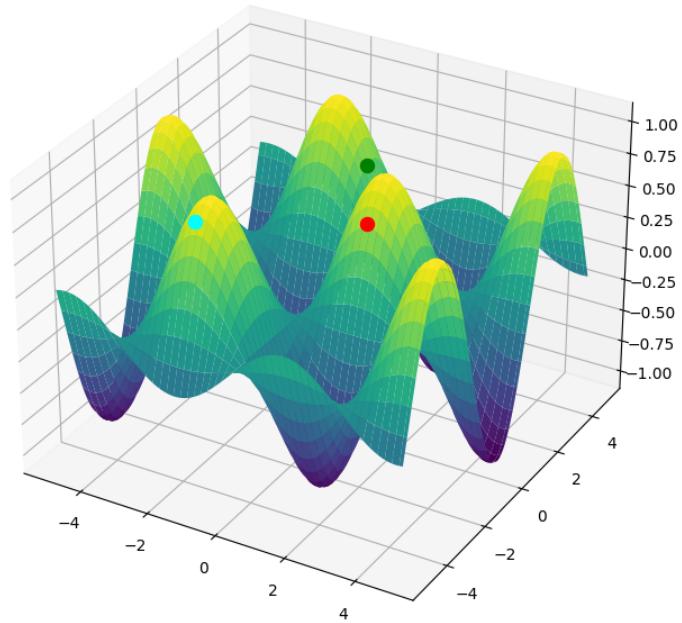
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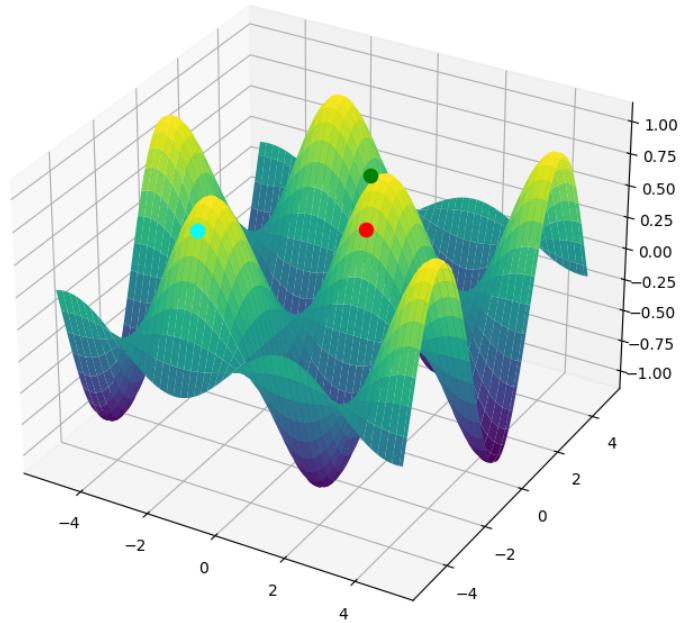
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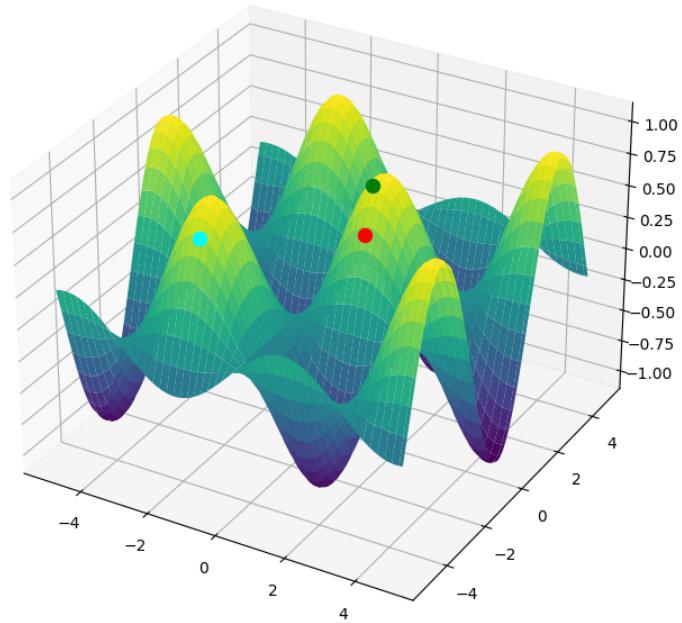
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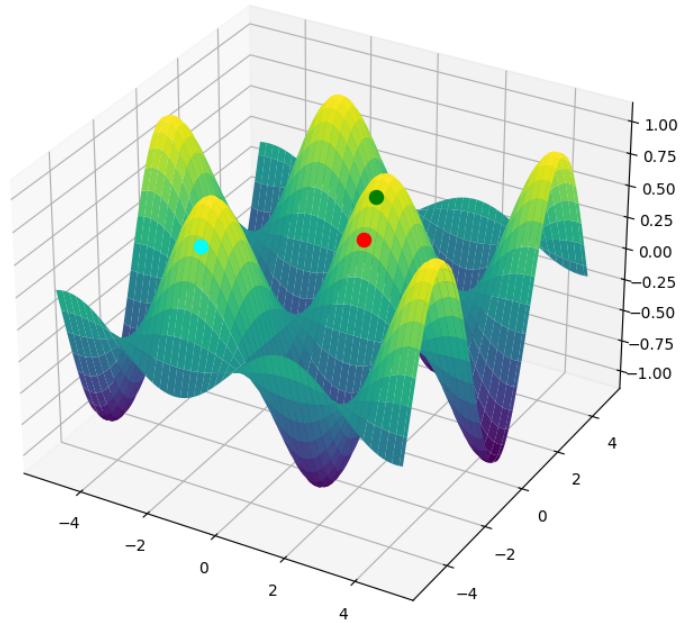
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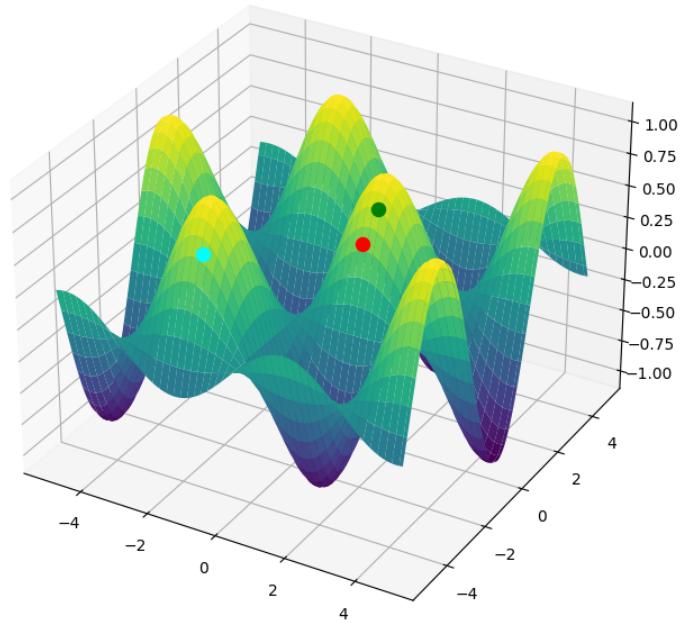
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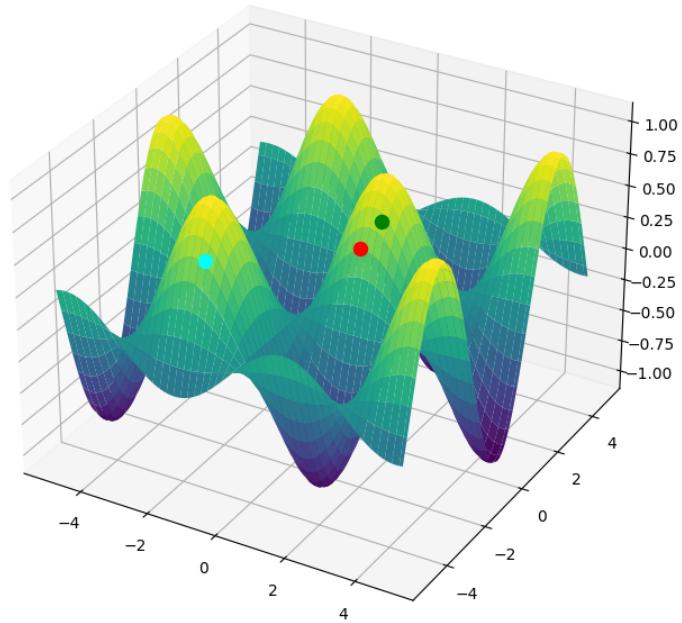
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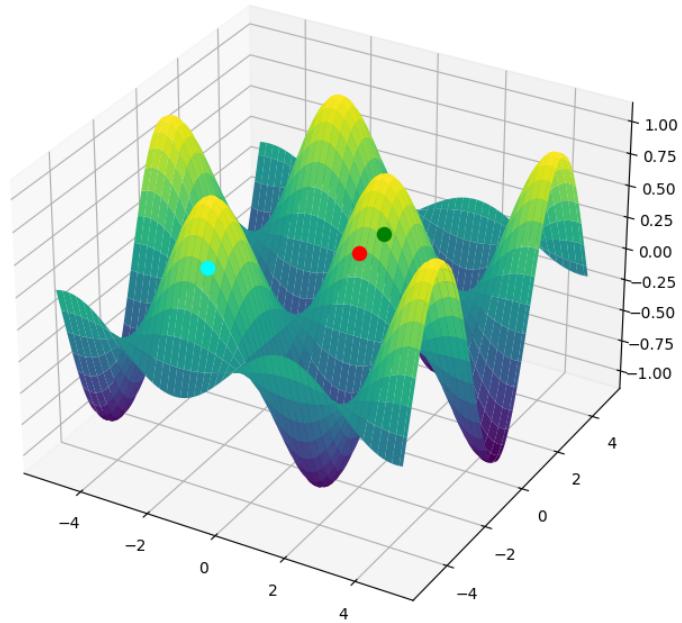
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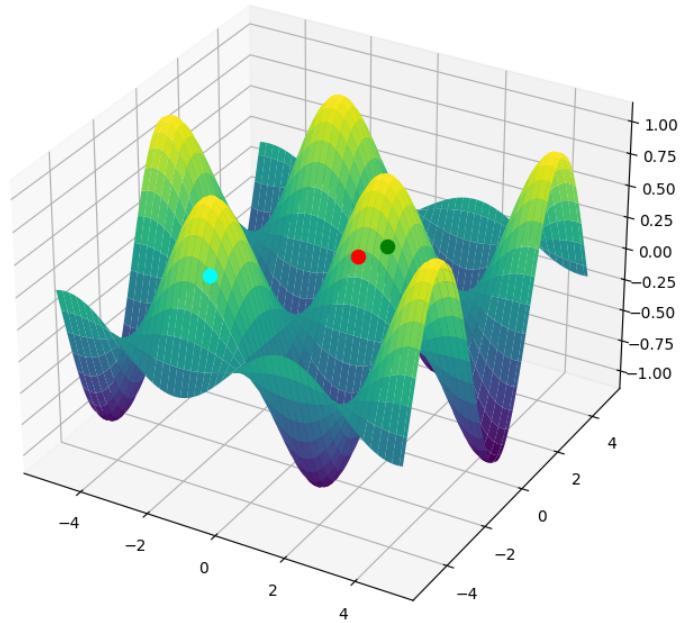
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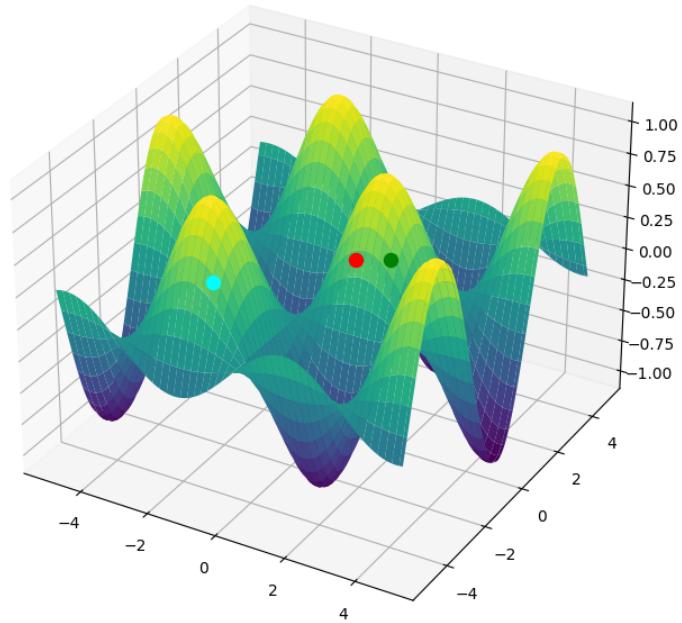
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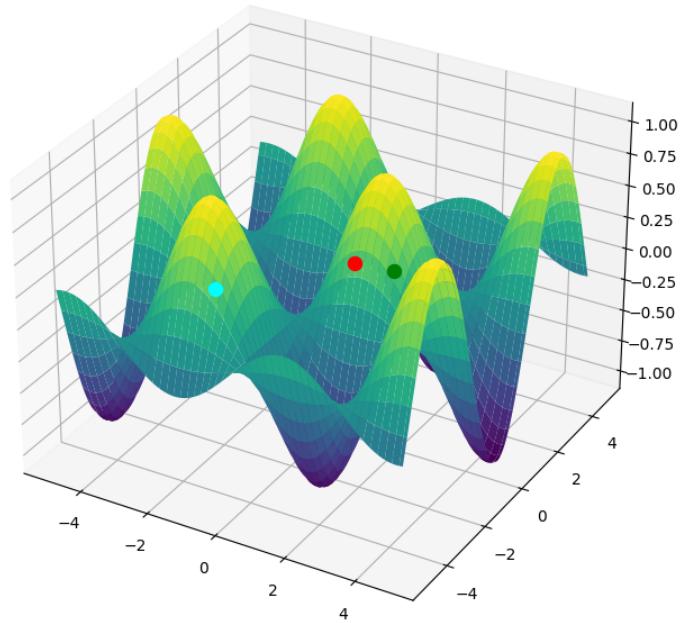
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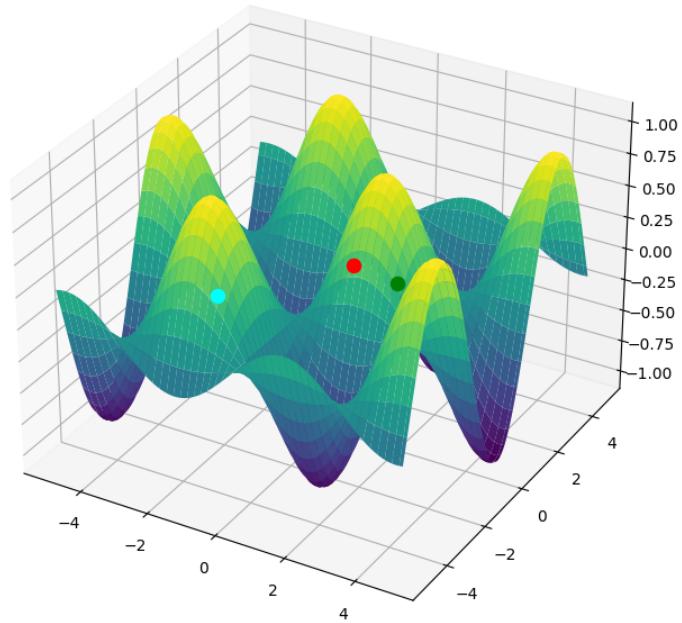
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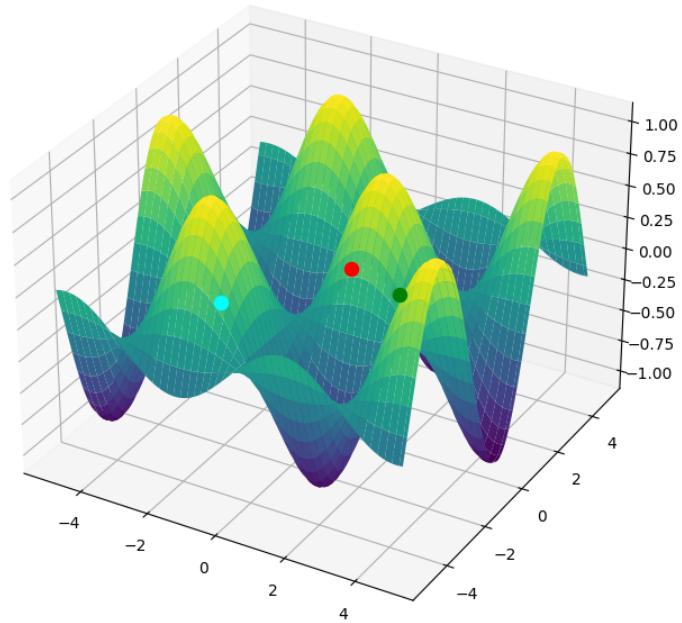
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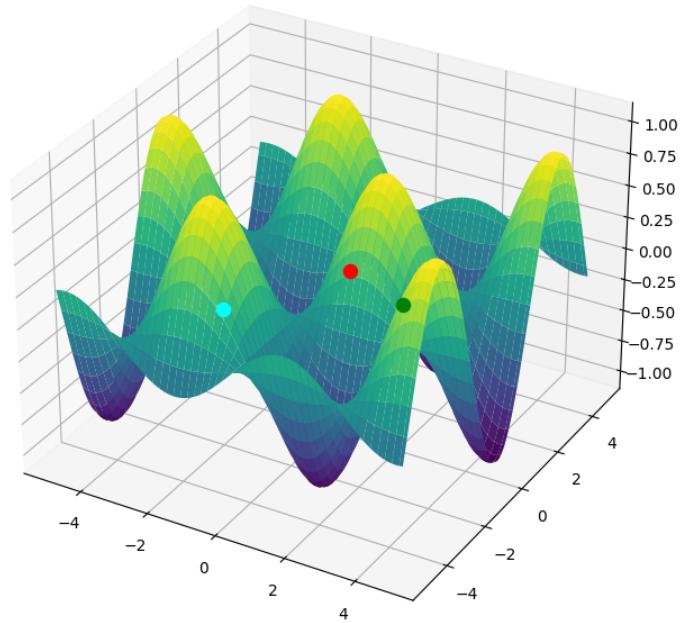
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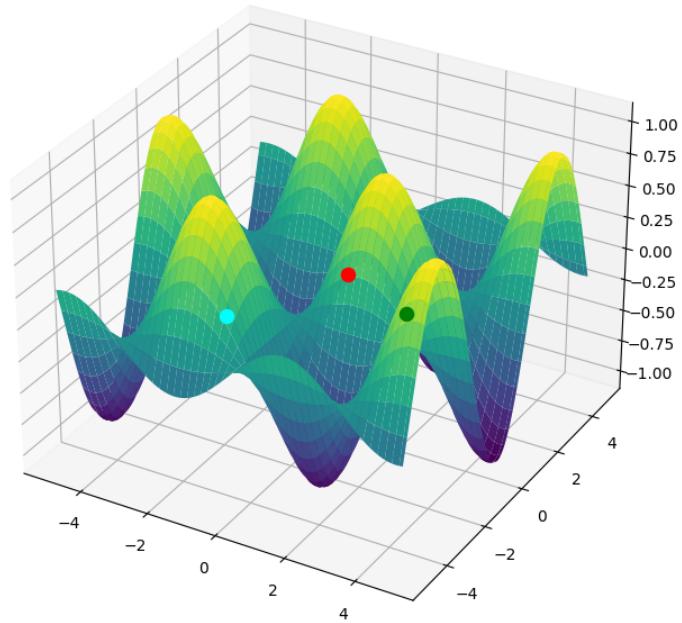
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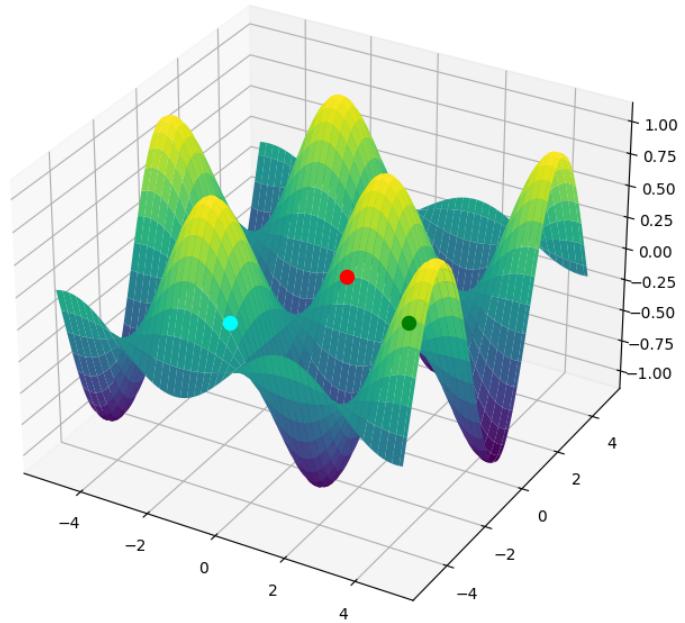
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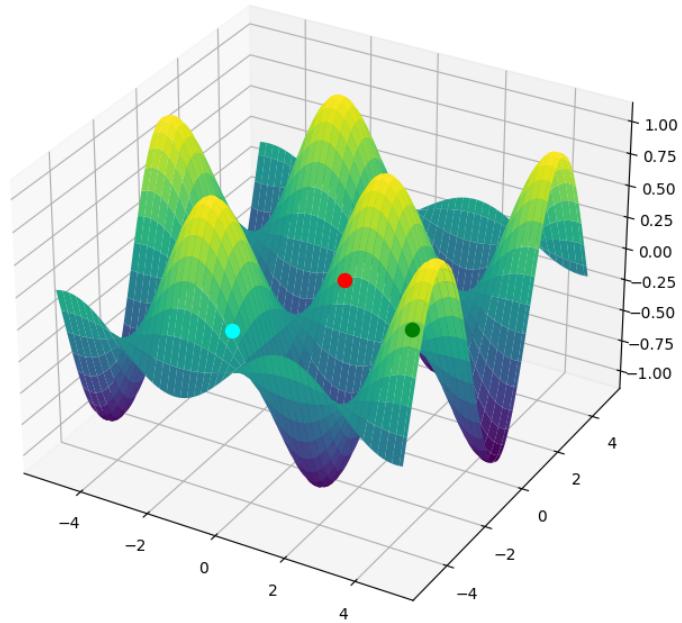
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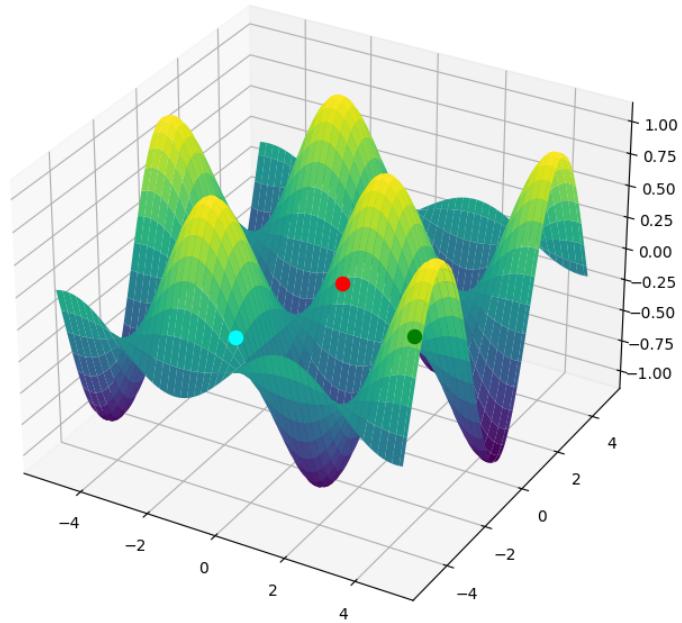
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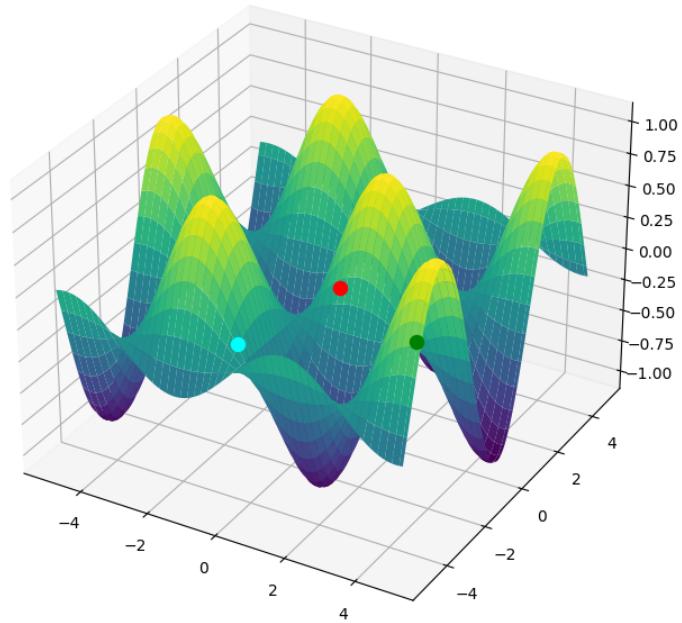
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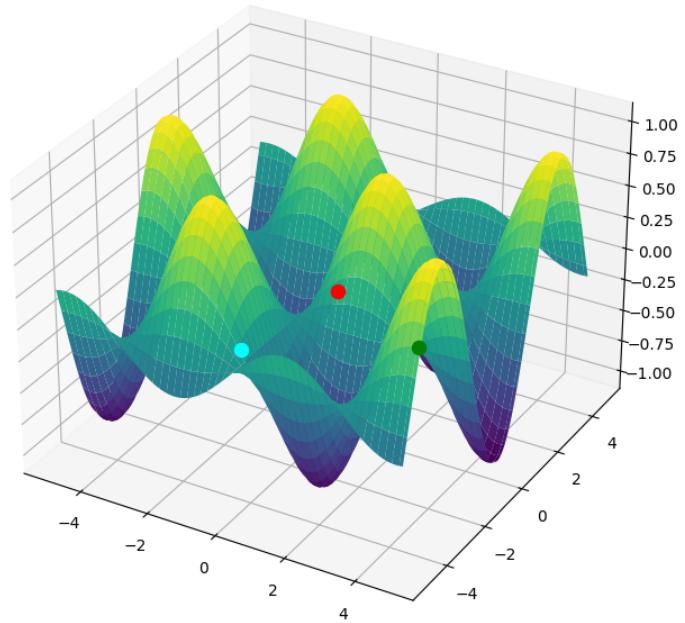
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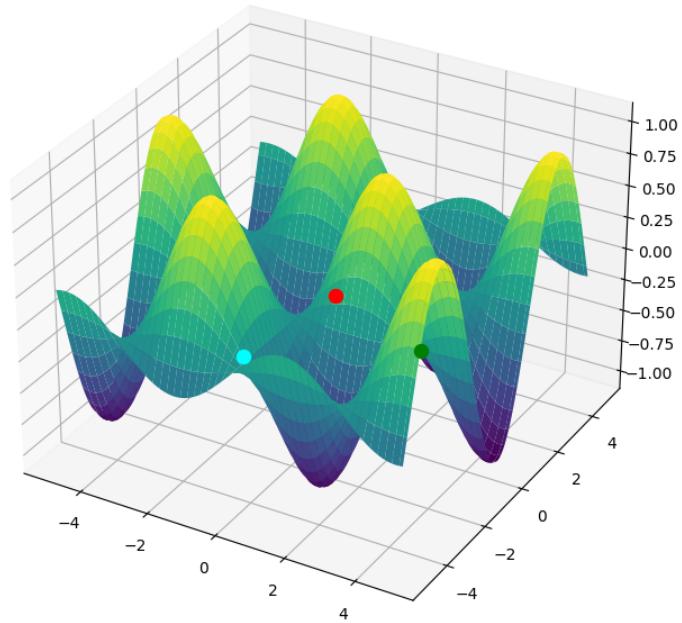
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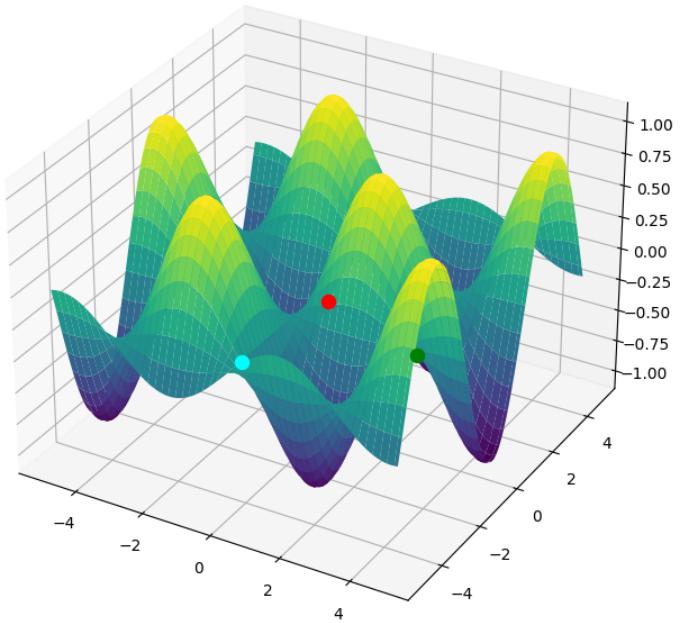
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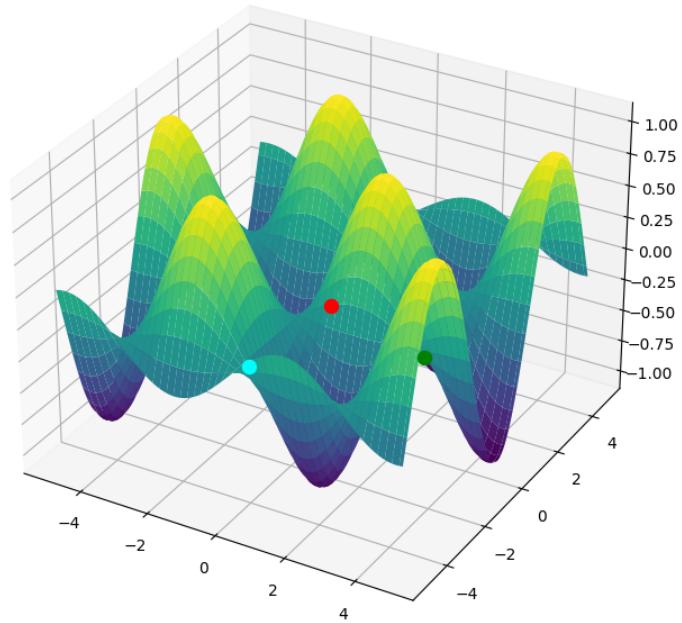
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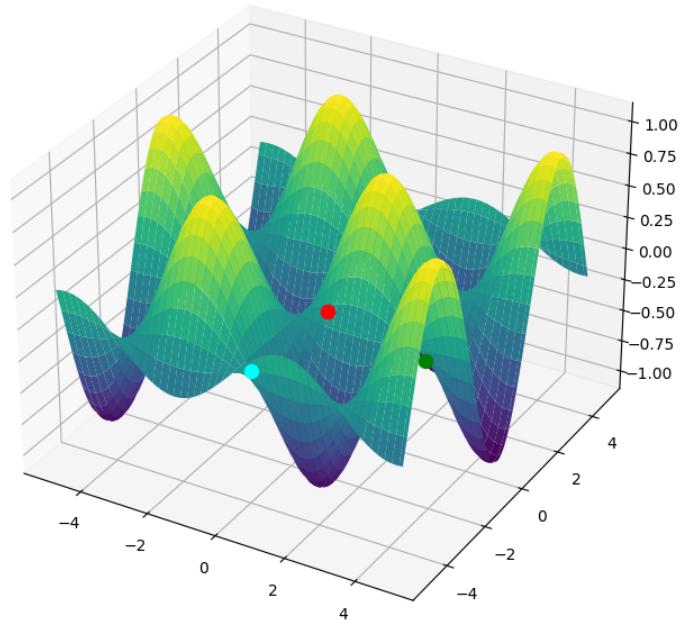
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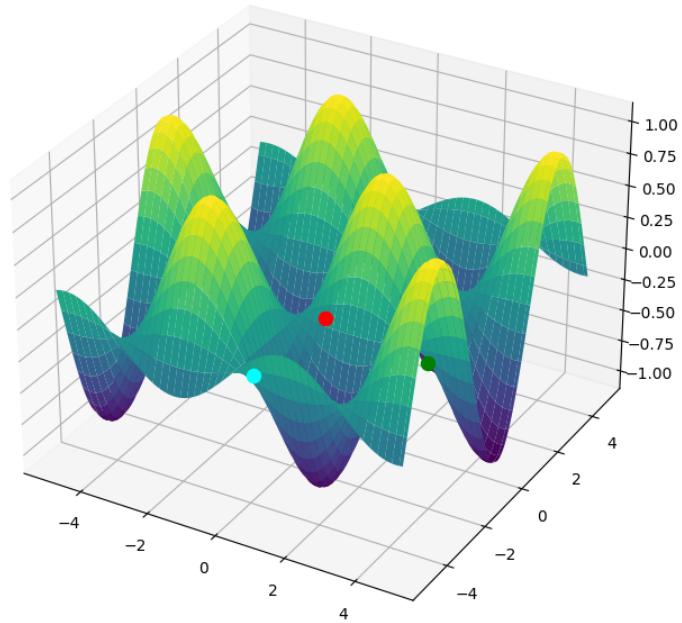
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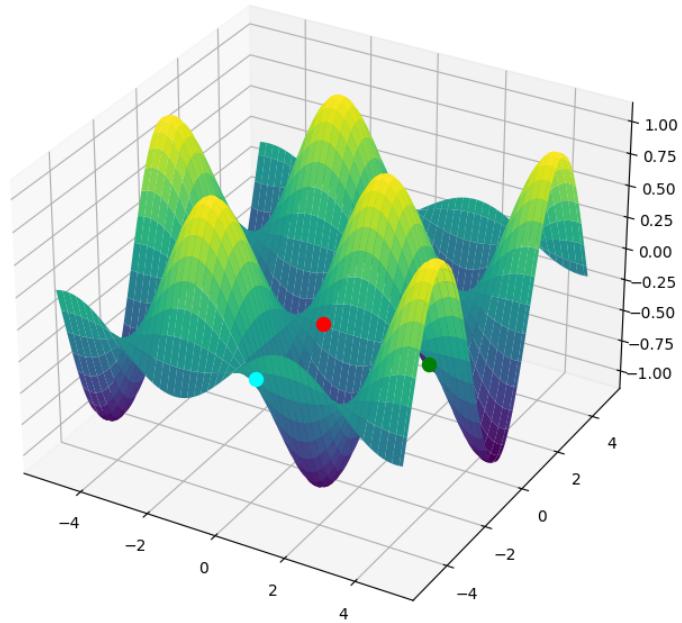
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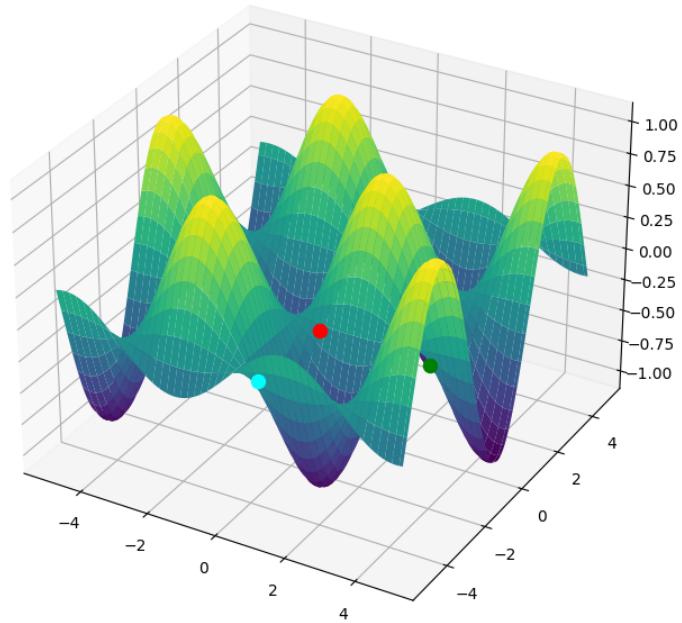
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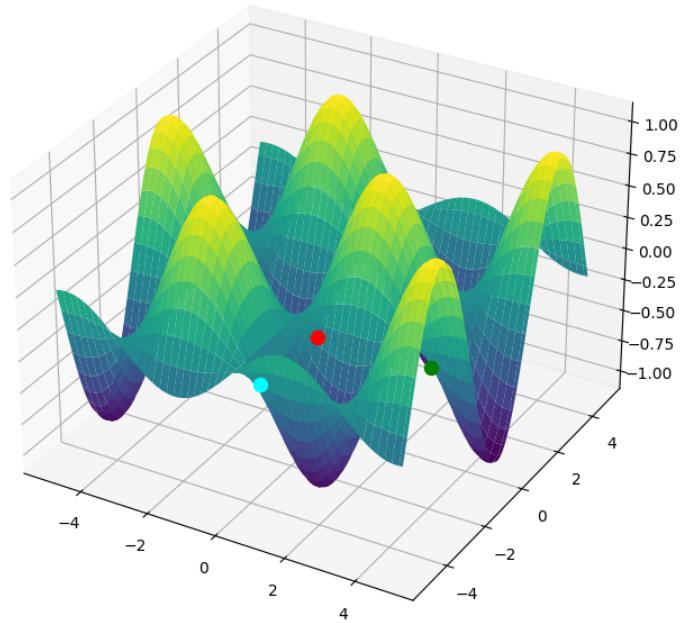
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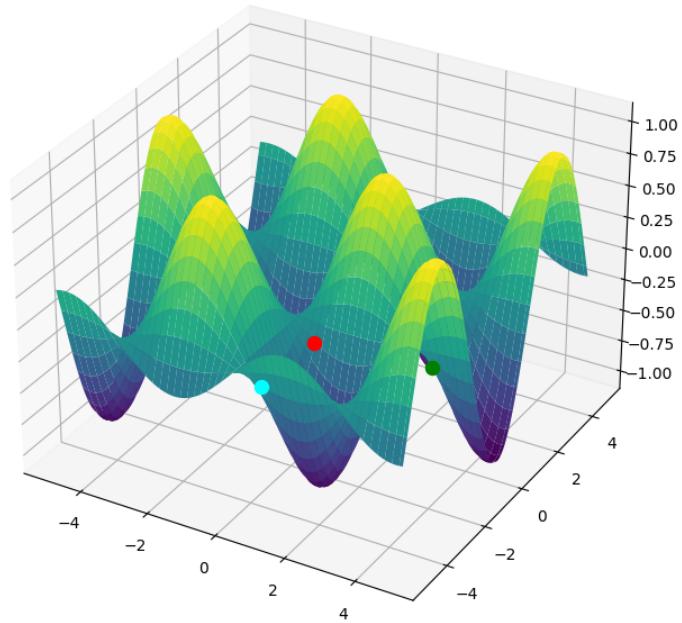
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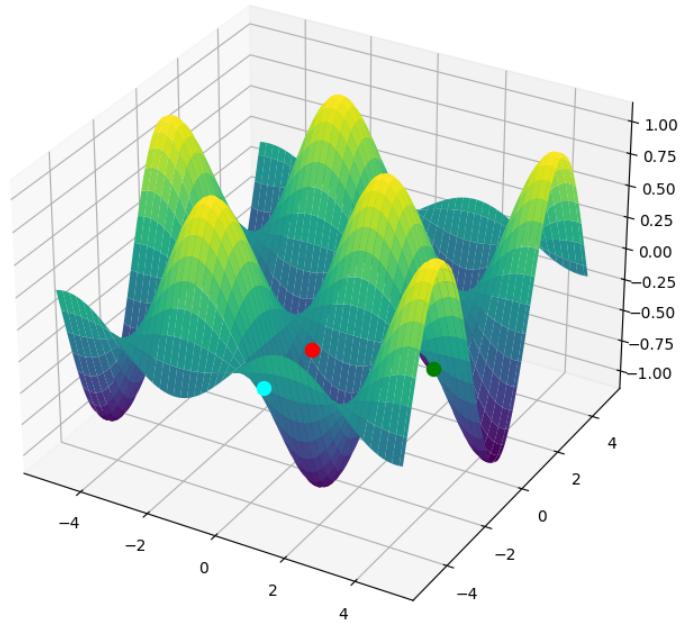
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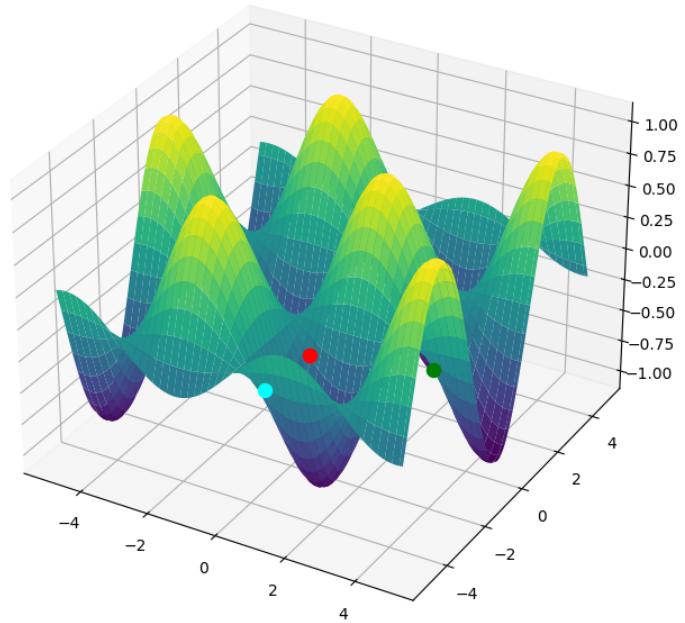
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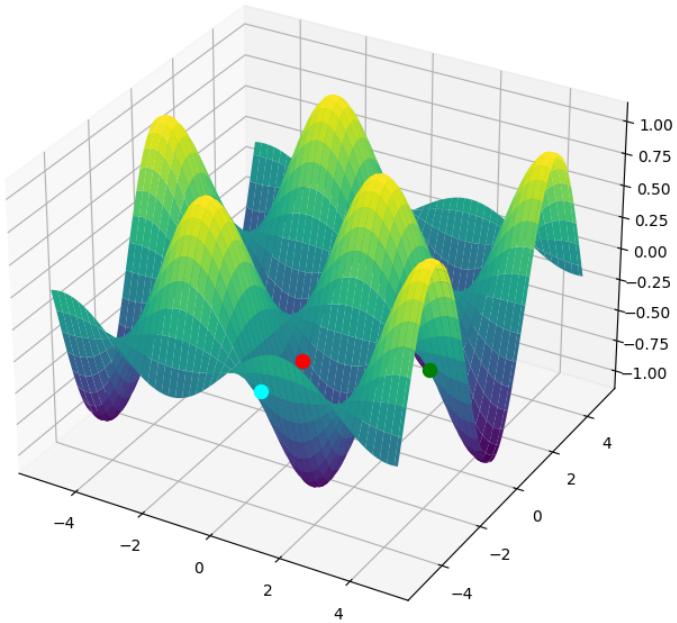
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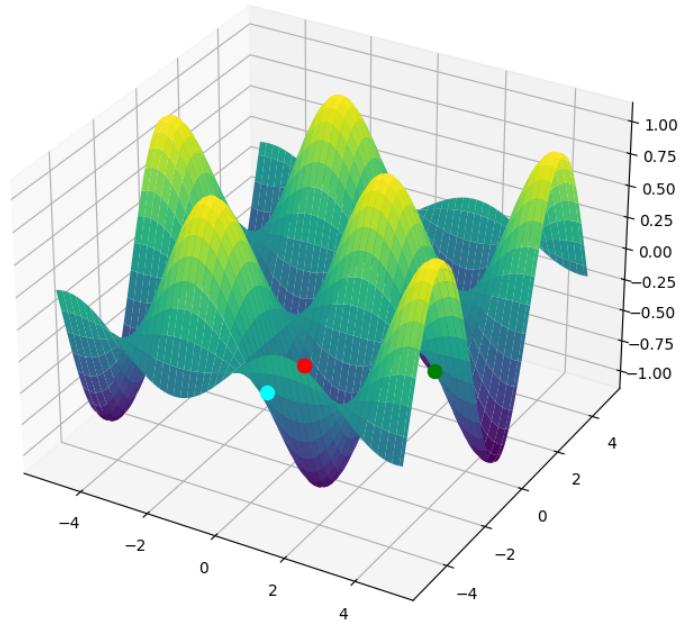
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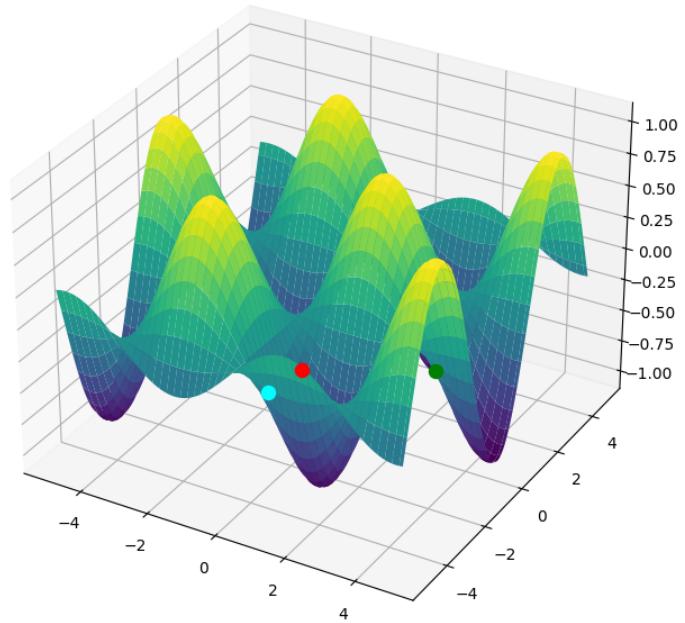
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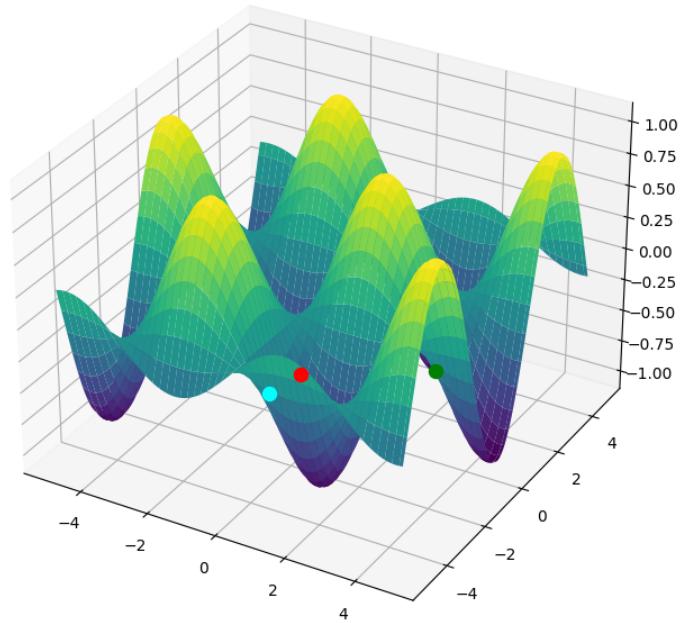
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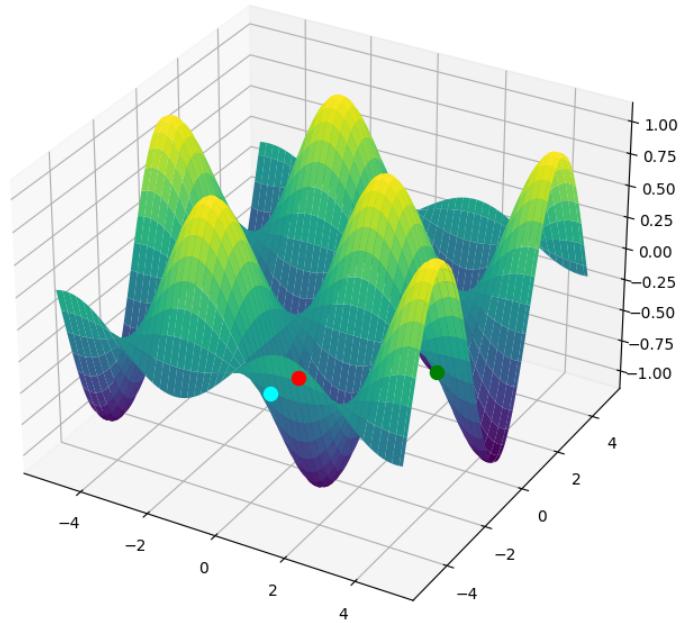
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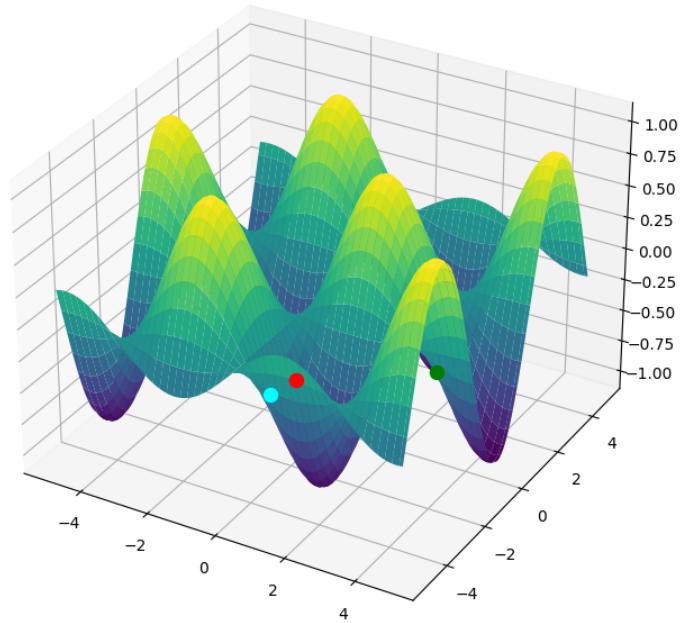
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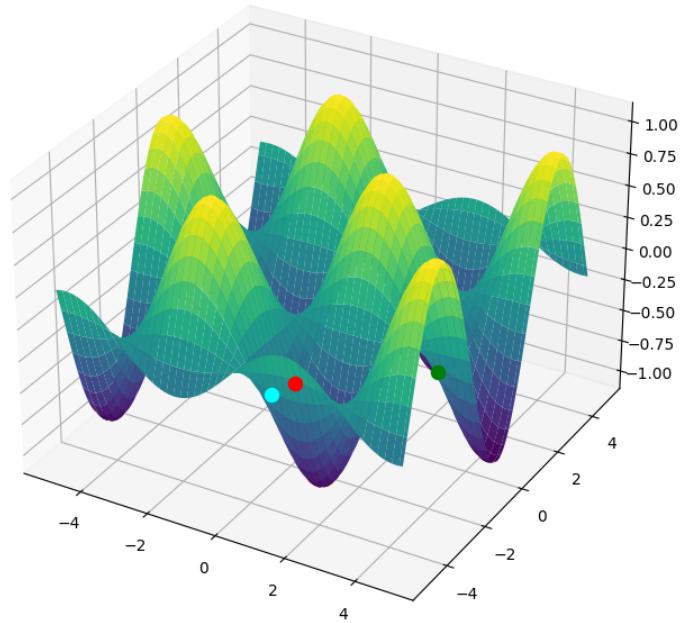
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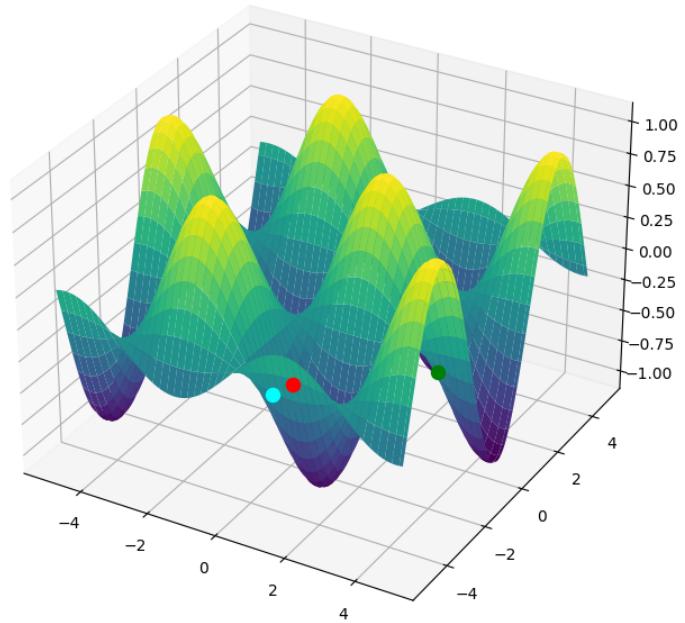
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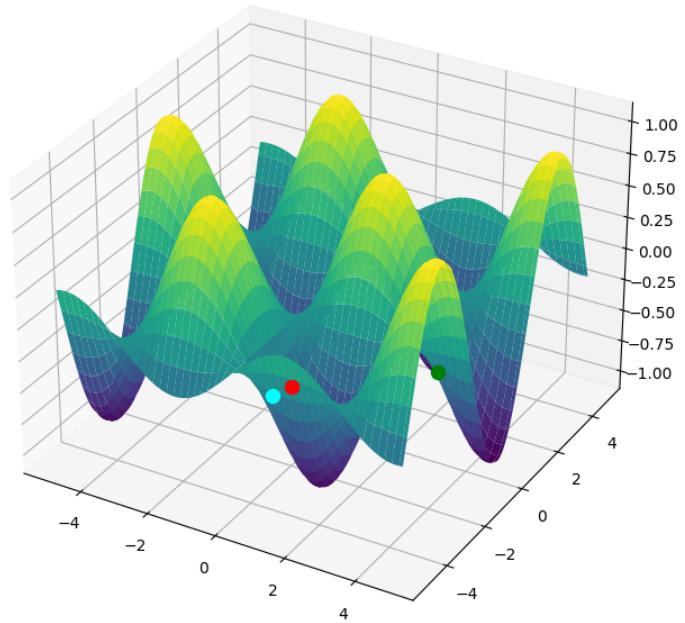
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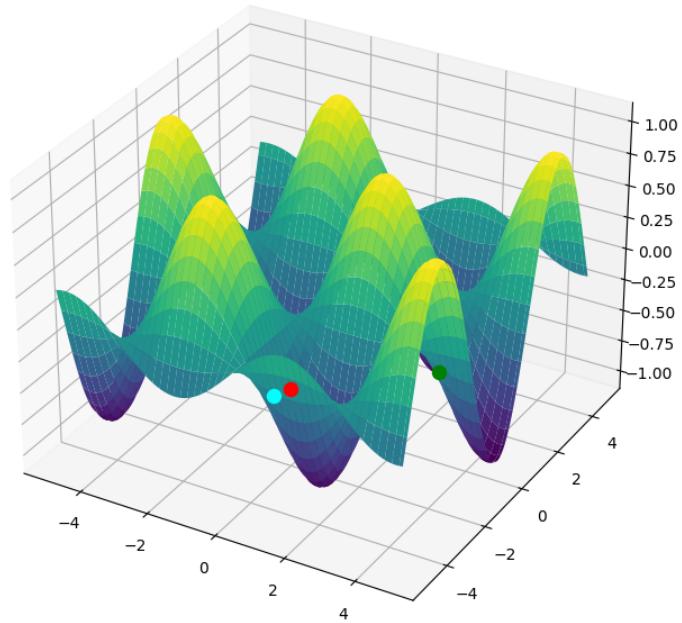
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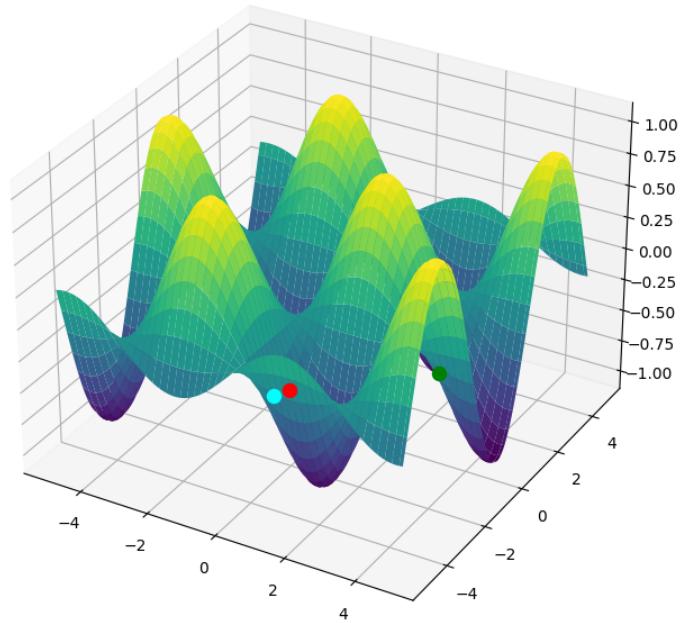
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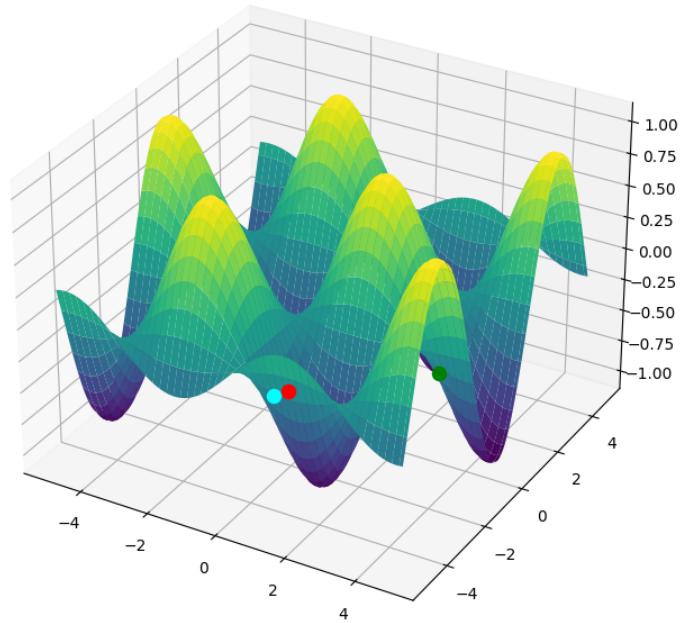
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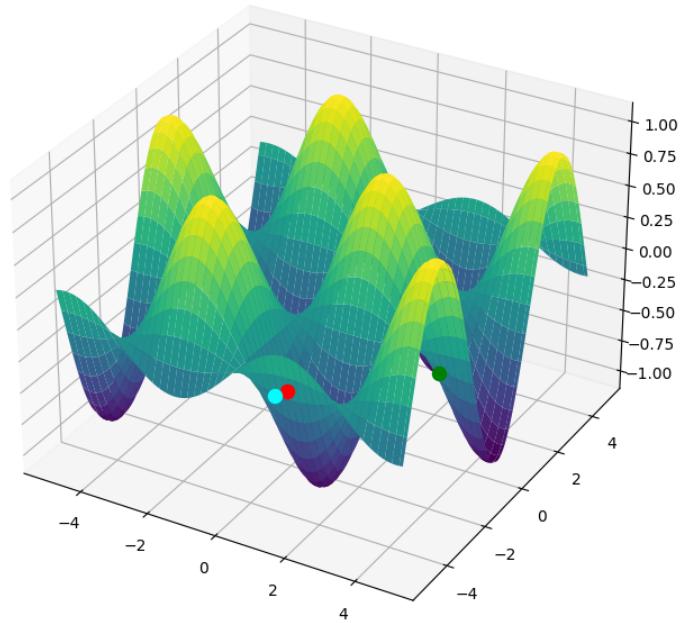
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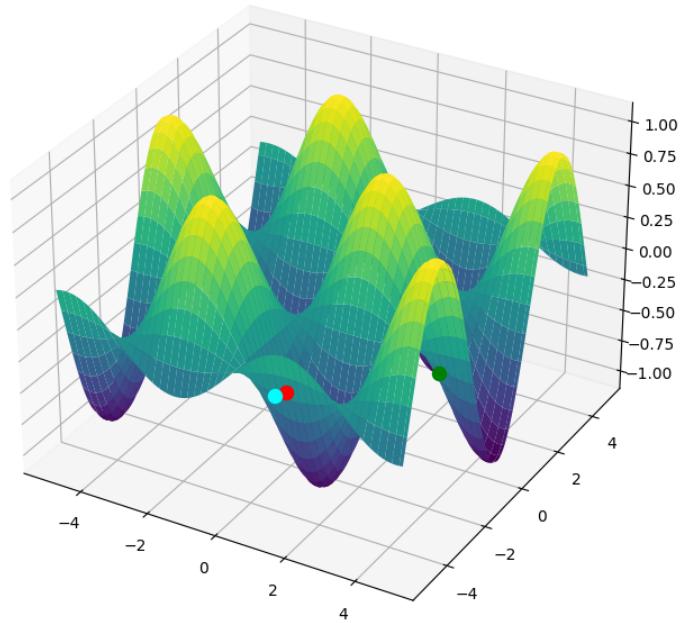
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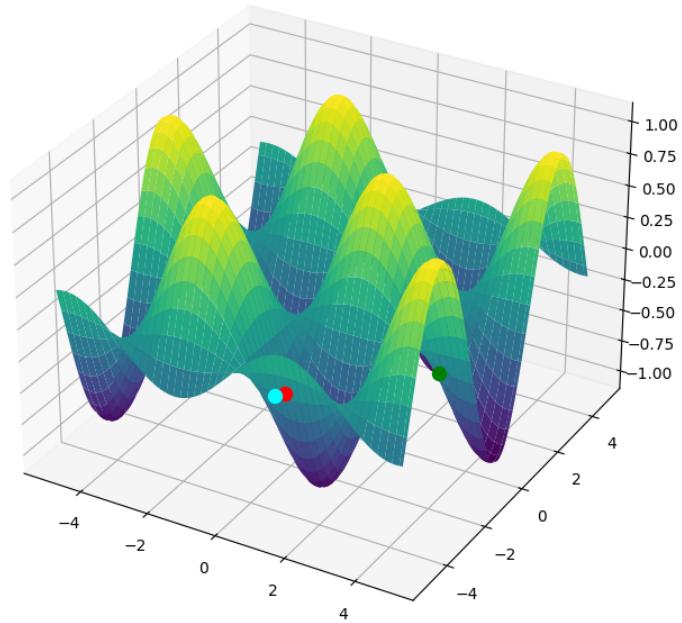
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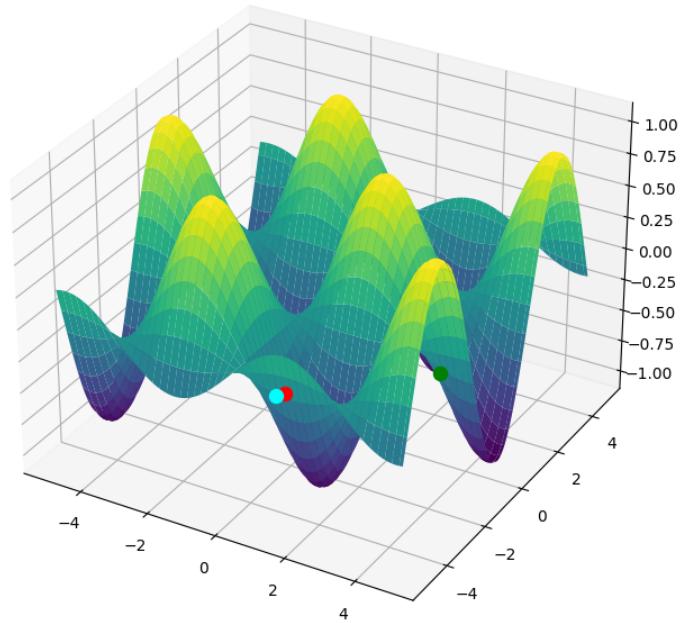
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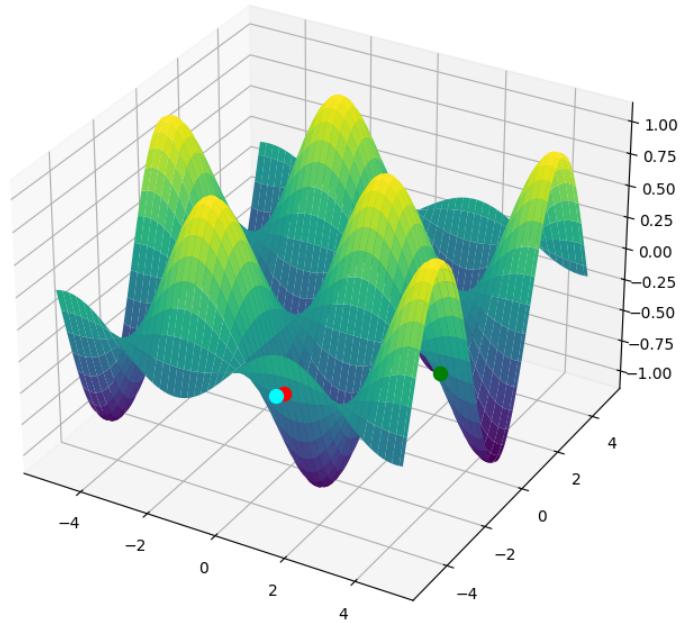
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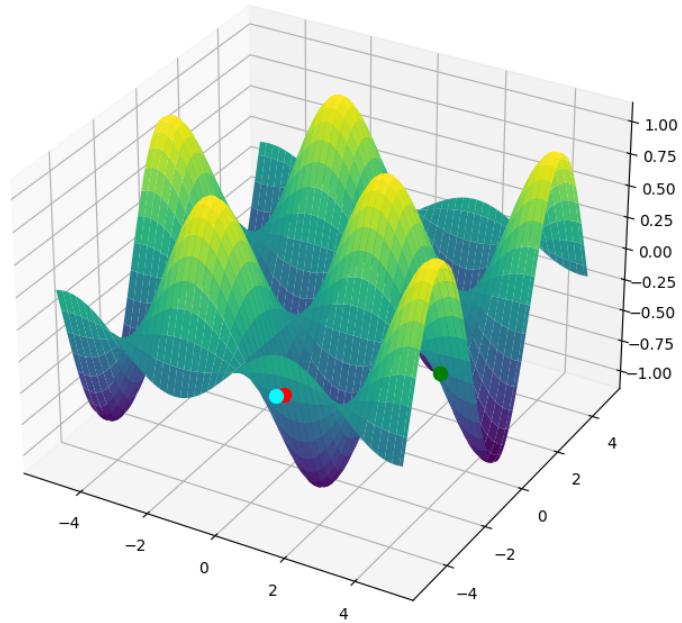
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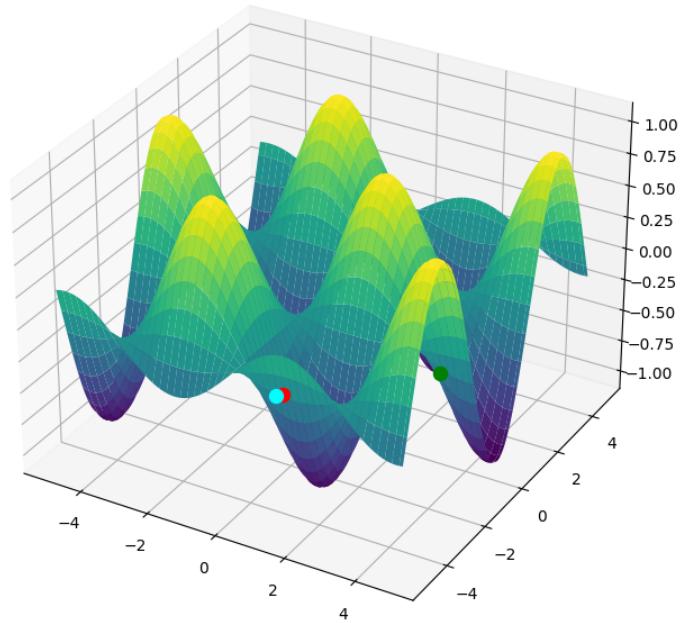
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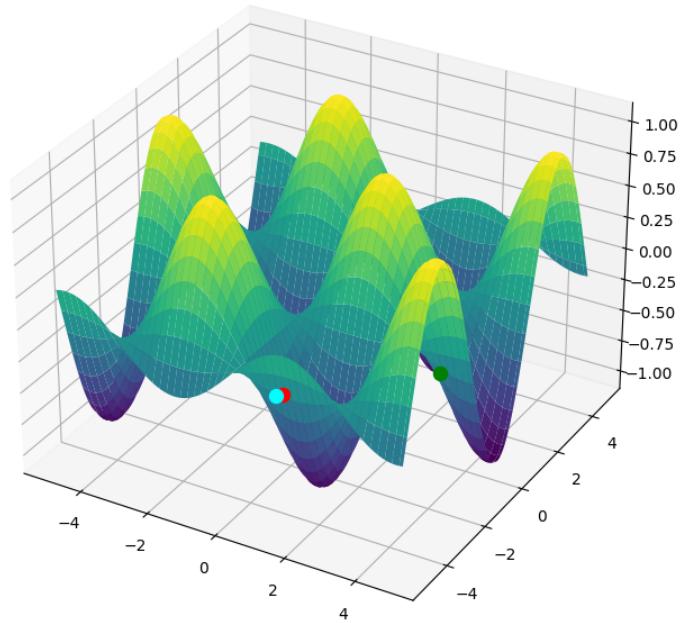
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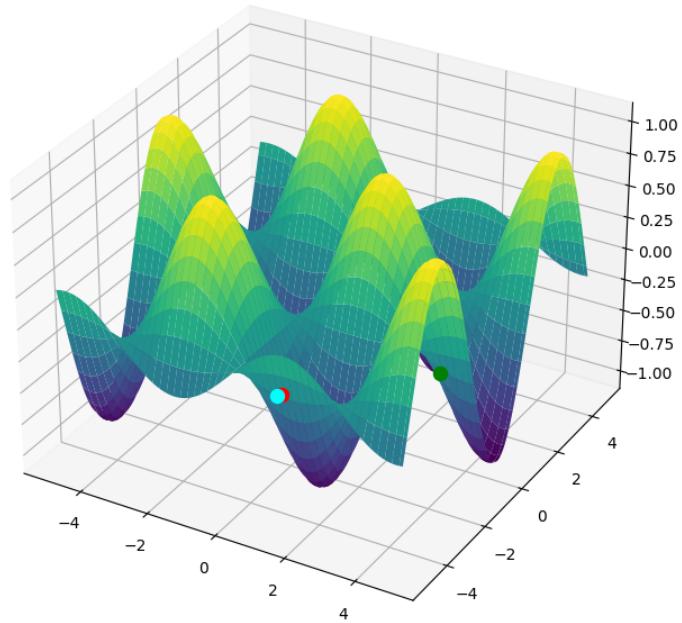
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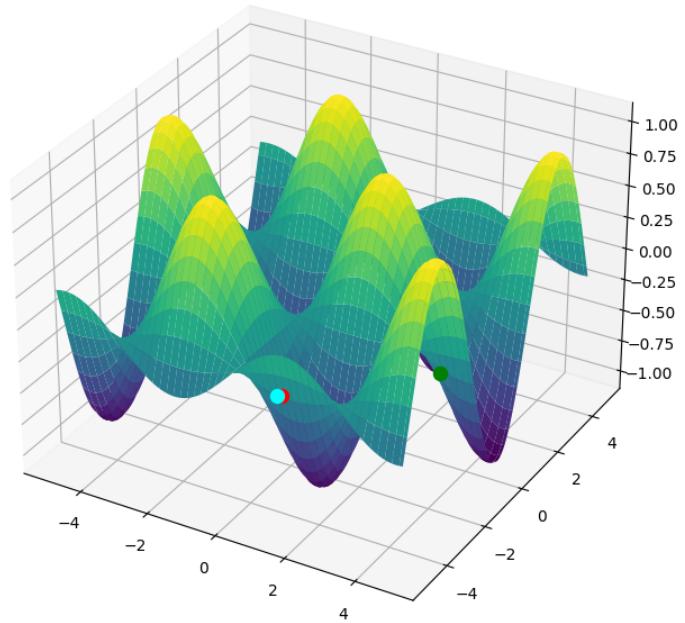
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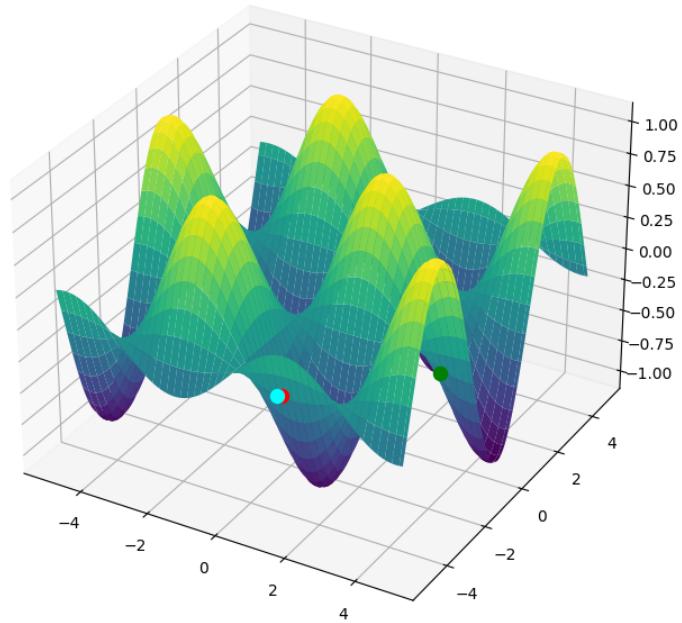
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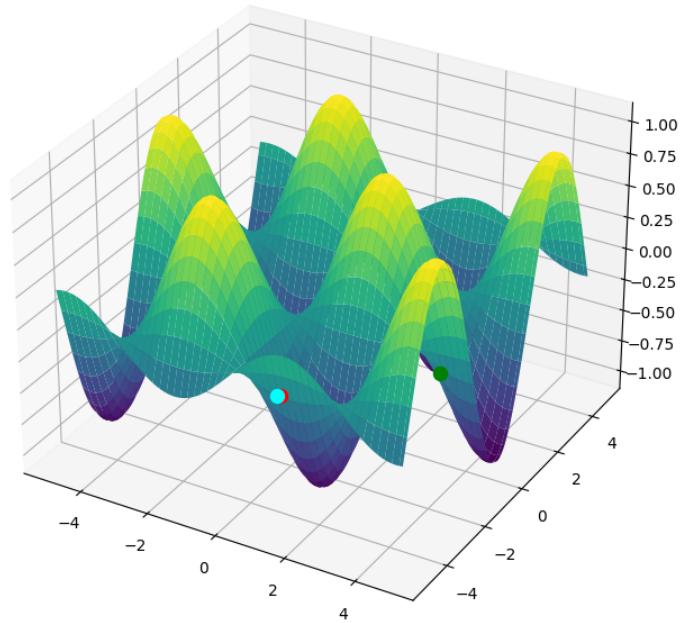
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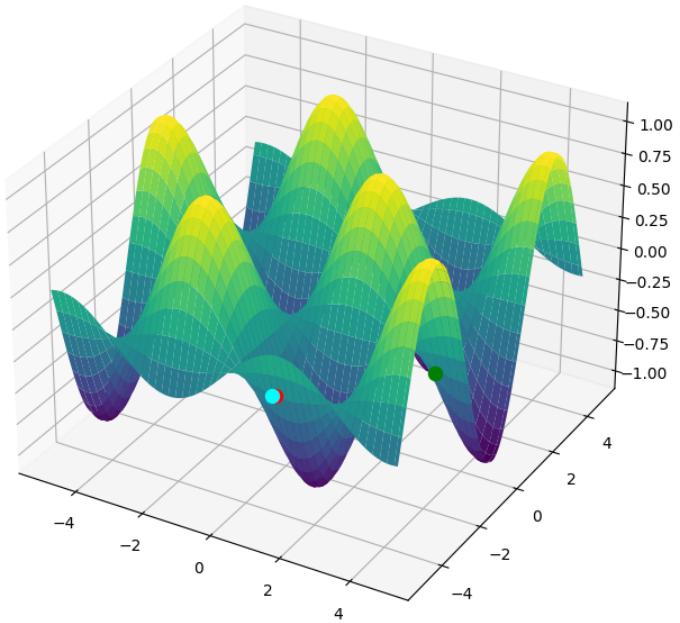
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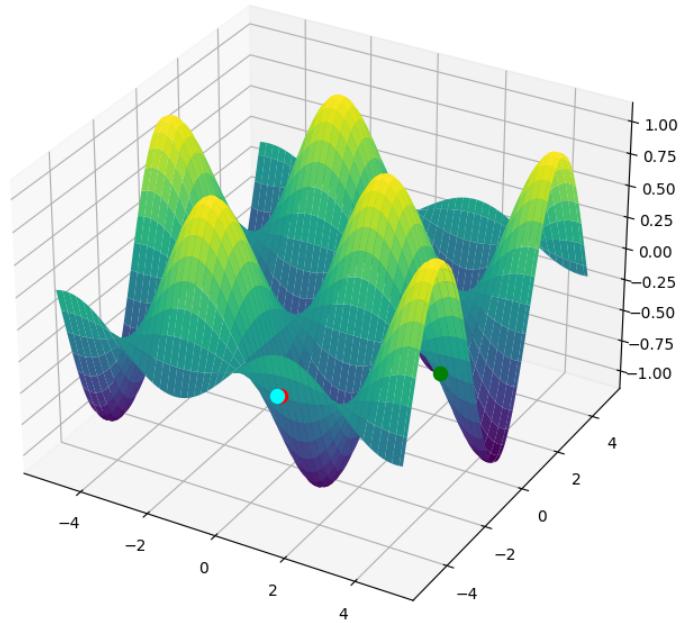
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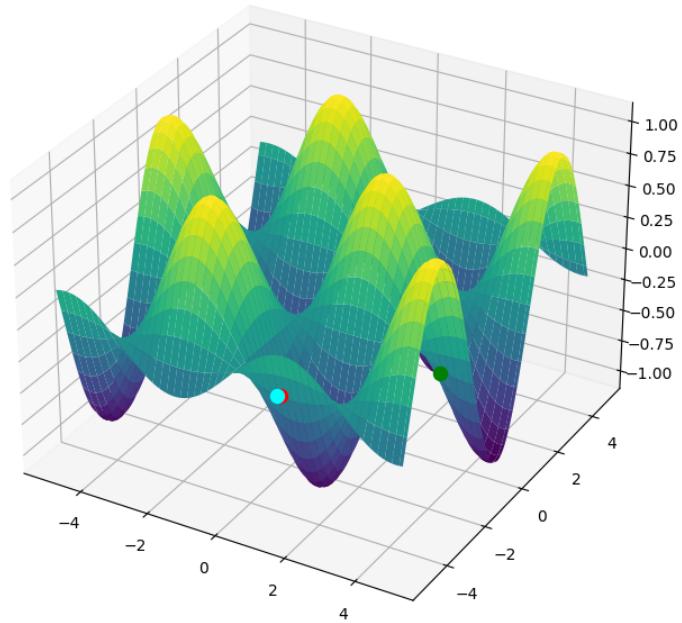
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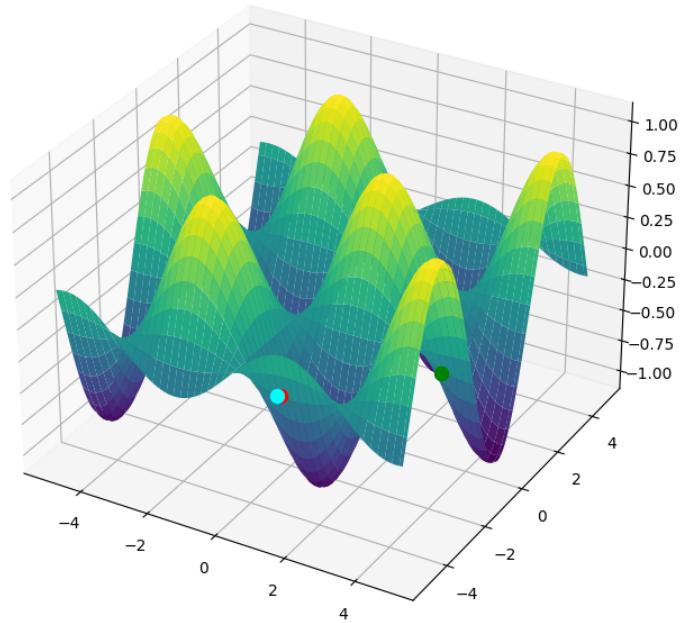
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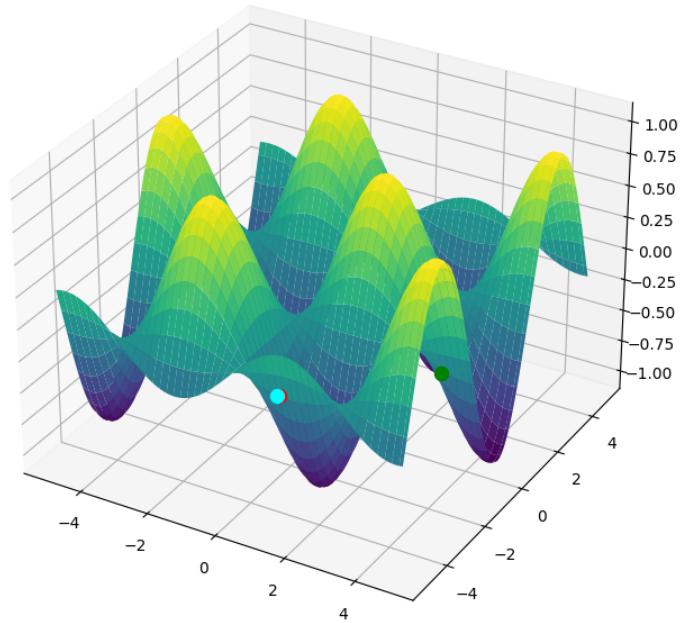
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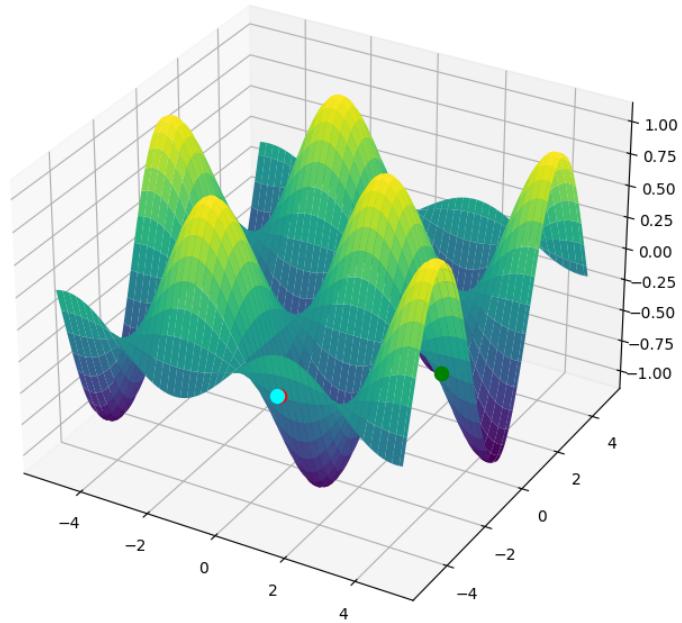
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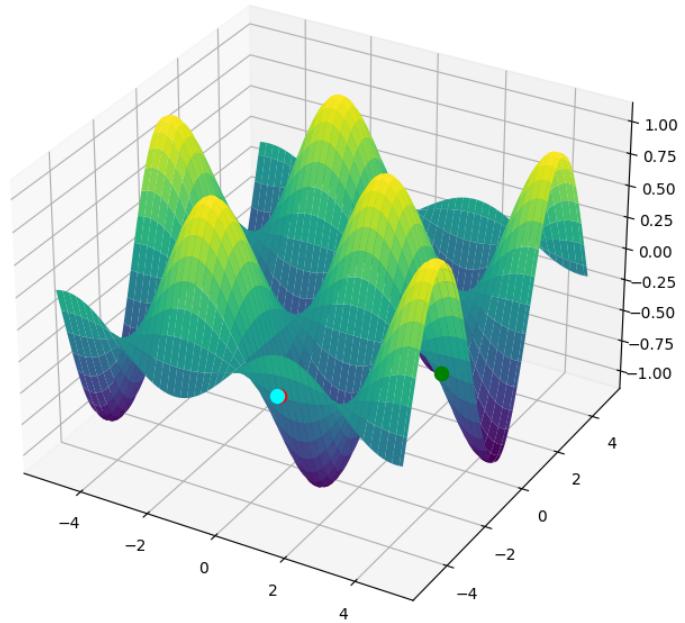
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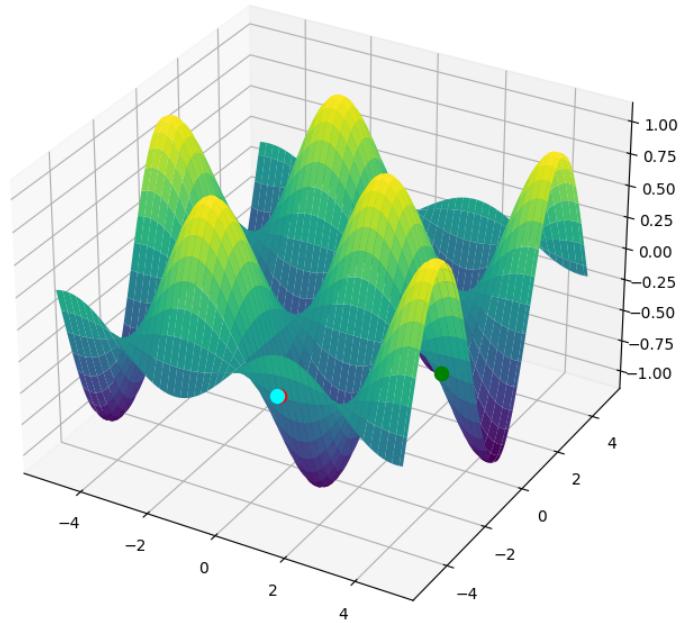
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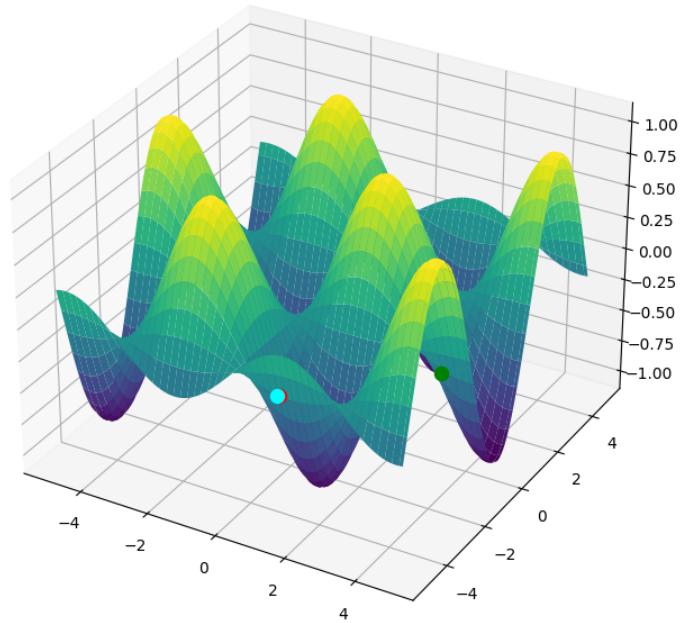
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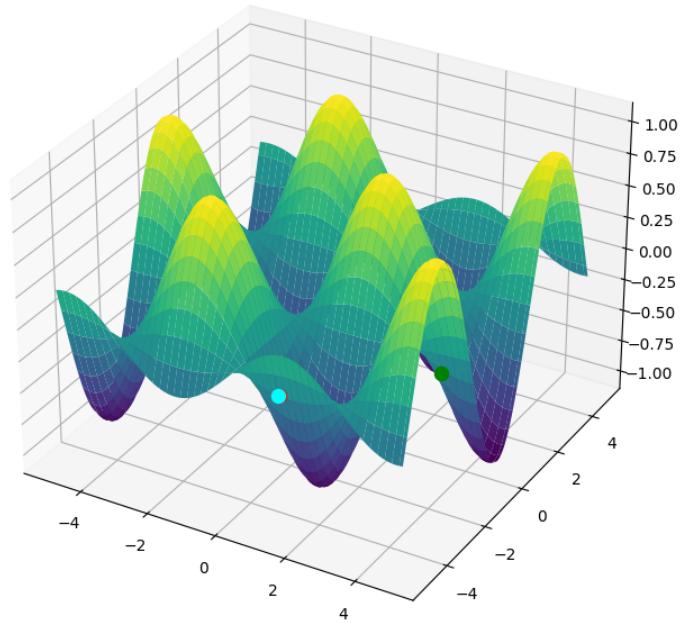
Epoch No: 66



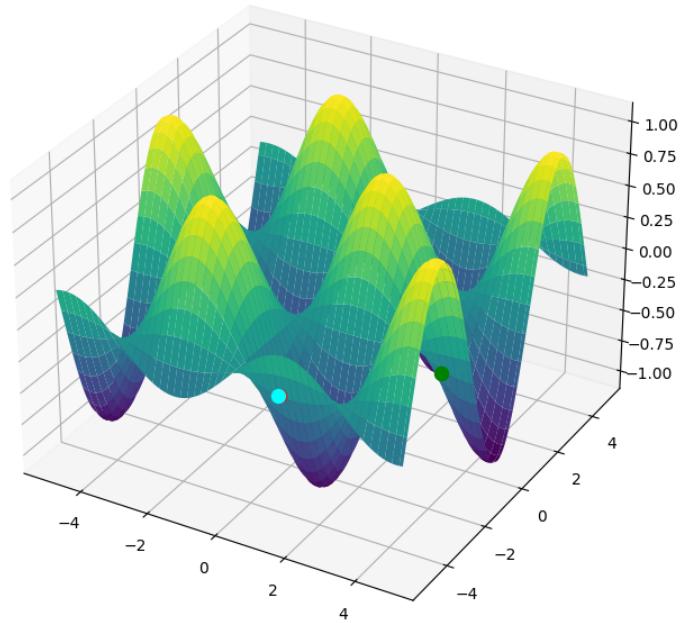
Epoch No: 67



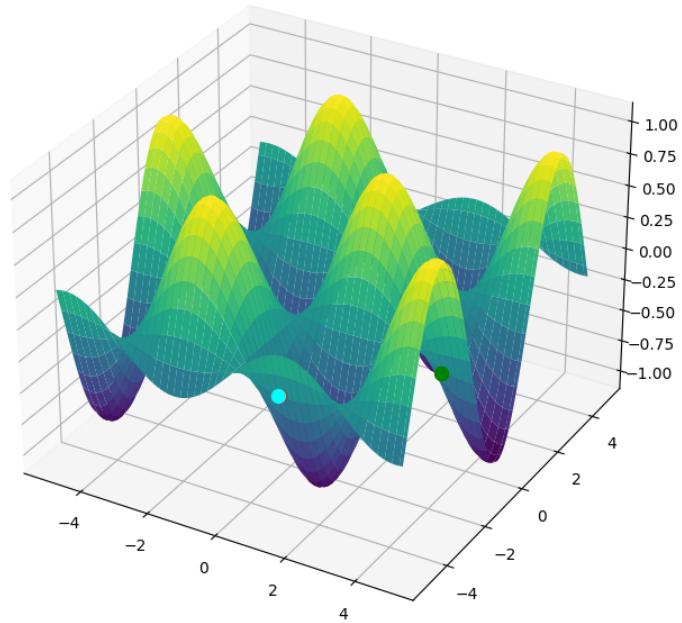
Epoch No: 68



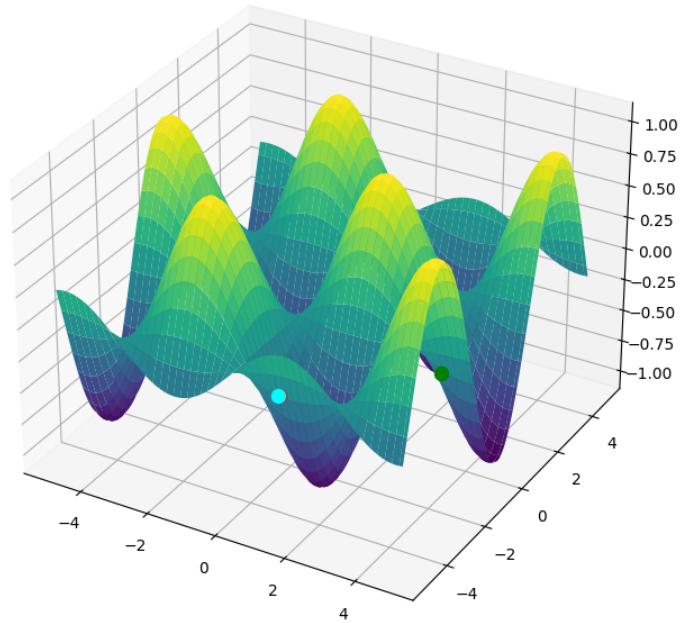
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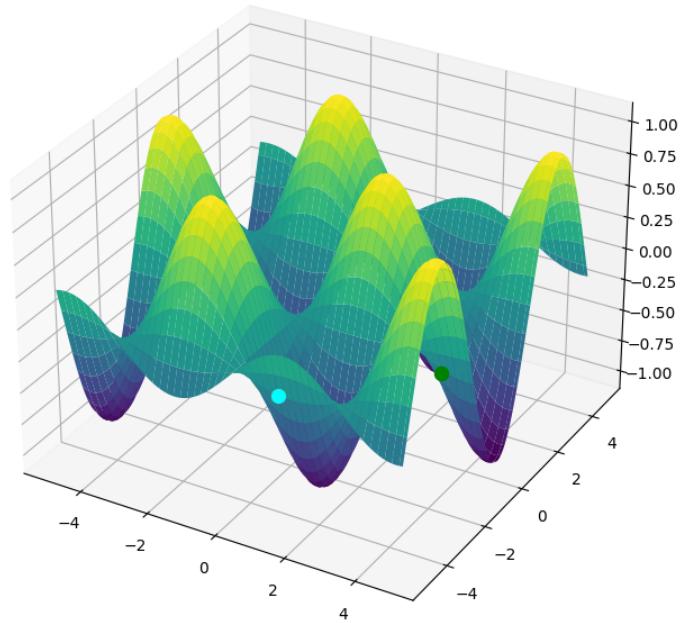
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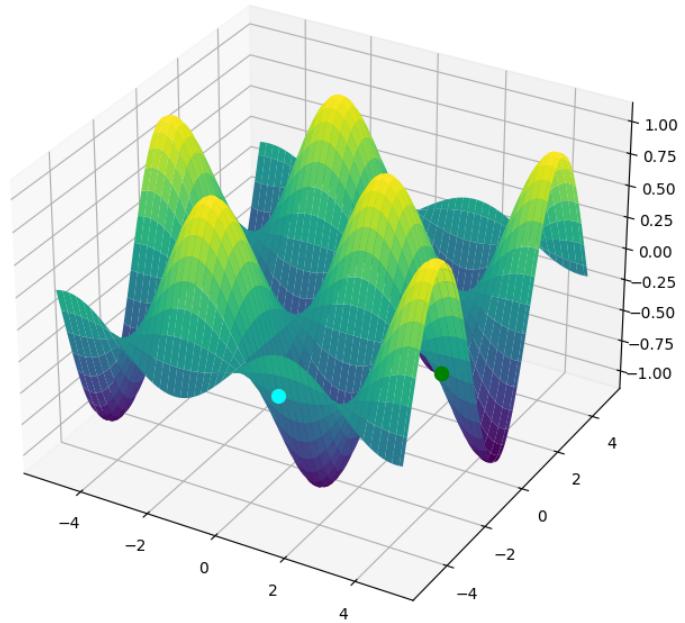
Epoch No: 71



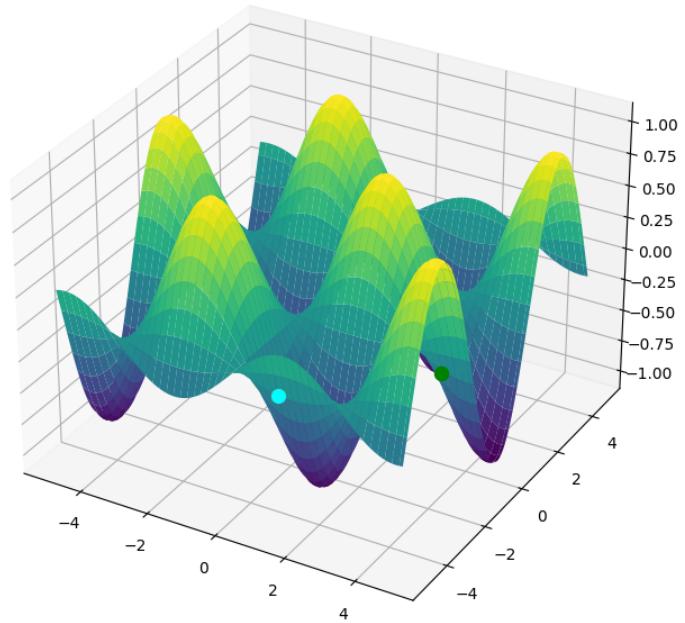
Epoch No: 72



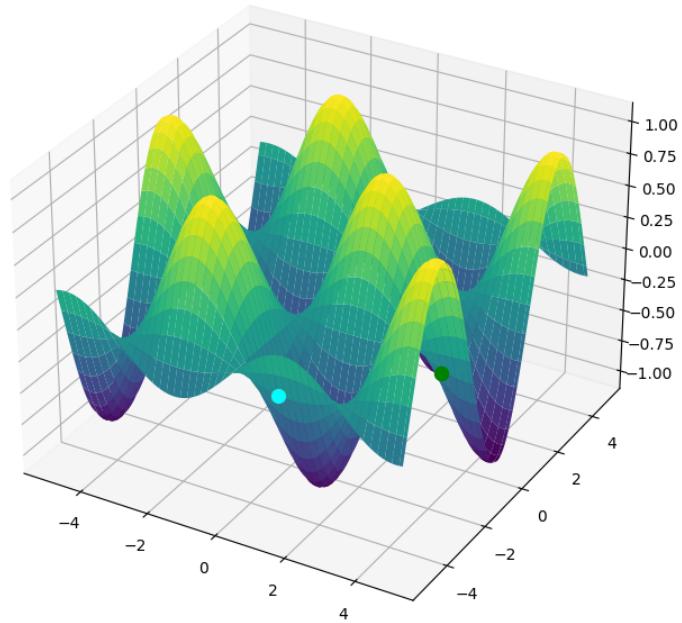
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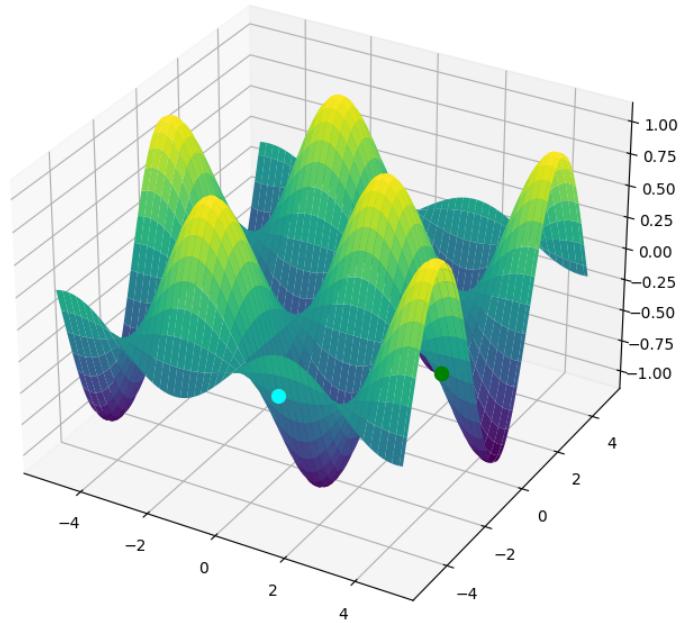
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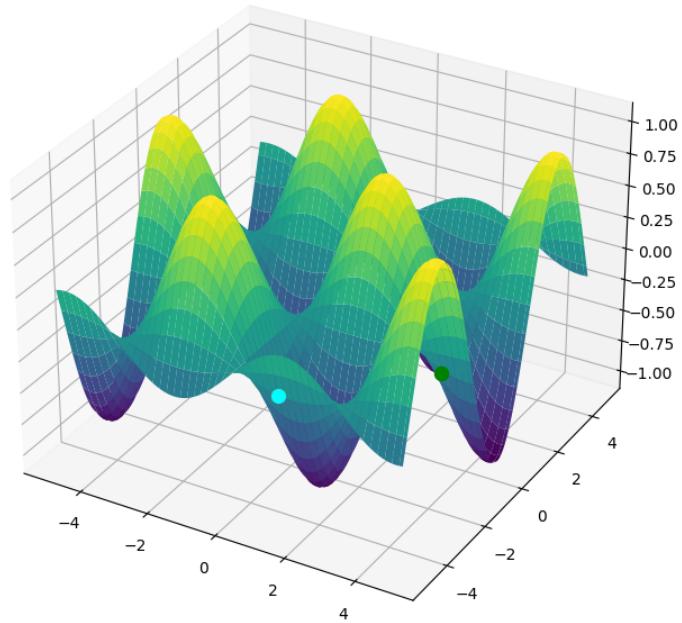
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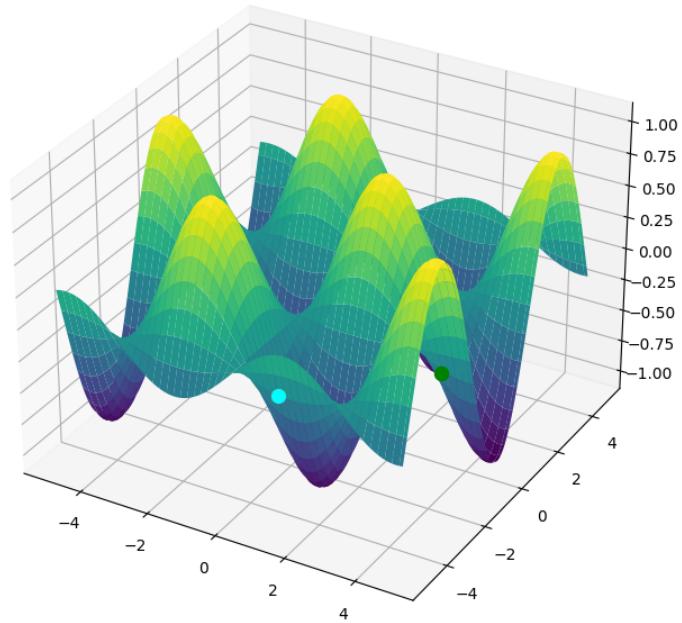
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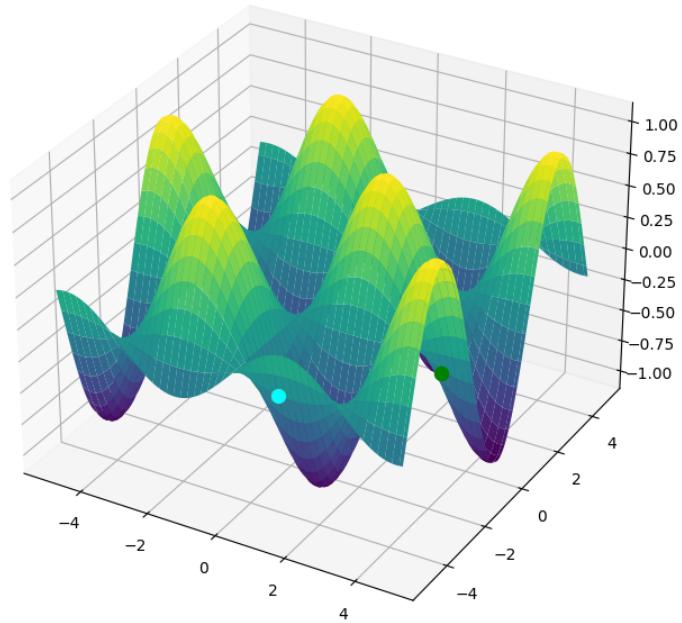
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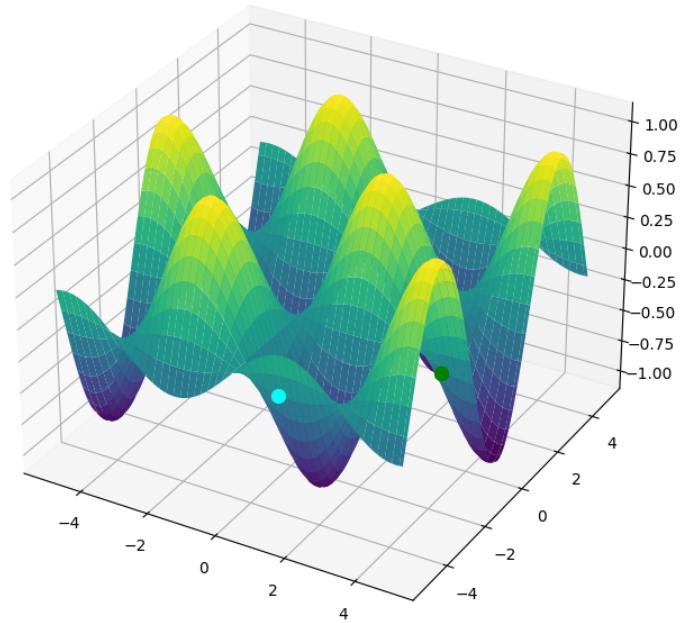
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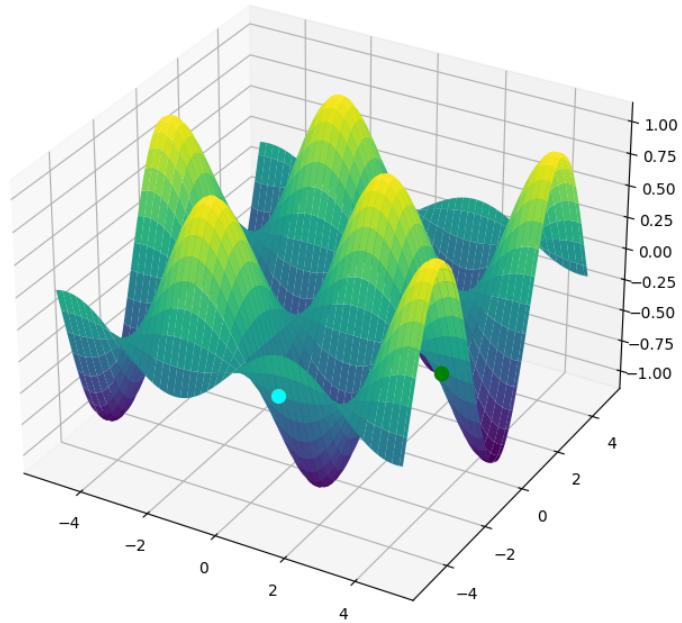
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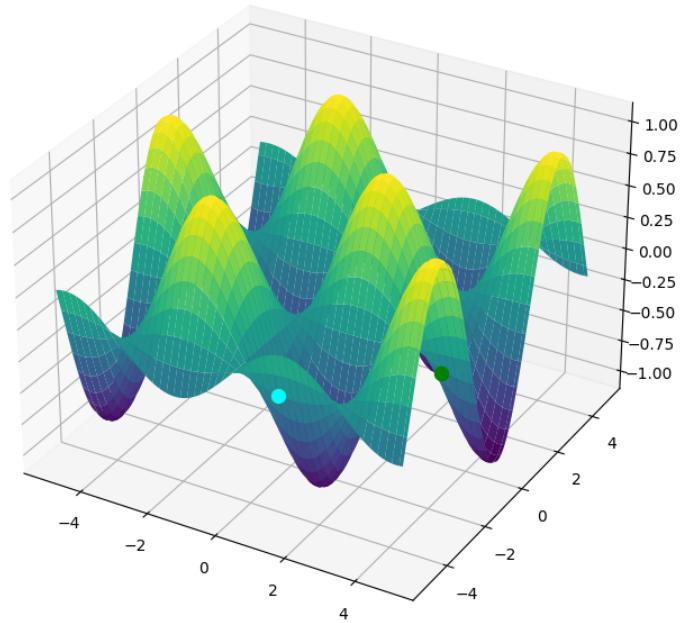
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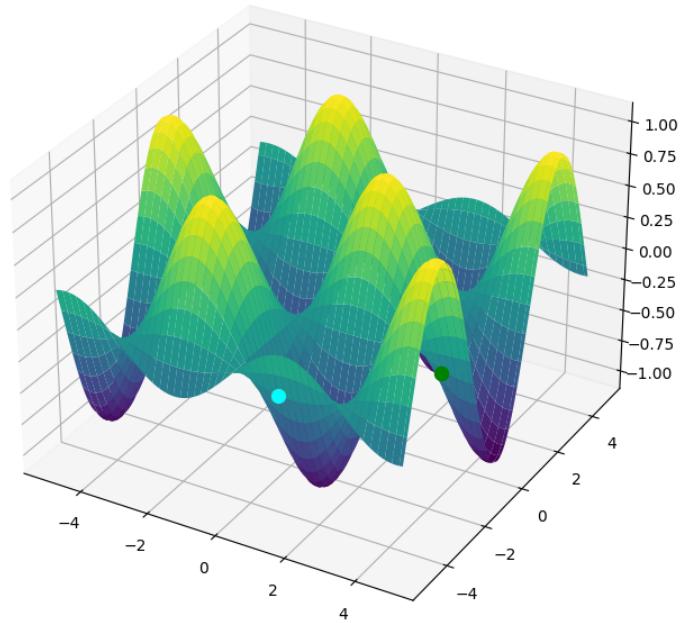
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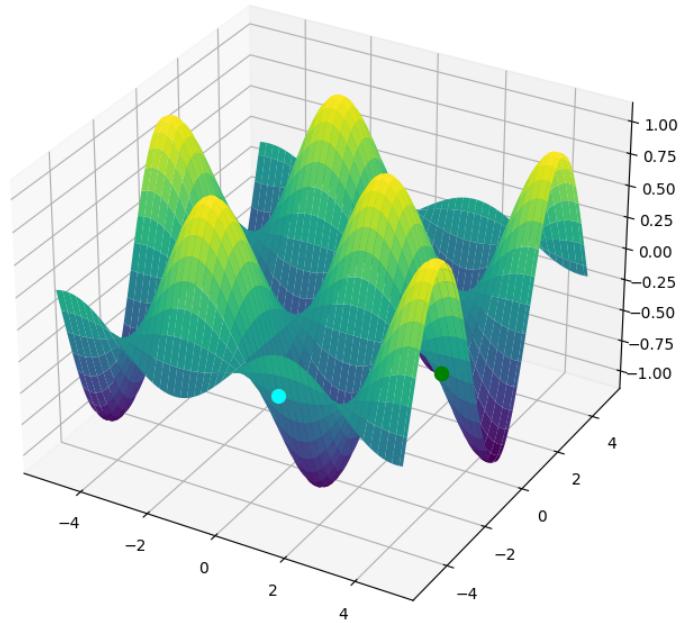
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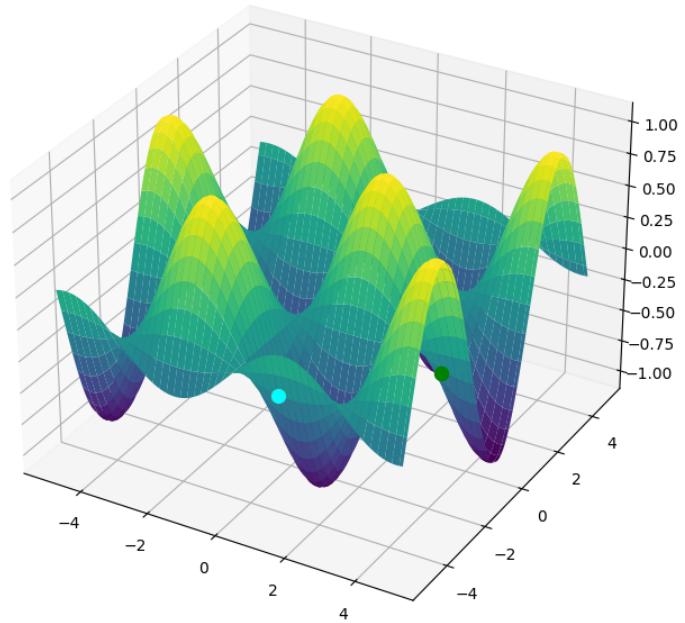
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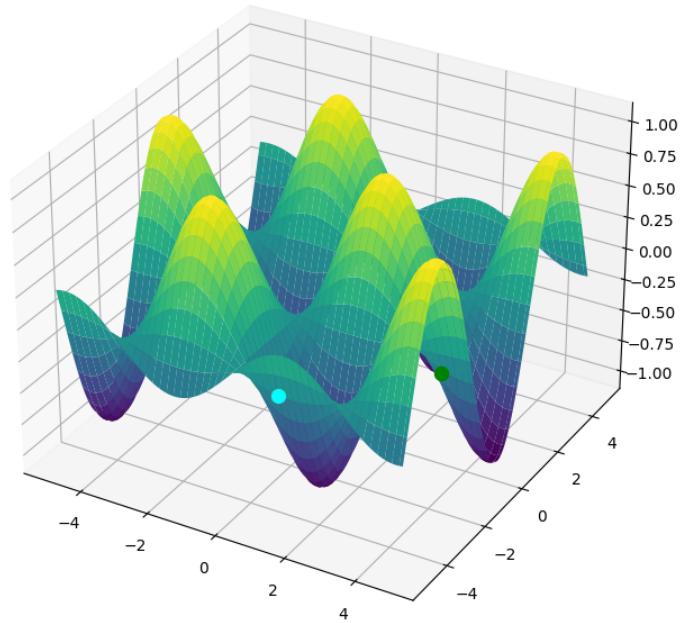
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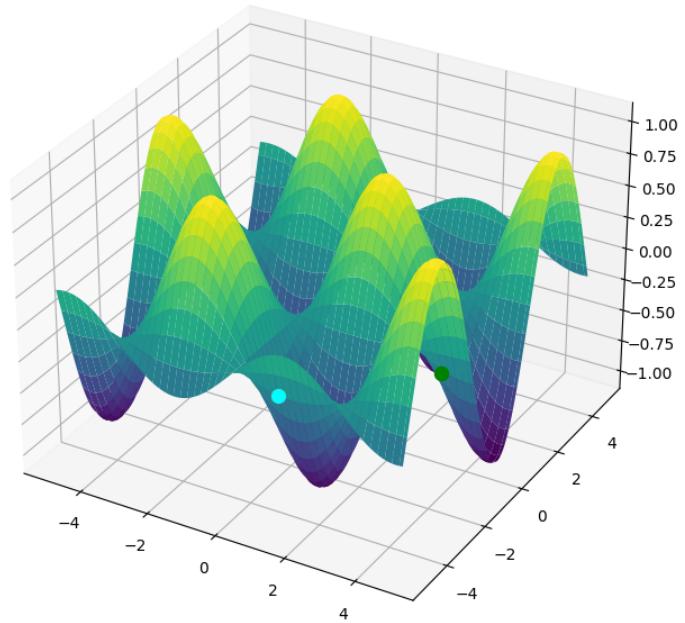
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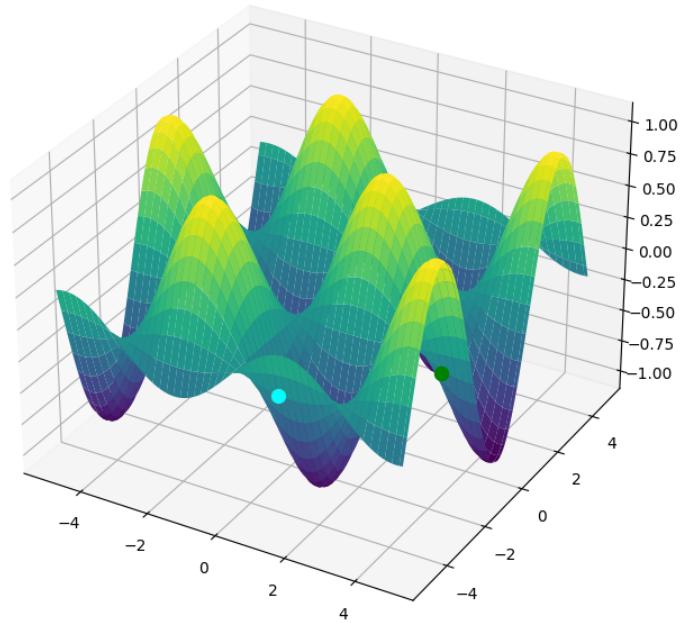
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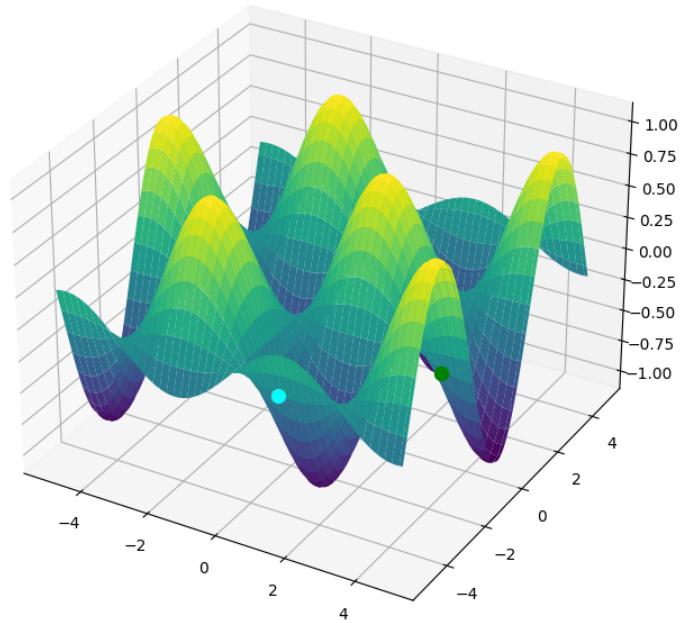
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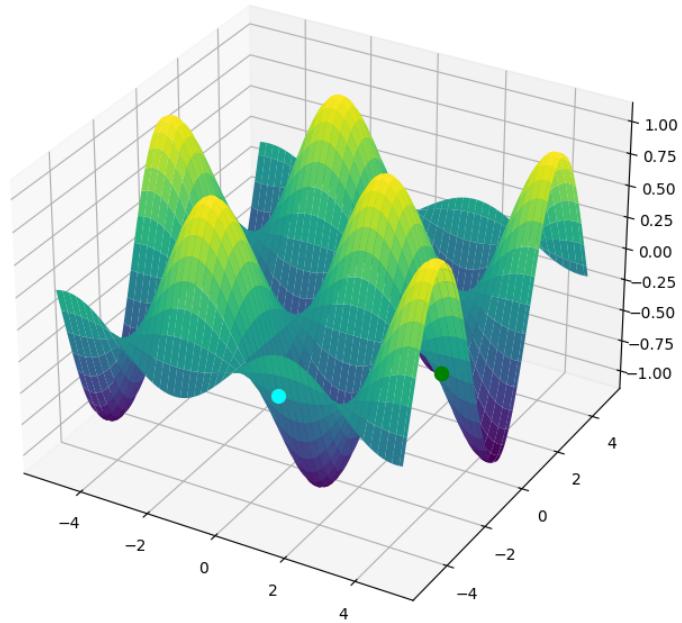
Epoch No: 88



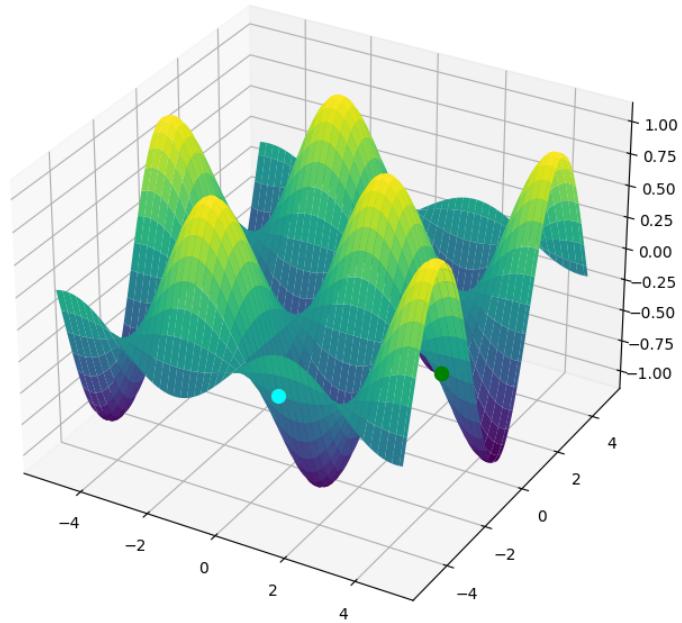
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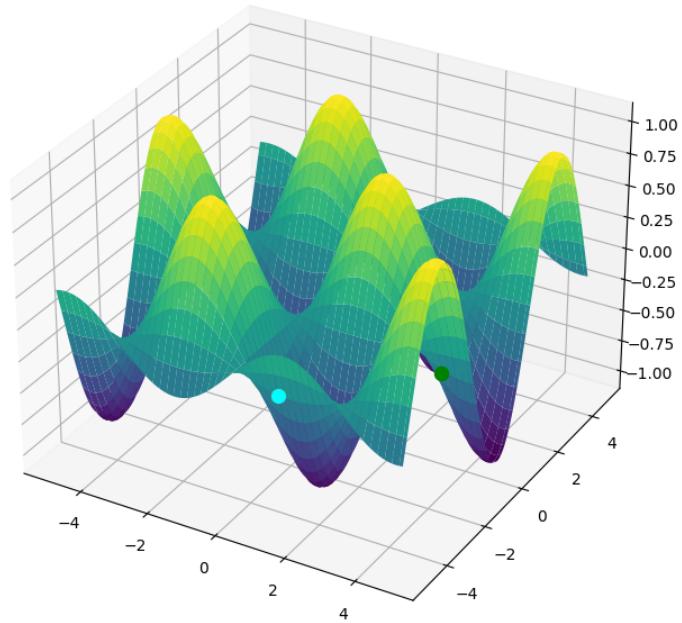
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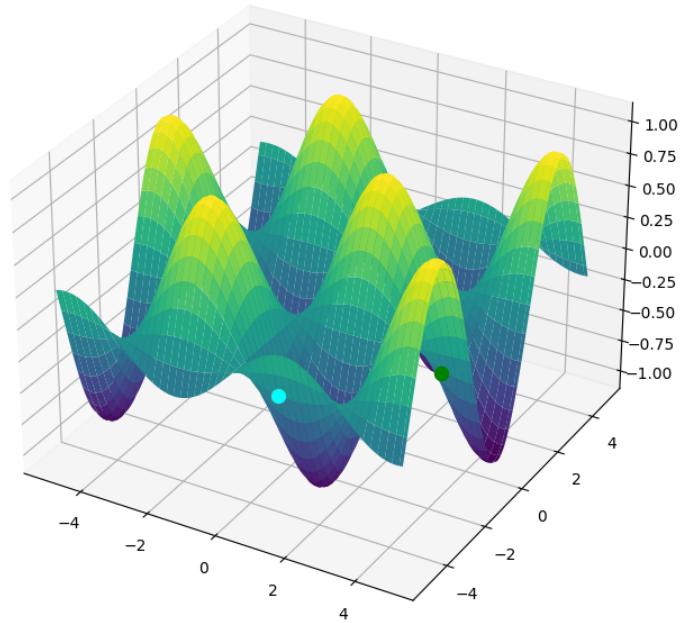
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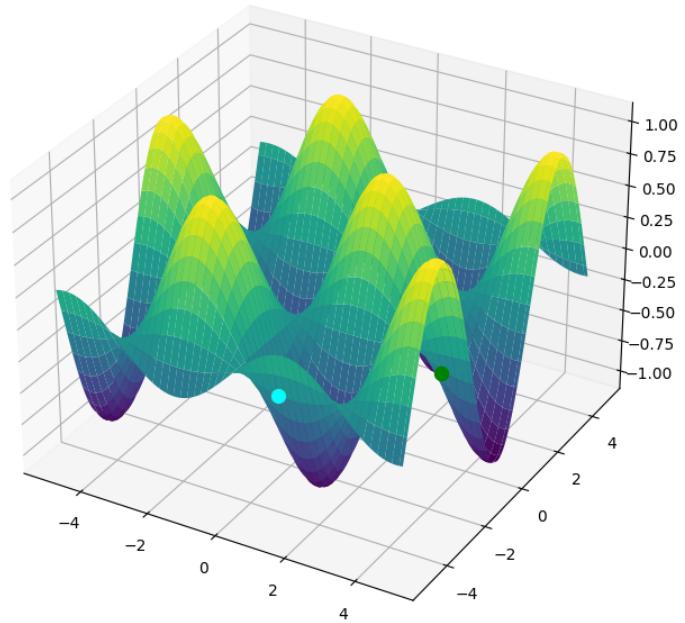
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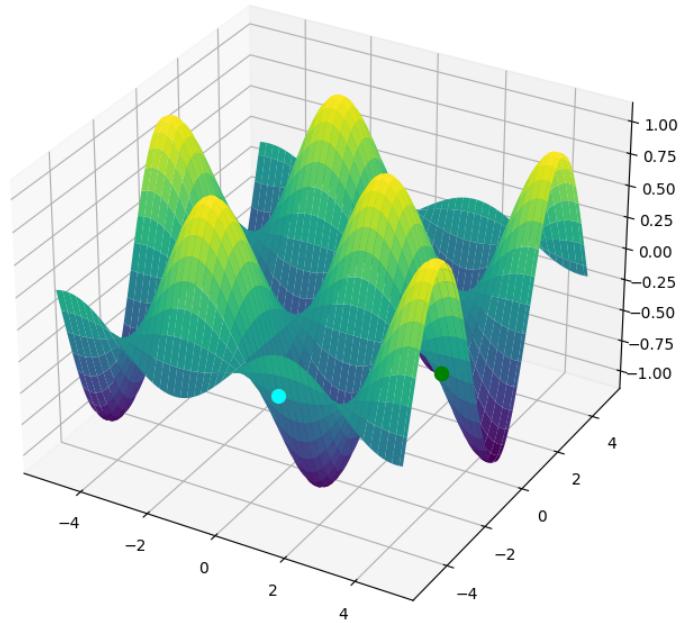
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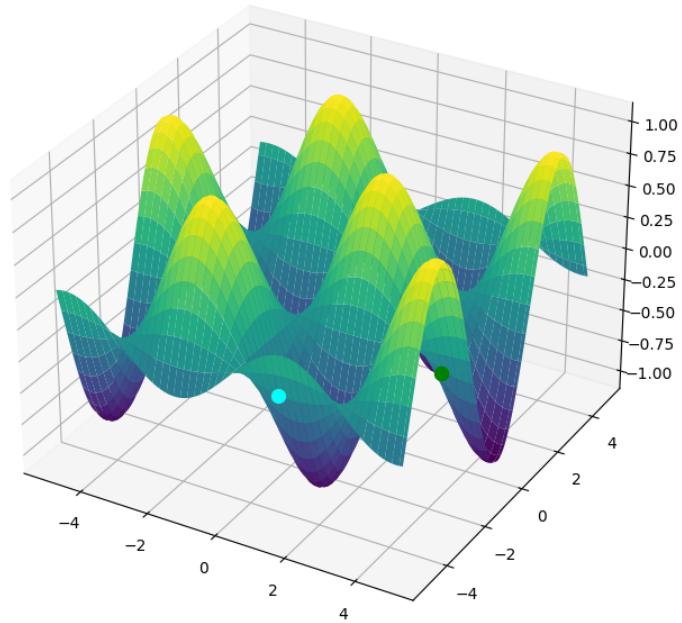
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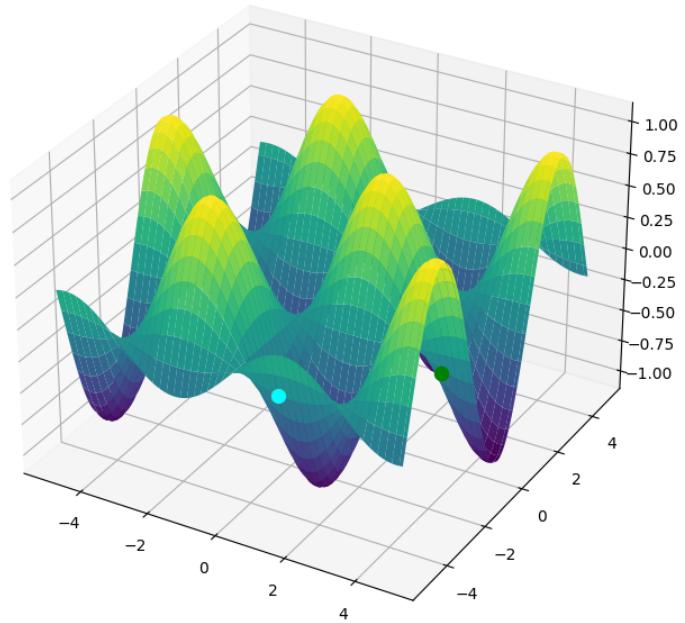
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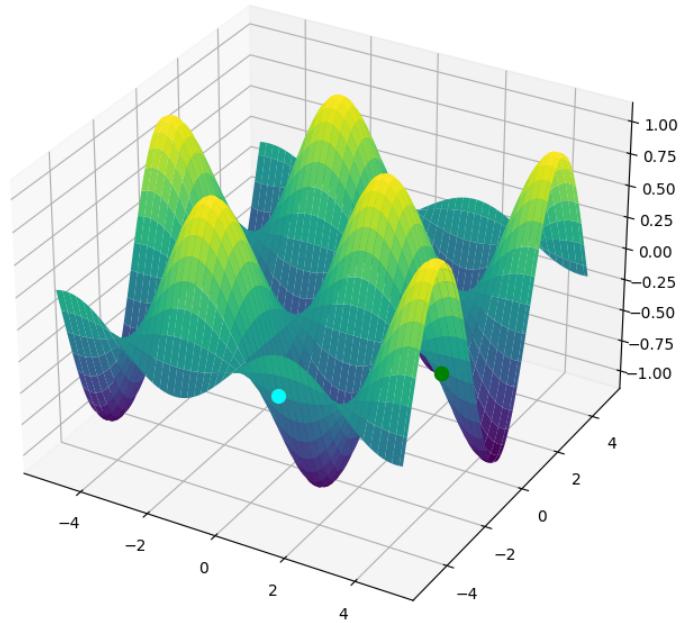
Epoch No: 96



Epoch No: 97



Epoch No: 98



Epoch No: 99

