ml-scratch-convolution

December 17, 2023

1 Convolution

For two functions $f(x)$ and $g(x)$, the convolution function $f(x) * g(x)$ is defined as:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) d\tau \tag{1}$$

for **discrete** samples that we deal with:

$$y[n] = f[n] * g[n] = \sum_{k=-\infty}^{\infty} f[k] \cdot g[n - k] \tag{2}$$

if f has N samples and g has M samples, then the convolved function has $N + M - 1$ samples. A basic rule: ***"flip any one of the functions, overlap it with the stationary one, multiply and add, and then traverse over."***

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

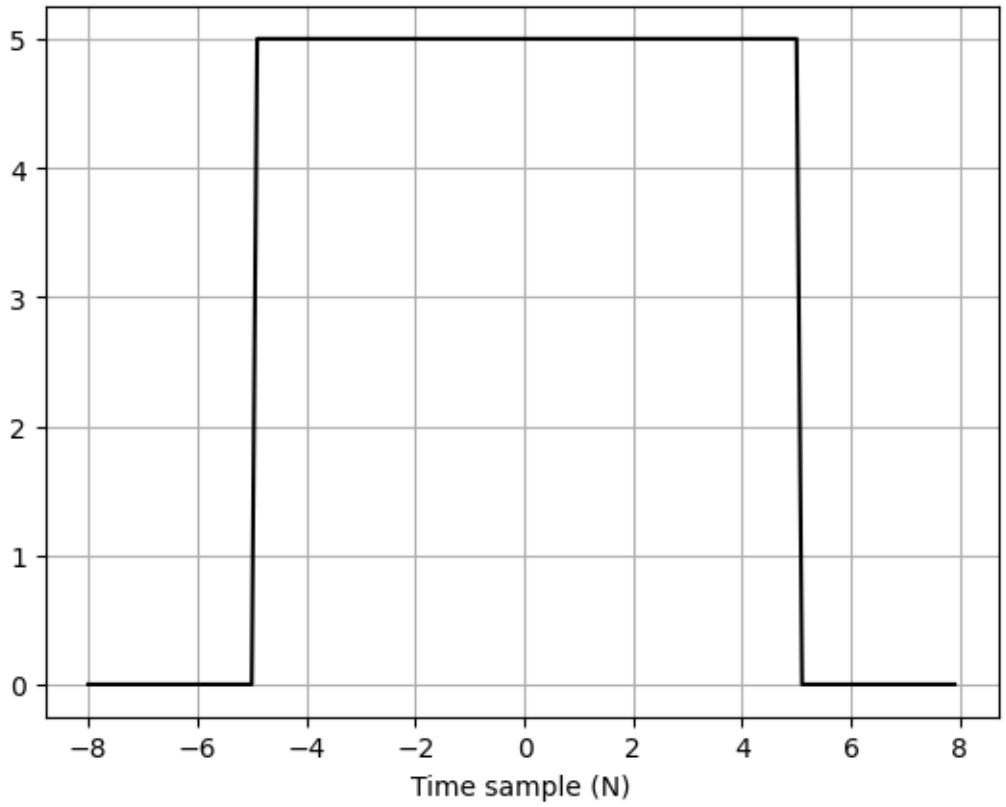
[24]:
```
def triangle(x):
  if x >= 0 and x <= 5:
    return 5 - x
  elif x < 0 and x >= -5:
    return 5 + x
  else:
    return 0

def rect(x):
  if -5 <= x <= 5:
    return 5
  return 0

def signal(f, x):
  return [f(i) for i in x]
```

[25]:
```
x = np.arange(-8, 8, 0.1)
f_x = signal(rect, x)
plt.plot(x, f_x, color = "black")
```

```
plt.xlabel("Time sample (N)")
plt.grid()
plt.show()
```



$O(N^2)$ complexity algorithm total $(N + M - 1) \cdot M$ **or** $(N + M - 1) \cdot N$ computations

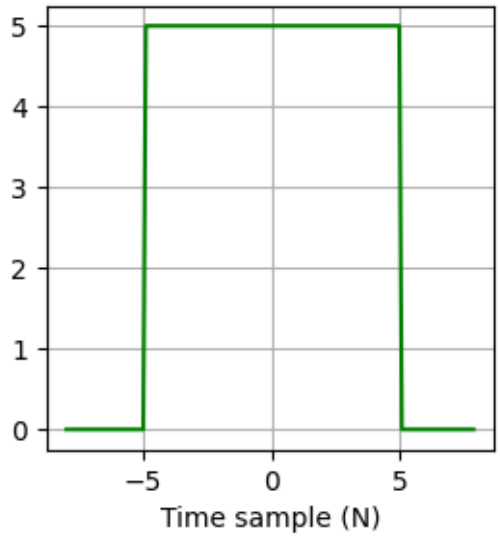
[28]:
```
A_signal, B_kernel = signal(rect, x), signal(rect, x)
N_signal, M_kernel = len(A_signal), len(B_kernel)
Y = np.zeros(N_signal + M_kernel - 1)
for i in range(len(Y)):
  for j in range(M_kernel):
    if i - j >= 0 and i - j < N_signal:
      Y[i] += A_signal[i - j] * B_kernel[j]
```

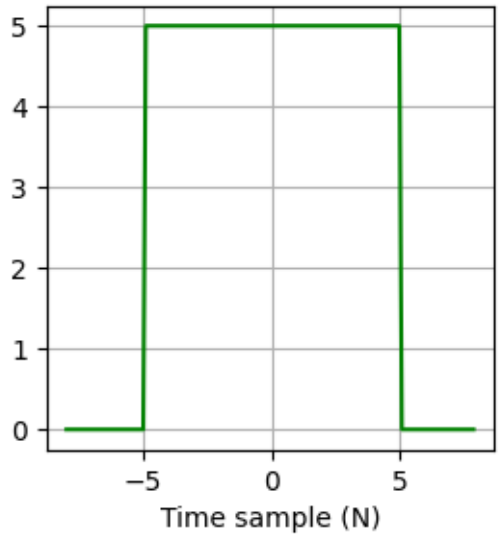
1.0.1 Convolution of two rect(x)

[32]:
```
plt.figure(figsize = (3, 3))
plt.plot(x, A_signal, color = "green")
plt.xlabel("Time sample (N)")
plt.grid()
plt.show()
```

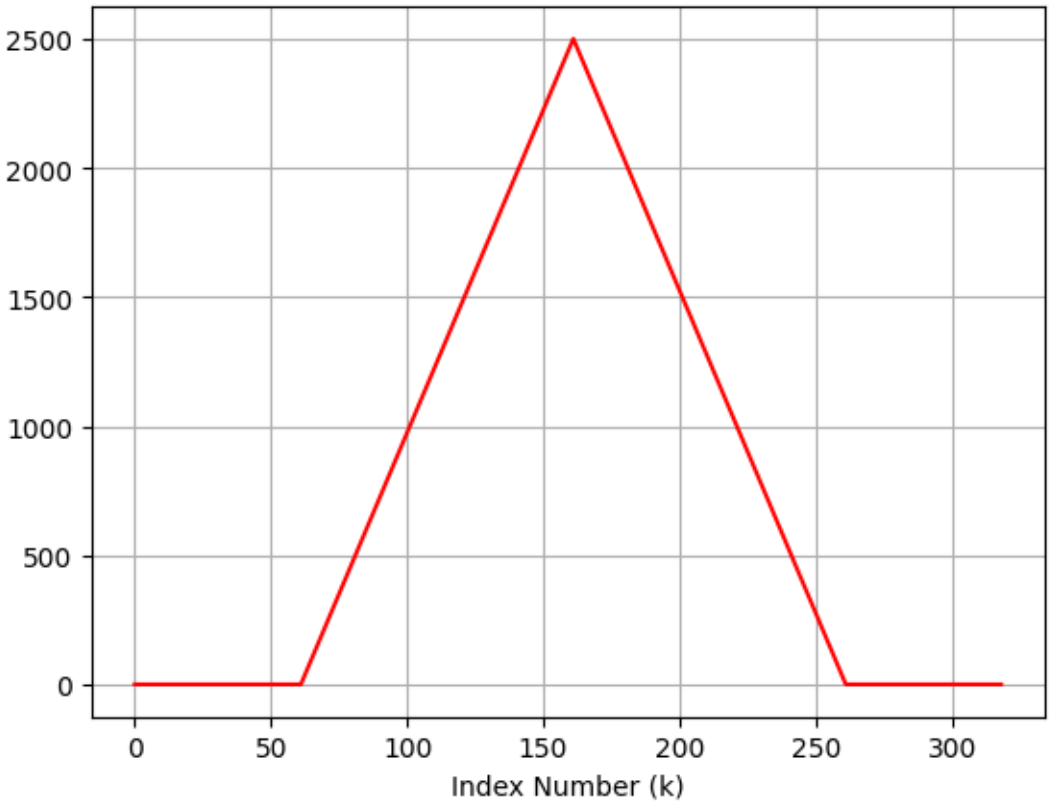
```
plt.figure(figsize = (3, 3))
plt.plot(x, A_signal, color = "green")
plt.xlabel("Time sample (N)")
plt.grid()
plt.show()

print(f"Signal Length: {N_signal}, Convolved Signal Lenght: {len(Y)}")
plt.plot(Y, color = "red")
plt.xlabel("Index Number (k)")
plt.grid()
plt.show()
```





```
Signal Length: 160, Convolved Signal Lenght: 319
```



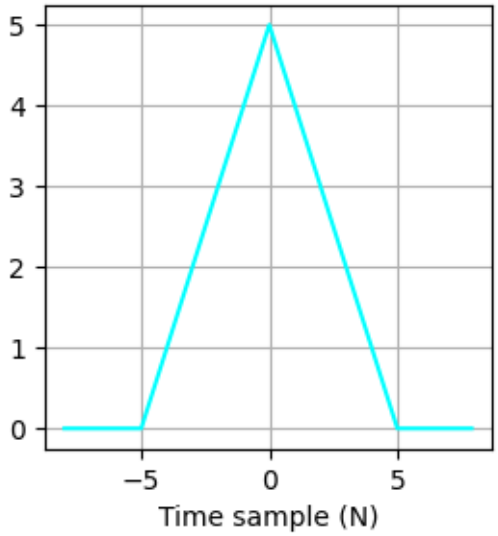
[33]:
```
A_signal, B_kernel = signal(triangle, x), signal(triangle, x)
N_signal, M_kernel = len(A_signal), len(B_kernel)
Y = np.zeros(N_signal + M_kernel - 1)
for i in range(len(Y)):
  for j in range(M_kernel):
    if i - j >= 0 and i - j < N_signal:
      Y[i] += A_signal[i - j] * B_kernel[j]
```

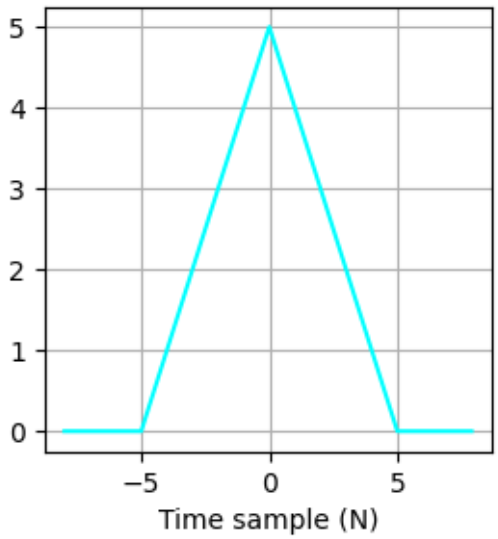
1.0.2 Convolution of two triangle(x)

[35]:
```
plt.figure(figsize = (3, 3))
plt.plot(x, A_signal, color = "cyan")
plt.xlabel("Time sample (N)")
plt.grid()
plt.show()
```

```
plt.figure(figsize = (3, 3))
plt.plot(x, A_signal, color = "cyan")
plt.xlabel("Time sample (N)")
plt.grid()
plt.show()

print(f"Signal Length: {N_signal}, Convolved Signal Lenght: {len(Y)}")
plt.plot(Y, color = "blue")
plt.xlabel("Index Number (k)")
plt.grid()
plt.show()
```





```
Signal Length: 160, Convolved Signal Lenght: 319
```