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Robust Optimization for Power Systems Capacity Expansion Under Uncertainty

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We develop a robust optimization model for planning power system capacity expansion in the face of uncertain power demand. The model generates capacity expansion plans that are both solution and model robust. That is, the optimal solution from the model is 'almost' optimal for any realization of the demand scenarios (i.e. solution robustness). Furthermore, the optimal solution has reduced excess capacity for any realization of the scenarios (i.e. model robustness). Experience with a characteristic test problem illustrates not only the unavoidable trade-offs between solution and model robustness, but also the effectiveness of the model in controlling the sensitivity of its solution to the uncertain input data. The experiments also illustrate the differences of robust optimization from the classical stochastic programming formulation.

Key words: stochastic programming, multi-objectives, electricity

INTRODUCTION

Planning large-scale systems in the presence of uncertain and incomplete information has been a central problem in the practice of management science and operational research. The domain of planning power utilities—either for generation or expansion purposes—has been one of the areas where uncertainty plays a key role. Several applications of management science techniques have been proposed for this broad problem class, starting in the early days of linear programming[1]. More recent work has concentrated on the development of models that explicitly take into account some aspect of uncertainty. In particular, stochastic programming formulations of diverse aspects of power utility planning have been proposed by many investigators[2–5].

In a recent paper Mulvey *et al.*[6] introduced the notion of robust optimization for planning large-scale systems that are subjected to noisy, uncertain or incomplete input data. In particular, they developed a general modelling (optimization) framework that controls the sensitivity of an optimal solution to the noise of the input data. The need for robustness in several applications had already been recognized. Paraskevopoulos *et al.*[7] proposed a capacity planning model for robust optimization under uncertainty of an application from the PVC industry. The model was proven to be effective in controlling the sensitivity of the model's recommendations to the uncertain data. Escudero *et al.*[8] proposed a robust optimization model for planning manufacturing and outsourcing decisions in a production system. Sengupta[9] discussed the notion of robustness in the context of stochastic programming models. However, practical applications of this methodology were not discussed. Mulvey *et al.*[6] have illustrated the effectiveness of their general robust optimization framework on a simple instance of the well-known diet problem.

In this report we develop a robust optimization model for the problem of planning capacity expansion of power systems under uncertain load forecasts. Our model is a generalization of the stochastic programming model of Murphy *et al.*[2] It is a special instance of the robust optimization framework. The model generates capacity expansion plans that are both solution and model robust. That is, the optimal solution from the model is 'almost' optimal for any realization of the demand scenarios (i.e. solution robustness). Furthermore, the optimal solution has 'almost' no excess capacity for any realization of the scenarios (i.e. model robustness).

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Extensive numerical experiences with a characteristic test problem illustrate the unavoidable trade-offs between solution and model robustness. Even more to the point, however, the experiments illustrate that the sensitivity of the model's solution to the problem's noisy data can be controlled. They also illustrate the differences of robust optimization from the classical stochastic programming formulation.

This paper serves a dual objective: we develop a model for robust planning of power system capacity expansion, and at the same time, however, we provide an example instance of the framework of robust optimization. We begin by reviewing the general framework of robust optimization. We follow this by developing the model for power system capacity expansion. Finally, numerical experiments and the relevance of robust planning for utility planning are discussed.

THE GENERAL MODELLING FRAMEWORK OF ROBUST OPTIMIZATION

We introduce first the general robust optimization modelling framework of Mulvey *et al.*[6] This will form the basis for the power capacity expansion model. We are dealing with optimization models that have two distinct components: a structural component that is fixed and free of any noise in its input data, and a control component that is subjected to noisy input data. In order to define the appropriate model we introduce two sets of variables:

$x \in \mathfrak{R}^{n_1}$, denotes the vector of decision variables that depend only on the fixed, structural constraints. These are the design variables whose optimal value is independent of any realization of the uncertain parameters.

$y \in \mathfrak{R}^{n_2}$, denotes the vector of control decision variables that could be adjusted once the uncertain parameters are observed. Their optimal value depends both on the realization of uncertain parameters, and on the value of the design variables.

We consider (non-robust) optimization models of the following general structure:

$$\text{[LP] minimize} \quad c^{\mathrm{T}}x + d^{\mathrm{T}}y \tag{1}$$

$$x \in \mathfrak{R}^{n_1}, \qquad y \in \mathfrak{R}^{n_2}$$

$$\text{subject to:} \quad Ax = b \tag{2}$$

$$Bx + Cy = e \tag{3}$$

$$x \geqslant 0, \qquad y \geqslant 0. \tag{4}$$

Equation (2) denotes the structural constraints that are fixed and free of noise. Equation (3) denotes the control constraints. The coefficients of this constraint set are subject to noise.

To define the robust optimization problem introduce a set of scenarios $\mathcal{S} = \{1, 2, 3, \ldots, S\}$. With each scenario $s \in \mathcal{S}$ associate the set $\{d_s, B_s, C_s, e_s\}$ of realizations for the coefficients of the control constraints, and the probability of the scenario p_s, $(\sum_{s=1}^{S} p_s = 1)$. The optimal solution of the mathematical program (1)–(4) will be robust with respect to optimality if it remains close to optimal for any realization of the scenario $s \in \mathcal{S}$. It is then termed solution robust. The solution is also robust with respect to feasibility if it remains almost feasible for any realization of s. It is then termed model robust.

Of course, it is unlikely that any solution to the mathematical program will remain both feasible and optimal for any realization of s. If the system that is being modelled has substantial redundancies built in, then it might be possible to find solutions that remain both feasible and optimal. Otherwise a model is needed that will allow us to measure the trade-off between solution and model robustness. The following model formalizes a way to measure this trade-off. It also controls both solution and model robustness.

Introduce a set $\{y_1, y_2, \ldots, y_S\}$ of control variables for each scenario $s \in \mathcal{S}$. Also, introduce a set $\{z_1, z_2, \ldots, z_S\}$ of error vectors that will measure the infeasibility allowed in the control constraints under scenario s. Consider now the following compact formulation of the robust optimization model:

$$\text{[Robust]} \quad \text{minimize} \quad \sigma(x, y_1, \ldots, y_S) + \omega\rho(z_1, \ldots, z_S), \tag{5}$$

$$\text{subject to:} \quad Ax = b \tag{6}$$

$$B_s x + C_s y_s + z_s = e_s \quad \text{for all } s \in \mathcal{S}, \tag{7}$$

$$x \geq 0, \quad y_s \geq 0 \quad \text{for all } s \in \mathcal{S}. \tag{8}$$

With multiple scenarios, the objective function $\xi = c^T x + d^T y$ becomes a random variable taking the value $\xi_s = c^T x + d_s^T y_s$ with probability p_s. Hence, there is no longer a single choice for an aggregate objective. Mulvey *et al.*[6] suggested several suitable alternatives. One such choice is to minimize the expected value of the objective function, plus a parameterized function of its variance. This is precisely the function we will be using in the capacity expansion model. For now we summarize all possible choices by simply calling the aggregate function $\sigma(x, y_1, \ldots, y_S)$.

The function $\rho(z_1, \ldots, z_S)$ is a feasibility penalty function. It is used to penalize violations of the control constraints under some of the scenarios, given a value of the design variable x, and the appropriate control variables under the same scenario. The model proposed above takes a multi-criteria objective form. The first term measures the optimality robustness, whereas the penalty term is a measure of model robustness. The goal programming weight ω is used to derive a spectrum of answers that trade-off solution for model robustness.

THE POWER SYSTEM CAPACITY EXPANSION MODEL

In its simplest form, the power system capacity expansion problem can be described as follows: 'Select design capacities over a given set of plants that minimize the capital and operating costs of the system, meet customer demand, and satisfy physical constraints'. Many other linear formulations are possible which feature different objectives and constraints. For instance, environmental impacts, fuel use restrictions or reliability requirements could be included.

Demand for electric power is not constant over time. Electricity use is much higher at noon than midnight, it changes with the season, and it exhibits long-term, yearly variations. A typical daily variation in demand is shown in Figure 1(a). The load duration curve (Figure 1(b)) is obtained by rearranging the instantaneous demands in decreasing order. To facilitate use of a mathematical programming model the continuous curve is approximated by step functions corresponding to a particular mode of operation (i.e. base, peak, cycling and so on). This is illustrated in Figure 2, where P_j and θ_j are the demand and duration of mode of operation j, respectively.

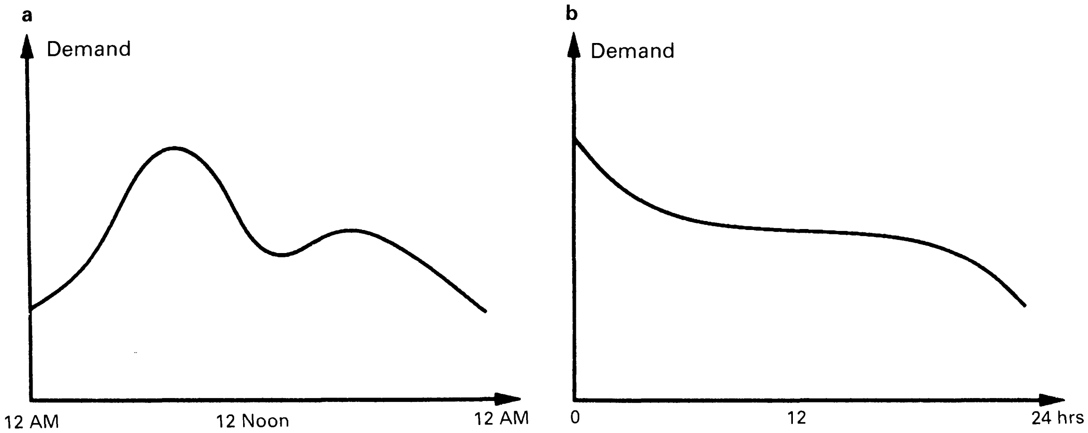

FIG. 1. *Daily and cumulative load duration curves.*

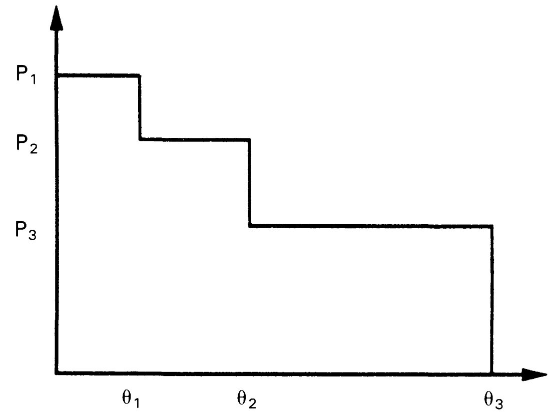

FIG. 2. *Piecewise linear load duration curve.*

For a single period, the power system planning model [PSP] is given by:

[PSP] minimize
$$\sum_{i\in I} c_i x_i + \sum_{j\in J}\theta_j \sum_{i\in I} f_i y_{ij} \tag{9}$$

subject to:
$$x_i - \sum_{j\in J} y_{ij} \geqslant 0 \qquad \text{for all } i \in I \tag{10}$$

$$\theta_j \sum_{i\in I} y_{ij} = d_j \qquad \text{for all } j \in J \tag{11}$$

$$x_i \geqslant 0, \qquad y_{ij} \geqslant 0 \qquad \text{for all } i \in I,\ j \in J. \tag{12}$$

I denotes the set of plant types (e.g. hydro, coal, etc), and J is the set of operation modes (e.g. base, peak). c_i and f_i are the annualized fixed cost (\$/MW) and operating cost (\$/MWh) respectively for plant $i \in I$. θ_j and d_j are the duration (h) and energy demand (MWh) of operation mode $j \in J$. Energy demands d_j are given by

$$d_j = (P_j - P_{j-1})\theta_j. \tag{13}$$

Decision variables x_i denote the total capacity assigned to plant $i \in I$. These are the design variables. Variables y_{ij} denote the allocation of capacity to operation mode j from plant type i. These are the control variables, and their values are determined after the capacities (x_i) have been constructed, and the demand levels (d_j) have been observed. Note there are no constraints of the form $Ax = b$. This is a result of the independence of the plants. Use of one plant type does not restrict the use of any other plant type. The underlying network structure of the problem is shown in Figure 3.

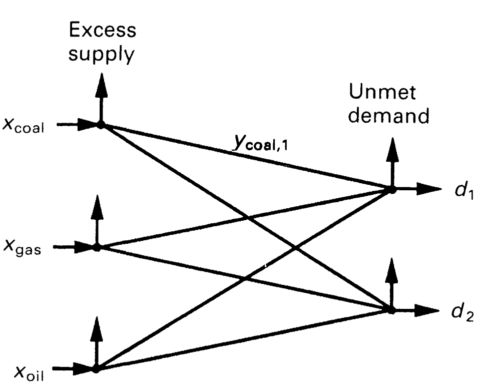

FIG. 3. *Network representation of power system.*

A decision is made today to build a set of plants I with capacities x_i. The capacities of each plant type are based on projections of future costs and demands. Since the future is not known with precision, the allocations y_{ij} (control variables that depend on revelation of future data) may differ from 'optimal' values determined by the model, possibly at great expense to the utility. It is desirable to make the chosen expansion plan insensitive to uncertainties in data. Insensitivity to uncertainty is not without cost, however. The following model provides an explicit method for quantifying the trade-offs between deviations from optimality (solution robustness) and feasibility (model robustness).

To generate the robust optimization formulation for the power systems planning model, a set of scenarios $s \in \mathcal{S}$ (with probability p_s) is introduced over the uncertain energy demands $\{d\}$. The control variables become scenario dependent and are denoted by y_{ij}^s. While uncertainty can be introduced into the model via costs, demand and load duration, for the purposes of this paper only uncertainty in demand will be considered. Of course, the model could be extended to address other forms of uncertainty as well. A robust optimization formulation of problem [PSP] is given by:

[R-PSP] minimize
$$\sum_{s\in\mathcal{S}} p_s \xi_s + \lambda \sum_{s\in\mathcal{S}} p_s \left(\xi_s - \sum_{s'\in\mathcal{S}} p_{s'}\xi_{s'}\right)^2 + \omega \sum_{s\in\mathcal{S}} p_s \left(\sum_{i\in I}(z_{1i}^s)^2 + \sum_{j\in J}(z_{2j}^s)^2\right), \tag{14}$$

subject to:
$$x_i - \sum_{j\in J} y_{ij}^s + z_{1i}^s = 0 \quad \text{for all } i \in I,\ s \in \mathcal{S} \tag{15}$$

$$\theta_j \sum_{i\in I} y_{ij}^s - d_j^s + z_{2j}^s = 0 \quad \text{for all } j \in J,\ s \in \mathcal{S} \tag{16}$$

$$x_i \geqslant 0, \quad y_{ij}^s \geqslant 0 \quad \text{for all } i \in I,\ j \in J,\ s \in \mathcal{S}. \tag{17}$$

The function ξ_s is defined as

$$\xi_s = \sum_{i\in I} c_i x_i + \sum_{j\in J} \theta_j \sum_{i\in I} f_i y_{ij}^s \tag{18}$$

and represents the total operating and capital cost of the system under scenario s. Uncertainty in demand can cause two types of error: z_i^s represents surplus capacity of plant type i given scenario s, and z_{2j}^s represents unmet demand of mode j given scenario s. Unmet demand clearly has an adverse effect on society by imposing shortages. Surplus capacity means capital resources are not being utilized efficiently, and may affect a utility's future expansion capability by committing it to a particular technology. With the introduction of additional constraints, different types of error relating to uncertainty in those constraints would also be present. The rest of the parameters are defined as in [PSP].

The objective function of the robust model is composed of three terms. The first term is the expected cost of the system over all possible scenarios. The second term is the variance of the cost ξ_s, weighted by a constant λ. The final term is a function that penalizes deviations from feasibility, weighted by ω. A measure of solution and model robustness is obtained by varying the penalty parameters, λ and ω, and observing the changes in expected value and expected infeasibility. Solutions in the multiobjective space defined by expected cost, variance of cost, and expected infeasibility are thus obtained by varying the penalty parameters.

RESULTS AND DISCUSSION

We now apply the robust optimization model to the test problem of Murphy *et al.*[2] A variety of experiments are carried out in order to illustrate the unavoidable trade-offs between solution and model robustness. The important observation in this respect is not that trade-offs exist, but that they can be quantified using a robust optimization model. Furthermore, we illustrate that the model does indeed produce solutions that are solution or model robust. The results of our model are also compared with the solution of the stochastic programming formulation of Murphy *et al.*[2] In this respect our experiments provide some insight into the differences between classical stochastic programming models and robust optimization.

The test problem

The robust optimization formulation is applied to the model of Murphy *et al.*[2] Four supply options are available, $\{A, B, C, D\}$. The cost parameters corresponding to these options are low capital cost/high operating cost, high capital cost/low operating cost, medium capital cost/medium operating cost, and zero capital cost/very high operating cost, used to meet possible shortage situations by outsourcing to other utility companies (Table 1). Four demand scenarios over two operating modes (peak and base) are considered, each scenario being equally likely. The load duration curve parameters for each of the scenarios are shown in Figure 4. The use of recourse supply option D is not penalized (i.e. $z_{1D}^{s} \equiv 0$), since it is not installed capacity and is only used whenever necessary to meet demand in excess of existing capacity. Also, as it is required to meet demand (possibly via use of external supply type D), unmet demand $z_{2j}^{s} \equiv 0$.

TABLE 1. *Supply options and associated costs*

Plant types	Capital cost	Operating cost
A	200	30
B	500	10
C	300	20
D	0	200

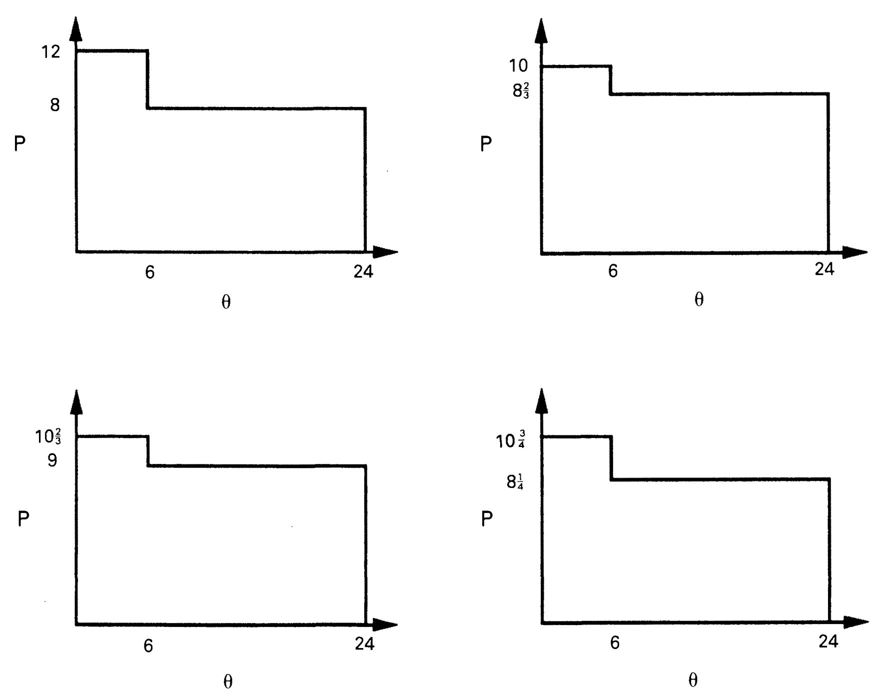

FIG. 4. *Cumulative load duration curves for four scenarios.*

Benchmark solutions

If it is known for certain that a particular demand scenario will occur, an optimal plan can be implemented. On the other hand, if a scenario occurs which does not match that assumed by a particular plan, a sub-optimal strategy will result. The costs of implementing the optimal plan for each scenario, given that particular scenario occurs, are shown in Table 2. The diagonal entries denote the cost of optimal policies. The off-diagonal entries reflect the cost of making an incorrect decision. The expected cost, standard deviation and expected excess capacity which result from implementing each strategy are also shown.

The stochastic programming model is obtained by setting λ and $\omega = 0$. The stochastic programming solution is shown in Table 3. Although this strategy has lower expected cost than the 'best optimal' strategy (i.e. Plan I), it will result in idle capacity in three of the scenarios, representing 75% of the possible demands. The stochastic solution emphasizes feasibility for the most restrictive scenario, regardless of the probability of occurrence.

TABLE 2. *Individual optimal solutions under differing scenarios. *denotes optimal plan for each scenario*

Plan	Scenario				Expected cost	σ	Excess capacity
	1	2	3	4			
I	7440*	7500	7740	7350	7508	144	1.145
II	9200	6920*	7900	7745	7941	816	0
III	8654	7114	7295*	8024	7772	613	0.188
IV	8510	7145	7445	7055*	7539	579	0.208

TABLE 3. *Stochastic programming solution* ($x_A = 3.33$, $x_B = 8.67$)

Scenario				Expected cost	σ	Excess capacity
1	2	3	4			
7560	7320	7620	7380	7470	124	1.145

Robust optimization solutions

The model [R-PSP] was run by varying λ and ω over suitable ranges. For each (λ, ω) pair, we computed expected cost, standard deviation and expected excess capacity. Note that since capacity type D provides complete recourse, unmet demand equals 0. It should be pointed out that expected costs are an upper bound as operating costs are based on values of the control variables determined from the model, which reflect goals other than only cost minimization. Once plants with the required design capacities are built, a less expensive allocation may be possible after a demand scenario becomes known.

For a given λ, increasing ω reduces expected error to zero (in the limit), with faster convergence for smaller λ (Figure 5). This result is intuitive. If little or no emphasis is placed on minimizing the variance of the solution, it is easier to reduce the error term. Conflict between objectives is reduced.

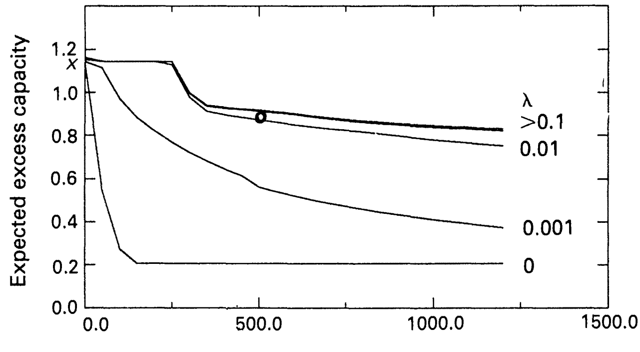

FIG. 5. *Controlling model robustness: excess capacity versus* ω. *(O denotes robust solution described in text, x denotes stochastic programming solution).*

Figure 6 shows the relationship of λ and ω to the variation in cost. With $\lambda > 0$, the standard deviation of cost is roughly constant and proportional to λ. Setting $\lambda = 0$ removes the penalty on cost variance over the scenarios. Standard deviation is thus unrelated to ω for $\lambda = 0$.

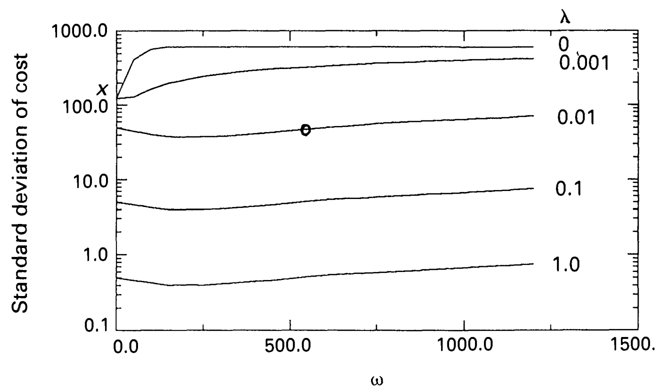

FIG. 6. *Controlling solution robustness: Standard deviation versus ω.*

Expected cost as a function of the penalty parameters is shown in Figure 7. Cost is near the stochastic programming solution for low ω, as excess capacity is not being penalized much. Expensive recourse capacity of type D is not required. Increasing λ for a given ω trades type D capacity for type C and A capacities. Having these capacities in place allows variations in demand to be met by these lower operating cost capacities rather than by the expensive type D capacity. This comes at the expense of higher fixed cost. As ω is increased further, type D is again brought on since types A, B and C are now required to be almost fully utilized.

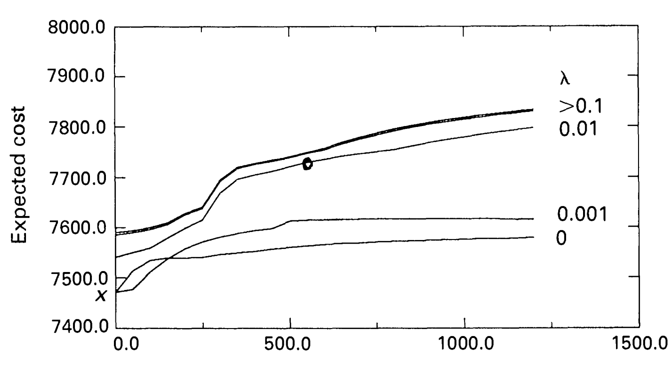

FIG. 7. *The cost of robustness for different levels of model (ω) and solution (λ) robustness.*

It can be seen in Figures 5–7 that the contours of constant λ become indistinguishable as λ increases. This suggests that there is a limit to how 'good' one attribute can get for fixed values of the others. Unattainable goals for plans can thus be identified. For example, suppose it is desired for a plan to have an expected cost < 7600, standard deviation < 50, and expected excess capacity < 0.8. The first condition requires $\lambda < 0.001$ or $\omega < 250$ for $\lambda > 0.001$, the second requires $\lambda > 0.01$, while the third requires $\lambda < 0.001$ or $\omega \gg 250$ for $\lambda > 0.001$. Clearly, no pair (λ, ω) can be found which meets these criteria.

An example of a possible robust solution is given by $\lambda = 0.01$ and $\omega = 512$. For these parameter values, the model produces the solution shown in Table 4. The points corresponding to this solution are circled (○) on Figures 5–7. While expected cost is higher than in the stochastic programming model (by 3.3%), the amount of unutilized capacity is reduced by 23%. The solution is therefore more model robust. The variation around the cost is also reduced (41 against 123), providing greater solution robustness. Whether or not these trade-offs are acceptable is dependent on the values of the decision maker.

TABLE 4. *A recommended robust programming solution,* $\lambda = 0.01$, $\omega = 512$ ($x_A = 3.21$, $x_B = 8.00$, $x_C = 0.452$), *and stochastic programming solution*

	Scenario				Expected cost	σ	Expected excess capacity
	1	2	3	4			
Robust	7772	7688	7730	7663	7713	41	0.890
Stoch. prog.	7560	7320	7620	7380	7470	124	1.145

Parameters λ and ω can be seen as reflecting the risk attitude of the decision maker. Driving ω high forces the maximum utilization of all installed capacity, but at the expense of purchasing power from external sources if a scenario emerges which requires extra capacity. High λ produces a set of design capacities and allocations whose cost is insensitive to whatever scenario evolves. Obviously, the more robustness desired, the higher the cost.

CONCLUSIONS

Incorporating uncertainty into power system planning and the search for robust solutions is becoming increasingly important, in the view of planning authorities[10,11]. The robust planning model can be used to address many issues of current importance to electric utilities. Utilities are penalized for excess capacity which is not 'used and useful' by not being able to include such capacity costs in their rate base. The robust model can help planners identify trade-offs between the inability to recover fully costs for excess capacity, and the need to purchase outside power or suffer shortages in the presence of uncertainty in demand. For utilities that have neighbours that generate reliable surplus electricity or that belong to a power pool, the robust model can assist planners by determining the indigenous capacity required. The model can also assist utilities to plan in environments where there is growth in self-generation among its customers. The robust model can help the utility to formulate plans which hedge against the uncertainty of this captive generation. While the model does not address the important issue of pricing of electricity, it may be considered indirectly. Plans can be obtained which are robust for a given price structure. In addition, expansion plans which are robust to fluctuations in price (as reflected by the elasticities of demand) can be generated.

We have presented a robust optimization model for determining power system capacity expansion plans. It was shown that the model produces solution- and model-robust plans. Non-robust least-cost formulations take advantage of the economies of scale present in large plants. Plans produced by these methods are often sensitive to uncertainty in demand, expensive to modify or abandon, and over-emphasize low probability/high-demand scenarios which drive the solution. Robust solutions can be made insensitive to uncertainty by exploiting the trade-offs between optimality and feasibility over the set of possible scenarios; the degree of insensitivity required is dependent on the attitude of the decision maker. The model we propose and develop here provides a mechanism for recognizing the trade-offs and generating solutions that are consistent with the preferences of the decision maker. This is also the first practical application of the robust optimization methodology of Mulvey *et al.*[6] and illustrates the value of this methodology.

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