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Invited Review

A survey of stochastic modelling approaches for liberalised electricity markets

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A R T I C L E I N F O

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A B S T R A C T

Liberalisation of energy markets, climate policy and the promotion of renewable energy have changed the framework conditions of the formerly strictly regulated energy markets. Generating companies are mainly affected by these changing framework conditions as they are exposed to the different risks from liberalised energy markets in combination with huge and largely irreversible investments. Uncertainties facing generating companies include: the development of product prices for electricity as well as for primary energy carriers; technological developments; availability of power plants; the development of regulation and political context, as well as the behaviour of competitors.

The need for decision support tools in the energy business, mainly based on operation research models, has therefore significantly increased. Especially to cope with different uncertain parameters, several stochastic modelling approaches have been developed in the last few years for liberalised energy markets. In this context, the present paper aims to give an overview and classification of stochastic models dealing with price risks in electricity markets.

The focus is thereby placed on various stochastic methods developed in operation research with practical relevance and applicability, including the concepts of:

- stochastic processes for commodity prices (especially for electricity);
- scenario generation and reduction, which is important due to the need for a structured handling of large data amounts; as well as
- stochastic optimising models for investment decisions, short- and mid-term power production planning and long-term system optimisation.

The approaches within the energy business are classified according to the above structure. The practical relevance of the different methods and their applicability to real markets is thereby of crucial importance. Shortcomings of existing approaches and open issues that should be addressed by operation research are also discussed.

1. Introduction

Until the mid-nineties, the monopoly of power supply companies was justified by the existence of a public energy supply and with the existence of a natural monopoly (cp. for definition e.g. Berg, 1988) in the field of energy supply. Regional markets have been assigned to utilities with a monopoly status by so-called concessional contracts. Prices for electricity have been approved on the basis of the cost structure of the utilities, the forecasted electricity sales and a reasonable profit margin for energy utilities.

In recent decades, most energy markets have been liberalised and privatised with the aim of obtaining more reliable and cheaper services for electricity consumers. A major step in Europe was the Directive of the European Commission at the end of 1996 (European Commission, 1997), requiring the stepwise opening of electricity markets in the European Union, ending with a fully competitive market in 2010 at the latest. In this new context, several wholesale electricity markets have been established in many places and energy utilities have been unbundled into generation, transmission and distribution companies (for an overview of the unbundling progress in Europe, see European Commission, 2005). With liberalisation and the introduction of energy markets, decision making no longer depends on centralised state- or utility-based procedures, but rather on decentralised decisions of energy utilities whose goals are to maximise their own profits. All firms compete to provide services at a price set by the market, as a result of all of their interactions.

However, energy supply companies are exposed to significantly higher risks than in regulated markets. California is often cited as the outstanding example of the risks and difficulties associated

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with liberalisation. Above all, generation companies are affected by these changing framework conditions, as they are exposed to the different risks from liberalised energy markets in combination with huge and generally irreversible investments. Uncertainties that generation companies face include the development of product prices for electricity as well as for primary energy carriers (e.g. oil, gas, coal and uranium), technological developments, availability of power plants, the development of regulation and the political context, as well as the behaviour of competitors.

The need for decision support tools in the energy business mainly based on operation research models has therefore significantly increased. Especially to cope with different uncertain parameters, several stochastic modelling approaches have been developed in the last few years for liberalised energy markets. In this context, the present paper aims to give an overview and classification of stochastic models especially dealing with price risks in electricity markets.[1] The diversity of these approaches makes it difficult to get a comprehensive overview of the field of stochastic models. Hence this survey should guide the way through the different approaches and describe the state-of-the-art in this research area, especially focusing on price risks in electricity markets. Many stochastic OR models for energy currently deal with fluctuating feed-in of renewable energies. However, we do not attempt to fully cover the stochastic issues in wind and renewable energies, which we only shortly mention in the paper.[2] Furthermore, we do not go into detail about coal, gas and oil price modelling, as we focus on general approaches for electricity markets. Thereby the focus is placed on stochastic methods developed in operation research and financial mathematics with practical relevance and applicability.

Electricity markets are characterised by some technical features which will be described in Section 2 and which determine the complexity of such models. Electricity market modelling usually requires the representation of the underlying characteristics and limitations of the production assets. As these models take the technical characteristics of the production system and the fundamental data into account, they are often called fundamental models. Beside these fundamental models, sophisticated financial and economic models can be used for modelling uncertain commodity prices in the short-term. In this survey, the various modelling approaches in the energy business are classified as follows:

- stochastic processes for electricity prices, commodity prices (for primary energy carriers) and other uncertain parameters (hydro inflow and wind distributions) (see Section 3);
- scenario generation and reduction (see Section 4), which is important for the practical relevance and applicability in energy markets due to the need for a structured handling of large data amounts; as well as
- stochastic optimising models for investment decisions, short- and mid-term power production planning and long-term system optimisation (see Section 5).

As the three fields cannot be examined separately from one another, they are illustrated by selected integrated models which represent a complete approach. Thereby the practical relevance of the different methods and their applicability to real markets is of crucial importance. In a conclusive summary, shortcomings of existing approaches and open issues that should be addressed by operation research are critically discussed (see Section 6).

[1] Beside stochastic models, deterministic models have been successfully used to give decision support in liberalised energy markets. A good overview of electricity market modelling trends with deterministic models can be found in Ventosa et al. (2005).

[2] A separate overview of models dealing with fluctuating feed-in of renewable energies would nevertheless be useful.

2. Decision problems in liberalised energy markets

Decision problems of utilities are characterised by special technical features of the commodity electricity and characteristics of the technical plants used to produce it. The product electricity is characterised by the following features (cp. Hensing et al., 1998; Wietschel, 2000; Stoft, 2002):

- Transportation of electricity requires a physical link (transmission lines).
- Electricity cannot be directly stored on a large scale, which necessitates that supply and demand are equalised at all times.
- Electricity can only be substituted to a limited extent, as the functioning of private, public and economic life in industrialised countries depends on a reliable electricity supply.
- As quality characteristics of electricity, such as voltage and frequency stability, are subject to strict regulations, it can be seen as an homogenous good. Furthermore, once electricity is fed into the grid, it cannot directly be assigned to a specific generator.

Electricity generation plants are characterised by:

- a long-term technical useful life between 40 and 60 years depending on power plant type;
- a high capital intensity for investment projects combined with long-term amortisation times;
- many different types of plants, which have to be taken into account in investment (and production) decisions and which significantly differ in technical, economic and environmental characteristics; and
- undesirable by-products such as CO_2, ash, fumes, heat, etc.

These technical features have a significant effect on decisions in the energy business and thus also on decision support tools.

In the past, several approaches have been developed to analyse and predict energy prices, especially electricity prices. These approaches can generally be divided into four classes. So-called *fundamental models* simulate the above-described technical characteristics of the electricity sector, especially the impact of power plant characteristics and capacities, and of restrictions in transmission capacities and demand variations. This kind of model is very popular; several approaches were developed during the oil crisis in the 1970s. Based on a (deterministic) linear (mixed-integer) optimisation approach, these models have become frequently-used tools for policy advisors and the corporate planning activities of electric utilities. Originally their application was motivated by the effort of industrialised nations to curb their dependence on imported mineral oil and elaborate strategies aimed at rearranging their national energy systems accordingly. More recently, these models have been adapted to the new market conditions in order to analyse the development of electricity prices and emission allowance prices. Most of these approaches are based on a few internationally known and widespread models, like MARKAL (Market Allocation Model – see Fishbone and Abilock, 1981), EFOM (Energy Flow Optimization Model – see Finon, 1974; Van der Voort et al., 1984), MESSAGE (Model for Energy Supply System Alternatives and their General Environmental Impact – see Agnew et al., 1979; Messner, 1984; Messner and Strugbegger, 2009), CEEM (Cogeneration in European Electricity Markets – see Starrmann, 2001), TIMES (The Integrated MARKAL EFOM system – see Remme, 2006) and PERSEUS (Program Package for Emission Reduction Strategies in Energy Use and Supply – see Möst, 2006; Fichtner, 1999), which was developed on the basis of EFOM.

Besides the well-known optimisation models, other approaches like agent-based simulation or system dynamic approaches are often used to simulate the fundamentals of energy markets. These approaches are mainly used when the problem under consideration is too complex to be addressed within a formal framework. Furthermore, the interaction of different market participants is the focus of these simulations. As analysis with simulations can cover a very broad field and the objectives of the particular analysis differ somewhat, we refrain from providing an overview of simulation models here. The interested reader can refer to Sensfuss et al. (2008), who give a detailed overview about agent-based simulation models.

Sophisticated *financial mathematical models* can be used for the modelling of commodity price paths without taking the above-mentioned fundamental characteristics into account. Originally developed for stock and interest rate markets, a number of these models have subsequently been applied to the energy field. These models have to take into account the characteristics of the spot prices resulting from technical characteristics, such as:

- daily, weekly and seasonal cycles;
- high volatility;
- mean-reversion; and
- spikes.

In general, these financial mathematical models are used for short-term planning horizons. They are more suited for coping with the volatility of energy prices so are often used for real option valuation and risk assessment purposes. However, results of these financial models are used as inputs for fundamental models, which combine a detailed representation of the physical system with rational modelling of the firms' behaviour on the basis of an optimisation problem.

A further class of models is *econometric time-series models*, which relate the fluctuations of energy prices, especially of electricity prices, to the impact of explanatory factors such as temperature, time of day, luminosity, etc. This approach is also often used to predict the electricity demand. Econometric time-series models are very similar to financial models, as they also apply statistical methods to historical time series. But in general these models focus on the impact of explanatory variables on electricity price fluctuations. They do not consider the stochastic processes of electricity and energy prices through their volatilities, in the same way that financial models do.

The final category of models consists of *game theoretic approaches*, which are particularly adequate for analysing the impact of strategic behaviour. These models of competitive electricity markets have mostly been developed to analyse longer-term equilibria on the wholesale market. Within the Cournot–Nash framework, electricity are assumed to be a homogenous good and market equilibrium is determined through the capacity setting decisions of suppliers. Smeers (1997) gives an overview of this kind of model as well as how game theoretical models can be used to explore relevant aspects of the design and regulation of liberalised energy markets. Furthermore, Hobbs (2001) presents an overview concentrating on Cournot-based models and Day et al. (2002) gives a survey of the power market modelling literature with emphasis on equilibrium models. In this paper, we explicitly exclude an overview of the game theoretic approaches as "most of these models developed are focusing on qualitative issues, taking quantitative results more as illustrative examples than as ultimate research objectives" (see Weber, 2005). As these results tend to be illustrative examples, most analysis aims at providing decision support more to the regulators than to the utilities.

Furthermore, decision problems can be classified according to the impact of the decision. Operational decisions have a short-term impact and affect only specific functional areas, while strategic decisions have a long-term impact and affect all functional areas within a firm. A survey of (mainly deterministic) short-term operation planning models, especially for cogeneration systems, can be found in Salgado and Pedrero (2008). Ventosa presents a survey of electricity generation market modelling, distinguishing optimisation models, equilibrium models and simulation models (see Ventosa et al., 2005), whereby one of the approaches compared in his survey is also based on stochastic programming. A survey of stochastic energy models is also carried out by Wallace and Fleten (2003), but this study concentrates only on the optimising models themselves; it does not introduce methods for the simulation of uncertain parameters and for the subsequent scenario generation.

To supplement the existing surveys, this paper firstly focuses on financial models and time-series models to simulate stochastic parameters in particular. Fundamental models on integrated approaches are subsequently introduced, which optimise an energy system or power plant output, based on scenario trees generated out of the simulation of uncertain parameters.

3. Modelling uncertainties in the electric power production

The first step of stochastic modelling is the analysis of temporal variation of the uncertain parameters. Whereas forecasting the uncertain load was the main challenge before liberalisation (for load forecasting see Hahn et al., 2009), new uncertain parameters now have to be considered in energy modelling, which are, inter alia, electricity prices, commodity prices (e.g. fuel, CO_2-certificates), fluctuant inflow to hydro reservoirs and uncertain wind power generation. These parameters are analysed using different stochastic processes, such as mean-reversion processes. Historical data, which is available from the power exchanges, is thereby necessary for the estimation of the main stochastic parameters, e.g. mean and volatility. In the following section, some of these stochastic processes are described as applied to different uncertain parameters and some selected models are listed in Table 1.

3.1. Electricity prices

Most of the existing stochastic energy market models focus on uncertain electricity prices in liberalised energy markets. Electricity prices are simulated with the aid of either deterministic fundamental models, especially linear optimising models for the calculation of long-term equilibrium prices, or stochastic processes, such as mean-reversion models (see Gibson and Schwartz, 1990) for short- and mid-term price analysis. While some of these stochastic models separate stochastic and deterministic parts (i.e. trend and seasonality) of the price process (see Karakatsani and Bunn, 2008), others consider both in a single closed approach (see Lucia and Schwartz, 2002). However, in some models, electricity prices are described only with a stochastic process (regardless of any deterministic component). But considering both deterministic and stochastic components delivers a more detailed and appropriate result. Firstly, this section focuses on the determination and removal of deterministic components from the original time-series of electricity prices. The time-series without deterministic components, also called stochastic residues, is then modelled with the help of econometric models. After the simulation of the stochastic components, the deterministic components are again added to them, forming the complete process of the price logs. The simulation ends with the retransformation of the simulated price logs into real prices (see Fig. 1).

3.1.1. Simulation of deterministic parts (trend and seasonality)

In most studies the logarithms (logs) of the electricity prices are simulated for variance stabilisation reasons, i.e. the simulation of the logarithms limits the volatility of the electricity prices to an

Table 1
Stochastic commodity price modelling – Overview.

	Uncertain parameter	Stochastic process	Trend/seasonality	Correlation
Tseng and Barz	Electricity and fuel prices	MR process	Considered	Correlation between electricity and fuel prices
Weron et al.	Electricity prices	MR process with jump diffusion, regime switching	Considered by a sinusoidal function	No correlation
Barlow	Electricity prices	Nonlinear OU process	Considered by trigonometric function	Dependencies from demand
Karakatsani and Bunn	Electricity prices	AR process with fundamental deterministic parts	Considered by demand curve	Dependencies from demand
Muche	Electricity, coal and CO_2-certificate prices	MR process/ARMA process with jump diffusion	Not considered	Correlation between CO_2-certificate and electricity prices
Schwartz	Electricity and commodity prices	MR process	Not considered	Not considered
Weber	Electricity and fuel prices	MR process	Considered by long-term equilibrium prices for electricity	Correlation between oil and other fuel prices
Schmöller	Electricity and fuel prices, inflow to hydro reservoirs	ARIMA process and peak regime	Considered by trigonometric function, no seasonality for coal	Correlation between coal and gas prices

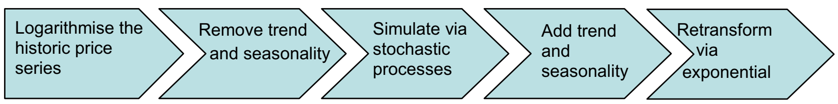

Fig. 1. The whole procedure for the electricity price simulation.

appropriate level. Besides, if electricity prices are assumed to be normally distributed, this would result in negative values for almost half of the time. To avoid this, the simulation of the logarithms instead seems to be a reasonable solution, as it also generates a left-skewed distribution for the prices. Since 2007, negative prices have been allowed to occur at the Energy Exchange in Leipzig. Especially with an increasing share of compensated (fluctuating) renewable energy feed-in, negative prices on the spot market will occur more frequently in future. This change in market design necessitates the adoption of the presented modelling approach to capture negative prices as well. However, this paper focuses on simulation of positive prices via electricity price logs.

Electricity prices p_t or their logarithms X_t can be seen as a mathematical composition of deterministic and stochastic components:

$$X_t = X_t^{trend} + X_t^{season} + X_t^{residue}. \tag{3.1}$$

The deterministic components of the price logs are the trend X_t^{trend} and annual seasonality X_t^{season}. There are some models which simulate the power prices by multiplying all components together (see Schmöller, 2005). In this case the stochastic residues of historic price logs form a weak stationary process, if the original price logs are divided by their deterministic components. A weak stationary process is necessary if the stochastic residues $X_t^{residue}$ are modelled by an autoregressive-mean average (ARMA) process. But before a stochastic process can be applied to the stochastic residues of the price logs, they have to be determined, removing the deterministic components trend from the original data series.

The first component, the *trend*, can be calculated via an exponential function

$$X_t^{trend} = X_0 \cdot e^{\gamma t}, \tag{3.2}$$

assuming a constant annual growth rate for electricity prices. Alternatively, a linear function

$$X_t^{trend} = X_0 + \gamma \cdot t, \tag{3.3}$$

can also be chosen for the trend component (see Schlittgen and Streitberg, 2001), if the modelled time steps are discrete. In both cases, the parameters X_0 and γ can be determined by applying a least-squares estimation to historical data. Moreover, it should be mentioned that the mean can also be derived on the basis of a fundamental model. This has the advantage that market developments influencing price levels are directly taken into account.

The model for the other deterministic component, *seasonality*, is a more complex one, as it should consider load variations and thus price variations within a day or on different day types, also according to specific loads on these days. A possible classification of day types, for example into four categories, can be made as follows:

- Monday or working day after holiday.
- Working day (Tuesday, Wednesday, Thursday).
- Friday or day before holidays.
- Weekend day or holiday.

For each of these day types d and each hour h a different seasonality function is defined using trigonometric functions for the seasonal oscillation. Although there are oscillation functions which consider harmonic oscillations of the price process (see Schlittgen and Streitberg, 2001), the basic oscillation describes a sufficient approach for the seasonal oscillation (see Eq. (3.4)):

$$X_{dh}^{season}(t) = \alpha_{dh} + \beta_{dh} \cos\left(2\pi \frac{t-\tau}{8760}\right) + \gamma_{dh} \sin\left(2\pi \frac{t-\tau}{8760}\right). \tag{3.4}$$

The periodic time of the oscillation function is 8760 hours, equal to the total number of hours per year. The parameter τ defines the phase-shift of the oscillation. The other parameters α_{dh}, β_{dh}, γ_{dh} are also estimated via least-squares method from historical data.[3] More detailed approaches describe the price seasonality as an overlay oscillation of the basic and first harmonic one (see Lucia and Schwartz, 2002).

However, if an electricity model has daily temporal resolution, e.g. if day ahead prices are simulated, then the seasonality function has to consider 365 days instead of the 8760 hours as the period length of the oscillation. In this case, the seasonality function is only formulated for the type days d and it has no inner-daily resolution (see Eq. (3.4)):

[3] Dividing a year into 96 time slots (24 hours * 4 type days) expands the parameter estimation, which has to be done 96 times in this case, while on the other side the data series used for parameter estimation is reduced strongly. That is why an enlarged database (including several years) should be used for parameter estimation.

$$X_d^{season}(t) = \alpha_d + \beta_d \cos\left(2\pi \frac{t-\tau}{365}\right) + \gamma_d \sin\left(2\pi \frac{t-\tau}{365}\right). \tag{3.5}$$

Having simulated the seasonality for each type day d and time hour h within a day, they are brought together in a unique function describing the seasonal component (see Eq. (3.5)). Therefore, identity functions are introduced, which determine the time interval h within type day d. The composition for the model without innerday resolution can be done analogously to Eq. (3.6), removing the sum for the hours:

$$X_t^{season} = \sum_{h'=1}^{24} \sum_{d'=1}^{4} 1_{h=h'} 1_{d=d'} X_{d'h'}^{season}(t). \tag{3.6}$$

Subtracting the deterministic components, determined via Eqs. (3.2/3.3) and (3.6), from the logs of historic time series delivers the stochastic residues of the electricity prices. These stochastic residues are used for parameter estimation of the appropriate stochastic process, which are applied to generate simulations of the stochastic components.

Beside the functions for deterministic components, there are other methods to capture trend and seasonal effects. If the stochastic residues are modelled with a mean-reversion process, then the use of a time-variant mean describes another approach to consider the deterministic components. Thereby the time-variant means can be estimated from historical data series (see Tseng and Barz, 2002), while e.g. monthly-varying means are calculated from relevant data. But there are also approaches which describe the deterministic components based on fundamental factors, such as demand or scarcity of supply (see Karakatsani and Bunn, 2008). These fundamental effects can be added via an exogenous function to the price process or as a time-varying mean after simulation by a fundamental model that describes a realistic picture of the expected generation costs and corresponding electricity prices within an energy system. The optimisation of the fundamental model, usually based on a mixed-integer objectives function, delivers the marginal costs of the production of one further unit of electricity at the model optimum, also known as shadow prices. These shadow prices contain seasonal effects, as their calculation is based on seasonally-varying load curves. They also include the trend of the price development, as fundamental models generally consider annually varying primary energy prices and technology developments in their input data. Hence the price expectations regarding the primary energy prices and technology development, but also demand expectations, form the marginal production costs of electricity and therefore its price curve.

3.1.2. Modelling stochastic price components

As it can be observed in Table 1, the most applied stochastic process for electricity prices is the *mean-reversion process*. According to this stochastic process, electricity prices are assumed to revert to their long-run mean μ with "reversion-speed" κ. As logarithms of the electricity prices ($X_t = \ln\, p_t$) are modelled to reach variance stabilisation, the mean-reversion process or the so-called Ornstein–Uhlenbeck process (see Uhlenbeck and Ornstein, 1930) can be formulated with the following stochastic differential equation:

$$dX_t = \kappa(\mu - X_t) \cdot dt + \sigma \cdot dW_t. \tag{3.7}$$

Thereby the variation component dW_t corresponds to that of the standard Brownian motion (also called Wiener process $dW_t = \varepsilon_t dt^{1/2}$).

The mean-reversion process is the continuous version of the time-discrete autoregressive process (AR(1) process). Therefore, the simulation is continued with the following AR(1) process, considering days as discrete time units for electricity price modelling (i.e. $dt^{1/2} = 1$):

$$X_{t+\Delta t} - X_t = \mu \cdot (1 - e^{-\kappa \cdot \Delta t}) + (e^{-\kappa \cdot \Delta t} - 1) \cdot X_t + \varepsilon_{t+\Delta t}, \tag{3.8}$$
$$\Longleftrightarrow X_{t+\Delta t} = a + b \cdot X_t + \varepsilon_{t+\Delta t}. \tag{3.9}$$

With the aid of linear regression and least-squares estimation (see Hartung et al., 1998) applied to historical prices, the parameters $a = \alpha(1 - e^{-\kappa \cdot \Delta t})$ and $b = e^{-\kappa \cdot \Delta t}$ can be determined easily. The error component $\varepsilon_{t+\Delta t}$ is assumed to be log-normally distributed. The parameters a and b of the AR(1) process can be estimated with the least-square method from historical data and used for the simulation of electricity prices.

There are also models which transform the mean-reversion process into a discrete one by replacing dt with Δt and choose for simplicity $\Delta t = 1$ (see Weron and Misiorek, 2008). In this case the discrete mean-reversion process (see Eq. (3.10)) can be applied directly, after its parameters κ, μ and σ are estimated with linear regression or ML-estimation from historical time series:

$$\Delta X_t = \kappa(\mu - X_t) + \sigma \cdot \varepsilon_t. \tag{3.10}$$

Mean-reversion jump diffusion MRJD models extend the mean-reversion process to capture price peaks, which occur in electricity price series very often. Therefore, the logarithm $\ln J$ of the jump height J is added to the mean-reversion process. While the Wiener process dW_t considers only small price deviations from the mean, $\ln J$ handles infrequent but large jumps of the price series. The logs of the jump height are, like the error term ε_t of the Wiener process, normally distributed with mean $\mu_{\ln J}$ and variance $\mu_{\ln J}$:

$$dX_t = \kappa(\mu - X_t) \cdot dt + \sigma \cdot dW_t + \ln J \cdot dq, \quad \ln J \sim N(\mu_{\ln J}, \sigma_{\ln J}). \tag{3.11}$$

The jump component $\ln J$ is multiplied with a Poisson factor dq, which incorporates price jumps into the mean-reversion process without jumps. The probability of dq_t is δ, the intensity of the price jumps, which corresponds to their share in historical price series:

$$dq = \begin{cases} 1 & \text{if } z_t \leqslant Pr(dq) = \delta \cdot dt, \\ 0 & \text{if } z_t > \delta \cdot dt. \end{cases} \tag{3.12}$$

The simulation of the Poisson process is done by generating a uniformly distributed random variable z_t and its comparison with the intensity derived from historical data series. If $z_t > \delta$, then dq equals "1" or "0".

MRJD models integrate price jumps into the mean-reversion approach, which describes the stochastic process of all electricity prices. But there are models which differ between the standard price process (base regime) and the one for price jumps (peak regime), the so-called *regime-switching models* (see Weron et al., 2004). Although there are multi-regime-switching models, two-regime-switching models have become more accepted in electricity price modelling. These regime-switching models distinguish between a base regime (e.g. mean-reverting regime $R_{1,t}$) and a second regime for price peaks (spike regime $R_{2,t}$). As electricity prices are assumed to follow either the base regime or the peak regime at each time within the modelling horizon, they can stay in the same regime or transit to the other regime in the next time step. A transition matrix Pr defines the probabilities for the transition from regime $R_{i\{1,2\}}$ to regime $R_{j\{1,2\}}$:

$$Pr = (p_{ij}) = \begin{pmatrix} p_{11} & p_{12} = 1 - p_{11} \\ p_{21} = 1 - p_{22} & p_{22} \end{pmatrix}. \tag{3.13}$$

While the stochastic process $Y_{1,t}$ for the base regime is, as mentioned above, a mean-reversion process, the peak regime process $Y_{2,t}$ is usually modelled via a Markov chain using a log-normally distributed random variable. The parameters of both regime processes are estimated via an Expectation–Maximization-Algorithm (EM-

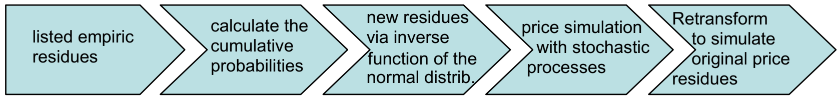

Fig. 2. The procedure to simulate the stochastic component by transforming into normal-distributed residues.

Algorithm), which uses an iterative approach to find the optimal solution for the process parameters (see Dempster et al., 1977).

The *autoregressive-moving average (ARMA) process* describes another method to simulate the stochastic residues, which can be applied to discrete time-series. While the autoregressive component of an ARMA process considers the last p-prices for the calculation of electricity price X_t in t, the moving average component takes the last q-error terms into account (see Swider and Weber, 2007). Therefore, an ARMA(p,q) process has the orders p and q representing the orders of each partial process. The calculation of the price in the t domain ultimately depends on a new error term ε_t, which can be normally distributed, for example,

$$X_t^R = \sum_{i=1}^{p} \alpha_i X_{t-i}^R + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j} + \varepsilon_t. \tag{3.14}$$

The use of the ARMA process presumes that the stochastic residues are weak stationary and that the distribution of the error terms is known because the errors are modelled as the random variables of the stochastic process.

3.1.3. Transformation and distribution of the random variables

The time-series of the stochastic residues are assumed to be (log-)normal or gamma distributed. But random variables, generated with these distribution functions, do not fulfil the Kolmogoroff–Smirnov-test as well as other statistical tests (see Schmöller, 2005). That is why the empirical residues are transformed into log-normally distributed residues. Therefore, they are listed in ascending order in a table, whilst their cumulative probabilities are calculated and added to the table. Thereby the cumulative probability corresponds to the quotient of the list number of the residue and the total number of residues. These probabilities are then input into the inverse function of the (log-)normal distribution, to calculate residues. The (log-)normal distribution can then be applied to these new residues to simulate the stochastic price process with the help of the models described in Section 3.1.2. Lastly, the simulated stochastic residues are retransformed to receive a simulation for the original stochastic price residues (see Fig. 2).

3.2. Commodity prices

In the electric power industry, modelling of commodity prices focuses on the price path simulation of prices of fuels, such as coal, gas and oil. Some models (e.g. Muche, 2007) also consider CO_2-certificate prices, since CO_2-certificate trading is established in electricity markets. But the main uncertainty for electric power producers remains the development of fuel prices. Different stochastic models have been explored in the last few years, to handle uncertain fuel prices. Again, they use mean-reversion and ARMA processes to describe the stochastic development of commodity prices.

Some of the commodity price models consider trend and seasonality of the price development similar to electricity price models (see Heydari and Afzal, 2008). Some financial models include a second factor, the convenience yield,[4] which also follows an MR process (see Schwartz, 1997). These models account for the correlation of the convenience yield and the commodity prices. More interesting is the correlation between different fuel prices, though, whereby the dependency of other fuel prices on the oil price plays a key role.

Analogous to electricity price simulations, the logarithms of the primary energy prices are generally modelled instead of the prices themselves ($X_f = \ln p_f$). Thereby f represents the index of fuel (primary energy carrier PEC) types. More precise approaches (e.g. Weber, 2005) model the derivatives of the price logs with the help of a mean-reversion process $d(dx_f) = \kappa_f(\mu_{df} - dx_f) + \sigma dW_f$, whereby $dW_f = \varepsilon_f\sqrt{dt}$ is again a Wiener process. The error term ε_f of the Wiener process dW_f is standard normal-distributed. For estimation purposes, the continuous model is again changed into a discrete one based on distinct time periods for the fuel price simulation; i.e. the marginal time interval is replaced by a discrete time period $\Delta t = 1$.

Based on the discrete approach, the oil price is modelled first, as oil is still the world's most important energy carrier. The oil price simulation is followed by the simulation of the other fuel prices considering correlations with the oil price:

$$\Delta\Delta x_{Oil} = \kappa_{Oil}(\mu_{\Delta x_{Oil}} - \Delta x_{Oil}) + \sigma_{\Delta x_{Oil}}\varepsilon_{\Delta x_{Oil}}, \quad \varepsilon_{Oil} \approx N(0,1). \tag{3.15}$$

For other energy carriers the mean-reversion model is extended by a term for the difference to the long-term equilibrium oil price $(x_{Oil} - \theta_f x_f)$ and oil price changes $(\Delta\Delta x_{Oil})$ as further explanatory variables, considering correlations and dependencies on the oil price:

$$\Delta\Delta x_f = \kappa_f(\mu_{\Delta f} - \Delta x_f) + \beta_f(x_{Oil} - \theta_f x_f) + \gamma_f \Delta\Delta x_{Oil} + \sigma_f\varepsilon_f,$$
$$\varepsilon_f \approx N(0,1). \tag{3.16}$$

The new parameters represent: the tendency of the oil price β_f; the price ratio θ_f between the specific fuel price x_f; and the oil price and finally a factor γ_f describing the dependency on the oil price change. After estimating these parameters from historical data via the least-squares method, the extended mean-reversion process can be applied for different fuel prices.

These mean-reversion models for fuel prices, however, do not take deterministic components as trend and seasonality into account. As mentioned above, there are models considering these components similar to electricity price modelling. The trend function again generally contains a constant growth rate (see Eq. (3.3)), whereas seasonality is described by a trigonometric function (see Eq. (3.5)). However, as coal prices are recorded quarterly, there are no noticeable seasonal effects, so the models for coal do not include seasonality functions. In contrast to coal prices, gas prices possess strong seasonal effects, which can be described by trigonometric functions.

After removing the deterministic components trend (and seasonality), the obtained stochastic residues of fuel prices are modelled via ARMA processes. The ARMA model can only be applied to a stochastic process if the process is a weak stationary one. However, the residue series of the normalised coal prices form a strong stationary process, so the stochastic process has firstly to be transferred into a weak stationary one using a filter (see Box et al., 1994). This filter can consist of the differences between two sequential residues forming a new residue series:

[4] The convenience yield can be defined as the surplus of holding the commodity itself instead of a future contract. It plays a major role in times of scarcity of resources.

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$$\Delta x_t = x_t - x_{t-1}. \tag{3.17}$$

Sometimes this filter has to be applied d-times to obtain a weak stationary process. Obtaining a weak stationary process which fulfils the Dickey–Fuller test, the resulting series can be simulated by an ARMA process with the orders p and q. The combination of filtering the stochastic price series d-times (according to Eq. (3.17)) and the proper ARMA(p,q) process is also called an autoregressive integrated mean average (ARIMA(p,d,q)) process. For example, coal prices are modelled with the help of an ARIMA(1,1,0) process, whilst the gas prices are described by an ARIMA(2,0,1) process.

These approaches describe independent models for coal and gas prices. In fact there is a correlation between both price processes. Therefore, the ARIMA(1,1,0) process for coal is extended, taking the correlation with the average gas price ϕ-times ago into account:

$$X^R_{coal,t} = \alpha X^R_{coal,t-1} + \gamma \overline{X}^R_{gas,t-\varphi} + \varepsilon_t. \tag{3.18}$$

As mentioned above, coal prices are recorded quarterly, so the coal price logs are not modelled on the basis of daily price changes. However, if future expected prices are required, e.g. for real option models (see Section 5), the AR(1) process for electricity prices can also be formulated for coal prices (see Eq. (3.19)), based on the expected value from the perspective of today's price logarithm X_0. As the risk-neutral process is required for real options, the AR(1) process is extended by a term $\lambda * \sigma/\kappa$, representing the market price of risk (see Hull, 2005):

$$E(X_{RN,t}) = e^{-\kappa \cdot t} \cdot X_0 + \left(\alpha - \frac{\lambda \cdot \sigma}{\kappa}\right)(1 - e^{-\kappa \cdot t});$$
$$Var_0(X_{RN,t}) = \frac{\sigma^2}{2 \cdot \kappa} \cdot (1 - e^{-2\kappa \cdot t}). \tag{3.19}$$

Due to the log-normal distribution assumption of the prices, the expected prices are calculated from their expected logs and variance as follows (see Jaillet et al., 2004):

$$E_0(p_{fu,RN,t}) = e^{E(X_{RN,t}) + \frac{1}{2} Var_0(X_{RN,t})}. \tag{3.20}$$

This model for the coal price is similar to the one factor model developed by Schwartz for the simulation of commodity prices. The one factor model is extended in the two factor approach by Gibson and Schwartz, which is based on two mean-reversion processes, one for the commodity spot prices and a second one for the convenience yield, regarding correlation between both parameters.

CO_2-*certificate prices* are also simulated with the aid of ARMA or mean-reversion processes, whereas *risk-neutral* processes are considered (see Wagner, 2007) if the simulated prices are used again in a real option model. As CO_2-certificates and coal are storable products (in contrast to electricity), no large price jumps are expected in their price process. It is therefore sufficient to apply a standard mean-reversion process without any jump component for CO_2-certificate prices.

At last, it is worth mentioning that the correlation of electricity prices and CO_2-certificate prices is also considered in the CO_2-certificate price model by Muche. The error term of the risk-neutral ARMA process of the CO_2-certificate prices is extended by the product of the correlation coefficient ρ_{ec} and the error term of the electricity prices:

$$\varepsilon_{c,t+1} = \varepsilon_{e,t+1} \cdot \rho_{ec} + \varepsilon'_{c,t+1} \cdot \sqrt{1 - \rho^2_{ec}}, \tag{3.21}$$

whereby $\varepsilon'_{c,t+1}$ represents the original error random variable of the CO_2-certificate price process, and $\varepsilon_{e,t+1}$ the error random variable of the electricity prices.

Only a few models simulate CO_2-certificate prices, however; the behaviour of this highly volatile market parameter should be further addressed in future research.

3.3. Other uncertain parameters

Other uncertain parameters which are considered in some electricity market models are specifically inflow to hydro reservoirs and wind electricity production. A lot of stochastic OR models for energy currently deal with fluctuating feed-in of renewable energies. However, we do not attempt to fully cover the stochastic issues in wind and renewable energies, which we only shortly mention in this paper. A separate overview of models dealing with fluctuating feed-in of renewable energies would be useful.

Some models integrate different uncertainties to their stochastic modelling approach (see Fleten et al., 2002). They describe uncertain parameters, like inflow to hydropower plants or the backup power for load balancing, with the help of ARIMA processes. As there is no deterministic part of the backup power process; it is directly analysed by an ARIMA(1,0,1) process. But the inflow to hydropower plants has a seasonal component. Thus the seasonal value for each month is determined by the average inflow of the same named months derived from historical series:

$$x^s_i = \frac{12}{T} \sum_{j=1}^{T/12} x_{i+12\cdot(j-1)}, \quad i = 1, \ldots, 12. \tag{3.22}$$

The stochastic residue process is defined by an ARIMA(2,0,2) process:

$$X^R_t = \varepsilon_t + \sum_{i=1}^{2} a_i X_{t-i} + \sum_{ji=1}^{2} b_j \varepsilon_{j-i}. \tag{3.23}$$

The error term ε_t in the ARIMA(2,0,2) process represents the "white noise", the simplest stochastic process, whose expected value equals zero and for which variance remains constant.

Another uncertain parameter which is often modelled in energy market models is the wind electricity generation depending on the forecast wind speed. Thereby the Weibull distribution, whose probability density and cumulative probability functions are defined as follows, fits the wind speed very well:

$$f(x) = \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}, \tag{3.24}$$
$$F(x) = 1 - e^{-\alpha x^\beta}. \tag{3.25}$$

The parameter α of the Weibull distribution is called the shape parameter, while β represents the scale parameter. Furthermore, ARMA models are again chosen to describe the stochastic process of the Weibull-distributed wind speed (see Torres et al., 2005). Before an ARMA model can be applied though, the wind data series are generally transformed (e.g. Box–Cox-Transformation) and standardised, because hourly wind data reveals cyclic behaviour. As the transformed and standardised data is not stationary, the ARMA process can be successfully applied. Other models use any unvaried stochastic process combined with a spectral power density function (see Olsina et al., 2007).

4. Scenario generation and reduction

The different stochastic processes are used to simulate the uncertain parameters and to generate future data for them. The simulations of each uncertain parameter at a time can be combined into a scenario. The generation of a large number of scenarios is a method to capture the uncertainties in energy markets. Thereby, two approaches to scenario generation have been successfully applied in energy market models: the analytical and the simulative.

4.1. Analytical scenario generation

Analytical scenario generation is based on binomial or trinomial trees, which integrate higher and lower values in $t + 1$ than the values in t for the uncertain parameters (see Pilipovic, 2007). Fig. 3 illustrates a trinomial tree:

The expected $E(x_t)$ value of the uncertain parameter follows a stochastic process, e.g. an ARMA process, but it can be extended by adding two more branches in each node (time step) to receive a trinomial tree. The value of the new leaves, belonging to the new branches, is calculated by adding or subtracting the same value Δx to the expected value. Δx is evaluated with the help of the variance of the stochastic process:

$$\Delta x = \frac{\sigma}{\sqrt{2p_{u/d}}}. \tag{4.1}$$

Thereby it is important to determine the probabilities of each branch. In a trinomial tree, the probability of the upper and the lower branches $p_{u/d}$ can be chosen as a constant corresponding to one-sixth, while the probability of the middle branch p_m would in this case equal two-thirds. The whole tree is built up by repeating this procedure forwards for each time step and for all existing nodes. Alternatively the stochastic behaviour of uncertain parameters can be described via binomial trees (see Göbelt, 2001).

4.2. Simulative scenario generation and scenario reduction

The more common approach in energy markets is the simulative scenario generation, however. Therefore, the uncertain parameters are simulated via the stochastic processes described above (see Section 3). With the help of Monte-Carlo simulation, based on a large number of scenario simulations via the described stochastic processes, the uncertainties can be handled very well. Thereby the number of scenario simulations has to be so large that the "law of large numbers" can be applied in order to obtain reasonable data for the uncertain parameters. If the generated scenarios are to be used in a stochastic optimising model, the large number of scenarios has to be reduced to a level at which the solution of the optimising problem can be calculated within an acceptable time. The scenario reduction of a stochastic optimisation problem is done with the help of different methodologies.

Recombining trees represents a feasible methodology to solve a stochastic optimisation problem. The idea of recombining trees involves the combination of different states $(s_1, s_2, s_3, \ldots)$ of a scenario tree with similar descendant subtrees into a single state s' at a time t. The probability of the appropriate branches of the former subtrees is cumulated and assigned to the respective new subtree branches. Furthermore, the former original states are summarised in a cluster of states, whose mean becomes the representative value for this state cluster at the appropriate time t. This procedure is repeated as long as the required number of states (or clusters) n_0 is reached in each time. The number of states does not need to be constant in each time step, but it simplifies the optimisation problem to apply a constant number of states to many stochastic models based on recombining trees.

Finally, the recombination procedure results in a scenario lattice with nodes representing the different clusters of each time and arrows illustrating the transition between clusters of time t and $t+1$ (see Fig. 4).

Before a stochastic optimisation problem based on such a lattice can be solved, the probabilities of each state transition have to be determined. This can be done via a Monte-Carlo simulation for the uncertain parameter, e.g. electricity price, whereby the whole price range is divided into intervals with equal ranges (see Tseng and Barz, 2002). These intervals represent the state clusters s of the scenario lattice. The probability $Pr_{s,t\to s',t+1}$ of a transition $s \in t$ to $s' \in t+1$ is defined as the ratio between the number of transitions from the state s to s' and the number of all transitions from s to all other states in $t+1$:

$$Pr_{s,t\to s',t+1} = \frac{card\left\{l|p_{l,t} \in \left[p_{s,t}^{\min}, p_{s,t}^{\max}\right] \wedge p_{l,t+1} \in \left[p_{s',t+1}^{\min}, p_{s',t+1}^{\max}\right]\right\}}{card\left\{l|p_{l,t} \in \left[p_{s,t}^{\min}, p_{s,t}^{\max}\right]\right\}}. \tag{4.2}$$

The transition probabilities and the means of the price intervals are used in the next step to solve the stochastic optimisation problem. For example, a profit maximising problem within a time period $[t_0, T]$, based on uncertain electricity prices $p_{El,s,t}$ and fuel prices $p_{f,s,t}$, can be solved by maximising the profit G_t in each time step t and stage s backwards from the leaves ($t = T$) to the root ($t = t_0$):

$$G^*_{t\to T}(X_{El,t}) = \max\left(\sum_s Pr_{s,t\to s't+1}\left[p_{El,s,t}X_{El,s,t} - p_{f,t}C_{Op,s,t}(X_{El,s,t}) - C_{St,s,t} + G^*_{s',t+1\to T}(X_{El,t\in[t+1,T]})\right]\right). \tag{4.3}$$

This function (Eq. (3.3)) maximises the profit from electricity production and sales in t, which consists of the optimal profit in $t+1$ and the revenues from electricity output $X_{El,s,t}$ minus the operating costs $C_{Op,s,t}$ and the plant start-up costs $C_{St,s,t}$ in t. A more detailed description of optimising models will be introduced below (see Section 5).

Besides recombining trees, there are other approaches to reduce a large number of scenarios generated with the help of a stochastic process. The received scenarios are connected with the same root, forming a scenario tree, whereby the root represents the initial value of the analysed uncertain parameters. In the next step, this "fan" of scenarios has to be transformed into a real scenario tree. The reduction of the scenario fan can be done by combining similar ancestor branches instead of the descendant subtrees for each time step. Therefore, the Kantorovic-Distance, often applied in power

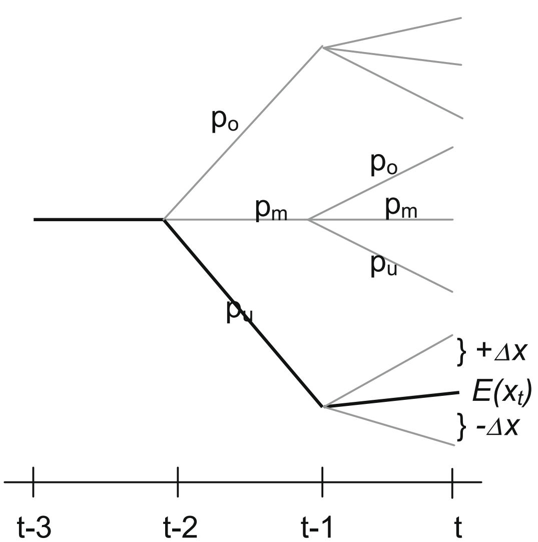

Fig. 3. Trinomial tree as an example of analytical generation.

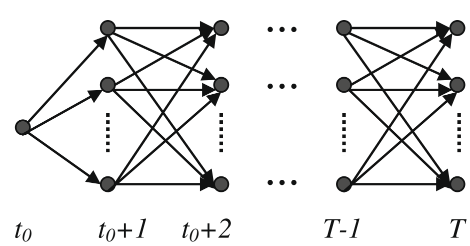

Fig. 4. Lattice with different states.

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production and sale models, leads to a transport problem, whose solution delivers the pair of scenario branches and the nodes which should be composed to a single branch (see Dupacova et al., 2003). However, minimising the Euclidean distance between each pair of branches would also deliver the most similarities. After determining the most similar two branches, one of them can be eliminated and its probability can be added to the other. This is possible because the new scenario tree is a subtree of the former tree (see Schmöller, 2005). It is worth mentioning that the Euclidean distance is calculated for all uncertain parameters in a common approach, however, assuming that the scenario tree simultaneously represents the forecast development of all considered uncertainties:

$$c_{t_j}(s_k, s_l) = \frac{1}{t_i}\sum_{t=1}^{t_i}\sqrt{\sum_j w_j\left|x_{j,t}^{s_k} - x_{j,t}^{s_l}\right|^2}$$
$$\forall t_i = t_1, t_2, \ldots, T;\ \forall s_k,\ s_l \in \{1, \ldots, n_0\}. \tag{4.4}$$

The uncertain parameters, indexed with j, are weighted with w_j, which can be derived from their importance in the electricity production. For example, the electricity price weight equals the whole installed capacity, while fuel prices are weighted only with the share of installed capacity based on each fuel. The values of each parameter $x_{i,t}$ have to be normalised because their co-domains differ from one other. As mentioned above, the distance values $c_{t_j}(s_k, s_l)$ have to be minimised to obtain the two scenarios with the most similar branches:

$$\min_{\substack{k,l,t_j \\ k \neq l}} pr_k c_{t_j}(\omega_k, \omega_l). \tag{4.5}$$

These two scenarios are combined into one, while one of them is deleted, partly adding its probability pr_k to the other. This step is repeated until the number of scenarios matches to the desired number n_0. The reduced scenario tree can afterwards be used for power generation optimisation or investment decision problems.

5. Applications of optimisation models

The reduced scenario tree or the scenario lattice which are generated with the methods described in Section 4, both form the base of a stochastic optimisation model. In electricity markets, these optimisation models concentrate on determining the optimal investment decision or optimal power production plan for a given time period. Some of these stochastic models even optimise whole energy systems from a long-term perspective (see Göbelt, 2001). Table 2 gives a brief overview of some stochastic models developed for energy markets in recent years.

For each of the main application fields – investment decision (IDM), short/mid-term power production planning (SMPP) and long-term system optimisation (LSO) – a different model is described in the following section to cover the main fields in energy markets for which stochastic optimisation models are used.

5.1. Short- and mid-term power production planning and portfolio management

Portfolio management and production planning models optimise an objective function, which can describe the total costs or profits relating to a whole energy system, particularly that of an energy company. The profit maximising approach fits the objectives of short- or mid-term analysis better than minimising approaches. In short-term power production models, the profit function G is defined as the difference between total expected revenues R from electricity or other energy sales and total expected costs C of generation. Some models also consider a correction term for stock changes, ΔS, if the company is dealing with other energy carriers, such as heat or fuel (see Weber, 2005). G is maximised to

Table 2
Overview of stochastic models for energy markets.

Model/author	Model type		Fundamental model/application			Uncertain parameter	Stochastic process
	Finan.	Fund.	SMPP	LSO	IDM		
Tseng and Barz (2002)	⊠	⊠	⊠	□	□	Electricity price and fuel prices	Mean-reversion (MR)
Muche (2007)	⊠	⊠	□	□	⊠	Electricity price, coal price and CO_2-certificate price	AR(1) process derived from MR
Weber (2005)	⊠	⊠	⊠	⊠	□	Electricity price and fuel prices	Mean-reversion process
Göbelt (2001)	□	⊠	□	⊠	□	Electricity price and fuel prices, energy demand	–
Fleten et al. (2002)	⊠	⊠	⊠	□	□	Electricity spot and contract prices, inflow into water reservoir	
Schmöller (2005)	⊠	⊠	⊠	□	□	Electricity and fuel prices, inflow into water reservoirs and reserve power	ARMA(p,q) process, ARIMA(0,1,0), VARIMA(0,2,0)
Swider and Weber (2006)	□	⊠	□	⊠	□	Wind intermittency	–
Olsina et al. (2007)	⊠	⊠	⊠	□	□	Load demand, available capacity, wind speed	Gauss–Markov process two-state Markov mo., univariate stochastic process
Hundt et al. (2008)	⊠	⊠	□	□	⊠	Electricity prices, coal and gas prices	Mean-reversion process (Hundt et al., 2008)
Dupacova et al. (2003)	⊠	⊠	⊠	□	□	(Load) demand, inflow to hydro-reservoirs and prices	Cluster-analytic approaches (Gröwe-Kuska et al., 2001)
Felix and Weber (2008)	⊠	⊠	□	□	⊠	Gas prices	Mean-reversion
Bowden and Payne (2008)	⊠	□	□	□	□	Electricity prices	ARIMA model, EGARCH model
Karakatsani and Bunn (2008)	⊠	□	□	□	□	Electricity prices	AR model, linear regression
Ladurantaye et al. (2009)	⊠	⊠	⊠	□	□	Electricity prices	Periodic AR(1) model
Krey et al. (2007)	⊠	⊠	□	⊠	□	Fuel prices	Multivariate AR(1) model
Fleten and Kristoffersen (2008)	⊠	⊠	⊠	□	□	Water inflow, electricity prices	ARMA model
Yang et al. (2008)	⊠	⊠	□	□	⊠	Electricity price, fuel prices and carbon price	Geometric Brownian motion
Kumbaroglu et al. (2008)	⊠	⊠	□	□	⊠	Electricity price, fuel prices	Geometric Brownian motion
Blyth et al. (2007)	⊠	⊠	□	□	⊠	Carbon price	Geometric Brownian motion
Kanudia and Loulou (1998)	□	⊠	□	⊠	□	CO_2-emissions, electricity and fuel supply, generation capacities	–

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find out the optimal solution for unit commitment and power production:

$$\max G = \max(R - C - \Delta S), \tag{5.1}$$

$$R = \sum_{t=1}^{T} \sum_{s \in S_t} Pr_s \left[R_{t,s}^{EL} + R_{t,s}^{HT} + R_{t,s}^{FU}\right]. \tag{5.2}$$

The total revenue of the power plant system consists of expected revenue from electricity sales $R_{t,s}^{EL}$, heat sales, $R_{t,s}^{HT}$ and (re)sales of fuels $R_{t,s}^{FU}$, for all scenario states s at all time steps t. The sales for each component can thereby be calculated from the quantities sold multiplied by the time- and state-dependent prices for them:

$$R_{t,s}^{EL} = \sum_{OTC} p_{OTC,t,s}^{EL} \cdot X_{OTC,t,s}^{EL} + p_{Spot,t,s}^{EL} \cdot X_{Spot,t,s}^{EL}, \tag{5.3}$$

$$R_{t,s}^{HT} = \sum_{OTC} p_{OTC,t,s}^{HT} \cdot X_{t,s}^{HT}, \tag{5.4}$$

$$R_{t,s}^{FU} = \sum_{f \in F} \sum_{OTC} p_{OTC\,f,t,s}^{FU} \cdot X_{OTC\,f,t,s}^{FU}. \tag{5.5}$$

The total system or company costs are made up of power plant operating costs $C_{t,s}^{PL}$ and contract bounded costs $C_{t,s}^{OTC}$ (for the purchase of fuel, heat or electricity). Both cost components can be further divided into variable and fixed costs. Thus, the total cost function is formulated as follows:

$$C = \sum_{t=1}^{T} \sum_{s \in S_t} \left(C_{t,s}^{PL,VAR} + C_{t,s}^{PL,FIX} C_{t,s}^{OTC,VAR} + C_{t,s}^{OTC,FIX}\right). \tag{5.6}$$

The variable operation costs $C_{t,s}^{PL,VAR}$ account for continuous operation as well as start-up and shut-down costs of power generation, whereas binary variables determine the operation mode, the start-up or shut-down action. The variable contract costs $C_{t,s}^{OTC,VAR}$ are determined by the mathematical product of the time-varying prices p_{OTC} and amount Y_{OTC} for each purchase contract of electricity, heat and fuel:

$$C_{t,s}^{OTC,VAR} = \sum_{OTC} p_{OTC,t,s}^{EL} \cdot Y_{OTC,t,s}^{EL} + \sum_{OTC} p_{OTC,t,s}^{HT} \cdot Y_{OTC,t,s}^{HT} + \sum_{OTC} p_{OTC,t,s}^{FU} \cdot Y_{OTC,t,s}^{FU}. \tag{5.7}$$

The last component of the objective function, the stock change ΔS, corresponds to the fuel storage changes, which can be determined by the difference between the storage level for each fuel type f at the beginning $S_f(1)$ and at the end of the planning period $S_f(T)$ multiplied with the appropriate fuel prices:

$$\Delta S = \sum_{f \in F} [S_f(1) \cdot p_f(1) - S_f(T) \cdot p_f(T)]. \tag{5.8}$$

Beside these more general models, which optimise trade portfolios combined with power generation planning, there are models which maximise the profit of electricity generation alone. The generation costs are described as the plant operation costs above. Some models are based on linear cost functions for the variable operation costs, but some consider a more detailed cost structure. Troncoso et al. use a genetic algorithm to solve a non-linear model for the optimal short-term electricity production. Thereby the total cost of electricity production cost is minimised assuming a non-linear cost function (see Troncoso et al., 2008). A quadratic cost function is also used by Tseng et al. instead of a linear function for the operation costs $C_{Op,u,t,s}$ of each plant u at time t and state s (see Tseng and Barz, 2002), whereby the start-up costs are no longer fixed ones, but depend on the time $SD_{u,t}$ passed since the beginning of the previous shut-down:

$$C_{Op,u,t,s} = p_{f,t,s} P_{f,u,t,s}(X_{EL,u,t,s}) = p_{f,t,s}\left(a_0 + a_1 X_{EL,u,t,s} + a_2 X_{EL,u,t,s}^2\right), \tag{5.9}$$

$$C_{ST,u,t,s} = \begin{cases} p_{f,t,s} a_{1,St,u}(1 - e^{SD_{u,t,s}}) + a_{2,St,u} & \text{if } U_{s,t} = 1, \\ 0 & \text{if } U_{s,t} = 0. \end{cases} \tag{5.10}$$

The first summand of the first term for the start-up costs represents the fuel costs in the start-up time; the second one $a_{St,u,2}$ covers other costs for start-up (e.g. labour). The binary variable $U_{s,t}$ indicates the shut-down status of a plant at time t and state s (1 = plant is offline, 0 = plant is online).

Based on these cost functions, Tseng and Barz maximise the total profit function, which can be formulated as follows:

$$G_{t_0 \to T}(O_{u,t \in [t_0,T]}, P_{El,u,t \in [t_0,T]}) = \sum_{t=t_0}^{T} \sum_{s \in S_t} Pr_s \Big(p_{El,u,t,s} X_{El,u,t,s} - p_{f,u,s}\left(a_0 + a_1 P_{EL,u,t,s} + a_2 P_{EL,u,t,s}^2\right) - C_{St,u,t,s}\Big). \tag{5.11}$$

This profit function can also be formulated as a recursive term, in which the profit $G_{t \to T}$ between time t and T is calculated with the help of the expected profit $G_{t+1 \to T}$ between $t + 1$ to T, adding the expected profit attained at time step t. Therefore, it is enough to maximise the profit in time step t and state s adding the expected profit $G_{t+1 \to T}^*$:

$$G_{s,t \to T}^*(X_{El,t}) = \max\left(\sum_{s' \in S_{t+1}} Pr_{s,t \to s',t+1}\Big[p_{El,s,t} X_{El,s,t} - p_{f,t} C_{Op,s,t}(X_{El,s,t}) - C_{St,s,t} + G_{s',t+1 \to T}^*(X_{El,t \in [t+1,T]})\Big]\right). \tag{5.12}$$

The calculation has to be done backwards with respect to the time axis. The recursive computation ends at the single root state at time t_0, resulting in the total profit maximum during the planning horizon t_0 to T.

Finally, it is worth mentioning that these approaches for short- and mid-term power production planning can be easily extended to optimise an energy system in the long-term planning horizon.

5.2. Long-term system optimisation

Long-term stochastic models can be solved – analogous to the others – with the help of single-stage or multi-stage modelling for the uncertain parameters. Whilst single-stage models handle uncertainties at time t_0, multi-stage models handle uncertainties in each stage separately. Like the one developed by Göbelt (see Göbelt, 2001) and derived from the MOTAD[5] approach (see Tauer, 1983), single-stage stochastic models use several input parameters adjusted via their standard deviations within the objective (profit) function. In energy modelling, these uncertain parameters are electricity prices p^{EL} on the income side and fuel prices p^{FU} on the cost side, as well as CO_2-certificate prices p^{cert} for the production of one unit electricity X^{EL}. In these models the variable costs c_u^{va}, the fixed costs c_u^{fix} of all plants and also investment costs c_{un}^{inv} for new plant capacities Cap_{un} are taken into account. Instead of using constant values for the stochastic parameters, as is the practice in deterministic models, the standard deviation of each parameter is subtracted on the income side or added on the cost side of the profit function. But before subtracting or adding the standard deviation σ of each parameter, the parameters are weighted with a risk aversion coefficient γ. Finally, it should be pointed out that uncertainties on the demand side are also modelled with the help of the standard deviation of the total demand D_t for time t. The standard deviation is weighted with a probability factor pr_D representing the probability that this constraint will be fulfilled:

[5] MOTAD means "minimisation of total absolute deviations". The approach was developed first by Hazel in 1971 to handle risks in agricultural economics and was extended by Tauer as Target MOTAD in 1983.

$$
\begin{aligned}
&\max \sum_{t\in T}(1+r)^{-t}\Bigg((p_t^{EL}-\gamma\sigma_{EL})X_t^{EL}\\
&\quad-\sum_{u_o\in U}\Big[\big((p_t^{FU}+\gamma\sigma_{FU})\eta_u+(p_t^{cert}+\gamma\sigma_{cert})carb_u+c_u^{va}\big)X_{u,t}^{EL}+c_{u,t}^{fix}\Big]\\
&\quad+\sum_{u_n\in U}(c_{u_n}^{inv}+c_{u_n}^{fix})Cap_{u_n,t}\Bigg),\\
&\sum_{u\in U_t}X_{u,t}^{EL}\geqslant D_t+pr_D\sigma_D,\\
&X_{u,t}^{EL}\leqslant Cap_u.
\end{aligned}
\tag{5.13}
$$

The advantages of single-stage modelling are the simple practicability and the low effort for additional data. The complexity of the model corresponds to that of the deterministic one, so that no extra computing time is necessary. The main disadvantage of single-stage stochastic modelling is that only the information about uncertain parameters is used, which exists at the beginning of the planning period because all decisions are made at the beginning.

As multi-stage models can handle information that might appear later, they are a much more robust approach than the single-stage approach and therefore their use is more widespread. But the long computing time of this kind of model limits their use for long-term problems. In this case a deterministic approach can be seen as a multi-stage decision tree consisting of only one path with a probability of 100%. So the complexity of a model based on a non-branched decision tree corresponds to the complexity of the deterministic model multiplied with the number of possible paths. This makes stochastic models with many stages and some thousand nodes impossible to compute in an appropriate time. Therefore, scenario reduction algorithms (see Section 4) have to be applied before the optimisation problem can be solved. The results of scenario reduction are recombining trees or usual decision trees (binomial, see Fig. 5, trinomial trees, etc.).

Long-term system optimisation models for energy markets usually consider a time horizon of more than 20 years. So they can be seen as the time-extended version of the profit maximising approach for short- or mid-term power production planning models, if the analysed system is an energy company. In this case, it is suggested to use the profit maximising approach in the long-term view. But if the analysed system is the whole energy system of a country or a region, it is wise to choose the cost minimising approach to maximise total welfare:

$$
\begin{aligned}
&\min \sum_{t\in T}(1+r)^{-t}\sum_{s\in S_t}pr_s\Bigg(\sum_{u_o\in U}\Big[\Big(p_{t,s}^{FU}\eta_u+c_u^{va}+p_{t,s}^{cert}carb_u\Big)X_{u,t,s}^{EL}+c_{u,t,s}^{fix}\Big]\\
&\quad+\sum_{u_n\in U}(c_{u_n}^{inv}+c_{u_n}^{fix})Cap_{u_n,t,s}\Bigg),\\
&\sum_{u\in U_t}\sum_{s\in S_t}pr_sX_{u,t,s}^{EL}\geqslant D_t,\\
&X_{u,t,s}^{EL}\leqslant Cap_u.
\end{aligned}
\tag{5.14}
$$

This simplified cost function considers the total costs of total electricity production, i.e. fuel costs, variable costs, fixed costs and investment costs regarding all available power plants u within the model horizon T. As emission trading has been established, the emission certificate costs are also taken into account in deterministic energy system models (see Barreto and Kypreos, 2004; Enzensberger, 2003), but also have to be integrated into stochastic energy models.

The main constraint ensures that the expected value of electricity production in each time t meets the demand in t .The second constraint ensures the availability of enough power plant capacity in each state s of each time step t. Further constraints can be added to this simplified multi-stage approach, if other market restrictions exist.

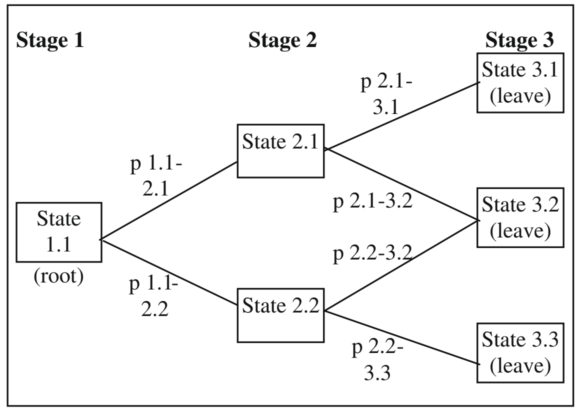

Fig. 5. Example of a two-stage binomial decision tree.

These kind of multi-stage models deliver a robust solution for long-term system optimisation and they are therefore usually applied in uncertain electricity markets. They can also be applied to other energy markets by adjusting the uncertain parameters and the cost structure of the analysed energy market.

5.3. Investment decision models

Investment decision models generally calculate the value of energy investments and deliver a strong decision base before starting huge and capital intensive investments in energy markets (power plants, gas storage, emission reduction technologies, etc.). Traditional investment decision models use the net present value approach and other basic evaluation methods. However, some advanced methods, such as the real options approach, have been applied in some evaluation models for energy investments in recent years. The real options approach is used to calculate the value of an investment, whereby some flexible mechanisms are taken into account. Generally one has the option to defer or to stop an investment at the beginning, and this option or flexibility has a value which should be regarded within the evaluation of the investment. Another option is the shut-down of the power plant, if this is the core of an investment, and running the plant only if a positive marginal return is expected. Actually this "real option" is evaluated with the help of a stochastic model for the evaluation of a coal power plant, developed by Muche (2007). In this real options model the electricity price, the coal price and the carbon price are modelled as stochastic parameters. They are simulated by mean-reverting processes (see Section 3). The price for electricity per unit (MWh) is modelled for every day t and is declared as $p_{e,t}$, which is a stochastic element varying each day due to its set up at the day ahead-market. Accordingly the price for CO_2-certificates is $p_{c,t}$ and has to be multiplied with a constant factor *carb*, which defines the number of CO_2-certificates needed to produce one unit of electricity. This product equals the costs of CO_2 emissions during the production of one unit of electricity. It is assumed that the plant is supplied with enough CO_2-certificates and the investors do not need to buy anymore, so the costs for CO_2-certificates have to be seen as opportunity costs, as they could be sold, e.g. on the EEX exchange. Another important cost component is the fuel cost, here the coal price $p_{coal,t}$, which is multiplied with the constant heating value *coal* to evaluate the fuel costs for 1 MWh of power production. Both the coal price and the CO_2-certificate price are stochastic parameters, which have to be estimated like the electricity prices, while the following cost components are considered as deterministic ones. The other variable operating costs are summarised in c_v (also a value for 1 MWh power output) and are considered as constant elements. In addition, the fixed costs c_f per MWh and the non-cash item depreciation c_d per MWh are also included in the model. As a simplification, the taxes are added via a tax rate s to the cash-flow extended by depreciation. According to these definitions the cash-flow per MWh at day t is calculated as follows:

$$\begin{aligned} z_t &= p_{e,t} - p_{c,t} \cdot carb - p_{coal,t} \cdot coal - c_v - c_f \\ &\quad - s \cdot \left(p_{e,t} - p_{c,t} \cdot carb - p_{coal,t} \cdot coal - c_v - c_f - c_d\right) \\ \Longleftrightarrow z_t &= (1-s) \cdot \left(p_{e,t} - p_{c,t} \cdot carb - p_{coal,t} \cdot coal - c_v - c_f\right) + s \cdot c_d. \end{aligned} \tag{5.15}$$

The definition of days as planning time periods makes the use of electricity, CO_2-certificate and coal prices in $t-1$ possible, which can be observed on the day ahead-market. That is why the marginal return $p_{e,t-1} - p_{c,t-1} \cdot carb - p_{coal,t-1} \cdot coal - c_v$ of production can be used for the real option approach to evaluate the investment in such a coal plant.

Depending on the optimal marginal return the operation of the power plant can be planned, whereas a plant operation in time period t only makes sense if the marginal return of this day t is positive. In this case the cash-flow in t can be calculated as a call option warrant on the underlying "electricity prices":

$$\begin{aligned} z_t &= \max[p_{e,t} - p_{c,t} \cdot carb - p_{coal,t} \cdot coal - c_v, 0] \cdot (1-s) \\ &\quad - c_f \cdot (1-s) + c_d \cdot s. \end{aligned} \tag{5.16}$$

Thereby the term $\max[p_{e,t} - p_{c,t} \cdot carb - p_{coal,t} \cdot coal - c_v, 0]$ of the cash-flow equation corresponds to the cash-flow structure of a European Call on the underlying "electricity prices" with a striking price amounting to the sum of all variable costs.

Thus, the evaluation of the coal power plant can be done on the basis of these Call Options for each day within the useful life of the plant. The risk-adjusted total value of all these Call Options in t_0 corresponds to the value of the coal power plant in t_0.

After simulating the electricity, CO_2-certificate and coal prices using the approaches from Section 3, the real options approach can be used for the evaluation of a coal plant. Eq. (5.2) estimates the daily cash-flow of the plant considering the option to operate the plant in t only if the marginal return is positive. This means that the cash-flow term in Eq. (5.16) illustrates a marginal return optimal plant operation. All daily expected cash flows z_t within the service life are adjusted by the risk-free interest, to calculate the net present value of the power plant:

$$C_0 = -I_0 + \sum_{t=1}^{T} E_{RN,0}(z_t) \cdot e^{-r_{f,s} \cdot t}. \tag{5.17}$$

Furthermore, it is worth mentioning that the risk-neutral process of the electricity prices is to be used if the real option approach is applied as an evaluation method. Therefore, the expected value of the cash flows is adjusted as $E_{RN,0}(z_t)$ and it is determined after $N = 1000$ simulations as the mean value of all simulations. Based on the risk neutral expected value of all cash flows throughout the lifetime of the plant, the risk-neutral net present value C_0 is calculated using the risk-free interest $r_{f,s}$ and the initial investments I_0. If the $NPV_{RN}C_0$ of a plant is positive, then the investment is executed; otherwise it should be cancelled analogously to the familiar NPV method.

In this section the evaluation of a coal plant has been described via the real options approach. But by making some adjustments, the evaluation method can also be applied for other plant types, especially gas plants, and even for other investments in energy markets, such as gas storage.

6. Summary and future research

Many models based on a deterministic approach can be found in energy modelling, which are suitable to cover several characteristics of today's markets. Stochastic approaches are especially useful for modelling uncertain parameters, and in recent years several approaches have been developed for application in energy markets. This paper presented a survey of stochastic models focusing on electricity market prices and also introduced selected integrated approaches which combine econometric models to simulate uncertainties with system optimisation models. The reason for this combination can be seen in the fact that many risks in electricity markets are fundamentally related to the underlying cost structures. This involves the application of integrated methods which combine the advantages of standard methods in financial markets with fundamental energy market models.

Firstly, different econometric models have been described, especially stochastic processes to simulate uncertain electricity and fuel prices. Some of these models consider deterministic trends and seasonality as well as stochastic components of the price processes. Others take price spikes, especially for electricity prices, into account. These approaches are especially used in energy trading companies to quantify price risks of trading position or of the power plant portfolio. Changing framework conditions such as the introduction of emissions trading or the change in market design necessitate the development of new and adapted methods. Models dealing with uncertain CO_2 emission allowance prices are still relatively rare and further efforts should be made in this particular field. The change in market design, allowing negative electricity prices, also necessitates some adaptations in energy models. The loss of an owner of long trading positions in electricity markets was limited until now. With the introduction of negative prices, owners of long trading positions were also exposed – at least in theory – to the risk of unlimited losses, which have to be taken into account in novel econometric models.

Econometric models are often used to simulate price paths, which serve as input to fundamental models. If deterministic models are solved with these simulated price paths, distributed computing can play a crucial role, as the models with different input price paths can be solved in parallel. Standardised tools, helping to distribute the generated models and aggregating procedures for the solutions, are necessary for successful implementation in the energy industry. If the simulated price paths are considered in an integrated stochastic optimisation approach instead, scenario reduction algorithms (see Section 4) are a reasonable method for solving the models within an acceptable amount of time. However, scenario reduction algorithms are only applied for a small but growing number of stochastic models. Several approaches have been developed and advanced in recent years, but further research is still necessary in this field, especially when the stochastic models are applied in the day-to-day business in energy trading.

The reduced scenario trees or the scenario lattice forms the basis for stochastic models. In electricity markets these models concentrate on determining the optimal investment decision or the optimal power production plan for a given period. Thereby the objective functions of these models include different simulated uncertain parameters, and they are optimised based on scenario states representing different values of the these parameters. Besides the system optimisation models, which take the total energy system into account, models determining the value of a single power plant or the optimal short- and mid-term plant dispatch of one energy supplier can be distinguished. Thereby it is important to stress that if the evaluation of a plant is done via a real options approach, then the stochastic processes, describing the uncertain parameters, have to be adjusted by a term for the market price of risk. If several uncertain parameters, such as e.g. gas prices, electricity prices and hydro inflow, are considered in such a real option approach, the correlation between the different price paths has a significant impact on the results and thus has to be adequately considered.

In general, the overview of stochastic modelling approaches for liberalised electricity markets has shown that a combination of fundamental market models with financial engineering approaches commonly used in banks and other trading companies provides an interesting and useful approach to derive electricity prices. The presented approaches can be used to derive both price forecasts and uncertainty ranges for the future development of prices. These can be used for the operational and strategic management of generation and trading portfolios as well as for assessing the risks associated with these portfolios. Further research in this field should aim at aggregating information, e.g. with the help of reduced scenario trees and at developing efficient decomposition approaches, which allow a broad range of price and quantity uncertainty to be dealt with in a reasonable computation time.

Interactions between energy prices and technology choice are analysed within the presented long-term optimisation models. These models can be developed further by also incorporating the impact of fluctuating generation uncertainty as well as load uncertainty, e.g. due to new consumers such as electric vehicles, and their impact on optimal investments. In this regard additional investigations are necessary to answer the questions concerning long-term price equilibriums and the robustness of investment decisions under uncertainty. In this regard questions of market design and market power are also of importance, so that supply adequacy can be assured at the lowest possible costs.

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