1 Math Introduction

General Model Formulation:

$$\min C = \sum_{p,t} mc_p \cdot G_{p,t} \tag{1}$$

$$\text{s.t.} \quad \sum_p G_{p,t} = d_t \qquad\qquad \forall t \in T \tag{2}$$

$$G_{p,t} \leq g_p^{max} \qquad\qquad \forall p \in P, t \in T \tag{3}$$

For a "system" of two time periods $t1, t2$ and two plants/generators/technology $p1, p2$ the equations above represent the following formulation:

$$\min C = mc_{p1} \cdot G_{p1,t1} + mc_{p1} \cdot G_{p1,t2}$$
$$+ mc_{p2} \cdot G_{p2,t1} + mc_{p2} \cdot G_{p2,t2} \tag{4}$$
$$\tag{5}$$

$$\text{s.t.} \quad -G_{p1,t1} - G_{p2,t1} \leq -d_{t1} \tag{6}$$
$$-G_{p1,t2} - G_{p2,t2} \leq -d_{t2} \tag{7}$$
$$G_{p1,t1} \leq g_{p1}^{max} \tag{8}$$
$$G_{p1,t2} \leq g_{p1}^{max} \tag{9}$$
$$G_{p2,t1} \leq g_{p2}^{max} \tag{10}$$
$$G_{p2,t2} \leq g_{p2}^{max} \tag{11}$$

Conventions for the mathematical formulation:

- Variables in capital letters

- Parameters in lower-case letters

- Indices as subscript

- Variation as superscript e.g. g_p^{max}, g_p^{min}

- Be clear for what indecies the equations are valid

2 Matrix Notation

$$\min\{c^T x \mid Ax \leq b\} \tag{12}$$

$$c = \begin{pmatrix} mc_{p1} \\ mc_{p2} \\ mc_{p1} \\ mc_{p2} \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} G_{p1,t1} \\ G_{p1,t2} \\ G_{p2,t1} \\ G_{p2,t2} \end{pmatrix} \leq \begin{pmatrix} -d_{t1} \\ -d_{t2} \\ g_{p1}^{max} \\ g_{p1}^{max} \\ g_{p2}^{max} \\ g_{p2}^{max} \end{pmatrix} \tag{14}$$

Essentially are the problems for $t1$ and $t2$ independant. Therefore this is the solution methodology for the timestep $t2$:

$$\begin{pmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} G_{p1,t2} \\ G_{p2,t2} \end{pmatrix} \leq \begin{pmatrix} -d_{t2} \\ g_{p1}^{max} \\ g_{p2}^{max} \end{pmatrix} \tag{15}$$

Example Data:

$$g^{max} = \{p1 \rightarrow 50, p2 \rightarrow 200\}$$
$$mc = \{p1 \rightarrow 5, p2 \rightarrow 10\}$$
$$d = \{t1 \rightarrow 100, t2 \rightarrow 150\}$$

2.1 Simplex Tableau

	x_1	x_2	s_1	s_2	s_3	RHS
s_1	-1	-1	1	0	0	$-d_{t2}$
s_2	1	0	0	1	0	g_{p1}^{max}
s_3	0	1	0	0	1	g_{p2}^{max}
z	mc_{p1}	mc_{p2}	0	0	0	0

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	x_1	x_2	s_1	s_2	s_3	RHS
s_1	-1	-1	1	0	0	-150
s_2	1	0	0	1	0	50
s_3	0	1	0	0	1	200
z	5	10	0	0	0	0

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