ECE53800 Digital Signal Processing I
Matlab Homework 1:
Radar Time-Delay Estimation

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3 Plots and explanations

In this Matlab homework, we were applying the discrete-time cross-correlation to the problem of time-delay estimation in radar.

The following model was used

$$y[n] = x[n-20] + a_2 x[n - D_2] + v[n]$$

where $x[n]$ is the input sequence of length $M = \{15, 63, 127\}$, $v[n]$ is additive Gaussian noise $\mathcal{N}(0,1)$, $a_2 = \{1, -1\}$ is the amplitude of the input sequence $x[n]$ dealyed by $D_2 = \{22, 21\}$.

Below are the results of "target" detection by means of cross-correlation obtained for different values of parameters M, a_2, and D_2.

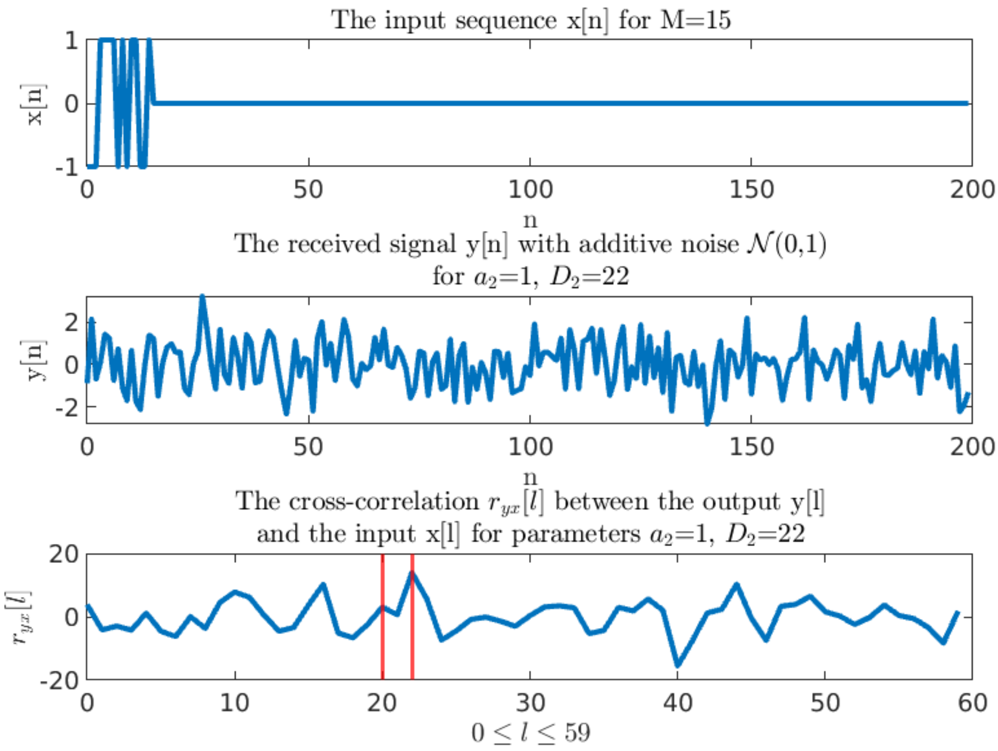

Figure 1: The generated maximal-length sequence $x[n]$ for $M = 15$, the received signal $y[n]$ with the amplitude $a_2 = 1$ and the delay $D_2 = 22$, and the cross-correlation $r_{yx}[l]$ between $y[l]$ and $x[l]$

From Figure 1, one can see that for the maximal-length sequence of length $M = 15$, the amplitude $a_2 = 1$ and the time delay $D_2 = 22$ we are not certain about "targets" to be detected at the expected locations $l = 20$ and $l = 22$ due to the noise presence and relatively short length of the input sequence $x[n]$.

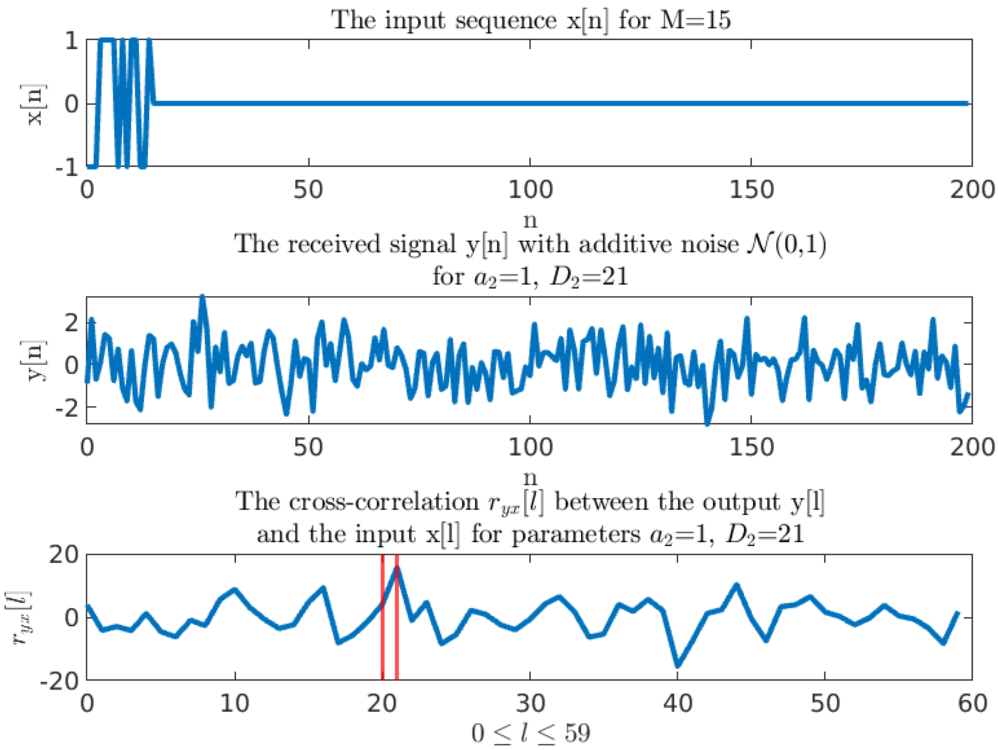

Figure 2: The generated Maximal-Length Sequences $x[n]$ for $M = 15$, the received signal $y[n]$ with the amplitude $a_2 = 1$ and the delay $D_2 = 21$, and the cross-correlation $r_{yx}[l]$ between $y[l]$ and $x[l]$

From Figure 2, for the same maximal-length sequence of length $M = 15$, the amplitude $a_2 = 1$ and the time delay $D_2 = 21$ one can observe the situation worsened as now it is even impossible to say whether there are 2 targets at the expected locations $l = 20$ and $l = 21$ or just one the location of which is undefined.

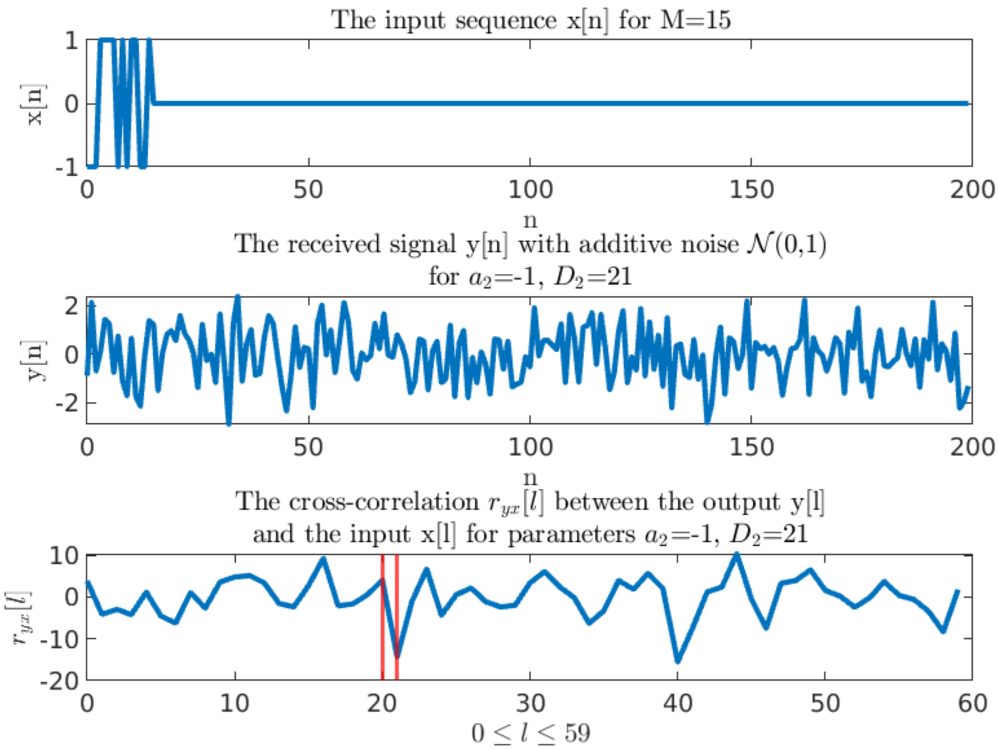

Figure 3: The generated Maximal-Length Sequences $x[n]$ for $M = 15$, the received signal $y[n]$ with the amplitude $a_2 = -1$ and the delay $D_2 = 21$, and the cross-correlation $r_{yx}[l]$ between $y[l]$ and $x[l]$

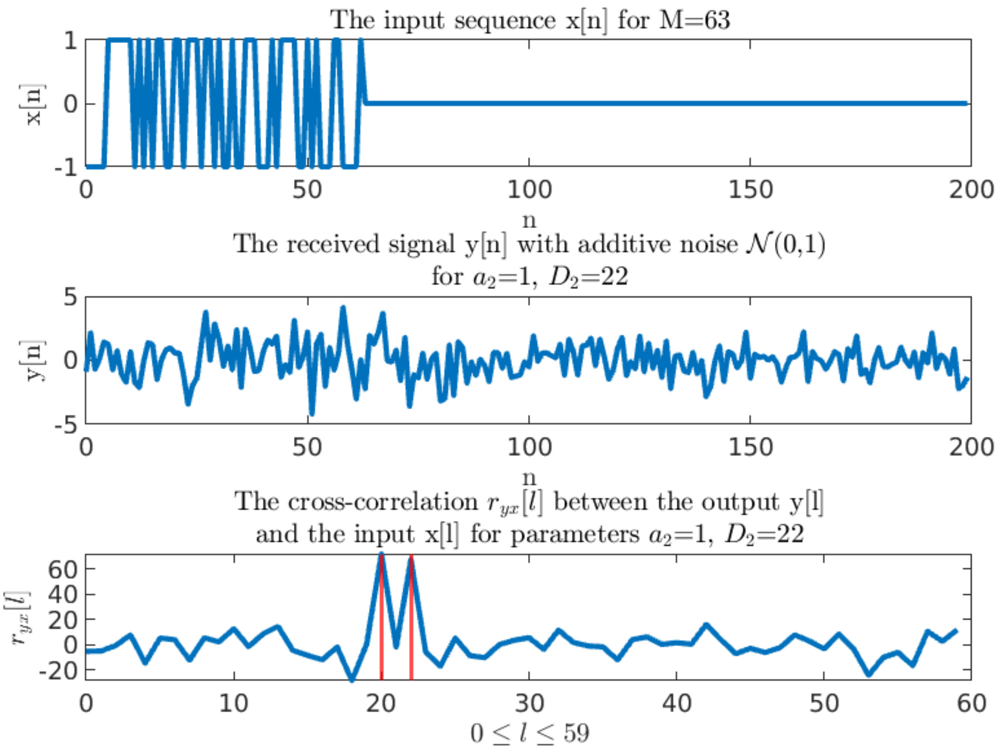

Figure 4: The generated Maximal-Length Sequences $x[n]$ for $M = 63$, the received signal $y[n]$ with the amplitude $a_2 = 1$ and the delay $D_2 = 22$, and the cross-correlation $r_{yx}[l]$ between $y[l]$ and $x[l]$

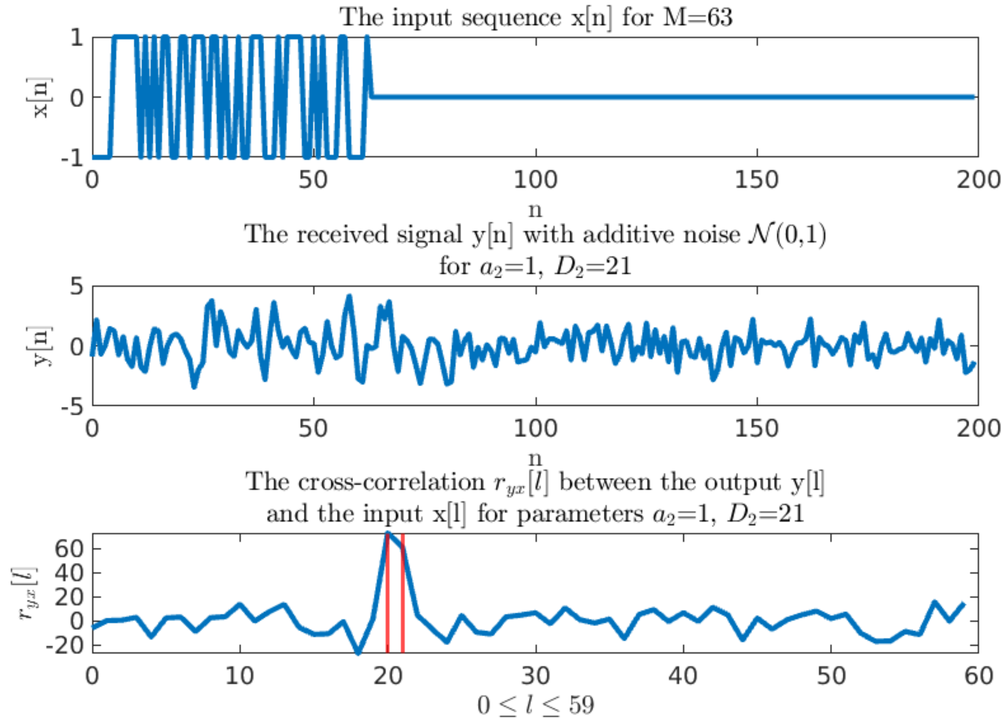

Figure 5: The generated Maximal-Length Sequences $x[n]$ for $M = 63$, the received signal $y[n]$ with the amplitude $a_2 = 1$ and the delay $D_2 = 21$, and the cross-correlation $r_{yx}[l]$ between $y[l]$ and $x[l]$

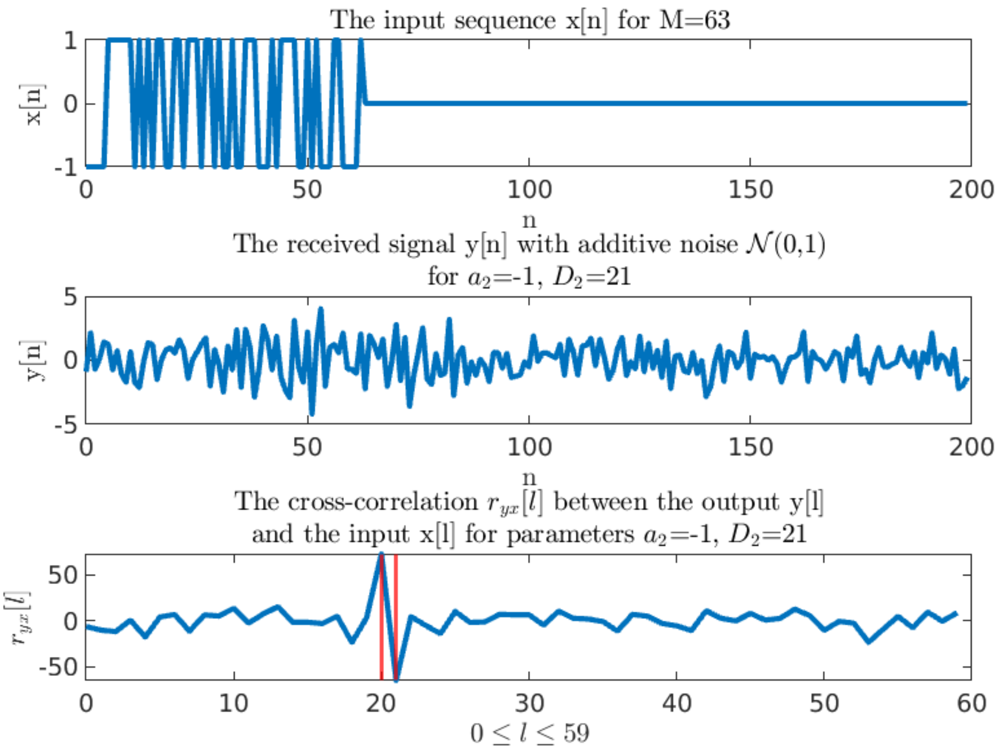

Figure 6: The generated maximal-length sequence $x[n]$ for $M = 63$, the received signal $y[n]$ with the amplitude $a_2 = -1$ and the delay $D_2 = 21$, and the cross-correlation $r_{yx}[l]$ between $y[l]$ and $x[l]$

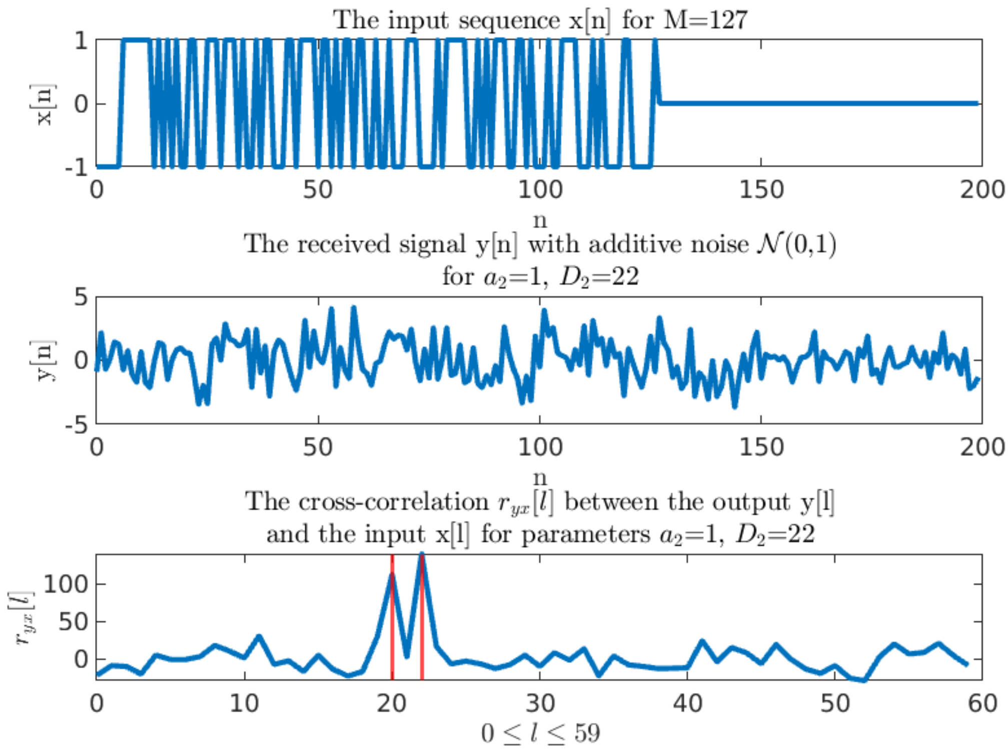

Figure 7: The generated maximal-length sequence $x[n]$ for $M = 127$, the received signal $y[n]$ with the amplitude $a_2 = 1$ and the delay $D_2 = 22$, and the cross-correlation $r_{yx}[l]$ between $y[l]$ and $x[l]$

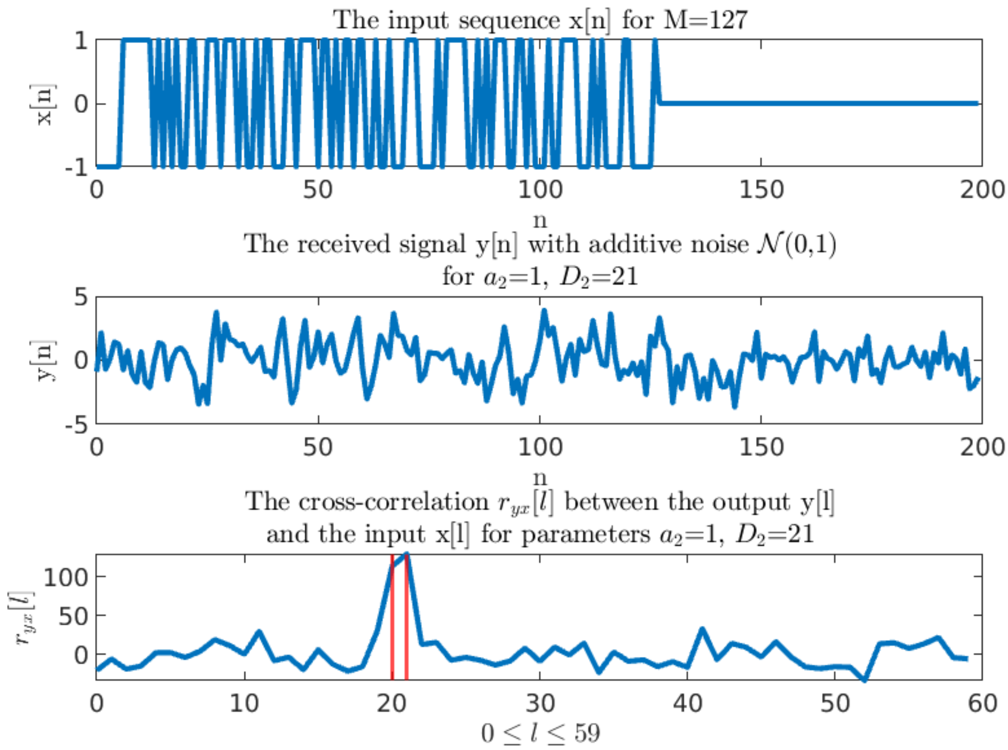

Figure 8: The generated maximal-length sequence $x[n]$ for $M = 127$, the received signal $y[n]$ with the amplitude $a_2 = 1$ and the delay $D_2 = 21$, and the cross-correlation $r_{yx}[l]$ between $y[l]$ and $x[l]$

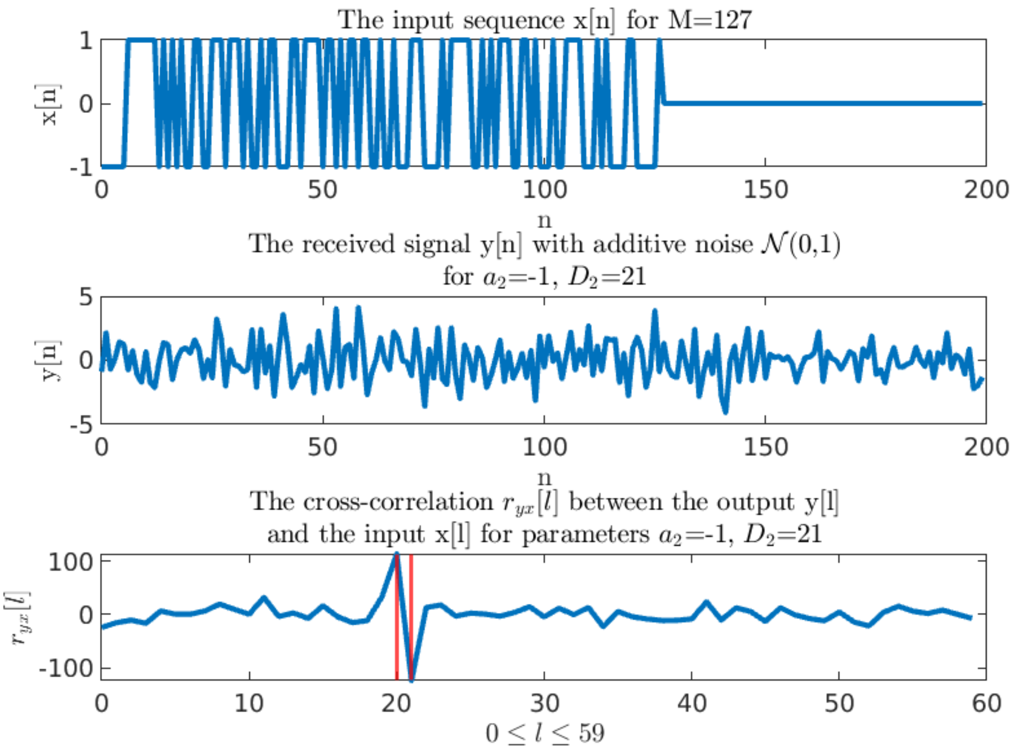

Figure 9: The generated Maximal-Length Sequences $x[n]$ for $M = 127$, the received signal $y[n]$ with the amplitude $a_2 = -1$ and the delay $D_2 = 21$, and the cross-correlation $r_{yx}[l]$ between $y[l]$ and $x[l]$

4 Conclusions

5 The Matlab code

```
clear all
clf

set(0,'defaultTextInterpreter','latex');

% Generate Gaussian noise
```

```
num_samples = 200;
v = randn(1, num_samples);

% Shift registers
reg_4 = [1 0 0 0];
reg_6 = [1 0 0 0 0 0];
reg_7 = [1 0 0 0 0 0 0];

regs = {reg_4, reg_6, reg_7};

% Set of parameters a_{2} and D_{2} and also time
    delay D_{1}
params = [1,22; 1,21; -1,21];
D1 = 20;

for k=1:numel(regs)
    reg = regs{k};
    % Generate the input sequence of dimension M
    x = gen_input(reg);
    M = length(x);
    x_pad = [x, zeros(1, num_samples-M)]

    for i=1:length(params)

        % Pick the amplitude a_{2} and the delay D_
            {2}
        % from the set of parameters
        a2 = params(i,1);
        D2 = params(i,2);
        % Calculate the output and the cross-
            correlation
        % between the output and the input
        [y, ryx] = calc(x, a2, D1, D2, v);

        % Plot the results
```

```
        fig=figure(i+3*(k-1));

        subplot(3, 1, 1);
        plot(0:num_samples-1,x_pad, 'Linewidth', 2)
        title("The input sequence x[n] for M="+M)
        xlabel("n");
        ylabel("x[n]");

        subplot(3, 1, 2);
        plot(0:num_samples-1, y, 'Linewidth', 2);
        title(["The received signal y[n] with
          additive noise $\mathcal{N}$(0,1)"...
        "for $a_{2}$="+a2+", $D_{2}$="+D2]);
        xlabel('n');
        ylabel('y[n]');

        subplot(3, 1, 3);
        plot(0:59, ryx(M:M + 59), 'Linewidth', 2);
        title(["The cross-correlation $r_{yx}[l]$
          between the output y[l]"...
        "and the input x[l] for parameters $a_{2}$
          ="+a2+", $D_{2}$="+D2]);
        xlabel("$0 \leq l \leq 59$");
        ylabel("$r_{yx}[l]$");
        xline(D1, 'r', 'Linewidth', 1.5);
        xline(D2, 'r', 'Linewidth', 1.5);

        saveas(fig, sprintf('fig%d.png', i+3*(k-1)))

    end;
end;

function x = gen_input(reg)
    N = 2^length(reg)-1;
    for ri=1:N
```

```
            x(ri)=reg(1,end);
            reg(2:end)=reg(1:end-1);
            reg(1,1)=rem((reg(1,1)+x(1,ri)),2);
        end
        % Transform 0s and 1s to -1s and 1s
        x = 2*x-1;
end

function [y, ryx] = calc(x, a2, D1, D2, v)
        x_D1 = [zeros(1, D1), x, zeros(1, length(v)-
            length(x)-D1)];
        x_D2 = [zeros(1, D2), x, zeros(1, length(v)-
            length(x)-D2)];
        y = x_D1 + a2 .* x_D2 + v;
        ryx = conv(y,x(end:-1:1));
end
```