ECE637 Digital Image Processing I
Laboratory work 5:
Eigen-decomposition of Images

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2 Multivariate Gaussian Distributions and Whitening

In this section, we generated independent Gaussian random vectors X_i having the following covariance

$$R_x = \begin{bmatrix} 2 & -1.2 \\ -1.2 & 1 \end{bmatrix}$$

The results are presented in Figures 1-3.

Then we used these generated samples of X_i to whiten them. The results of whitening along with numerical listings of the covariance estimates $\hat{R}_X$ and $\hat{R}_W$ are given in chapter Covariance Estimation and Whitening.

2.1 Generating Gaussian random vectors

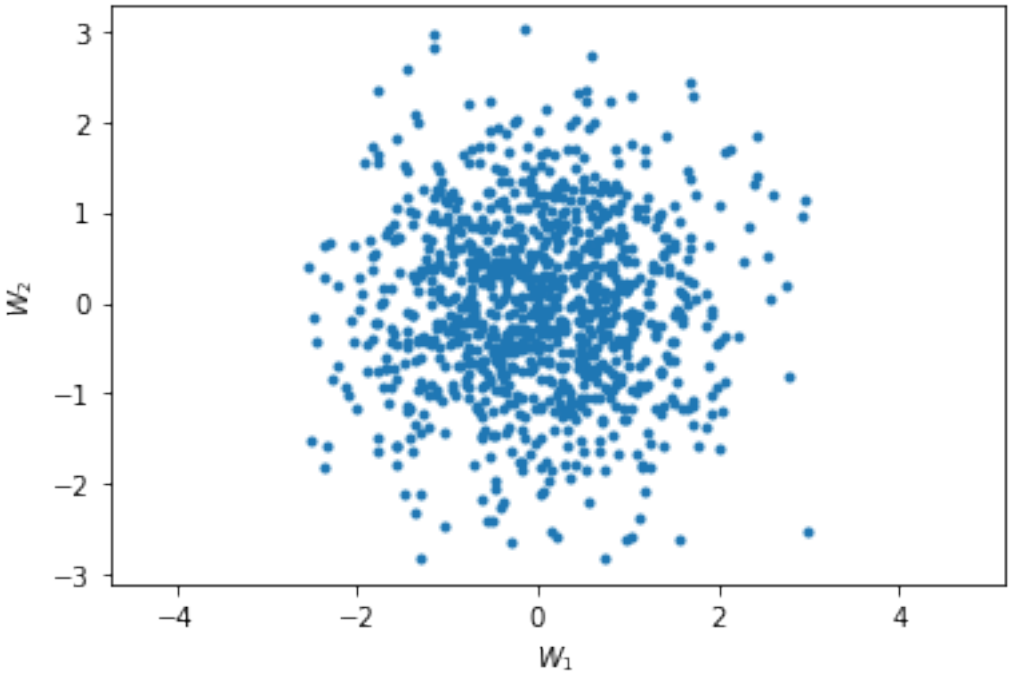

Figure 1: The scatter plot for W

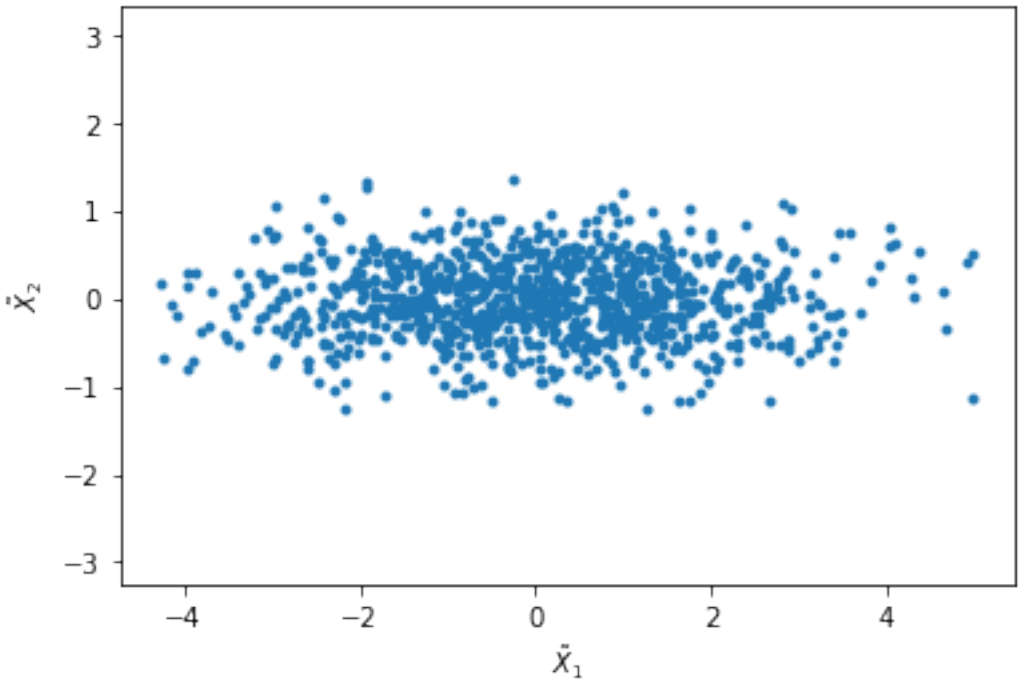

Figure 2: The scatter plot for $\tilde{X}$

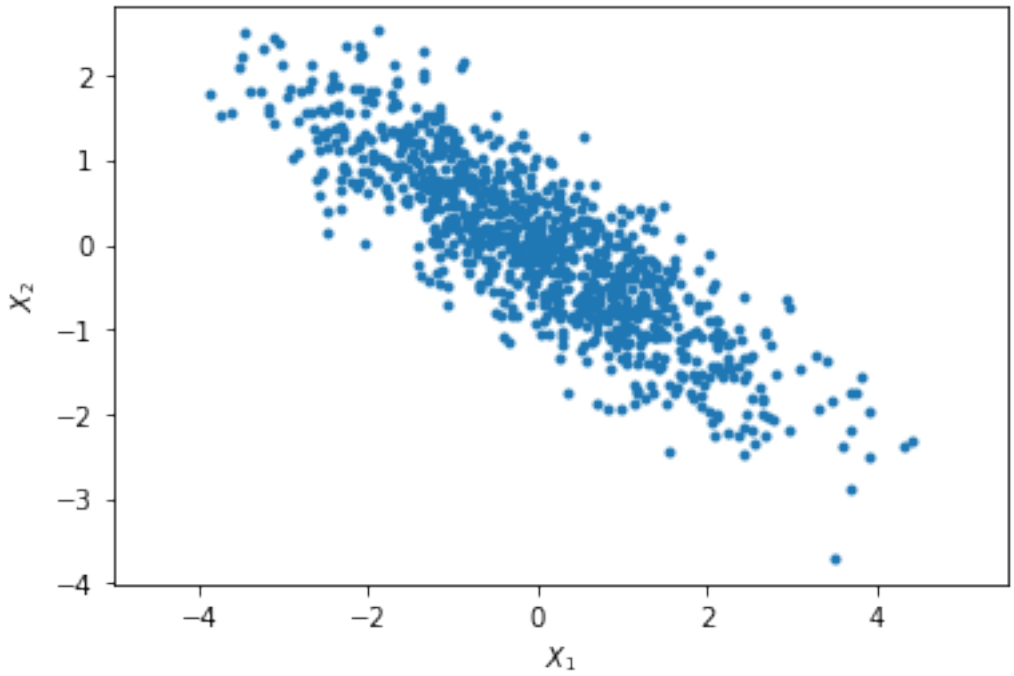

Figure 3: The scatter plot for X

2.2 Covariance Estimation and Whitening

2.2.1 The theoretical value of R_x

$$R_x = \begin{bmatrix} 2 & -1.2 \\ -1.2 & 1 \end{bmatrix} \tag{1}$$

2.2.2 The estimated value of $\hat{R}_x$

$$\hat{R}_x = \begin{bmatrix} 1.95 & -1.17 \\ -1.17 & 0.97 \end{bmatrix} \tag{2}$$

2.2.3 Scatter Plots for $\tilde{X}_i$ and W_i

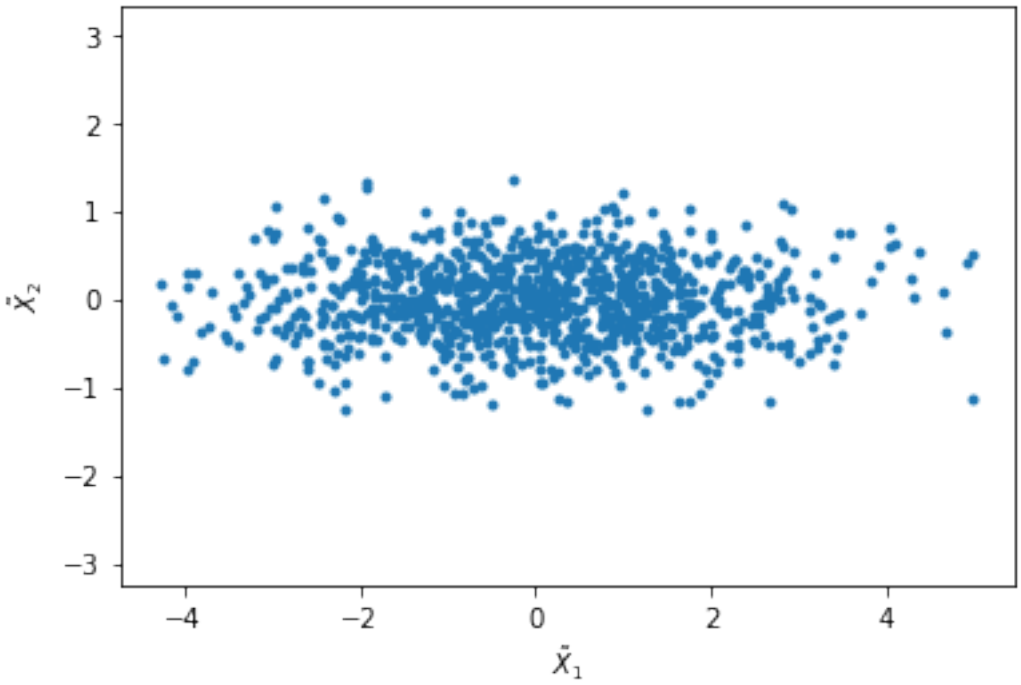

Figure 4: The scatter plot for $\tilde{X}_i$

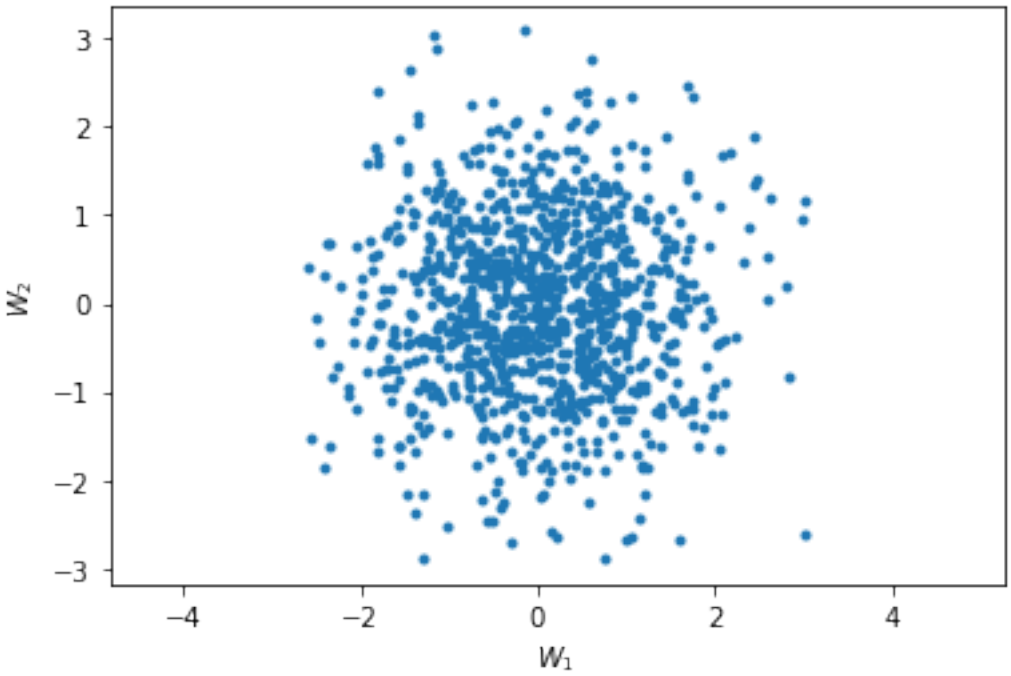

Figure 5: The scatter plot for W_i

2.2.4 Numerical Listing of the Covariance Estimate $\tilde{R}_W$

$$\hat{R}_W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4 Eigenimages, PCA, and Data Reduction

In this part of the laboratory work, we designed a lower dimensional representation of images from the set *training_data* by means of SVD.

The eigenimages associated with the 12 largest eigenvalues are shown in Figure 6. The first 10 projection coefficients for the first 4 images in the set *training_data* are depicted in Figure 7.

Finally, the result of synthesizing the original image $X[:, 0]$ (shown in Figure 8) using the first m eigenvectors is demonstrated in Figure 9

4.1 The first 12 eigenimages

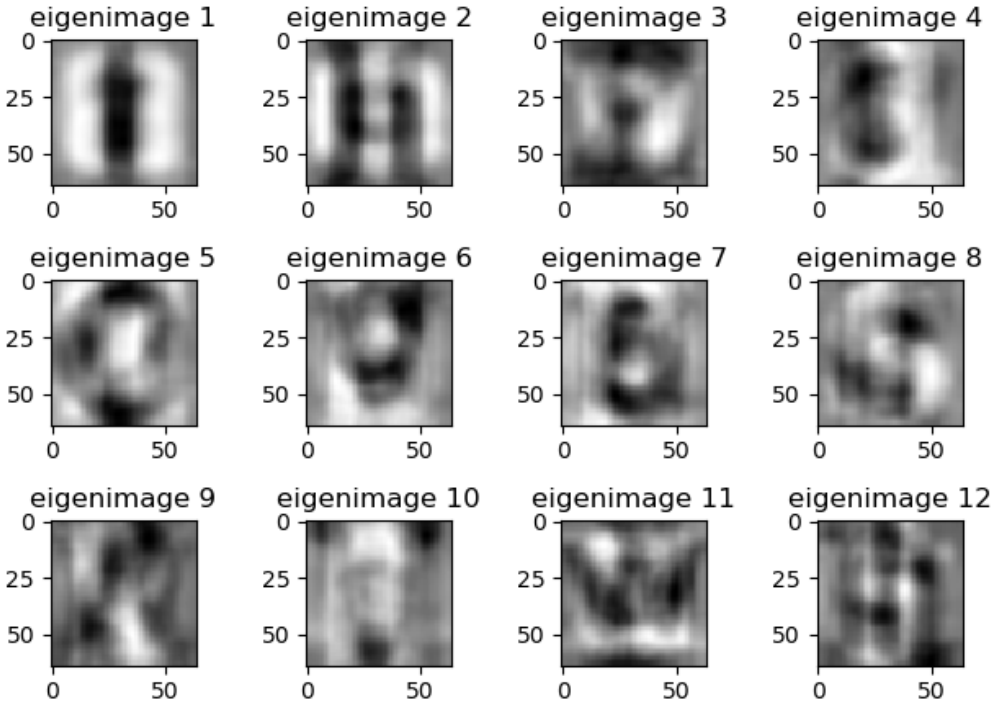

Figure 6: The first 12 eigenimages

4.2 The plots of projection coefficients vs. eigenvector number

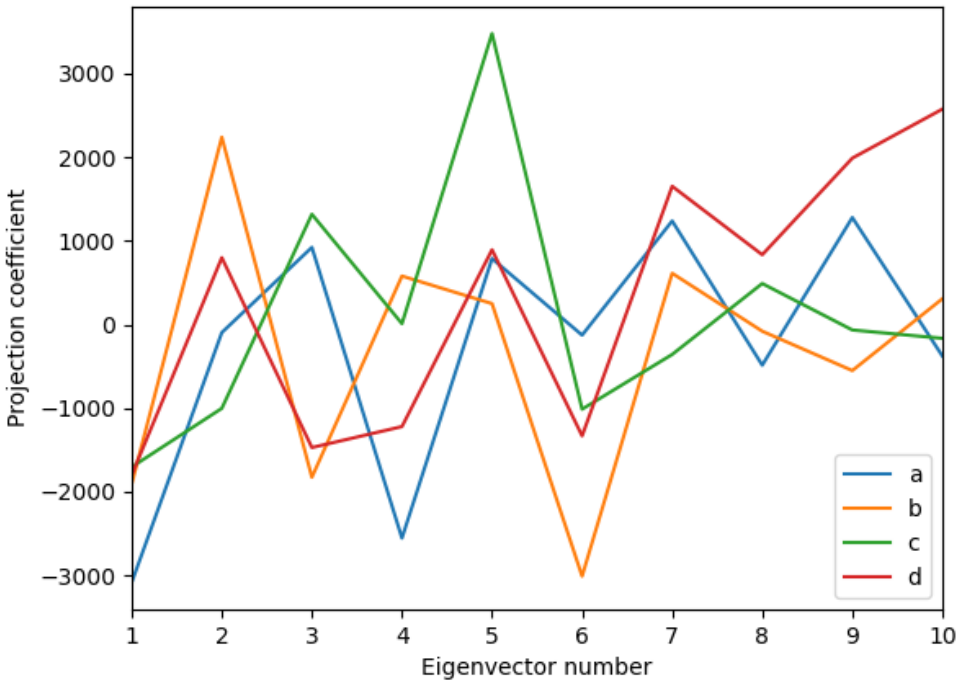

Figure 7: Projection coefficients vs. eigenvector numbers

4.3 The original image, and its 6 resynthesized versions

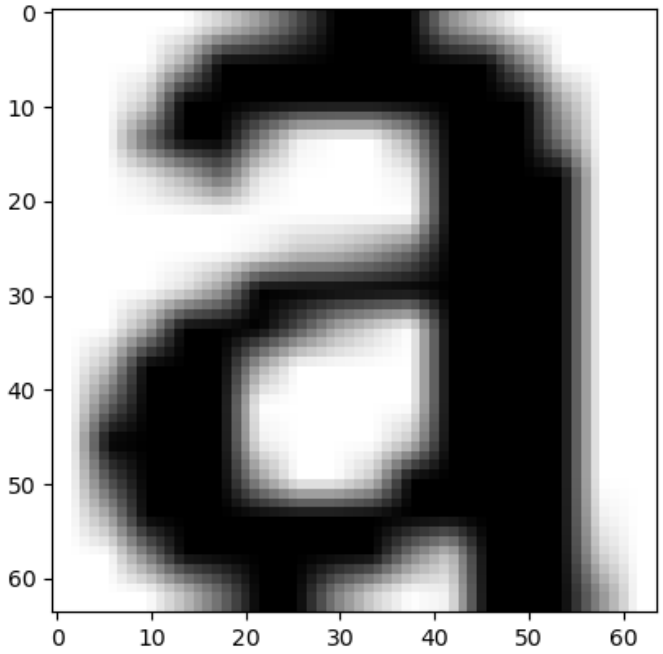

Figure 8: The original image $X[:, 0]$

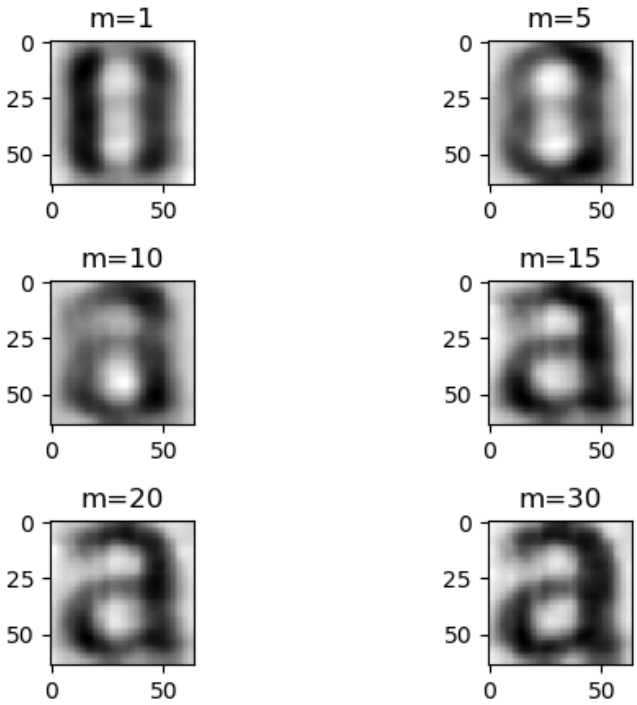

Figure 9: 6 versions of the original image resynthesized from first m eigenvectors

5 Image Classification

In this section, we classified images of the text characters from the set *test_data*. Different classifiers were used. The results are presented in chapter Classification and PCA

5.1 Classification and PCA

5.1.1 Classification errors for R_k

Input character	Output from the classifier
d	a
j	y
l	i
n	v
p	e
q	a
u	a
y	v

5.1.2 Classification errors for $B_k = \Lambda_k$

Input character	Output from the classifier
i	l
u	v

5.1.3 Classification errors for $B_k = R_{wc}$

Input character	Output from the classifier
g	q
y	v

5.1.4 Classification errors for $B_k = \Lambda$

Input character	Output from the classifier
f	t
y	v

5.1.5 Classification errors for $B_k = I$

Input character	Output from the classifier
f	t
g	q
y	v

5.2 Answers to questions

1. The best results are observed for 3 classifiers, $B_k = \Lambda_k$, $B_k = R_{wc}$, and $B_k = \Lambda$. For all the listed classifiers, there were 2 misclassified inputs.

2. In constraining the covariance, it can be concluded from the results that the higher is the complexity (accuracy) of the model, the poorer is the accuracy of the estimates.