The legendary Rome halo displays

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The two Rome halo displays of 1629 and 1630 are prominent in the early halo literature, and the 1629 display is still cited today for having contained a 28° circular halo. We have examined seventeenth century correspondence and publications in order to learn as much as possible about the existing documentation of the two displays. We find the documentation to be too weak to support a definitive interpretation of either display, and we see little evidence for a 28° halo or for other rare halos. The two displays remain important for their role in initiating modern halo science. © 2008 Optical Society of America

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Five suns shone over St. Peter's Basilica on a late March day in 1629. In those days celestial displays were often remembered for the number of their suns, and this display was therefore seen as special. But only ten months later, again in Rome, there was an even more dazzling display, this time with seven suns.

The "suns," other than the true sun, were atmospheric halos. Halos of various shapes are possible, not necessarily circular, and indeed many halos were present in the Rome displays. The suns, or parhelia, as they were also called, were the halos that were relatively localized, somewhat spot-like.

Today we know that halos are due to the interplay of sunlight with ice crystals in the atmosphere, but at the time of the Rome displays there was no such understanding. Halo displays were widely regarded as omens or signs from God, usually unfavorable. It was said that the five suns over St. Peter's portended five years of upheavals within the Church.

Beginning about 1658, however, Christiaan Huygens devised an elegant theory of halos which was modern in spirit and which, despite being wrong about the shape of the atmospheric ice particles that were responsible, was largely correct in its predictions for the common halos. The transition from ignorance to real halo science, largely due to Huygens, was dramatic. Because Huygens' work relied heavily on the Rome halo displays of 1629 and 1630, these displays mark the beginning of that transition.

The 1629 display is remembered for another reason as well. The display has been interpreted as containing a 28° halo, that is, a circular halo having an angular radius of 28°. This has led to speculation about the existence of cubic ice in the atmosphere, since, as pointed out by Whalley [1], this unusual form of ice might theoretically give rise to a 28° halo.

For this article we have examined seventeenth century correspondence and publications in order to clarify the historical circumstances and the significance of the two Rome displays. We find that the displays were important to the beginnings of halo science but that as contributions to the halo record they have only marginal value today. We also find little evidence for a 28° halo. We do find a fascinating detective story, much of it still unsolved.

1. The 1629 Rome display

The first of the two Rome displays occurred in the early afternoon of 20 March 1629. From Rome, Cardinal Francesco Barberini, nephew of Pope

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Urban VIII, sent a diagram and description of the display to Nicolas Claude Fabri de Peiresc, in Aix-en-Provence. Peiresc made copies and distributed them to various acquaintances, including Pierre Gassendi (Fig. 1). It was Gassendi who wrote the first published account of the display, the ten-page *Phaenomenon rarum* of 1629 (Fig. 2).

The information that reached Gassendi left a lot to be desired. Gassendi expressed his frustration in a 15 June 1629 letter to Peiresc:

> I wish that the observer had made note of several things: the diameters of all the halos, their thicknesses, the distances between the suns, their sizes,…

But Gassendi had no idea who the observer was that he was admonishing. A year and a half later, on learning that Christoph Scheiner had been the observer of the 1630 Rome display, Gassendi wrote Scheiner and asked whether he had seen the 1629 display. Scheiner eventually responded in the affirmative, in a letter of 3

Fig. 1. (Color online) Report of the 1629 Rome halo display, perhaps one of the copies made under Peiresc's direction. The diagram and text here are virtually identical to the diagram and *Explication* in both *Phaenomenon rarum* [2] and *Parhelia* [3]. [Bibliothèque nationale de France (BnF), DUPUY 5, fol 54. The manuscript DUPUY 488, fol 169 is similar.]

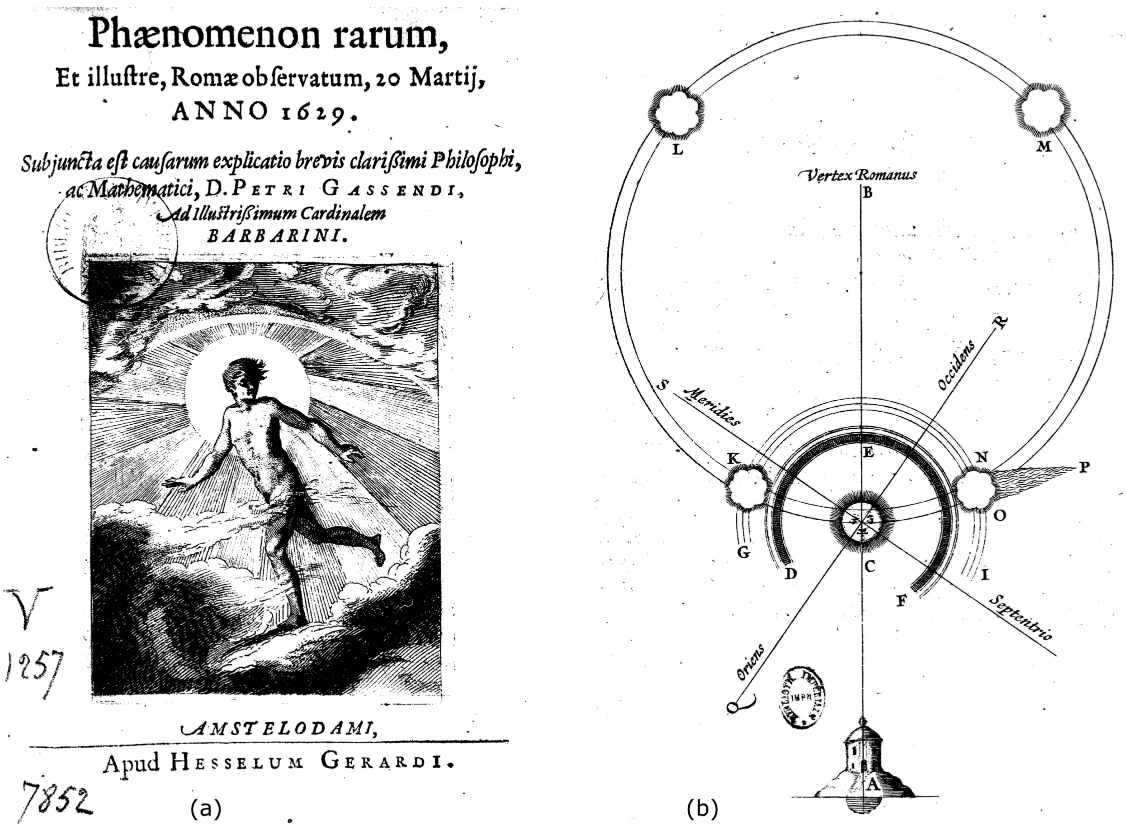


Fig. 2. (a) Title page of Gassendi's *Phaenomenon rarum* [2], which contains the earliest published description of the 1629 Rome halo display. (b) Diagram of the display, from *Phaenomenon rarum*. The five suns are K, L, M, N, and C, the latter being the true sun (Courtesy BnF).

December 1631, but he never did say, as far as we know, that the description of the display that appeared in *Phaenomenon rarum* was his, or that he had been in contact with Barberini. And the diagram of the display apparently was *not* his, though it seems to have been based on his (no longer existing) sketches, according to this same letter. Modern halo literature routinely names Scheiner as the observer of the 1629 display, and so did Gassendi in his later writings, but there was no such understanding at the time of the display, at least not in the correspondence that we have examined. We cannot help wondering where Barberini got the halo report.

The printing of *Phaenomenon rarum* had proceeded without Gassendi's direct supervision, and Gassendi was not happy with the outcome. He eventually arranged for a revision, the result being *Parhelia* [3], which appeared in 1630 and which is today much easier to get than *Phaenomenon rarum*. We are not sure what upset Gassendi about *Phaenomenon rarum*. Gassendi wrote to Peiresc that the article had been dressed up in bad taste, but was there something else, something specific to the science? It seems to have been neither the halo description itself (the *Explication*) nor the diagram; they are virtually the same in *Parhelia* as in *Phaenomenon rarum*, and, for that matter, as in the handwritten report of Fig. 1.

2. The 1630 Rome display

The second Rome display occurred on 24 January 1630. This time Scheiner's name was associated with the display from the start, but Scheiner himself never published anything on the display. What we know about it comes from the above mentioned letter from Scheiner to Gassendi, dated 3 December 1631 and hence written nearly two years after the display. Referring to a diagram of the 1630 display that was supposed to be included with the letter, Scheiner described the display in detail. But Scheiner complicated matters by first remarking that, although he was unsure whether Gassendi had wanted information on the 1629 or on the 1630 display, it really did not matter much, since the two displays were about the same. It was a puzzling comment, since the description that Scheiner gave is inconsistent with the 1629 display as we understand it (Fig. 1).

In any event, the diagram mentioned in Scheiner's letter is now lost and in fact had been lost already by 1658, according to the editors of Gassendi's *Opera Omnia*, where the letter was published. Huygens in 1659 asked J. Chapelain for help in locating the diagram, but by September of 1660 Huygens still did not have it. Huygens did, however, have the *Opera Omnia*. Using the halo description in the published letter, Huygens reconstructed the diagram, as he freely admitted (Appendix A).

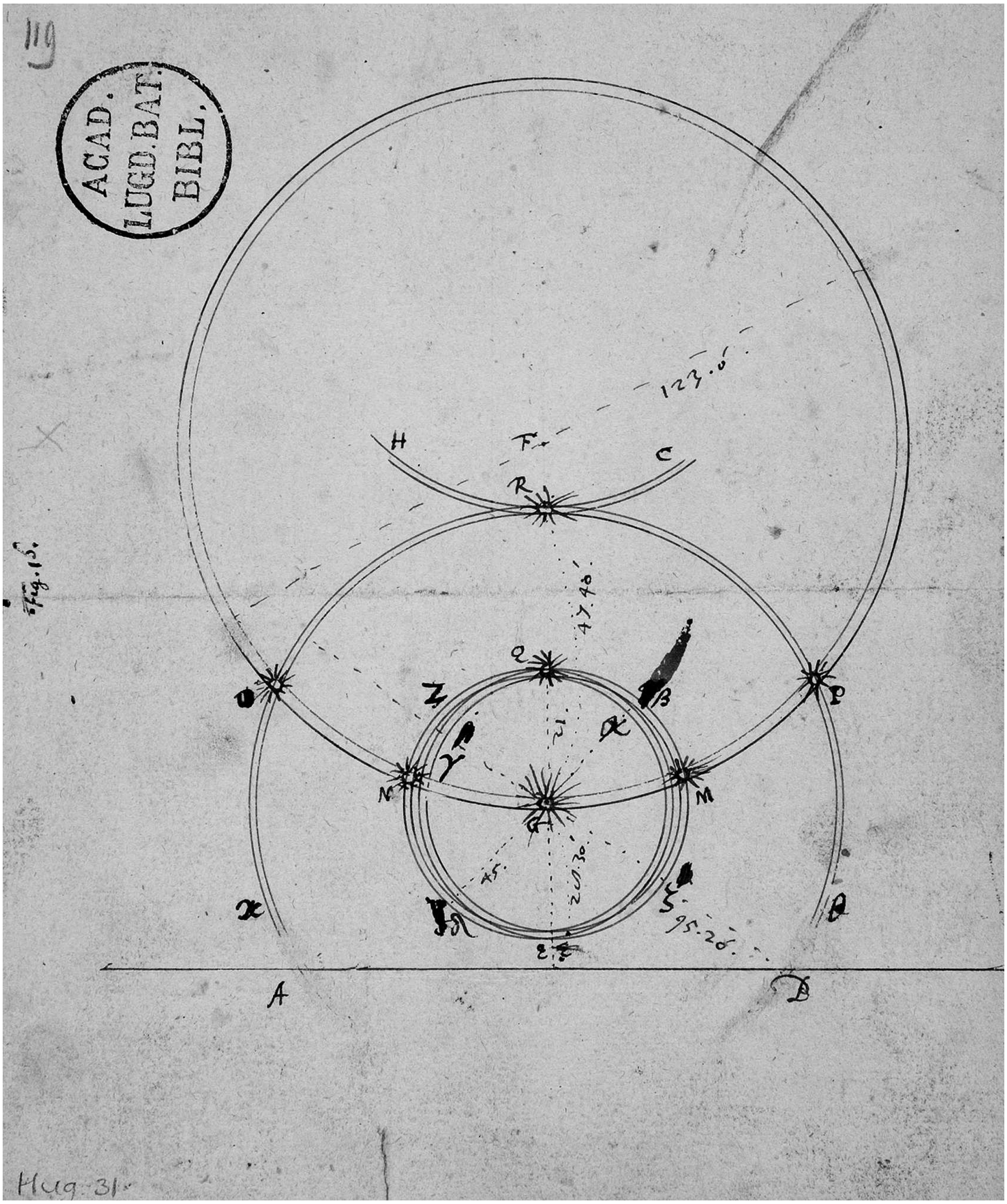


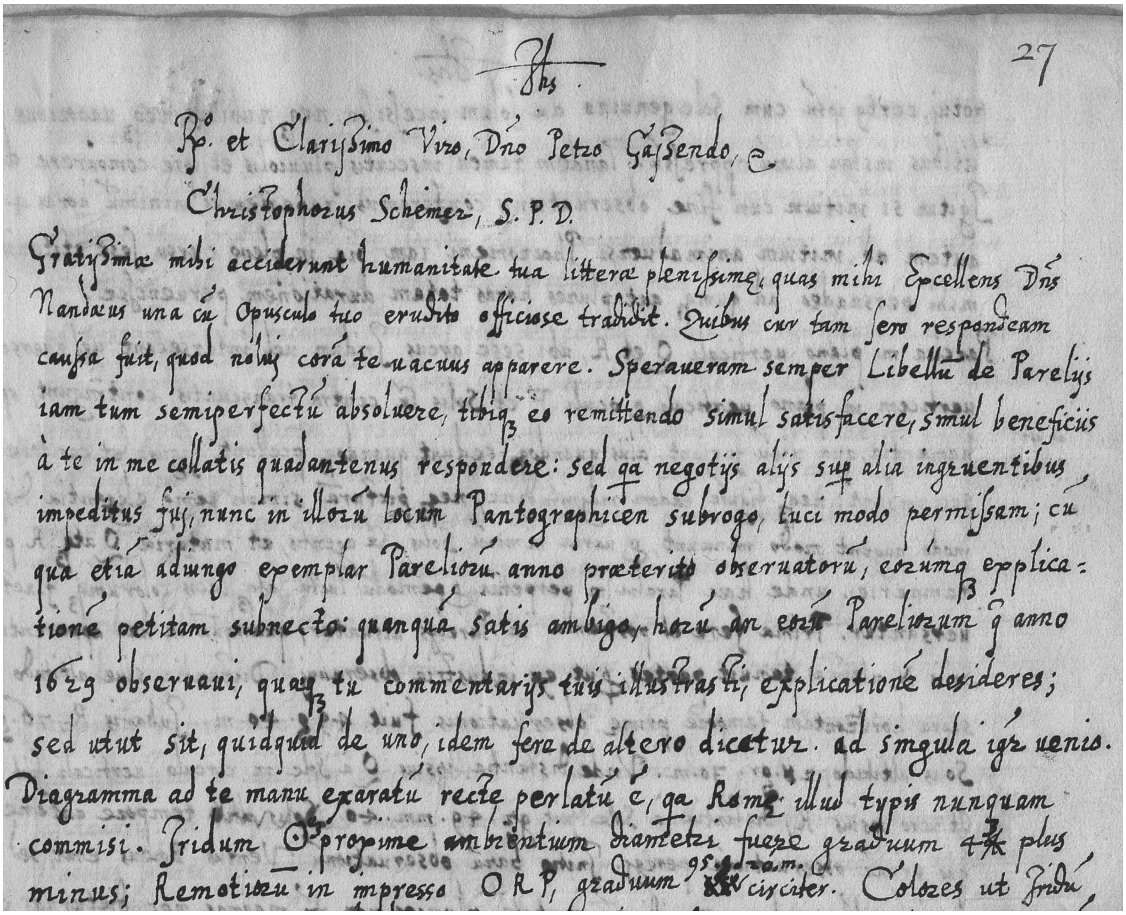
Fig. 3. “The” diagram of the 1630 Rome halo display. The seven suns, marked by sun symbols, are the true sun G, then M and P to the right of it, N and O to the left, and Q and R above. The dashed lines are where angular distances are indicated. The diagram was drawn by Huygens, not by the observer of the display. The figure number was probably added by de Volder and Fullenius [4]. (Leiden University Library, HUG 31, fol 119 r.)

The diagram in Fig. 3, which is today found among Huygens’ surviving manuscripts, is almost certainly the one that Huygens drew (Appendix D). The diagram sometimes appears in the halo literature as “the” diagram of the 1630 Rome display. Seldom is it mentioned that it was drawn not by the observer, but by Huygens, three decades after the display.

Thus we really know very little about the two Rome displays. For the 1629 display we have the diagram of Fig. 1 and its associated text (the same as the *Explication* in Appendix B). For the 1630 display we have the text of Scheiner’s letter (Fig. 4 and Appendix C). Hellmann [8], Meyer [9], and Pernter and Exner [10] all stressed the inadequacy of the existing documentation of the two displays, but their warnings have not always been heard.

3. The place of the two Rome displays in halo science

> The Roman phenomenon, observed by Scheiner, is so famous, on account of its having been the first appear-



Ihs. 27

Rᵒ. et Clarissimo Viro, Dno Petro Gassendo, &c.

Christophorus Scheiner, S. P. D.

Gratissimae mihi acciderunt humanitate tua litterae plenissimae, quas mihi Excellens Dns Naudaeus una cū Opusculo tuo erudito officiose tradidit. Quibus cur tam sero respondeam caussa fuit, quod nolui corā te uacuus apparere. Speraueram semper Libellū de Pareliis iam tum semiperfectū absoluere, tibiq eo remittendo simul satisfacere, simul beneficiis a te in me collatis quadantenus respondere: sed qa negotiis aliis sup alia ingruentibus, impeditus fui, nunc in illorū locum Pantographicen subrogo, luci modo permissam; cū qua etiā adiungo exemplar Pareliorū anno praeterito obseruatorū, eorumq explicatione petitam subnecto: quanquā satis ambigo, horū an eorū Pareliorum q anno 1629 obseruaui, quae tu commentariis tuis illustrasti, explicationē desideres; sed utut sit, quidquid de uno, idem fere de altero dicatur. ad singula igr uenio. Diagramma ad te manu exaratū recte perlatū e, qa Romae illud typis nunquam commisi. Iridum ☉ proxime ambientium diametri fuere graduum 45 plus minus; Remotiorū in impresso ORP, graduum 95 circiter. Colores ut Iridū,

Fig. 4. Opening lines of Scheiner's 3 Dec 1631 letter to Gassendi, in which Scheiner describes the 1630 Rome display. Except for the date and time of the display, which come from Gassendi [5,6] and Braunmühl [7], almost everything that we know about the 1630 display comes from this letter. (BnF, NAF1637, fol 27.)

ance of the kind that engaged the attention of philosophers (Joseph Priestley [11], in 1772).

Gassendi was travelling in the Low Countries when he received word of the 1629 display from Peiresc. There Gassendi passed the report on to Nicolaas van Wassenaer, Hendrik Reneri, and others. Reneri in turn passed on the information to René Descartes. The display made a big impression on Descartes and turned his attention to halos, rainbows, and the like, with the eventual result being Descartes' *Les Météores* [12] of 1637. The 1629 Rome display is the only halo display treated at length in *Les Météores*, though others are mentioned in passing.

Descartes' explanations of halos were only qualitative, and they were far off the mark. Christiaan Huygens fared better. Beginning about two decades after *Les Météores*, Huygens ([13] Vol. 17) constructed a quantitative and remarkably successful theory of halos, many of whose ideas still survive in modern halo theory.

To illustrate his theory, Huygens concentrated on three displays—the two Rome displays together with Hevelius' [14] famous Danzig display of 1661. Many subsequent writers have discussed these displays, and the displays became a staple of the early halo literature.

There are halo reports that long predate the two Rome displays (e.g., [15–19]), but the Rome displays came at a time when science was beginning to be seen as a real possibility. A handful of people, especially Kepler, had a hunch that natural phenomena could be observed carefully, perhaps even measured, and that physical causes could be found to explain them quantitatively. Probably it is no accident that halo science began when it did, at the time of the Rome displays.

4. Scheiner's halo

As mentioned earlier, the 1629 display is sometimes said to have contained a 28° circular halo, often referred to as Scheiner's halo. What is the evidence for a 28° halo in the display?

The 22° circular halo is common. Seven other circular halos, though rare, do exist and are today understood [20]. They are the 9, 18, 20, 23, 24, 35, and 46° halos. But any circular halo other than these eight would be considered highly exotic, perhaps anomalous.

The diagram of the 1629 display (Fig. 1) shows two concentric halos. The inner halo is probably the 22°

halo, in which case a 28° halo would be roughly consistent with the outer halo as drawn. But the report of the display—the *Explication* in Appendix B—said nothing quantitative about the radius of the halo, and it was not until Bravais' *Mémoire* [21] of 1847 that the number 28° appeared. Bravais accepted the *Explication* statement that the (ordinary) parhelia N and K were on the outer halo, and he then calculated the theoretical angular distance of the parhelia from the sun for the time and location of the display. He thus found the outer halo radius to be between 25°15′ and 27°40′, corresponding to sun elevations of 32° and 40°. He then used his (incomplete) knowledge of the crystallography of ice to infer a theoretical halo radius of 27°45′. Scheiner's halo—the 28° halo—was born.

Bravais' argument was made at a time when there was no firm idea of what halo radii, other than 22° and 46°, could reasonably be expected. It depended on the parhelia being on the outer halo, and it depended on the outer halo being a circular halo and not some other arc.

The 28° halo turns out to be extraordinarily rare. There is only one unequivocal sighting of it [20,22]—in the skies above the Chilean altiplano, a place so exotic that we sometimes wonder whether the halo-making conditions there might be unique. To make sense of the report of the 1629 display, we can believe that there was a 28° halo in the display, or we can believe that the report contains an incorrect detail—the location of the parhelia with respect to the outer halo, or perhaps the shape of the outer halo. For us—the authors—it is easier to believe that the observer, putting his recollections to paper after the display was over, got it wrong.

Several other features of the 1629 halo diagram suggest that it is not completely correct. The right parhelion tail P is shown below the parhelic circle KLMN, whereas it should run along the parhelic circle. The zenith—the vertex Romanus B—should be at the center of the parhelic circle, not offset [23]. The lines QR and ST also seem peculiar. Whether we interpret them as ordinary compass directions or as great circles on the celestial sphere—in the latter case indicating the celestial equator and the hour circle through the sun—they look incorrect to us. The diagram is beautiful, but it looks more like the work of a professional illustrator than the report of a scientist. Obviously the diagram cannot be taken seriously in all its details.

5. What halos were they seeing?

We do not know exactly what halos they were seeing, whether in 1629 or in 1630. The uncertainties surrounding the two halo reports simply do not permit a definitive answer. Many halo enthusiasts, however, find irresistible the temptation to speculate here, and so we, too, offer our own best guesses—one for each display. But they are indeed only guesses, and neither is completely satisfactory.

For the 1629 display we think the least unlikely interpretation is that the halos DEF and GKNI were the 22° halo and the circumscribed halo. This requires us to believe that the circumscribed halo, which in reality is oval-shaped, was mistakenly reported as a circle. The mistake would not have been unusual; it was common practice for observers of that era to depict all arcs as being circular, regardless of the reality. In any case, the remaining halos are clear: the ordinary parhelia K and N, the 120° parhelia L and M, and the parhelic circle KLMN.

In the 1630 display four of the halos are readily identifiable, whether from Scheiner's letter or from Huygens' diagram: The suns M and N are the ordinary parhelia, the arc HRC is the circumzenith arc, the largest circle OMNP is the parhelic circle (M and N are probably interchanged in the diagram), and the circle ORP is the 46° halo or, more likely, the supralateral arc. The confused band of nearly circular arcs closest to the sun in the diagram looks wrong as drawn, but we can redraw them so as to give a theoretically correct upper tangent arc and 22° halo that are consistent with Scheiner's letter; we think that the arcs $ZQ\alpha$ and $\beta Q\gamma$ in the letter were the upper tangent arc, and that $\delta\epsilon\zeta$ was the 22° halo. The redrawn diagram (not shown) looks reasonable, except for the suns O and P. They remain a serious problem. The best that we can offer is to speculate that O and P were 120° parhelia, drawn long after the display was over, incorrectly remembered and grossly misplaced.

Halo experts may be tempted to invoke various rare halos to explain the Rome displays, especially the 1629 display. Given the weakness of the documentation, however, we think it more likely that the surviving information is inaccurate and that no rare halos are required. We think the Rome displays got special attention not because they contained rare halos but because they happened to be seen by some influential people who were sympathetic toward the budding scientific enterprise of the time. Both displays were good halo displays, but neither was a great display. Both have a place in halo history, but today neither has much value as a record of rare halos.

6. Where might the Rome halo records be?

Scheiner's letters show that he was preparing to publish a pamphlet (*libellus*) of some sort on halos. The *Bibliotheca Scriptorum Societatis Iesu* [24,25] gives the title as *Parelia in quibus multa de Iridibus, Halonibus, Virgis, Chasmatis*. Scheiner's pamphlet would be an obvious place to look for the Rome halo records, but it was never published, and the existence and whereabouts of the manuscript are unknown.

Anton von Braunmühl [7], pursuing research on Scheiner in the Munich University Library in the 1890's, came across a bound volume of material that had been put together by Scheiner's assistant Johann Baptist Cysat. Some of it had been written by Cysat, and some by Scheiner himself. The first item in the collection was apparently Scheiner's diagram—thought to be lost already in 1658—of the 1630 Rome display: a copper engraving, claimed by Braunmühl to be from Scheiner's unpublished work on halos, and bearing the Latin caption, here rendered in English as

> Parhelia seen, observed and recorded at Rome by Fr. Christoph Scheiner S. J., Profess House, 24 January 1630…between about 10:30 AM and 2:30 PM.

The diagram was accompanied by a description. Today neither the diagram nor the description can be found, and in fact the entire Cysat volume has disappeared, probably [26] as a result of the massive bombing of the library in 1943. The library record has survived, however, leaving a tantalizing glimpse of what we might have had (Fig. 5).

A manuscript of Peiresc's entitled *Halones seu circuli et coronae solares et lunares* [27] contains Peiresc's halo observations from 1630. We had hoped that it might also have information on the Rome displays, but it does not.

Massimo Ceresa, Reference Librarian at the Vatican Library, kindly agreed to look in the library for material relating to the 1629 Rome display, especially for the letter from Barberini or Suares to Peiresc that contained the report of the display. His search was not successful, but the library was undergoing renovation at the time, and not all of the relevant material was accessible.

According to Daxecker [28], some of Scheiner's letters were preserved for a time at Neisse but were probably destroyed in a fire in 1807. Some of the other letters relevant to the Rome displays are also probably gone forever. But massive amounts of correspondence from that era remain, and it is unlikely that we—the authors—have managed to find everything relevant to our story. And professional historians, though in general more proficient than we are, might easily have overlooked a critical halo diagram or description, not realizing its significance. At least it is fun to think so.

Appendix A. Chronology of correspondence

The literature bearing on the Rome displays is unusually prone to error. There are two halo displays, both in Rome, a year apart, supposedly reported by the same observer. There are two publications, both by Gassendi, again a year apart, both treating the first display. The potential for confusion is obvious.

We have based this article on old correspondence and on the original publications, so far as possible, rather than on secondary sources. Below is a list of some of the existing letters most relevant to the history and reliability of the reports of the two Rome halo displays.

16 Apr 1629, Aix From Peiresc to Du Puy [29].
Peiresc says that (Joseph Maria) Suarès, on behalf of Cardinal Barberini, has sent Peiresc a diagram of a remarkable display of five suns seen over St. Peter's on 20 March. Peiresc would be glad to send the diagram to Du Puy, but some other people have it at the moment.

20 Apr 1629, Aix From Peiresc to Du Puy [29].
Peiresc has had a copy of the halo display made for Du Puy.

12 May 1629, Aix From Peiresc to Gassendi [30].
Peiresc speculates at length on the halo display, whose report he has recently sent to Gassendi. Much attention is given to the chronology of various halo displays said to have occurred in Rome in the years immediately following the murder of Julius Caesar.

19 May 1629, Aix From Peiresc to Gassendi [31].
A continuation of the 12 May letter.

19 May 1629, Aix From Peiresc to Du Puy [29].
Peiresc is astonished to learn that Du Puy did not find the diagram of the halo display in the packet that Peiresc had sent. Peiresc had made a copy expressly for Du Puy, at the same time that he had made one for Gassendi. Perhaps, he fears, his assistant inadvertently put both in the same envelope. This evening Peiresc will see the copyist and have him make

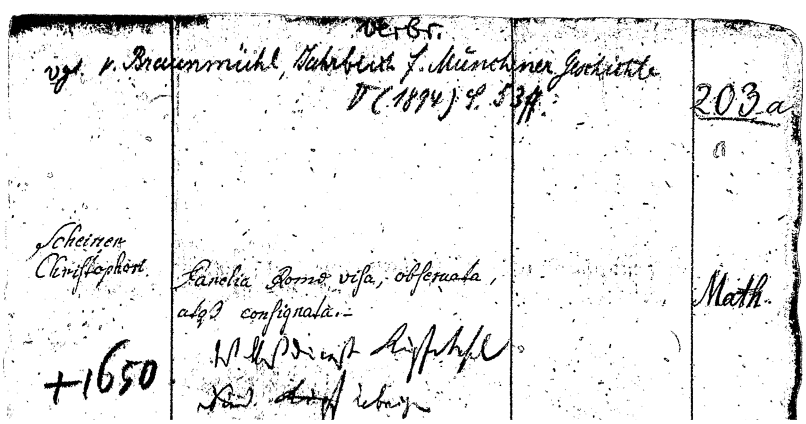


Fig. 5. Munich University Library record (Scheiner, 2° Math 203) of Cysat's volume containing a diagram of the 1630 Rome display probably made by Scheiner himself. The comment in the first row refers to Braunmühl's 1894 description of the volume. The next entry reads *Scheiner Christophori, Parelia Romae visa, observata, atque consignata*.

another. Peiresc mentions, with some skepticism, a prediction that the halo display signifies major changes within the Church in the coming five years.

21 May 1629, Louvain From Gassendi to Godefroy Wendelin [5].
Gassendi tells Wendelin that on 18 May he received a letter from Peiresc, through the Du Puy brothers, containing a report of the halo display. Gassendi will include an accurate copy with this letter.

15 Jun 1629, Brussels From Gassendi to Peiresc [32].
Gassendi stresses that, contrary to the common view, he does not consider halo phenomena to be omens of any kind. He expresses his regret at the lack of quantitative information regarding the halo display, then goes on to speculate at length about the causes of the display. Much of this letter is paraphrased in *Phaenomenon rarum* [2].

14 Jul 1629 From Gassendi to Hendrik Reneri [5].
This letter contains the text that later appeared as *Parhelia* [3].

4 Sep 1629, Paris From Gassendi to Peiresc [29].
Gassendi again wishes for more information on the halo display.

8 Oct 1629 From Descartes to M. Mersenne [33].
Mersenne had apparently written Descartes about the Rome halo display, but Descartes had also gotten the information elsewhere. Descartes says that the news of the halo display has prompted him to interrupt what he has been doing and to turn his attention to all "meteors," that is, to halos, rainbows, coronas, etc. He resolves to write a small treatise on them.

13 Nov 1629, Amsterdam From Descartes to Mersenne [33].
Descartes says that he has expanded the topic of his planned treatise to include not just meteors but all natural phenomena.

11 Dec 1629, Paris From Gassendi to Peiresc [29].
Gassendi has received what appears to be his *Phaenomenon rarum* from the printer in Holland. (The title is not given.) He does not like what he sees.

21 Jul 1630, Paris From Gassendi to Peiresc [29].
In an earlier letter Peiresc had apparently warned Gassendi about possible political repercussions from *Phaenomenon rarum*. Gassendi is concerned as well, and he expresses apprehension lest some copies reach Rome. He has arranged to publish a revised version. (This will be *Parhelia*.)

27 Aug 1630, Paris From Gassendi to Wilhelm Schickard [5].
Gassendi again expresses his pain over *Phaenomenon rarum*, and he says that *Parhelia* is in press, though neither publication is mentioned by name.

6 Sep 1630, Paris From Gassendi to Reneri [5].
Gassendi sends a copy of his newly published *Parhelia* to Reneri, in Holland.

13 Jan 1631, Paris From Gassendi to Scheiner [5].
Gassendi introduces himself, in what is clearly his first contact with Scheiner. Gassendi writes that he has received from Deriennes a diagram of a new halo display that Scheiner observed at Rome the previous year. (Thus this is the 1630 display.) Gassendi asks for clarification. Gassendi sends a copy of what is probably his *Parhelia*, which deals with the 1629 display. He remarks that he does not know whether Scheiner saw the display!

17 Mar 1631, Paris From Gassendi to Peiresc [29].
Gassendi tells Peiresc of the new display. Gassendi says that he has written to Scheiner, the observer, and will let Peiresc know if Scheiner responds.

21 Aug 1631, Paris From Gassendi to Gabriel Naudé [5].
Gassendi asks Naudé for more precise information on the Roman parhelia. (This would be the new display, the 1630 display.)

3 Dec 1631, Rome From Scheiner to Gassendi. (See Appendix C.)
Scheiner says that he is sending a copy of his recently published *Pantographice* [34] and a diagram of the halo display. (Gassendi, however, did not receive the diagram with Scheiner's letter; see the letters of 6 Mar, 13 Apr, and 11 May 1632.) Responding to Gassendi's 13 Jan 1631 request for more information on the display, Scheiner gives a long description that he says applies more or less equally to the two Rome displays. He acknowledges receipt of Gassendi's "Commentary" on halos. (This would be *Parhelia*, treating the 1629 display.) Scheiner seems to approve of the rendition of the display in the Commentary, but the context—a colleague politely offering his opinion of a book to its author—allows room for doubt.

13 Jan 1632, Rome From Naudé to Gassendi [5].
Naudé had apparently long since delivered Gassendi's 13 Jan 1631 letter to Scheiner, with its request for information on the 1630 display. Only now has Naudé received a response from Scheiner, prefaced with apologies for the long delay. This is presumably the above 3 Dec 1631 letter.

6 Mar 1632, Rome From Naudé to Gassendi [5].
This is probably the letter that contained Scheiner's diagram of the 1630 halo display.

13 Apr 1632, Paris From Gassendi to Naudé [5].
Gassendi acknowledges receipt of Scheiner's letter containing *Pantographice*. (This would be the letter of 3 Dec 1631.) But the promised diagram of the 1630 display was not included.

13 Apr 1632, Paris From Gassendi to Scheiner [5].

Gassendi acknowledges receipt of Scheiner's letter. He is pleased to learn that Scheiner had been an observer of the 1629 display.

11 May 1632, Paris From Gassendi to Naudé [5].

Gassendi acknowledges receipt of the diagram of the halo display.

23 Feb 1633, Rome From Scheiner to Gassendi [5].

Scheiner mentions that his publication on halos is still not finished.

21 Apr 1641, Endegeest From Descartes to Mersenne [33].

Descartes defends himself for not crediting anyone when treating the 1629 Rome display in *Les Météores*.

11 Sep 1659 From Huygens to Chapelain [13].

Huygens asks Chapelain for help in locating Scheiner's diagram of the 1630 Rome display, that is, the diagram referred to in Scheiner's 3 December 1631 letter to Gassendi. (From Huygens' letter it is not so clear that the halo display in question is the 1630 Rome display, but all doubts are removed in subsequent letters. See Huygens [13] Vol. 2, pp. 496, 529, and Vol. 3, pp. 12, 81, 82, 120.

2 Sep 1660, The Hague From Huygens to Chapelain [13] Vol. 3, pp. 118–120.

Huygens mentions that he has recreated Scheiner's missing diagram himself, using the description in Scheiner's letter: … *l'observation Romaine de l'an 1630, de la quelle j'ay trouuè l'entiere description dans la lettre de Scheinerus à Monsieur Gassendi, et mesme j'ay assez bien restituè la figure qui y manque.*

25 Jul 1662, The Hague From Huygens to Hevelius [13] Vol. 4, pp. 181, 182.

Huygens says the same to Hevelius.

Other letters that touch on the two Rome displays can be found in: *Lettres de Peiresc* [29] Vol. 2, pp. 99–105, 117–120; Vol. 4, pp. 198–202, 236–238. *Correspondance du P. Mersenne* [35] Vol. 2, pp. 241–249; Vol. 3, p. 51. *Opera Omnia* [5] Vol. 6, pp. 16, 29–31, 41, 42, 54, 55, 395, 396.

Appendix B. Excerpt from *Phaenomenon rarum* [2]

[The main body of text in *Phaenomenon rarum* consists of the *Explication* and the *Opinion*. According to Gassendi, the *Explication* is the halo report as he received it, hence not written by Gassendi. The *Opinion* is Gassendi's analysis of the display. Most of the *Explication* is reproduced here, but only a small part of the *Opinion*. The translation is from Smith's *Opticks* [36] and from Quintus' Latin Translation Service, with modifications by us].

If ancient Rome saw three suns shining in the sky, the new Rome now sees four.

If the unhappy city then endured three savage masters, now happy Rome reveres four peaceful rulers.

If Augustus, father of his native-land, became master of the world he had pacified only after dreadful battles,

Now Urban, father of his native-land, will be lord of all the world, now that he has finally calmed so much unrest.

BALDASSAR BONIFACIO.

In the last civil war, after which the Triumvirate was brought to an end and Augustus made himself master of the Empire and established universal peace, three suns were seen in Rome, as Pliny [37] related. This year in Rome four of them were seen. That is the subject of the above epigram.

Explication of the Figure (our Fig. 2(b)).

A the place of the observer at Rome, B the vertex or point over his head, C the true sun, AB a vertical plane passing through the observer's eye, the true sun and the vertex B; which are all projected in the straight line ACB. About the sun C there appeared two concentric irises not complete, but diversified with colors. The lesser and inner of them DEF, was fuller and more perfect; and though it was open from D to F, yet these ends D and F were perpetually endeavoring to unite; sometimes they did unite and complete the ring and then opened again. The other exterior and fainter and scarce discernable iris was GHI; it had a variety of colors but was very inconstant. The third iris KLMN was very large and all over of a white color, such as are often seen with paraselenae about the moon. This was an eccentric circle passing through the middle of the sun, at first entire, but towards the end of the appearance it was weak and ragged and scarce discernable from M towards N. In the common intersection of this circle and of the outward iris GHI, there broke out two parhelia N and K not entirely perfect, K was somewhat weak, but N shone brighter and stronger. The brightness in the middle of them both resembled that of the sun, but towards their edges they were tinged with colors like those of the rainbow. They were not perfectly round and even at their edges, but uneven and ragged. The parhelion N was a little wavering, and sent out a spiked tail NOP of a color somewhat fiery, which had a continual reciprocation. The parhelia at L and M, beyond the zenith B, were not so bright as the former, but rounder and white like the circle which they were placed in; they resembled milk or clean silver … At the time of this phenomenon the true sun passed through the vertical circles which, in respect of the observer, cross Montorio, and in respect of others cross St. Peter's Basilica and other localities facing the Castel Sant'Angelo. The duration of this phenomenon was in my judgment at least two hours, for at the 20th hour, or, if you will, the second astronomic hour, some men in the Roman College saw four suns appearing, which were very lively and bright, just like the true sun. But also some people from Tusculum who were busy clearing a vineyard describe how four exceedingly lively suns had been seen by them in addition to the true sun, which could not have been the case unless it were around 2 PM or earlier.…

…tokens of all these
The sun will give thee. Who dare charge the sun
With leasing? He it is who warneth oft
Of hidden broils at hand and treachery,
And secret swelling of the waves of war.
Virgil I *Georgics*

The Opinion of that Most Illustrious Scientist and Mathematician Master Petrus Gassendus Concerning this Phenomenon.

This sketch of the parhelia was sent from Rome to Aix-en-Provence by the most illustrious Cardinal Barbarini [38] to the noble Nicolaus Fabricius, the governor of Peyruis and Councillor of Aix. He at once wrote a letter to Petrus Gassendus, a Canon at Dijon, who was at that time touring Belgium, and sent an outline of this kind to him. But when Gassendus was in Amsterdam around the beginning of July, and was visiting the celebrated doctor Nicolaus à Wassenaer, he showed this same document to him too, and shared it with him. Then Gassendus was asked, in his capacity as a professed scientist, what he personally felt about a phenomenon of this kind. He replied that, as far as prophecies are concerned, especially of those events which are commonly believed to be foretold by unnatural occurrences, there does not seem to be any underlying reason why men should fear anything from them. Indeed, he continued, this is a purely natural effect, and if it were a sign of any event it could not signify anything contrary to the order of nature.… As far as the origin of this beautiful phenomenon is concerned, he stated that in the observation that had been described he needed in particular a statement of the diameters of all the circles, and the distance of all the suns, especially the ones at the back, both from one another and other objects,…

Appendix C. Excerpt from Scheiner's 3 Dec 1631 letter to Gassendi [39]

(The translation is by Quintus' Latin Translation Service, with modifications by us.)

Christoph Scheiner sends friendly greetings to that most Reverend and Illustrious Man, Master Peter Gassendus.

I have received your most welcome letter, most full of kindness, which that excellent Master Naudaeus dutifully handed over to me along with your learned little work. The reason why I have taken so long to reply is that I did not wish to appear empty-handed in your presence. I had always hoped to finish off my pamphlet about parhelia, which was even then half complete, and, by sending it back to you, to satisfy you and to some extent to return the kindnesses you have done me. But because I was prevented by one task piling up on top of another, in its place I am now sending *Pantographice* [34] which has just recently been made publicly available. With this I am sending a copy of a picture of the parhelia observed last year and I am attaching the explanation of them which you requested, although I am somewhat uncertain as to whether you are seeking an explanation of these or of those which I observed in the year 1629, and which you illustrated in your Commentaries. But however that may be, one may make more or less the same observations about both. Now to the details: the rough draft of the diagram was sent to you because I have not yet given the diagram to the printers in Rome. [That is, a professional engraving has not yet been made. Here Scheiner is responding to Gassendi's 13 Jan 1631 letter, in which Gassendi suggested that his confusion stemmed partly from the fact that Scheiner's diagram was only a freehand version, not yet engraved on copper.] The diameter of those irises which went most closely round the sun was more or less 43°; of those further away, ORP on the print, it was about 95°20′. The first solar irises had a scarlet or red color nearest the sun, with the other colors in their usual order and manner. The thickness or breadth of all these arcs was consistent and equal, but less than the diameter of the sun by about a third, as the diagrams correctly show, although I could not deny that the milky-colored circle which was parallel to the horizon was somewhat broader than the other arcs; its apparent diameter is to be calculated in accordance with the sun's altitude, after the manner of circles which are smaller than those which are the largest on a sphere. The parhelia of the year 1629 seem well enough explained in the paper that was conveyed, but those of the year 1630 were as follows: Two of them, M and N, were quite lively, but the other two, O and P, were less so. M and N glowed with a purplish redness where they faced the sun, and with a radiant white hue where they faced away from it; O and P were all white. The durations of all these images were different,… I am easily convinced that the total duration was five or more hours.

The parhelia Q and R in the vertical plane, which are where the arcs of the irises intersect each other, or where the arcs, inverting themselves near the point of the vertical plane through the eye F and the sun G, touch, as they cross, are of a startling brightness as they strike the sight,… The distance or altitude of Q above the horizon at the time of the first observation was 49°40′, and that of R was 76°10′. The altitude of the sun was 28°30′, from which the distance of Q itself from the sun in the vertical circle was 21°10′. The distance of R itself from the sun was 49°40′ before noon, when I completed my first observation. At the beginning of these observations the wind was in the north,… The iris ORP seems to have been one portion of a circular arc, and for that reason a semicircle concentric with the sun, although near θ and κ it has not made complete contact with the horizon AB, and the portions Oκ and Pθ were of a changing length, now longer, now shorter. The arcs ZQα, βQγ, $\delta\epsilon\zeta$, which go closest round the sun, displayed to the sight almost one single circular belt, all blended together, with neither a consistent width nor an unchanging appearance, but billowing out as though by a continual ebb and flow, and varying; in truth however it was a combination of the arcs depicted in the diagram, as I observed in my customary manner in my very precise notes. The horns HRC appear to be the segment of a smaller semi-circle which touches the larger one ORP where it lies facing it at the common point R. The arcs of irises ZQα and βQγ intersect one another at the point Q, and there they make the parhelion Q. The two suns N and M were produced at the common knots or intersections M and N which the iris $\delta\epsilon\zeta$ makes with the milky circle OMNP. The northern part of the sky was clearer than the southern, which experienced thin and denser mists that made this phenomenon more noticeable and impressive. The above information is what I now consider should be given in reply to your request. In my little work on parhelia perhaps more facts will be revealed which are to your taste. I was exceptionally pleased by that learned Commentary [3] of yours on my parhelia. I can certainly not expect a more careful hand and pen, and I am delighted that so great an artist has taken over my sketches. Everything which you discuss about the origin, nature, and meaning of this phenomenon is to my taste, as you will clearly see from my Treatise. My work on sunspots and solar flares, which was published under the title of *Rosa Ursina* [40], is now on sale in Rome, and that excellent master Naudaeus will send it to you.…

Rome, Profess House, 3 December 1631.

Appendix D. The diagram of the 1630 Rome display (Fig. 3)

Although Huygens mentioned in his correspondence that he had drawn a diagram of the 1630 Rome display using the text of Scheiner's letter (Appendix C), and although the diagram of Fig. 3 is found today among the Huygens manuscripts at Leiden University, we wondered initially whether the diagram in the figure was really the one that Huygens had drawn. The diagram was never published during Huygens' lifetime [41], and today it is found loose among Huygens' manuscripts, just a single small sheet unattached to any text. So conceivably it had come to Huygens from elsewhere. Thus the editors of Huygens' Oeuvres were able to suggest that the diagram had come from Antoine Poteria, who was Gassendi's assistant [42].

Karin Scheper, Conservator of Special Collections at the Leiden University Library, nevertheless believes that the diagram is almost certainly due to Huygens [43]. At one time the diagram was attached to a certain manuscript bifolium, as shown by matching glue marks on the diagram and bifolium. In this bifolium Huygens had copied Scheiner's letter, hence the bifolium is the natural location for the diagram. Ms. Scheper concluded that Huygens drew the diagram in one of his workbooks (Fig. 6) and then cut it out and glued it to the bifolium, and that he did so at about the same time that he wrote the text in the bifolium. She based her conclusion on a careful examination of the diagram [44], the bifolium [45], and the workbook [46]. She found the paper characteristics of all three to be identical; these include distinctive watermarks, chain and laid lines, and paper weight. She also found the handwriting to be the same in all three. [Ms. Scheper, together with one of us (Können), searched the workbook for a vacant rectangle that would accommodate the diagram. No such rectangle was found, but the workbook is so thoroughly cut up, with many pages gone entirely, that we were not surprised.]

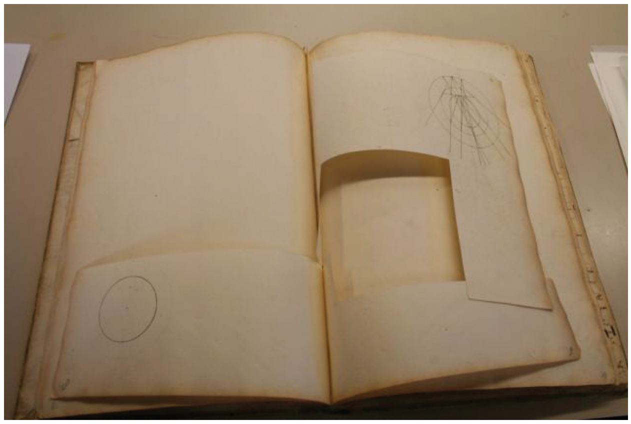


Fig. 6. (Color online) One of Huygens' workbooks, probably the source of the diagram of the 1630 Rome display. Huygens often drew diagrams in the workbooks and then cut them out to be placed elsewhere [54]. (Leiden University Library, HUG 5; photo by Karin Scheper.)

Long after Ms. Scheper had convinced us that the diagram was due to Huygens, we realized that we ourselves had overlooked a clue that pointed to the same conclusion. In the diagram, at the 7:30 position with respect to the sun, a tiny number 45 gives the diameter for the inner band of halos. Scheiner's letter in Gassendi's *Opera Omnia*, from which Huygens said he drew his diagram, gives 43° for the diameter. At first glance this discrepancy suggests that our diagram is not the one made by Huygens. The value 45°, however, was introduced by Huygens, as can be seen from Fig. 7; in copying Scheiner's letter from the *Opera Omnia* to his own notes in the bifolium, Huygens changed 43° to 45°, inadvertently or not. As far as we know, the value 45° appears in connection with the 1630 display only in Huygens' notes and in the diagram of the display. (The value in the handwritten Scheiner letter is 43°, seen on the next to last line of Fig. 4.) The agreement between Huygens' notes and the diagram is compelling evidence that the diagram is due to Huygens [47]. The tiny 45 on

(a)

(b)

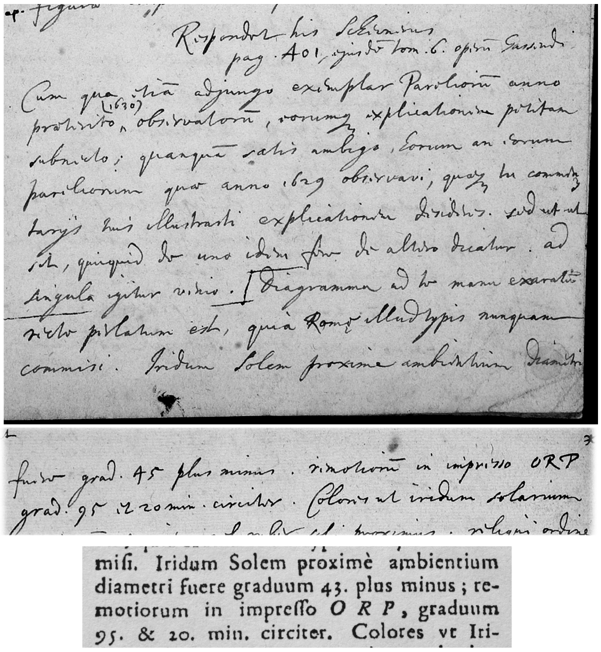


Fig. 7. (a) Excerpt from the bifolium in which Huygens copied Scheiner's 3 Dec 1631 letter to Gassendi. The first two lines are *Respondet his Scheinerus pag. 401. ejusdem tom. 6 operum Gassendi.*, leaving no doubt that Huygens copied the letter from the published version in Gassendi's *Opera Omnia* [5] Vol. 6, p. 401. Next comes the text of the letter, but with the opening civilities omitted. On the next to last line is the crucial *grad. 45*, that is, 45°. (Leiden University Library, HUG 31 fol 77 r.v. A printed rendering of this text is in [13] Vol. 17, pp. 468, 469.) (b) The corresponding *graduum 43* in the *Opera Omnia*. The fact that 45°, not 43°, is the value used in the halo diagram (Fig. 3) is evidence that the diagram is due to Huygens.

the diagram is Huygens' figurative, subtle, and unintended signature.

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39. Bibliothèque nationale de France, NAF1637, fol 27 r.v. and fol 28 r.v. A published version, with minor transcription errors, is in [5] Vol. 6, pp. 400–402.
40. C. Scheiner, *Rosa Ursina*… (Apud Andream Phaeum Typographum Ducalem, Bracciani, 1626–1630).
41. Essentially the same diagram appears in Huygens' *Opuscula Postuma* [4] of 1703 and much later in his *Oeuvres*. Kuhse [53] in 1848 added a dotted curve through Q to suggest an upper tangent arc, and his curve survives in some later publications.
42. Their thinking is puzzling to us. They base their suggestion ([13] Vol. 17, pp. 358, 439) on an 18 Dec 1659 letter from Chapelain to Huygens. But subsequent letters show that Huygens, much later, still did not have the diagram.
43. Personal communication, August 2007.
44. Leiden University Library, HUG 31, fol 119 r., corresponding to Fig. 15 of Huygens' *Oeuvres* [13] Vol. 17, pp. 411, 469.
45. Leiden University Library, HUG 31, fol 77/78, corresponding to Huygens' *Oeuvres* [13] Vol. 17, pp. 468–471.
46. Leiden University Library, HUG 5.
47. Much the same can be said for the value 47°40′ as was said for 45°: Both in Huygens' notes and in the diagram the angular distance GR is 47°40′, but in Scheiner's letter it is 49°40′. However, the reasoning is complicated by the fact that in the notes the 7 in 47°40′ has been changed back to a 9 (probably by de Volder and Fullenius). Moreover, the value $47°\,40' = 76°\,10' - 28°\,30'$ follows from Scheiner's values for the elevations of G and R, so the change from 49°40′ to 47°40′ is one that might reasonably have been made by anyone.
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