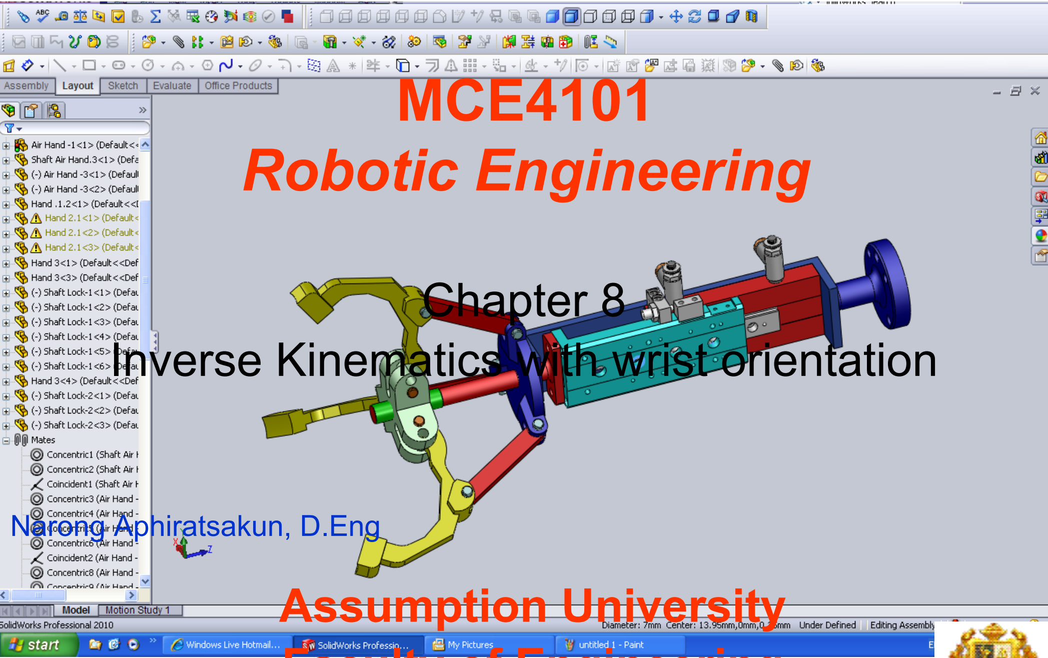
MCE4101

Robotic Engineering



Chapter 8

Inverse Kinematics with wrist orientation

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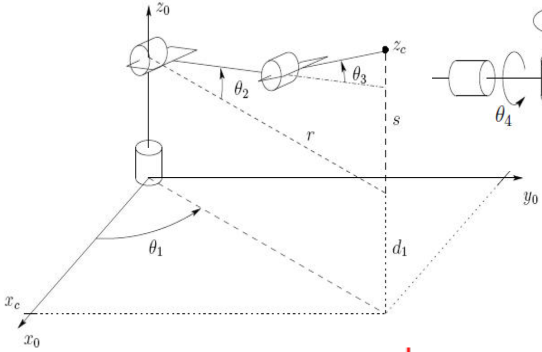
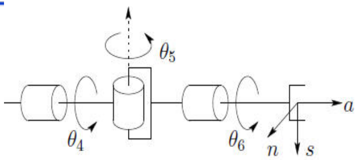
Inverse Kinematics with Full System



RRR



Wrist Mechanism

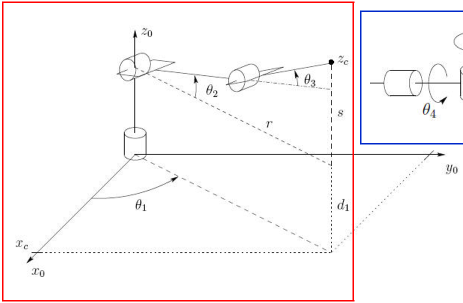
Refer to Homogenous Transformation

Homogenous transformation is a matrix representation of a rigid motions : combine of rotation and translation.

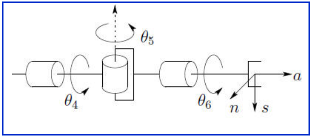
$$T\,(or\,H) = \left[\begin{array}{ccc:c} 3 & & & 3 \\ & * & & * \\ & & 3 & 1 \\ \hdashline 1 & * & 3 & 1*1 \end{array}\right] = \left[\begin{array}{c:c} Rotation & Translation \\ \hdashline 0 & 1 \end{array}\right]$$

$$\boxed{T = \left[\begin{array}{c:c} R & d \\ \hdashline 0 & 1 \end{array}\right]}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist



RRR



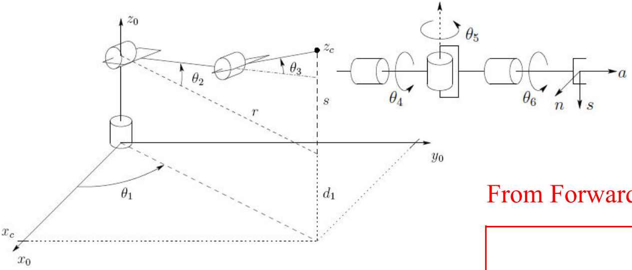
Wrist Mechanism

Obtain $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ *and* θ_6

θ_1, θ_2 *and* θ_3 are obtained from Chapter 6 geometric

θ_4, θ_5 *and* θ_6 let's do by Analytical method! analytic

Inverse Kinematics Examples : RRR with 3 DOF Wrist



RRR

Link	a_i	α_i	d_i	θ_i
1	0	90	d_1	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	0	0	θ_3^*

From Forward Kinematic

RRR

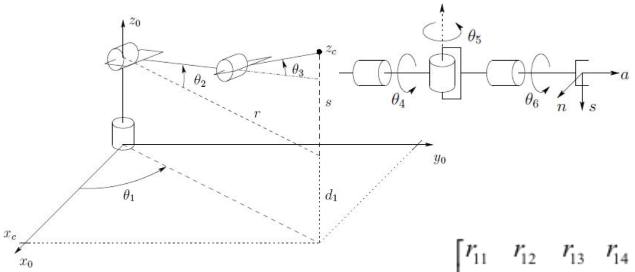
$$R_3^0 = A_1A_2A_3 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 \\ s_1c_{23} & -s_1s_{23} & -c_1 \\ s_{23} & c_{23} & 0 \end{bmatrix}$$

From Forward Kinematic

Wrist

$$R_6^3 = A_4A_5A_6 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist



From Forward Kinematic

$$T_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix}$$

So

$$R_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist

r11 = - cos(th6)*(sin(th1)*sin(th5) + cos(th5)*(cos(th4)*(cos(th1)*sin(th2)*sin(th3) - cos(th1)*cos(th2)*cos(th3)) + sin(th4)*(cos(th1)*cos(th2)*sin(th3) + cos(th1)*cos(th3)*sin(th2)))) - sin(th6)*(cos(th4)*(cos(th1)*cos(th2)*sin(th3) + cos(th1)*cos(th3)*sin(th2)) - sin(th4)*(cos(th1)*sin(th2)*sin(th3) - cos(th1)*cos(th2)*cos(th3)))

r12 = sin(th6)*(sin(th1)*sin(th5) + cos(th5)*(cos(th4)*(cos(th1)*sin(th2)*sin(th3) - cos(th1)*cos(th2)*cos(th3)) + sin(th4)*(cos(th1)*cos(th2)*sin(th3) + cos(th1)*cos(th3)*sin(th2)))) - cos(th6)*(cos(th4)*(cos(th1)*cos(th2)*sin(th3) + cos(th1)*cos(th3)*sin(th2)) - sin(th4)*(cos(th1)*sin(th2)*sin(th3) - cos(th1)*cos(th2)*cos(th3)))

r13 = cos(th5)*sin(th1) - sin(th5)*(cos(th4)*(cos(th1)*sin(th2)*sin(th3) - cos(th1)*cos(th2)*cos(th3)) + sin(th4)*(cos(th1)*cos(th2)*sin(th3) + cos(th1)*cos(th3)*sin(th2)))

r14 = d6*(cos(th5)*sin(th1) - sin(th5)*(cos(th4)*(cos(th1)*sin(th2)*sin(th3) - cos(th1)*cos(th2)*cos(th3)) + sin(th4)*(cos(th1)*cos(th2)*sin(th3) + cos(th1)*cos(th3)*sin(th2)))) + a2*cos(th1)*cos(th2) + a3*cos(th1)*cos(th2)*cos(th3) - a3*cos(th1)*sin(th2)*sin(th3)

r21 = cos(th6)*(cos(th1)*sin(th5) - cos(th5)*(cos(th4)*(sin(th1)*sin(th2)*sin(th3) - cos(th2)*cos(th3)*sin(th1)) + sin(th4)*(cos(th2)*sin(th1)*sin(th3) + cos(th3)*sin(th1)*sin(th2)))) - sin(th6)*(cos(th4)*(cos(th2)*sin(th1)*sin(th3) + cos(th3)*sin(th1)*sin(th2)) - sin(th4)*(sin(th1)*sin(th2)*sin(th3) - cos(th2)*cos(th3)*sin(th1)))

r22 = - sin(th6)*(cos(th1)*sin(th5) - cos(th5)*(cos(th4)*(sin(th1)*sin(th2)*sin(th3) - cos(th2)*cos(th3)*sin(th1)) + sin(th4)*(cos(th2)*sin(th1)*sin(th3) + cos(th3)*sin(th1)*sin(th2)))) - cos(th6)*(cos(th4)*(cos(th2)*sin(th1)*sin(th3) + cos(th3)*sin(th1)*sin(th2)) - sin(th4)*(sin(th1)*sin(th2)*sin(th3) - cos(th2)*cos(th3)*sin(th1)))

r23 = - cos(th1)*cos(th5) - sin(th5)*(cos(th4)*(sin(th1)*sin(th2)*sin(th3) - cos(th2)*cos(th3)*sin(th1)) + sin(th4)*(cos(th2)*sin(th1)*sin(th3) + cos(th3)*sin(th1)*sin(th2)))

r24 = a2*cos(th2)*sin(th1) - d6*(cos(th1)*cos(th5) + sin(th5)*(cos(th4)*(sin(th1)*sin(th2)*sin(th3) - cos(th2)*cos(th3)*sin(th1)) + sin(th4)*(cos(th2)*sin(th1)*sin(th3) + cos(th3)*sin(th1)*sin(th2)))) + a3*cos(th2)*cos(th3)*sin(th1) - a3*sin(th1)*sin(th2)*sin(th3)

r31 = sin(th6)*(cos(th4)*(cos(th2)*cos(th3) - sin(th2)*sin(th3)) - sin(th4)*(cos(th2)*sin(th3) + cos(th3)*sin(th2))) + cos(th5)*cos(th6)*(cos(th4)*(cos(th2)*sin(th3) + cos(th3)*sin(th2)) + sin(th4)*(cos(th2)*cos(th3) - sin(th2)*sin(th3)))

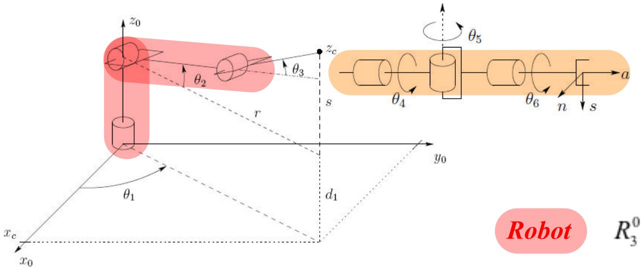
r32 = cos(th6)*(cos(th4)*(cos(th2)*cos(th3) - sin(th2)*sin(th3)) - sin(th4)*(cos(th2)*sin(th3) + cos(th3)*sin(th2))) - cos(th5)*sin(th6)*(cos(th4)*(cos(th2)*sin(th3) + cos(th3)*sin(th2)) + sin(th4)*(cos(th2)*cos(th3) - sin(th2)*sin(th3)))

r33 = sin(th5)*(cos(th4)*(cos(th2)*sin(th3) + cos(th3)*sin(th2)) + sin(th4)*(cos(th2)*cos(th3) - sin(th2)*sin(th3)))

r34 = d1 + a2*sin(th2) + a3*cos(th2)*sin(th3) + a3*cos(th3)*sin(th2) + d6*sin(th5)*(cos(th4)*(cos(th2)*sin(th3) + cos(th3)*sin(th2)) + sin(th4)*(cos(th2)*cos(th3) - sin(th2)*sin(th3)))

r41 = 0
r42 = 0
r43 = 0

Inverse Kinematics Examples : RRR with 3 DOF Wrist



Equation can be formed as

$$R_6^3 = R_0^3 R_6^0 = \left(R_3^0\right)^T R_6^0$$

Robot

$$R_3^0 = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 \\ s_1 c_{23} & -s_1 s_{23} & -c_1 \\ s_{23} & c_{23} & 0 \end{bmatrix}$$

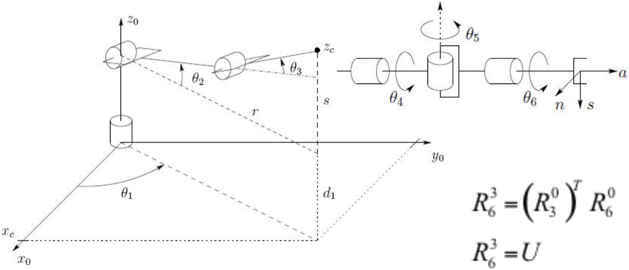
Robot and Wrist

$$R_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Wrist

$$R_6^3 = \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_4 s_5 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_4 s_5 \\ -s_5 c_6 & s_5 s_6 & c_5 \end{bmatrix}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist



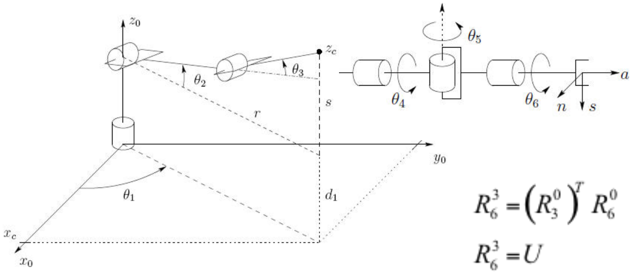
$$R_6^3 = \left(R_3^0\right)^T R_6^0$$

$$R_6^3 = U$$

After manipulate and compare

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist



$$R_6^3 = \left(R_3^0\right)^T R_6^0$$

$$R_6^3 = U$$

After manipulate and compare

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist

$$\left(R_3^0\right)^T R_6^0 = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix} =$$

[r31*s23 + c1*c23*r11 + c23*r21*s1, r32*s23 + c1*c23*r12 + c23*r22*s1, r33*s23 + c1*c23*r13 + c23*r23*s1;
c23*r31 - c1*r11*s23 - r21*s1*s23, c23*r32 - c1*r12*s23 - r22*s1*s23, c23*r33 - c1*r13*s23 - r23*s1*s23;
r11*s1 - c1*r21, r12*s1 - c1*r22, r13*s1 - c1*r23]

Inverse Kinematics Examples : RRR with 3 DOF Wrist

$$
\begin{aligned}
c_4 s_5 &= c_1 c_{23} r_{13} + s_1 c_{23} r_{23} + s_{23} r_{33} \\
s_4 s_5 &= -c_1 s_{23} r_{13} - s_1 s_{23} r_{23} + c_{23} r_{33} \\
c_5 &= s_1 r_{13} - c_1 r_{23}
\end{aligned}
= \begin{bmatrix} \cdots\cdots & u_{13} \\ \cdots\cdots & u_{23} \\ \cdots\cdots & u_{33} \end{bmatrix}
$$

CASE 1

If u13 and u23 both are NOT ZERO or at least one is NOT ZERO

$\therefore s_5 \neq 0$

$and\ c_5 \neq 1$

$so \quad c_5 = u_{33}$

$$
\begin{aligned}
\boxed{c_4 s_5} &= c_1 c_{23} r_{13} + s_1 c_{23} r_{23} + s_{23} r_{33} \\
\boxed{s_4 s_5} &= -c_1 s_{23} r_{13} - s_1 s_{23} r_{23} + c_{23} r_{33} \\
\boxed{c_5} &= s_1 r_{13} - c_1 r_{23}
\end{aligned}
$$

$\theta_5 \neq 0 \Rightarrow \theta_5 = \text{answer}$

$$c_5 = u_{33}$$

$$s_5 = \pm\sqrt{1-(u_{33})^2}$$

$$\theta_5 = A\tan 2(u_{33}, \pm\sqrt{1-(u_{33})^2})$$

θ_5 +ve → θ_4 → θ_6

θ_5 -ve → θ_4 → θ_6

$$\boxed{\theta_5 = A\tan 2(s_1 r_{13} - c_1 r_{23}, \pm\sqrt{1-(s_1 r_{13} - c_1 r_{23})^2})}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist

$$\begin{aligned} c_4 s_5 &= c_1 c_{23} r_{13} + s_1 c_{23} r_{23} + s_{23} r_{33} \\ s_4 s_5 &= -c_1 s_{23} r_{13} - s_1 s_{23} r_{23} + c_{23} r_{33} \\ c_5 &= s_1 r_{13} - c_1 r_{23} \end{aligned} = \begin{bmatrix} \cdots\cdots & u_{13} \\ \cdots\cdots & u_{23} \\ \cdots\cdots & u_{33} \end{bmatrix}$$

CASE 1

A. If positive θ_5 is chosen

$$\therefore s_5 > 0$$

$$+u_{13} = c_4 \; and \; +u_{23} = s_4$$

$$\begin{aligned} c_4 s_5 &= c_1 c_{23} r_{13} + s_1 c_{23} r_{23} + s_{23} r_{33} \\ s_4 s_5 &= -c_1 s_{23} r_{13} - s_1 s_{23} r_{23} + c_{23} r_{33} \\ c_5 &= s_1 r_{13} - c_1 r_{23} \end{aligned}$$

$$\therefore \theta_4 = A\tan 2(u_{13}, u_{23})$$

$$\boxed{\theta_4 = A\tan 2(c_1 c_{23} r_{13} + s_1 c_{23} r_{23} + s_{23} r_{23}, -c_1 s_{23} r_{13} - s_1 s_{23} r_{23} + c_{23} r_{33})}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix}$$

CASE 1

A. If positive θ5 is chosen

$$\therefore s_5 > 0$$

$$+u_{13} = c_4 \; and \; +u_{23} = s_4$$

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix}$$

$$\therefore \theta_6 = A\tan 2(-u_{31}, u_{32})$$

$$\boxed{\theta_6 = A\tan 2(-r_{11}s_1 + r_{21}c_1, r_{12}s_1 - r_{22}c_1)}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist

$$
\begin{aligned}
c_4 s_5 &= c_1 c_{23} r_{13} + s_1 c_{23} r_{23} + s_{23} r_{33} \\
s_4 s_5 &= -c_1 s_{23} r_{13} - s_1 s_{23} r_{23} + c_{23} r_{33} \\
c_5 &= s_1 r_{13} - c_1 r_{23}
\end{aligned}
= \begin{bmatrix} \cdots\cdots\; u_{13} \\ \cdots\cdots\; u_{23} \\ \cdots\cdots\; u_{33} \end{bmatrix}
$$

CASE 1

B. If negative θ5 is chosen

$$\therefore s_5 > 0$$

$$+u_{13} = c_4 \; and \; + u_{23} = s_4$$

$$
\begin{aligned}
c_4 s_5 &= c_1 c_{23} r_{13} + s_1 c_{23} r_{23} + s_{23} r_{33} \\
s_4 s_5 &= -c_1 s_{23} r_{13} - s_1 s_{23} r_{23} + c_{23} r_{33} \\
c_5 &= s_1 r_{13} - c_1 r_{23}
\end{aligned}
$$

$$\therefore \theta_4 = A\tan 2(-u_{13}, -u_{23})$$

$$\boxed{\theta_4 = A\tan 2(-c_1 c_{23} r_{13} - s_1 c_{23} r_{23} - s_{23} r_{23}, c_1 s_{23} r_{13} + s_1 s_{23} r_{23} - c_{23} r_{33})}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix}$$

CASE 1

B. If negative θ5 is chosen

$$\therefore s_5 > 0$$

$$+u_{13} = c_4 \; and \; +u_{23} = s_4$$

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix}$$

$$\therefore \theta_6 = A\tan 2(-u_{31}, u_{32})$$

$$\boxed{\theta_6 = A\tan 2(r_{11}s_1 - r_{21}c_1, -r_{12}s_1 + r_{22}c_1)}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist

$$\begin{aligned} c_4 s_5 &= c_1 c_{23} r_{13} + s_1 c_{23} r_{23} + s_{23} r_{33} \\ s_4 s_5 &= -c_1 s_{23} r_{13} - s_1 s_{23} r_{23} + c_{23} r_{33} \\ c_5 &= s_1 r_{13} - c_1 r_{23} \end{aligned} = \begin{bmatrix} \ldots\ldots\ldots u_{13} \\ \ldots\ldots\ldots u_{23} \\ \ldots\ldots\ldots u_{33} \end{bmatrix}$$

CASE 2

If u13 and u23 both are ZERO

$$\therefore s_5 = 0$$

$$and\ c_5 = \pm 1$$

$$so \quad c_5 = u_{33} = \pm 1$$

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & \boxed{c_4s_5} \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & \boxed{s_4s_5} \\ \boxed{-s_5c_6} & \boxed{s_5s_6} & c_5 \end{bmatrix}$$

$$\boxed{\theta_5 = 0^\circ\ or\ 180^\circ}$$

This is orthogonal case $\therefore \begin{bmatrix} u_{11} & u_{12} & 0 \\ u_{21} & u_{22} & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$

Inverse Kinematics Examples : RRR with 3 DOF Wrist

CASE 2

A. u33 = +1

$$\theta_5 = 0^\circ$$

$\therefore s_5 = 0$

$and\ c_5 = \pm 1$

$so \quad c_5 = u_{33} = \pm 1$

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix}$$

180

$\theta_5 = 0$

$\theta_4 + \theta_6$

$\theta_4 - \theta_6$

Let $\theta_4 = 0$ find θ_6

Let $\theta_4 = 0$ find θ_6

$$\therefore \begin{bmatrix} c_4c_6 - s_4s_6 & -c_4s_6 - s_4c_6 & 0 \\ s_4c_6 + c_4s_6 & -s_4s_6 + c_4c_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{46} & -s_{46} & 0 \\ s_{46} & c_{46} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore \begin{bmatrix} u_{11} & u_{12} & 0 \\ u_{21} & u_{22} & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$$

$\theta_6 = 0$ " θ_4

$\theta_6 = 0$ " θ_4

$$\theta_4 + \theta_6 = A\tan 2(u_{11}, u_{21}) = A\tan 2(u_{11}, -u_{12})$$

$$\boxed{\theta_4 + \theta_6 = A\tan 2(r_{31}s_{23} + c_1c_{23}r_{11} + c_{23}r_{21}s_1,\ c_{23}r_{31} - c_1r_{11}s_{23} - r_{21}s_1s_{23})}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist

CASE 2

A. u33 = +1

$$\boxed{\theta_5 = 0°}$$

$\therefore s_5 = 0$

$and\ c_5 = \pm 1$

$so \quad c_5 = u_{33} = \pm 1$

$$\boxed{\theta_4 + \theta_6 = A\tan 2(r_{31}s_{23} + c_1c_{23}r_{11} + c_{23}r_{21}s_1, c_{23}r_{31} - c_1r_{11}s_{23} - r_{21}s_1s_{23})}$$

In this case $\theta_4 + \theta_6$ there are infinitely many solutions. θ_4 can be choose arbitrary, one solution is to take

$$\boxed{\theta_4 = 0°}$$

Then θ_6 can be determined.

(Or $\theta_4 = 0°$, then obtained θ_6)

Inverse Kinematics Examples : RRR with 3 DOF Wrist

CASE 2

B. u33 = -1

$$\boxed{\theta_5 = 180^\circ}$$

$$\therefore s_5 = 0$$

$$and\ c_5 = \pm 1$$

$$so \quad c_5 = u_{33} = \pm 1$$

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -c_4c_6 - s_4s_6 & c_4s_6 - s_4c_6 & 0 \\ -s_4c_6 + c_4s_6 & s_4s_6 + c_4c_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -c_{4-6} & s_{4-6} & 0 \\ s_{4-6} & c_{4-6} & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & 0 \\ u_{21} & u_{22} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\theta_4 - \theta_6 = A\tan 2(u_{11}, u_{21})$$

$$\boxed{\theta_4 + \theta_6 = A\tan 2(\mathrm{r}_{31}\mathrm{s}_{23} + \mathrm{c}_1\mathrm{c}_{23}\mathrm{r}_{11} + \mathrm{c}_{23}\mathrm{r}_{21}\mathrm{s}_1, \mathrm{c}_{23}\mathrm{r}_{31} - \mathrm{c}_1\mathrm{r}_{11}\mathrm{s}_{23} - \mathrm{r}_{21}\mathrm{s}_1\mathrm{s}_{23})}$$

Inverse Kinematics Examples : RRR with 3 DOF Wrist

CASE 2

B. u33 = -1

$$\boxed{\theta_5 = 180^\circ}$$

$\therefore s_5 = 0$

and $c_5 = \pm 1$

so $c_5 = u_{33} = \pm 1$

$$\boxed{\theta_4 + \theta_6 = A\tan 2(r_{31}s_{23} + c_1 c_{23} r_{11} + c_{23} r_{21} s_1, c_{23} r_{31} - c_1 r_{11} s_{23} - r_{21} s_1 s_{23})}$$

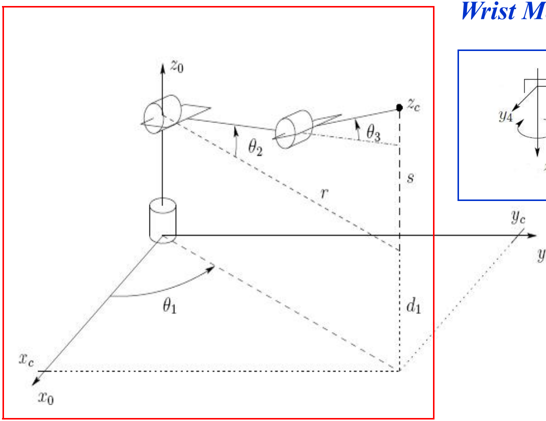
In this case $\theta_4 + \theta_6$, there are infinitely many solutions. θ_4 can be choose arbitrary, one solution is to take

$$\boxed{\theta_4 = 0^\circ}$$

Then θ_6 can be determined.

(Or $\theta_4 = 0^\circ$, then obtained θ_6)

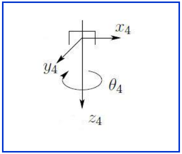
Inverse Kinematics Examples : RRR with 1 DOF Wrist



RRR

Make actuation direction as in base z0 axis

Wrist Mechanism



RRR

Link	a_i	α_i	d_i	θ_i
1	0	90	d_1	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	0	0	θ_3^*

1 DOF Wrist Mechanism

Link	a_i	α_i	d_i	θ_i
θ_4^*	0	90	0	0
4	0	0	d_4	θ_4^*

Q = ikine(ROBOT, T, Q, M)

Thursday, September 9, 2021 10:09 AM

Robot

robot({ })

DH Table

Position desired (4×4)

inital guess

mask matrix according [DOF]

2DOF [1 1 0 0 0 0]

3DOF [1 1 1 0 0 0]

⋮

6DOF [1 1 1 1 1 1]

Thursday, September 9, 2021 10:16 AM

```
Px = 7.5; Py = 2; Pz = 0;
```

given

ikine

fkine

Window

```
end =

    7.5000
    2.0000
         0
    1.0000
```

Robot end point.