MCE4101
Robotic Engineering

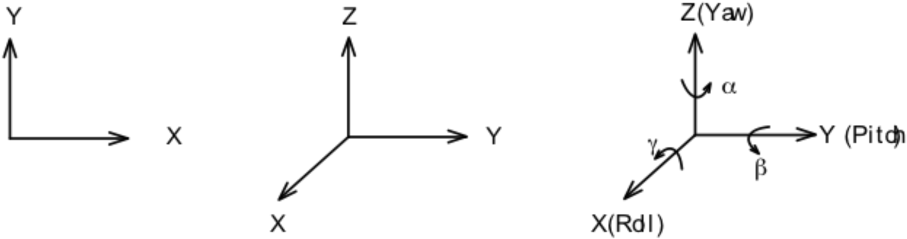
Chapter 3
Coordinate Transform

Narong Aphiratsakun, D.Eng

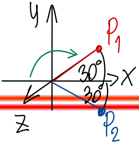
Assumption University
Faculty of Engineering

Cartesian Coordinate

The position of an object in space is described with Cartesian coordinates. Coordinate axes for 2-3 dimensional system (2D and 3D) are normally labeled as shown in the figure below. There is no standard in computer graphics and axis labeling varies from books to books. But consistence of direction is important in the analysis. In this class, we would follow a Roll-Pitch-Yaw axis (R-P-Y).



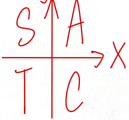
Cartesian Coordinate

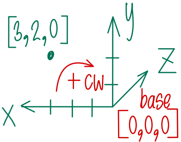


$P_{x_1} = 1\cos 30° = 0.86$
$P_{y_1} = 1\sin 30° = 0.5$
$P_{z_1} = 0$

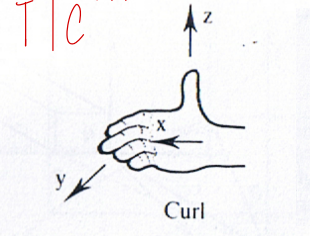
$P_{x_2} = 1\cos 30° = 0.86$
$P_{y_2} = 1\sin(-30°) = -0.5$
$P_{z_2} = 0$

Here the chosen coordinate frame forms a right hand set vectors. The positive direction of rotation is chosen in accordance with the right hand as illustrated.

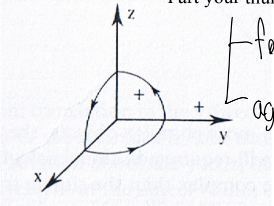




(1) axis orientation
Right hand rule
(2) axis of rotation
Curl rule
(3) Reference



(1) RH RULE
(2) ORIENTATION-> Curl Rule
Part your thumb (axis of rotation)

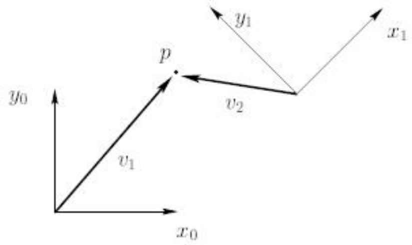


follow curl
+ve.
against curl
-ve

Hold your right hand open, with your thumb pointing in the direction of the axis of rotation, and your fingers pointing in the direction of a second axis. Curl your fingers through 90 degree until they point in the direction of the third axis. This is a positive direction.

(for axis above) ***Take CW : -ve and CCW : +ve***

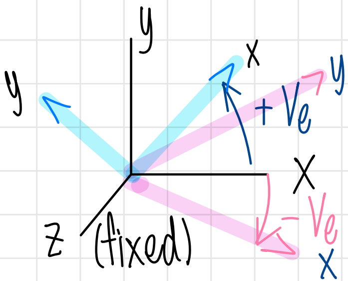
Position



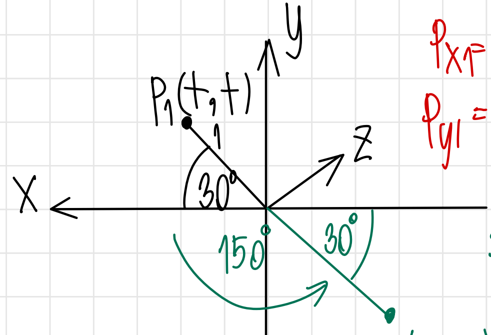
Coordinate of point ***p*** can be specify with respect to frame 0 or frame 1.

For example

$$p^0 = \begin{bmatrix} 5 \\ 6 \end{bmatrix}, \; p^1 = \begin{bmatrix} -2.8 \\ 4.2 \end{bmatrix}$$



2nd rule thumb



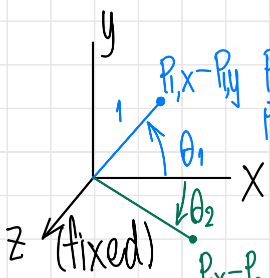
$P_{x1} = 1\cos(+30°) = 0.86$ (+ve)

$P_{y1} = 1\sin(+30°) = 0.5$ (+ve)

(or) 210°

$P_{x2} = 1\cos(-150°) = -0.86$

$P_2(-,-)$ $P_{y2} = 1\sin(-150°) = -0.5$



$P_{1,x}-P_{1,y}$

$P_{1,x} = 1\cos(+\theta_1)$

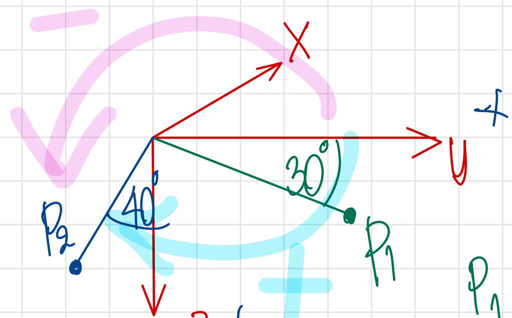
$P_{1,y} = 1\sin(+\theta_1)$

$P_z = 0$ (rotation axis)

$P_{2,x} = 1\cos(-\theta_1)$

$P_{2x}-P_{2y}$ $P_{2,y} = 1\sin(-\theta_1)$

$P_z = 0$ (rotation axis)



$P_x = 0$

$P_{1y} = 1\cos(+30°)$

$P_{1y} = 1\sin(+30°)$

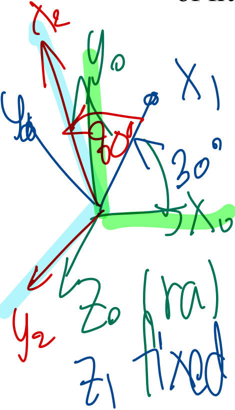
$P_{2x} = 0$ z+

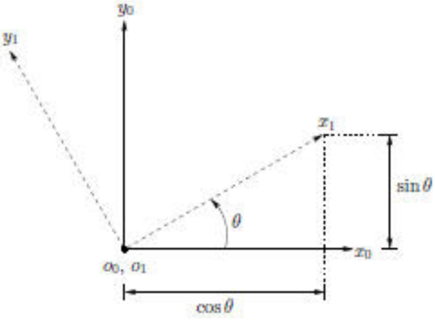
$P_{2y} = 1\cos(+130°)$

$P_{2z} = 1\sin(+130°)$

Rotation

R_1^0 This notation specify rotation of coordinates vectors for the axes of frame ***1*** with respect to coordinate frame ***0***.

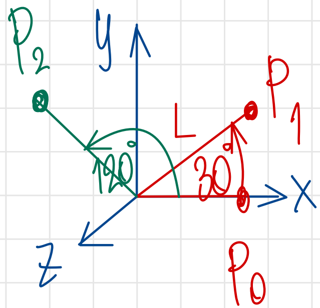




initial frame

HOW TO OBTAIN THIS ROTATION MATRIX $R_1^0 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

final frame or next



L = Robot arm

$P_0\ (\theta = 0°)$

$P_1\ (\theta = +30°)$

P_1 from P_0

$R_1^0 \quad P_1 = [\text{factor}]\, P_0 \rightarrow 30°$

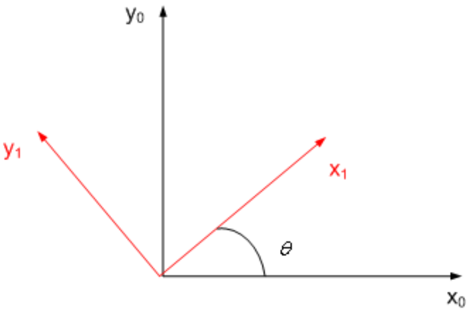
$R_2^0 \quad P_2 = [\text{factor}]\, P_0 \rightarrow 120°$

$R_2^1 \quad P_2 = [\text{factor}]\, P_1 \rightarrow 90°$

(R) (P)

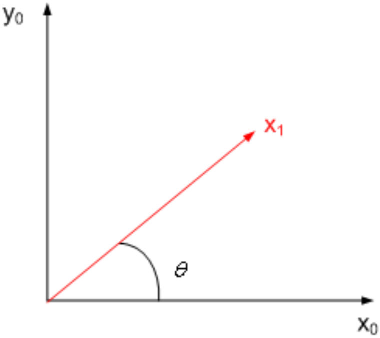
Rotation, Translation, Matrix

Rotation in the plane



$$R_1^0 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Rotation in the plane

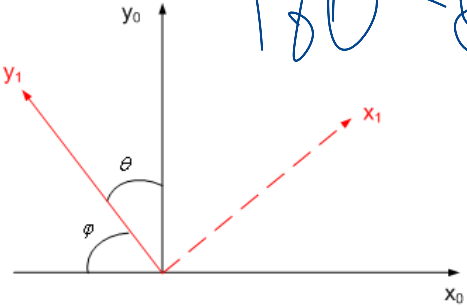


$$x_1^0 = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} x_1 . x_0 \\ x_1 . y_0 \end{bmatrix}$$

$$x_1^0 : x_1 \; with\, reference\, to\, 0 - axis$$

Rotation in the plane

$180 - \theta^\circ = -\phi^\circ$

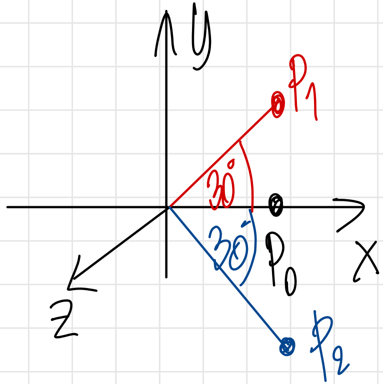


$$y_1^0 = \begin{bmatrix} \cos\varphi \\ \sin\varphi \end{bmatrix}$$

$y_1^0 : y_1$ *with reference to* $0 -$ *axis*

$$y_1^0 = \begin{bmatrix} -\cos\varphi \\ \sin\varphi \end{bmatrix} = \begin{bmatrix} -(\cos(90^\circ - \theta)) \\ \sin(90^\circ - \theta) \end{bmatrix} = \begin{bmatrix} -(\cancel{\cos 90^\circ \cos\theta} \;\; + \sin 90^\circ \sin\theta\,) \\ \sin 90^\circ \cos\theta \;\; - \;\; \cancel{\cos 90^\circ \sin\theta} \end{bmatrix}$$

$$y_1^0 = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} = \begin{bmatrix} y_1 . x_0 \\ y_1 . y_0 \end{bmatrix}$$



$$P_0 = [1, 0]$$

$$P_1 = [R^0_1] P_0$$

$$X = \begin{bmatrix} \cos(+30°) & -\sin(+30°) \\ \sin(+30°) & \cos(+30°) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.86 \\ 0.5 \end{bmatrix}_{2\times 1}$$

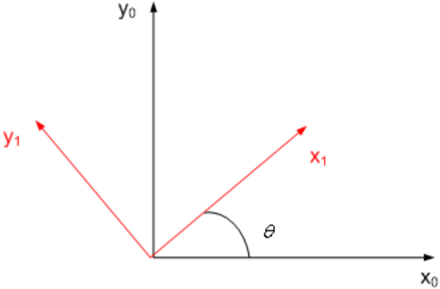
$$P_2 = [R^0_2] \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(-30°) & -\sin(-30°) \\ \sin(-30°) & \cos(-30°) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.86 \\ -0.5 \end{bmatrix}_{2\times 1}$$

$$= [R^1_2] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(-60°) & -\sin(-60°) \\ \sin(-60°) & \cos(-60°) \end{bmatrix} \begin{bmatrix} 0.86 \\ 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.86 \\ -0.5 \end{bmatrix}$$

Rotation in the plane

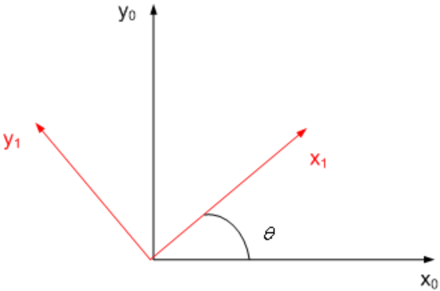


$$R_1^0 = \begin{bmatrix} x_1 . x_0 & y_1 . x_0 \\ x_1 . y_0 & y_1 . y_0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

x1relative to frame ***x0y0***.

R_1^0 This notation specify rotation of coordinates vectors for the axes of frame ***1*** with respect to coordinate frame ***0***.

Rotation in the plane



$$R_0^1 = \begin{bmatrix} \boxed{\begin{matrix} x_0 . x_1 \\ x_0 \cdot y_1 \end{matrix}} & \begin{matrix} y_0 . x_1 \\ y_0 \cdot y_1 \end{matrix} \end{bmatrix}$$

x0 relative to frame ***x1y1***.

R_0^1 This notation specify rotation of coordinates vectors for the axes of frame ***0*** with respect to coordinate frame ***1***.

Rotation in the plane

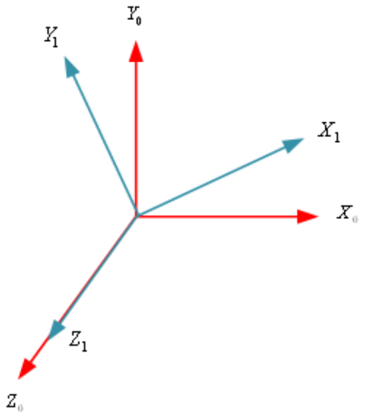
$$R_1^0 = \begin{bmatrix} x_1 . x_0 & y_1 . x_0 \\ x_1 . y_0 & y_1 . y_0 \end{bmatrix} \qquad R_0^1 = \begin{bmatrix} x_0 . x_1 & y_0 . x_1 \\ x_0 . y_1 & y_0 . y_1 \end{bmatrix}$$

dot product is commutative : $x_i . y_j = y_j . x_i$

$$R_1^0 = \left(R_0^1\right)^T$$

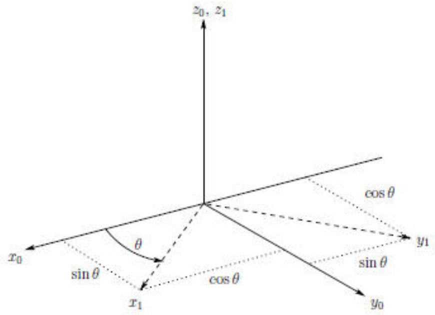
Rotation in 3D

The projection technique before scale to the 3 dimensional case. The matrix is given as



$$R_1^0 = \begin{bmatrix} x_1 . x_0 & y_1 . x_0 & z_1 . x_0 \\ x_1 . y_0 & y_1 . y_0 & z_1 . y_0 \\ x_1 . z_0 & y_1 . z_0 & z_1 . z_0 \end{bmatrix}$$

Rotation in 3D : Rotation about z-axis



$z_0 . z_1 = 1$

and

$x_1 . z_0 = 0$

$y_1 . z_0 = 0$

$z_1 . x_0 = 0$

$z_1 . y_0 = 0$

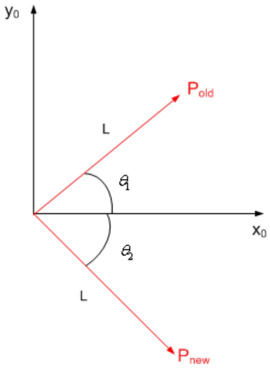
$$R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation in 3D : Rotation about x-axis and y-axis

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad x_0 . x_1 = 1$$

$$R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad y_0 \cdot y_1 = 1$$

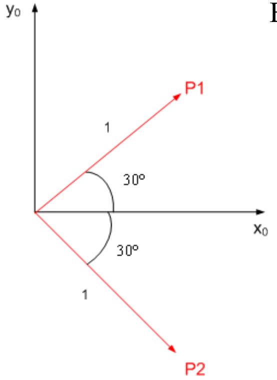
Rotational Transformation : 2D



Many times, we would like to know the new coordinates from the rotation from old coordinates.

$$P^{new} = \left[A\right]_{new}^{old} P^{old}$$

Rotational Transformation : 2D



If L =1, $\theta_1 = 30°, \theta_2 = -30°$

By normal convention, we know that

$$P^1 = \left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]^T \qquad P^2 = \left[\frac{\sqrt{3}}{2}, -\frac{1}{2}\right]^T$$

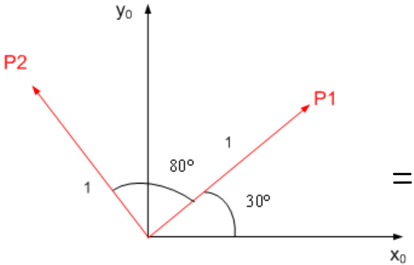
Let's apply the formula: $P^2 = [A]P^1$

$$A_2^1 = \begin{bmatrix} \cos(-60) & -\sin(-60) \\ \sin(-60) & \cos(-60) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$P_2 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix}$$

Rotational Transformation : 2D

EX: If L =1, $\theta_{1\rightarrow 2} = 80^\circ$ $P^1 = \left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]^T$



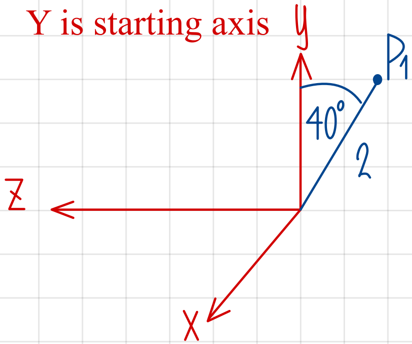
=

$$A_2^1 = \begin{bmatrix} \cos(80^\circ) & -\sin(80^\circ) \\ \sin(80^\circ) & \cos(80^\circ) \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.174 & -0.985 \\ 0.985 & 0.174 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} -0.342 \\ 0.94 \end{bmatrix}$$

Y is starting axis



$$R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-40^\circ) & -\sin(-40^\circ) \\ 0 & \sin(-40^\circ) & \cos(-40^\circ) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.766 & 0.643 \\ 0 & -0.643 & 0.766 \end{bmatrix}$$

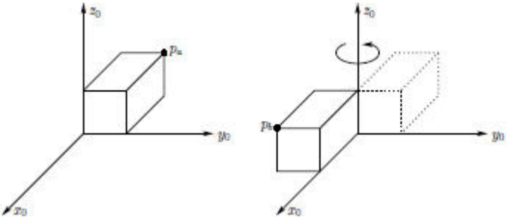
$$p_0 = \begin{bmatrix} 0 & 2 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$p^1 = R_1^0 p_0$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.766 & 0.643 \\ 0 & -0.643 & 0.766 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1.532 \\ -1.286 \end{bmatrix}$$

Rotational Transformation : 3D

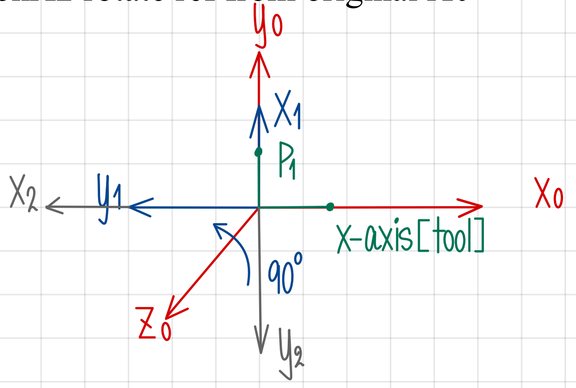


In a similar way, $P^{new} = [A] P^{old}$

$$A_1^0 = \begin{bmatrix} x_1 . x_0 & y_1 . x_0 & z_1 . x_0 \\ x_1 . y_0 & y_1 . y_0 & z_1 . y_0 \\ x_1 . z_0 & y_1 . z_0 & z_1 . z_0 \end{bmatrix}$$

P1 onX1 rotate ref from original X0

P2 onX2 rotate ref from original X0



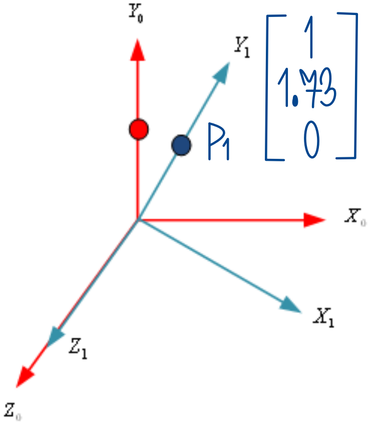
Rotational Transformation : 3D

EX: P0 is at [0 2 0]T position of the reference frame. Compute the vector P1 obtained by rotating this vector about Z-axis by 30 degrees CW.

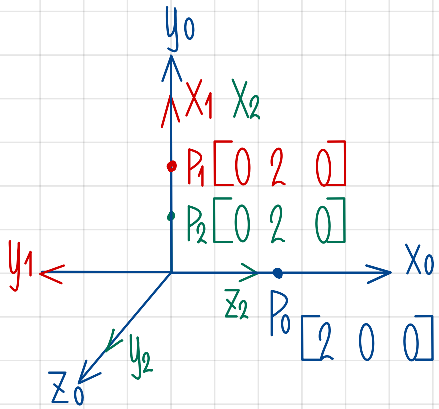
$$p^1 = \left[R_{z,-30^\circ} \right] p^0$$

$$R_{z,-30^\circ} = \begin{bmatrix} 0.86 & 0.5 & 0 \\ -0.5 & 0.86 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p^1 = \begin{bmatrix} 0.86 & 0.5 & 0 \\ -0.5 & 0.86 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.73 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 \\ 1.73 \\ 0 \end{bmatrix}$$



(1) rotz(90’)
(2) rotx(90’)

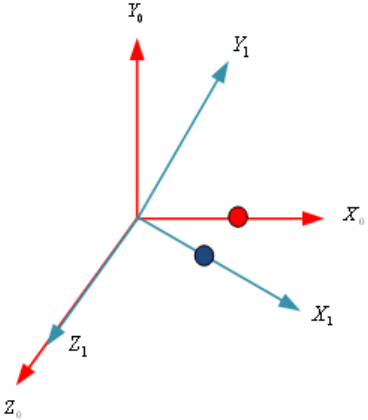
Rotational Transformation : 3D

EX: If P0 is at [2 0 0]T position of the reference frame. Compute the vector P1 obtained by rotating this vector about Z-axis by 30 degrees CW.

$$p^1 = \left[R_{z,-30^\circ}\right] p^0$$

$$R_{z,-30^\circ} =$$

$$p^1 =$$



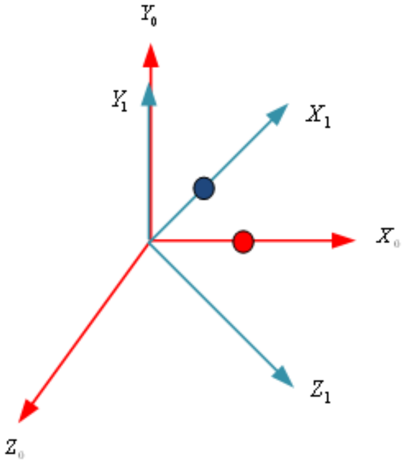
Rotational Transformation : 3D

EX: If P0 is at [2 0 0]T position of the reference frame. Compute the vector P1 obtained by rotating this vector about Y-axis by 30 degrees CCW.

$$p^1 = \left[R_{y,30^\circ}\right] p^0$$

$$R_{y,30^\circ} =$$

$$p^1 =$$



Rotation RotX

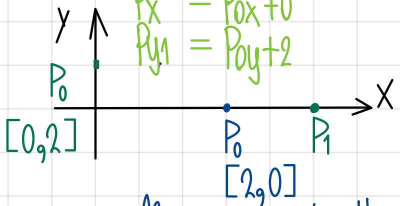
RotY

RotZ

Linear ⟶ Prismatic

$P_x = P_{0x} + 0$

$P_{y1} = P_{0y} + 2$



$P_{x1} = P_{0x} + 2$

$P_y = P_{0y} + 0$

Move +2 unit in the x-axis

Move +2 unit in the y-axis

2-Dimensional Transformations with Translation

Translation can not be produced by assigning values to the elements of a 2x2 matrix.

$$x_1 = (x\cos(\phi) - y\sin(\phi)) + q$$
$$y_1 = (y\cos(\phi) + x\sin(\phi)) + r$$

Translation parameter

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & q \\ \sin(\phi) & \cos(\phi) & r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3-Dimensional Transformations with Translation

For 3D matrix, rotation and translation about z-axis will produced

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 & q \\ \sin(\phi) & \cos(\phi) & 0 & r \\ 0 & 0 & 1 & s \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

for inverse matrix

Look at the structure The structure is 4x4 matrix

Homogenous Transformation

Homogenous transformation is a matrix representation of a rigid motions : combine of rotation and translation.

$$H = \left[\begin{array}{c|c} 3 * 3 & 3 * 1 \\ \hline 1 * 3 & 1 * 1 \end{array}\right] = \left[\begin{array}{c|c} Rotation & Translation \\ \hline 0 & 1 \end{array}\right]$$

Homogenous Transformation

For Rotation Homogeneous transformation

Rotation

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogenous Transformation

Translation in 3-D is more simple as there are three translation components for each direction. The general transformation matrix, corresponding to a translation by a vector q,r,s becomes

$$D(x,y,z)=\begin{bmatrix}1 & 0 & 0 & q\\ 0 & 1 & 0 & r\\ 0 & 0 & 1 & s\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Homogenous Transformation

For Translation Homogeneous transformation

Translation

$$D(x,q) = \begin{bmatrix} 1 & 0 & 0 & q \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D(y,r) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & r \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D(z,s) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & s \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

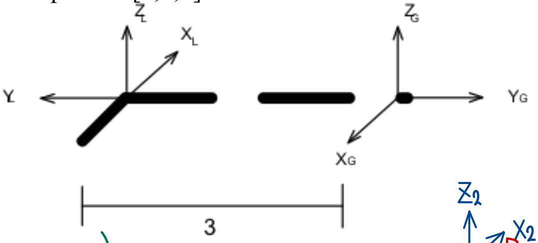
$$M = \begin{bmatrix} 1 & 0 & 0 & q \\ 0 & C & -S & 0 \\ 0 & S & C & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

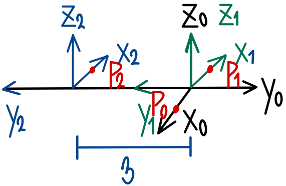
$\left.\begin{matrix} R.D \\ \text{or } D.R \end{matrix}\right\}$ order

Homogenous Transformation

Current

EX: Let [x,y,z]G be the global frame and [x,y,z]L be the local frame of the system. Here rotate the local frame by 180 degrees CCW about z-axis and translate along y axis by 3 unit. Find the transformation matrix from local frame to global frame and test the result whether you have obtained the correct result with pold as [1,0,0].





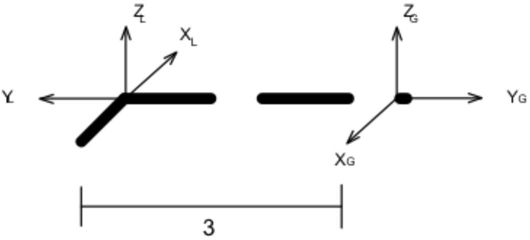
R_1^0 1st: Rot(z, +180)

D_2^1 2nd: D(y, +3)

$$P^0_{new} = {}^G_L T * P_{old}$$

$T_2^0 = R_1^0 \cdot D_2^1$ $\quad P^2 = T_2^0 P^0$

Homogenous Transformation



$$ {}^G_L T = [R][D] = [Rot(z, 180°)] * [D(y, 3)] $$

$$ P_{old} = [1\ 0\ 0\ 1]^T $$

$$ P^0_{new} = {}^G_L T * P_{old} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} $$

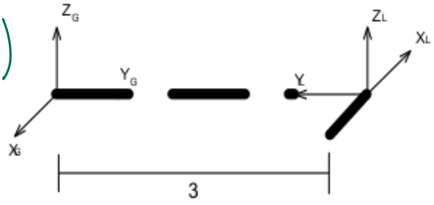
$$ P^0_{new} = [-1\ \ -3\ \ 0\ \ 1]^T $$

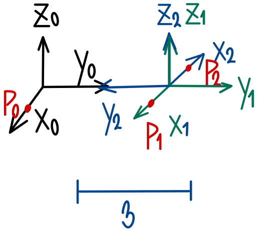
Homogenous Transformation

EX: Let [x,y,z]G be the global frame and [x,y,z]L be the local frame of the system. Here translate along y axis by 3 unit and rotate frame by 180 degrees CCW about z-axis. Find the transformation matrix and test the result whether you have obtained the correct result with pold as [1,0,0].

$D_1^0=(y_0+3)$

$R_2^1=Rot(z_0+180)$

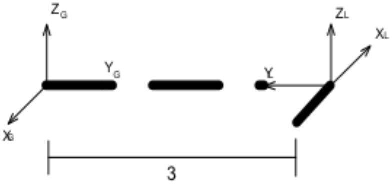




$$P^0_{new} = {}^G_L T * P_{old}$$

$T_2^0 = D_1^0 * R_2^1$

Homogenous Transformation



$$ {}^G_L T = [D][R] = [D(y,3)]*[Rot(z,180°)] $$

$$ P_{old} = [1\ 0\ 0\ 1]^T $$

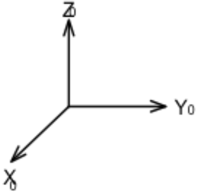
$$ P^0_{new} = {}^G_L T * P_{old} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} $$

$$ P^0_{new} = [-1\ 3\ 0\ 1]^T $$

Homogenous Transformation

EX: Perform the below transformations with a = 2 and b = 3. Draw the following frames. Frame 0 is shown.

Given p0 = [0,0,0]T, where is p4 wrt frame 0.



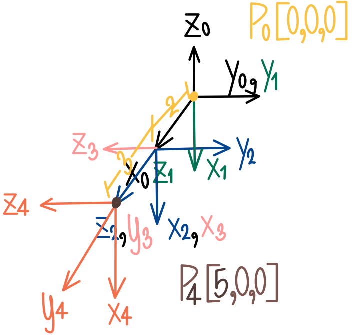
$$H_4^0 = Rot(y,90^\circ)Trans(z,a)Rot(x,90^\circ)Trans(y,b)$$

R_1^0 D_2^1 2 R_3^2 D_4^3 3

where $H_4^0 = H_1^0 H_2^1 H_3^2 H_4^3$

Homogenous Transformation

$H_4^0 =$



Homogenous Transformation

$$p_4^0 = H_4^0 p^0$$

$$p_4^0 =$$

Rotating with respect to CURRENT FRAME

$$H_2^0 = H_1^0 H_2^1$$

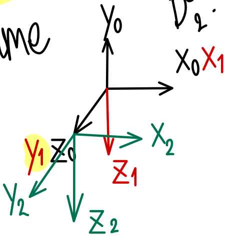
H_1^0 Represent rotations relative to the frame x0y0z0

H_2^1 Represent rotations relative to the frame x1y1z1

Current Frame

R_2^0 : Rotx(90°)

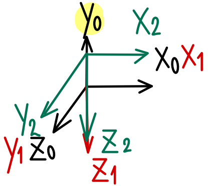
D_2^0 : Trans(y,3



Fixed Frame

R_2^0 : Rotx(90°)

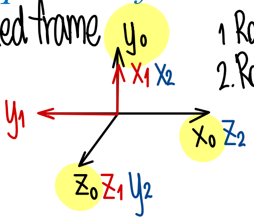
D_2^0 : Trans(y,3



Rotating with respect to FIXED FRAME

Many times it is desired to perform a sequence of rotations about a given fixed coordinate frame. Fixed frame

1 Rot z 90°
2. Rot y 90°



$$H_2^0 = H_1^0\left[\left(H_1^0\right)^{-1} H_2^1 H_1^0\right]$$

From Similarity transformation as $B = \left(H_1^0\right)^{-1} A H_1^0$

Where H_1^0 is the coordinate transformation between frame 0 and frame 1

$$\boxed{H_2^0 = H_2^1 H_1^0}$$

Rotating with respect to FIXED FRAME

$$\therefore H_2^0 = H_2^1 H_1^0$$

H_1^0 Represent rotations relative to the frame x0y0z0

H_2^1 Represent rotations relative to the frame x1y1z1

x0y0z0 as a FIXED FRAME
0

Examples on current frame and fixed frame

Fixed

EX: Let [x,y,z]G be the global frame and [x,y,z]L be the local frame of the system. Here rotate frame by 180 degrees CCW about z-axis and translate 3 unit with y-axis. Find the transformation matrix

i. If rotation about current frame.

ii.If rotation about fixed frame.

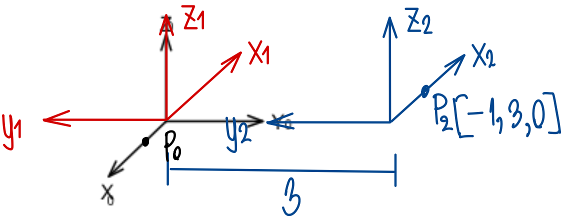
iii. test the result whether you have obtained the correct result with pold as [1,0,0].

$T_1^0 = Rot(z, +180^\circ)$

$T_2^1 = D(y,3)$

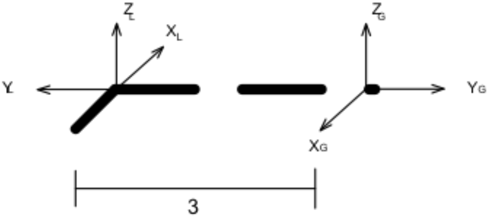
$T_{2F}^0 = T_2^1 T_1^0$

$P_{2F} = T_{2F}^0 P^0$



Examples on current frame and fixed frame

Current Frame



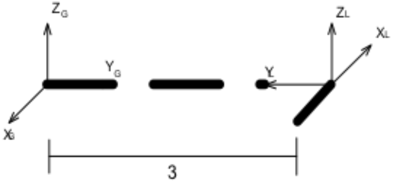
$${}^{G}_{L}T = [R][D] = [Rot(z,180°)]*[D(y,3)]$$

$$P_{old} = [1\ 0\ 0\ 1]^T$$

$$P^0_{new} = {}^{G}_{L}T * P_{old} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P^0_{new} = [-1\ \ -3\ \ 0\ \ 1]^T$$

Fixed Frame



$${}^{G}_{L}T = [D][R] = [D(y,3)]*[Rot(z,180°)]$$

$$P_{old} = [1\ 0\ 0\ 1]^T$$

$$P^0_{new} = {}^{G}_{L}T * P_{old} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P^0_{new} = [-1\ \ 3\ \ 0\ \ 1]^T$$

Examples on current frame and fixed frame

Current Frame

EX:

1. Rotate 90□ in the current y axis then

2. Rotate 90□ in the current z axis

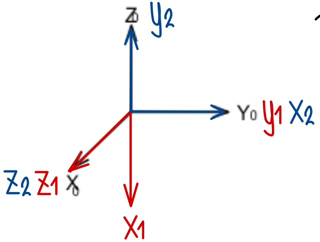
Find H_2^0

Plot the rotating axes (0,1,2). If a point is fixed on the x axis at (1,0,0), find the coordinate of this point when reference to origin.

$T_1^0 = Rot_y(90^\circ)$

$T_2^1 \; Rot_z(90^\circ)$

$T_2^0 = T_1^0 T_2^1$



$P^2 = T_2^0 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Examples on current frame and fixed frame

Current Frame

$$H_1^0 =$$

$$H_2^1 =$$

$$H_2^0 = H_1^0 H_2^1 =$$

$$P_2^0 = H_2^0 P^2 =$$

Examples on current frame and fixed frame

Fixed Frame

EX:

1. Rotate 90□ in the fixed y0 axis then
2. Rotate 90□ in the fixed z0 axis

Find H_2^0

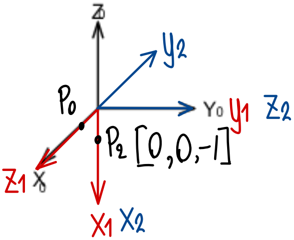
Plot the rotating axe frames : 0,1,2. If a point is fixed on the x axis at (1,0,0), find the coordinate of this point with reference to origin.

$T_1^0 = Roty(90^\circ)$

$T_2^1 = Rotz(90^\circ)$

$T_{2P}^0 = T_2^1 T_1^0$

$P_2 = T_{2P}^0 P_0$



Rotating and translation with respect to FIXED FRAME

Do the translation term first and follow by rotation term:

1st Rule

$$\therefore H_4^0 = D_4^3 D_2^1 (H_3^2 H_1^0)$$

$T_{4F}^0 = D_4^3 H_3^2 D_2^1 H_1^0$

2nd Rule

$T_{4F}^0 = (D_4^3 D_2^1)(H_3^2 H_1^0)$

H_1^0 Represent rotations relative to the frame x0y0z0

D_2^1 Represent rotations relative to the frame x1y1z1

H_3^2 Represent translation relative to the frame x2y2z2

D_4^3 Represent translation relative to the frame x3y3z3

x0y0z0 as a FIXED FRAME