MCE4101

Robotic Engineering

Assignment 2

Due: 29 July 2021

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Assignment: 2

Q1. The 3 links RPP robot is shown.

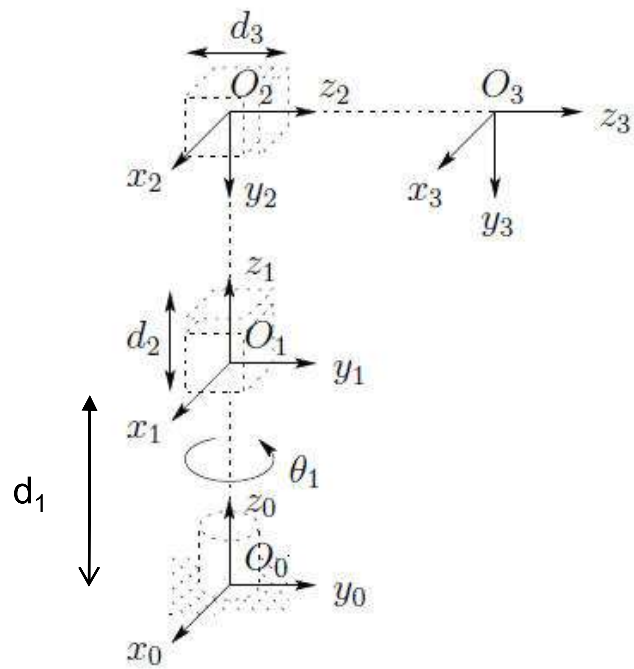
a) Obtain the forward kinematic **equation** T^0_3 with given DH table. Where d_1 is link offset.

b) Given $d_1 = 1$, $\theta_1 = 0°$, $d_2 = 1$ and $d_3 = 2$, obtain transformation matrix T^0_3.

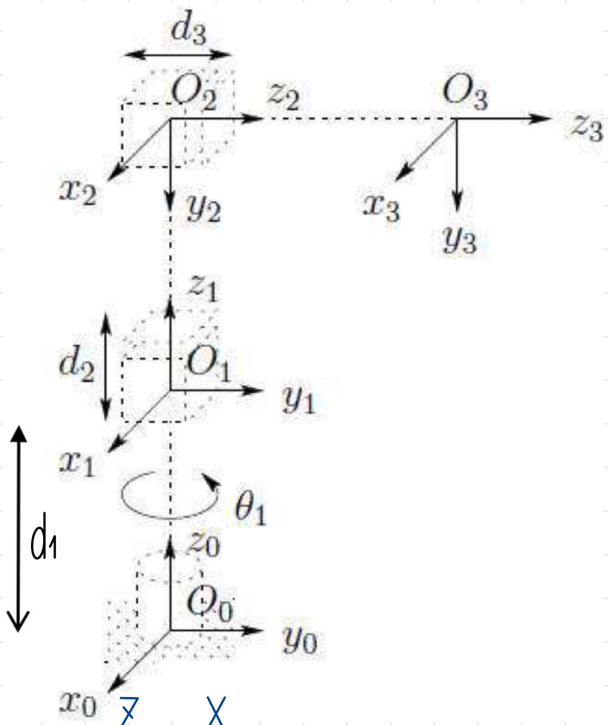
c) Given $d_1 = 1$, $\theta_1 = 0°$, $d_2 = 1$ and $d_3 = 2$, obtain end point location P_3.

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1 (offset)	θ_1^*
2	0	-90°	d_2^*	0
3	0	0	d_3^*	0

*: *denote variables*



Q1



Z X

Links	θ	d	a	α
1	θ_1^*	d_1	0	0
2	0	d_2^*	0	-90°
3	0	d_3^*	0	0

a) $T_3^0 = A_1 A_2 A_3$

$$= \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & -d_3\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & d_3\cos\theta_1 \\ 0 & -1 & 0 & d_1+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P_{end} = T_3^0 P_0$

$$= \begin{bmatrix} -d_3\sin\theta_1 \\ d_3\cos\theta_1 \\ d_1+d_2 \\ 1 \end{bmatrix}$$

$\theta_1 = 0^\circ \quad d_1 = 1 \quad d_2 = 1 \quad d_3 = 2$

b) $T_3^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

c) $P_{end} = T_3^0 P_0$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

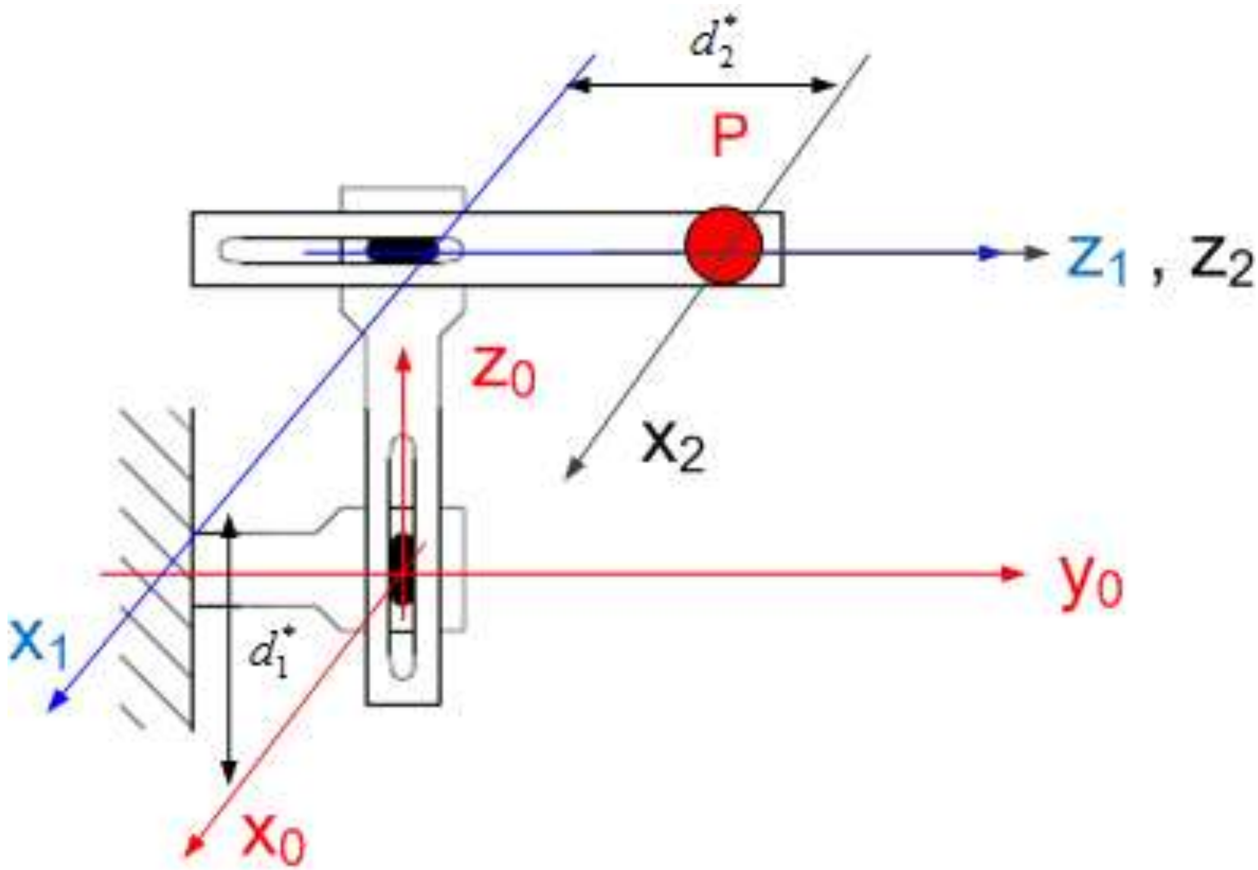
$$= \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

Assignment: 2

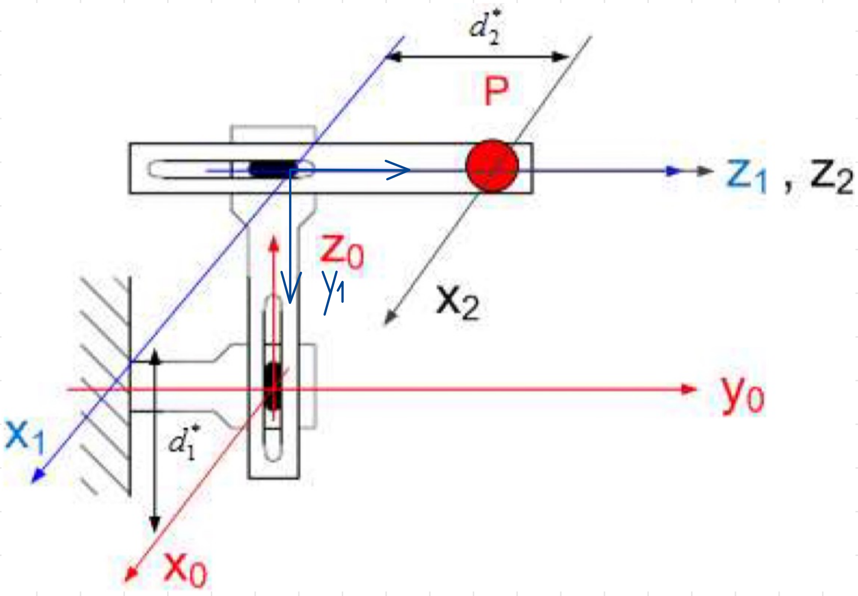
Q2.

a). Obtain the forward kinematic **equation** for the 2 links Cartesian manipulator using **DH convention**. Use link variables as d_1 and d_2.

b). Find the transformation matrix and P position for $d_1 = 1.5$ and $d_2 = 2$.



Q_2



Links	θ	d	a	α
1	0	d_1^*	0	-90°
2	0	d_2^*	0	0

$Z_0 \longrightarrow Z_1$

a) $T_2^0 = A_1 A_2$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$d_1 = 1.5 \quad d_2 = 2$

b) $T_2^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

c) $P_{end} = T_2^0 P_0$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ d_2 \\ d_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 2 \\ 1.5 \\ 1 \end{bmatrix}$$

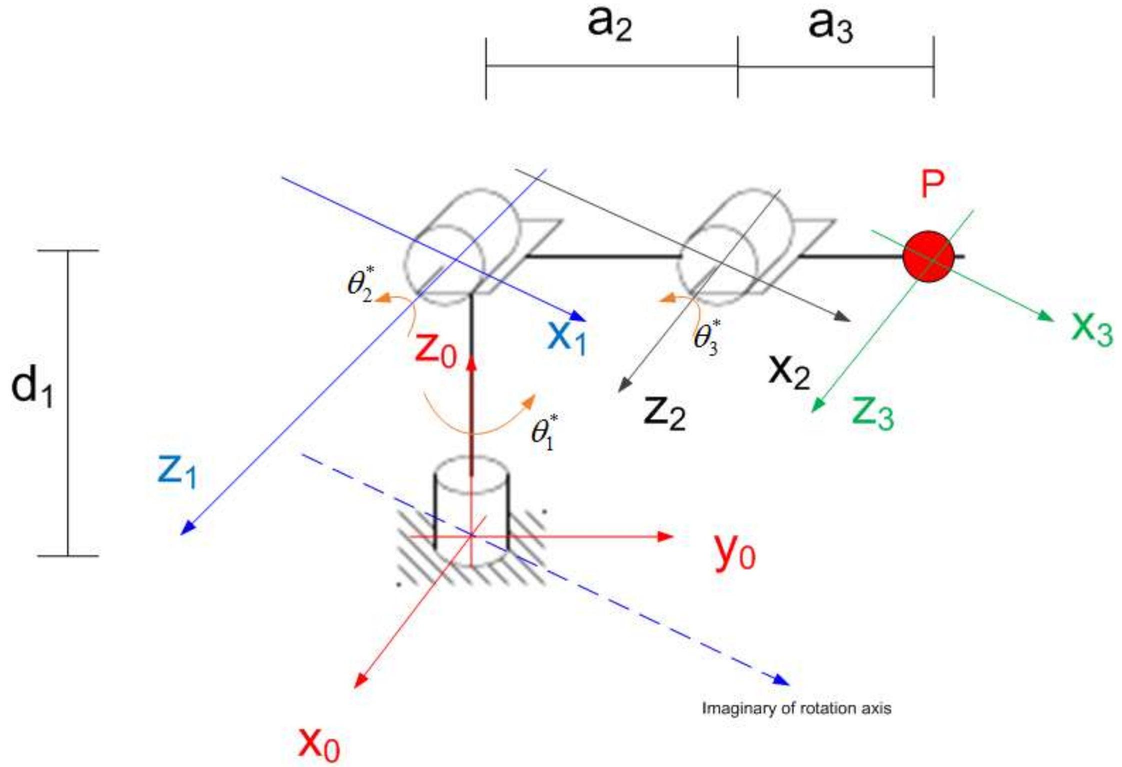
Assignment: 2

Q3.

a). Obtain the forward kinematic **equation** for the 3 links articulated (RRR) robot using **DH convention**. Where a_2 and a_3 are length of link 2 and 3 respectively. d1 is link offset. Use joint variables as θ_1, θ_2 and θ_3.

b). Find the transformation matrix and P position for θ_1=90°, θ_2=0°, θ_3 = 0°, a_2 = 2, a_3 = 1.5 and d_1 = 3.

c). Find the transformation matrix P position for θ_1=0°, θ_2=30°, θ_3 = -20°, a_2 = 2, a_3 = 1.5 and d_1 = 3.



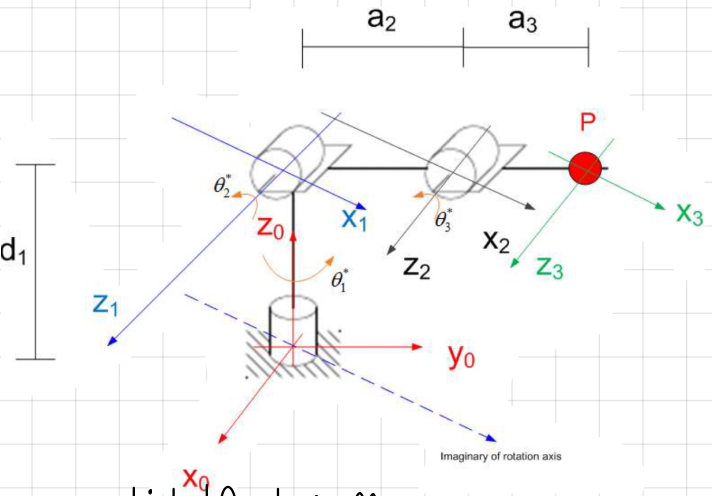
Q_3 a) $T_3^0 = A_1 A_2 A_3$

$S_1 S_2 S_3$

$$= \begin{bmatrix} \text{sin(th1)*sin(th2)*sin(th3) - cos(th2)*cos(th3)*sin(th1),} & \text{cos(th2)*sin(th1)*sin(th3) + cos(th3)*sin(th1)*sin(th2),} & \text{a3*sin(th1)*sin(th2)*sin(th3) - a3*cos(th2)*cos(th3)*sin(th1) - a2*cos(th2)*sin(th1)} \\ \text{cos(th1)*cos(th2)*cos(th3) - cos(th1)*sin(th2)*sin(th3),} & \text{- cos(th1)*cos(th2)*sin(th3) - cos(th1)*cos(th3)*sin(th2),} & \text{a2*cos(th1)*cos(th2) + a3*cos(th1)*cos(th2)*cos(th3) - a3*cos(th1)*sin(th2)*sin(th3)} \\ \text{cos(th2)*sin(th3) + cos(th3)*sin(th2),} & \text{cos(th2)*cos(th3) - sin(th2)*sin(th3),} & \text{d1 + a2*sin(th2) + a3*cos(th2)*sin(th3) + a3*cos(th3)*sin(th2)} \\ \text{0,} & \text{0,} & \text{1} \end{bmatrix}$$

$P_{end} = T_3^0 P_0$

$$= \begin{bmatrix} \text{a3*sin(th1)*sin(th2)*sin(th3) - a3*cos(th2)*cos(th3)*sin(th1) - a2*cos(th2)*sin(th1)} \\ \text{a2*cos(th1)*cos(th2) + a3*cos(th1)*cos(th2)*cos(th3) - a3*cos(th1)*sin(th2)*sin(th3)} \\ \text{d1 + a2*sin(th2) + a3*cos(th2)*sin(th3) + a3*cos(th3)*sin(th2)} \\ \text{1} \end{bmatrix}$$



Links	θ	d	a	α
0_1	90°	0	0	0
1	θ_1^*	d_1	0	90°
2	θ_2	0	a_2^*	0
3	θ_3	0	a_3^*	0

$\theta_1 = 90^\circ$ $\theta_2 = 0^\circ$ $\theta_3 = 0^\circ$

$d_1 = 3$ $a_2 = 2$ $a_3 = 1.5$

b) $T_3^0 = \begin{bmatrix} -1 & 0 & 0 & -3.5 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $P_{end} = \begin{bmatrix} -3.5 \\ 0 \\ 3 \\ 1 \end{bmatrix}$

$\theta_1 = 0^\circ$ $\theta_2 = 30^\circ$ $\theta_3 = -20^\circ$

$d_1 = 3$ $a_2 = 2$ $a_3 = 1.5$

c) $T_3^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.9848 & -0.1736 & 0 & 3.2093 \\ 0.1736 & 0.9848 & 0 & 4.2605 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $P_{end} = \begin{bmatrix} 0 \\ 3.2093 \\ 4.2605 \\ 1 \end{bmatrix}$

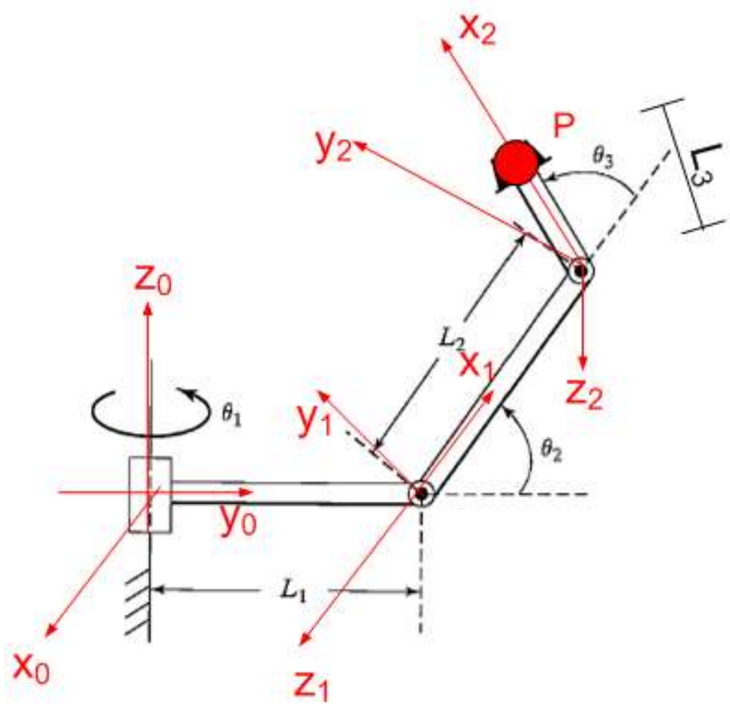
Assignment: 2

Q4.

a). Obtain the forward kinematic **equation** for the 3 links articulated (RRR) robot using **DH convention**. Where L_1 , L_2 and L_3 are length of link 1, 2 and 3 respectively. Use joint variables as θ_1, θ_2 and θ_3.

b). Find the transformation matrix and P position for θ_1=90°, θ_2=0°, θ_3 = 0°, L_1 = 1, L_2 = 2 and L_3 = 3.

c). Find the transformation matrix and P position for θ_1=0°, θ_2=90°, θ_3 = 0° , L_1 = 1, L_2 = 2 and L_3 = 3.



Q4 a) $T_3^0 = A_1 A_2 A_3$

=
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cos(th1)*cos(th2)*cos(th3) - cos(th1)*sin(th2)*sin(th3), - cos(th1)*cos(th2)*sin(th3) - cos(th1)*cos(th3)*sin(th2),  sin(th1), L1*cos(th1) + L2*cos(th1)*cos(th2) + L3*cos(th1)*cos(th2)*cos(th3) - L3*cos(th1)*sin(th2)*sin(th3)]
cos(th2)*cos(th3)*sin(th1) - sin(th1)*sin(th2)*sin(th3), - cos(th2)*sin(th1)*sin(th3) - cos(th3)*sin(th1)*sin(th2), -cos(th1), L1*sin(th1) + L2*cos(th2)*sin(th1) + L3*cos(th2)*cos(th3)*sin(th1) - L3*sin(th1)*sin(th2)*sin(th3)]
                    cos(th2)*sin(th3) + cos(th3)*sin(th2),                       cos(th2)*cos(th3) - sin(th2)*sin(th3),         0,                                           L2*sin(th2) + L3*cos(th2)*sin(th3) + L3*cos(th3)*sin(th2)]
                                                       0,                                                           0,         0,                                                                                                       1]
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$P_{end} = T_3^0 P_0$

=
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L1*cos(th1) + L2*cos(th1)*cos(th2) + L3*cos(th1)*cos(th2)*cos(th3) - L3*cos(th1)*sin(th2)*sin(th3)
L1*sin(th1) + L2*cos(th2)*sin(th1) + L3*cos(th2)*cos(th3)*sin(th1) - L3*sin(th1)*sin(th2)*sin(th3)
                                         L2*sin(th2) + L3*cos(th2)*sin(th3) + L3*cos(th3)*sin(th2)
                                                                                                 1
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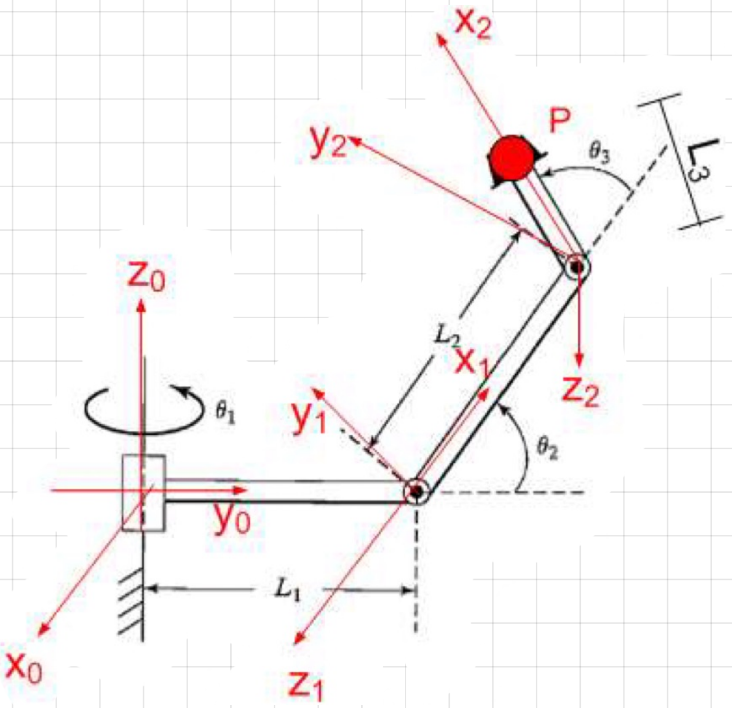

Links	θ	d	a	α
	Z		X	
1	θ_1^*	0	L_1	90°
2	θ_2^*	0	L_2	0
3	θ_3^*	0	L_3	0

$\theta_1 = 90^\circ$ $\theta_2 = 0^\circ$ $\theta_3 = 0^\circ$
$L_1 = 1$ $L_2 = 2$ $L_3 = 3$

b) $T_3^0 = \begin{bmatrix} -1 & 0 & 0 & -6 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $P_{end} = \begin{bmatrix} -6 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$\theta_1 = 0^\circ$ $\theta_2 = 90^\circ$ $\theta_3 = 0^\circ$
$L_1 = 1$ $L_2 = 2$ $L_3 = 3$

c) $T_3^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $P_{end} = \begin{bmatrix} 0 \\ -1 \\ 5 \\ 1 \end{bmatrix}$