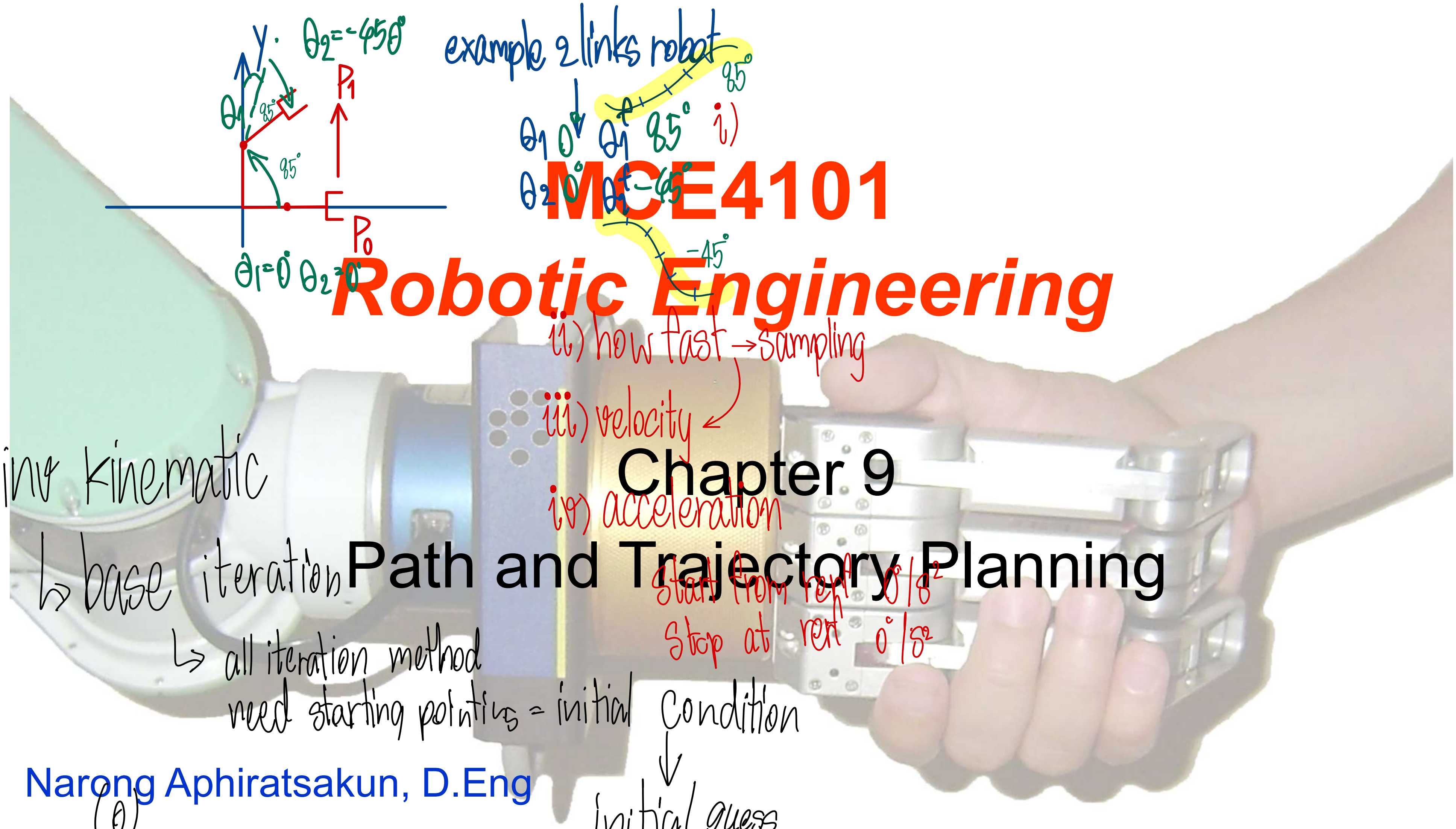

MCE4101

Robotic Engineering

Chapter 9

Path and Trajectory Planning

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(θ)

$(\theta!)$

$\theta \Rightarrow (l_i/_{10})$

Trajectory Planning

A Trajectory is a function of time $q(t)$ such that $q(t_0)$ is the starting point, and $q(t_f)$ is the final point, where t_f-t_0 is the amount of time taken to execute the trajectory.

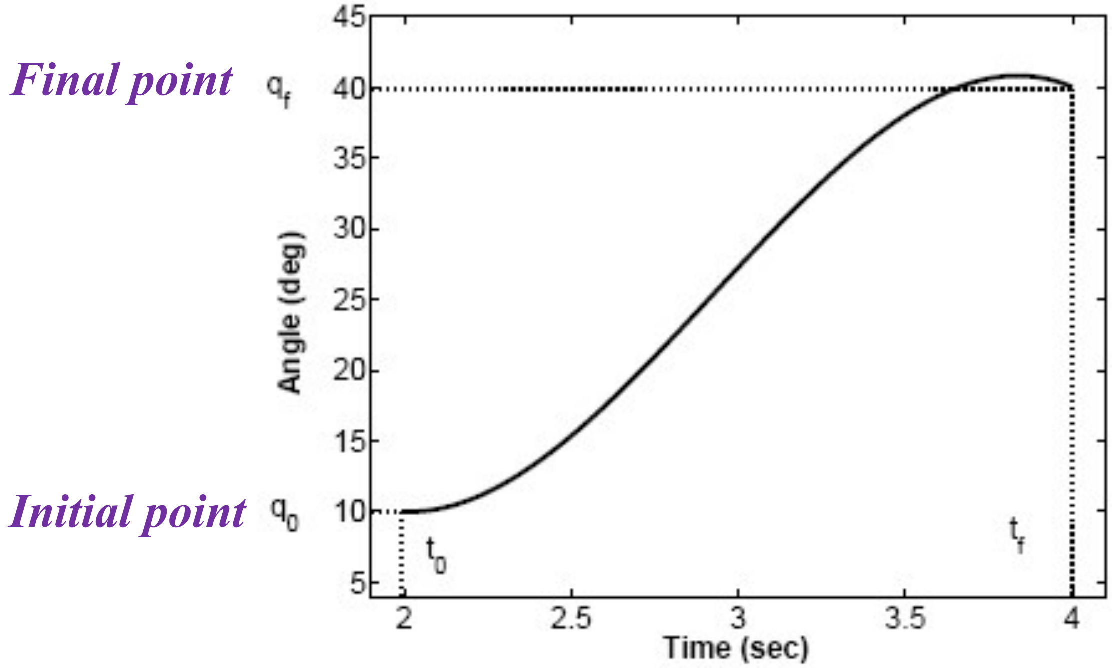
A common way to specify paths for industrial robots is to physically lead the robot through the desired motion with a TEACH pendant, so-called TEACH and PLAYBACK mode.

In static environment, the same path will be executed many times, the desired motion is recorded as set of joints angles. In such case, there is no need for calculation of the inverse kinematics.

Trajectories for Point to Point Motion

A Trajectory from an initial configuration $q(t_0)$ to a final configuration $q(t_f)$ is to be planned.

The trajectory will be planned for a single join, since the trajectories for remaining joints will be created independently and exactly the same way.



Trajectories for Point to Point Motion

A Trajectory is parameterized by time, velocities and acceleration can be computed by differentiation.

$$q(t_0)=q_0 \quad \textit{initial position}$$

$$q(t_f)=q_f \quad \textit{final position}$$

$$\dot{q}(t_0)=v_0 \quad \textit{initial velocity}$$

$$\dot{q}(t_f)=v_f \quad \textit{final velocity}$$

$$\ddot{q}(t_0)=\alpha_0 \quad \textit{initial acceleration}$$

$$\ddot{q}(t_f)=\alpha_f \quad \textit{final acceleration}$$

There are several ways to compute the trajectories using low order polynomials.

Trajectories: Cubic polynomials

If we wish to find the trajectory between two points where we specify the starting and ending velocities. There will be <u>four</u> constraints and therefore polynomial with <u>four</u> independent coefficients must be chosen

Cubic trajectory is in the form of

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

And desired velocity is in the form of

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

Trajectories: Cubic polynomials

With four constraints yield four equations

$$q_0 = a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3$$
$$v_0 = a_1 + 2a_2 t_0 + 3a_3 t_0^2$$
$$q_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3$$
$$v_f = a_1 + 2a_2 t_f + 3a_3 t_f^2$$

These four equations can be combined into matrix equation as

$$\begin{bmatrix} q_0 \\ v_0 \\ q_f \\ v_f \end{bmatrix} = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 0 & 1 & 2t_0 & 3t_0^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Trajectories: Cubic polynomials

$$\begin{bmatrix} q_0 \\ v_0 \\ q_f \\ v_f \end{bmatrix} = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 0 & 1 & 2t_0 & 3t_0^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Suppose we want to move from initial to final points in 1s, and initial and final velocities are 0

Therefore we have $t_0 = 0s$, $t_f = 1s$, $v_0 = 0$ and $v_f = 0$.

$$\Rightarrow \begin{bmatrix} q_0 \\ 0 \\ q_f \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Trajectories: Cubic polynomials

$$\Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} q_0 \\ 0 \\ 3q_f - 3q_0 \\ 2q_0 - 2q_f \end{bmatrix}$$

Trajectories: Cubic polynomials

Cubic trajectory, velocity and acceleration is in the form of

$$q(t) = q_0 + 3(q_f - q_0)t^2 + 2(q_0 - q_f)t^3$$

$$\dot{q}(t) = 6(q_f - q_0)t + 6(q_f - q_0)t^2$$

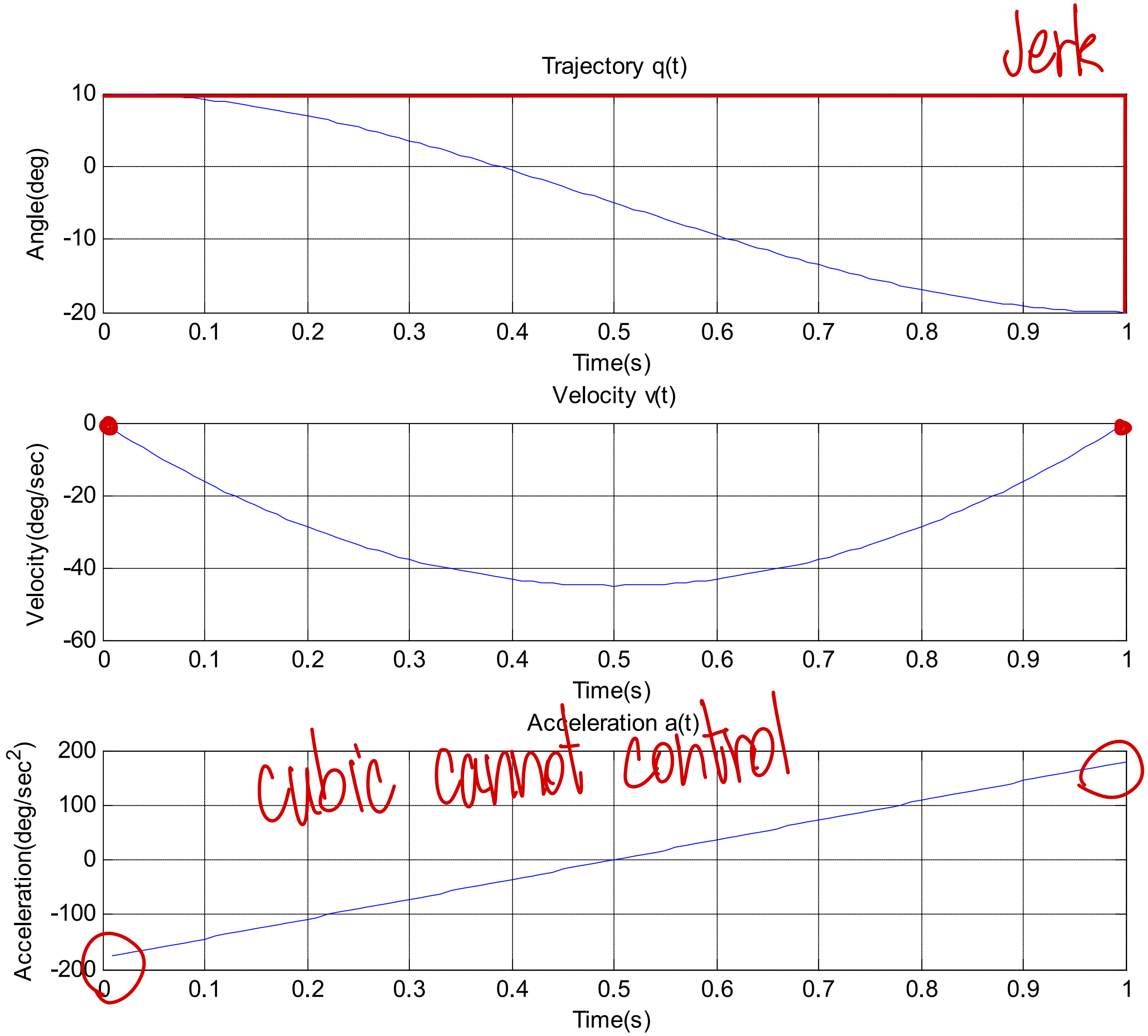
$$\ddot{q}(t) = 6(q_f - q_0) + 12(q_f - q_0)t$$

Trajectories: Cubic polynomials

$$q_0 = 10°,$$

$$q_f = -20°,$$

$$time\, t = 0\, to\, 1s$$



Trajectories: Quintic polynomials

In cubic trajectory, acceleration isn't take into consideration. Derivative of acceleration will result in a jerk to the system. We may specify the starting and ending acceleration as well. Therfore, there will be <u>SIX</u> constraints and therefore polynomial with <u>SIX</u> independent coefficients must be chosen

Quintic trajectory is in the form of

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

And desired velocity is in the form of

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

And desired acceleration is in the form of

$$\ddot{q}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

Trajectories: Quintic polynomials

With six constraints yield six equations

$$q_0 = a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 + a_4 t_0^4 + a_5 t_0^5$$

$$v_0 = a_1 + 2a_2 t_0 + 3a_3 t_0^2 + 4a_4 t_0^3 + 5a_5 t_0^4$$

$$\alpha_0 = 2a_2 + 6a_3 t_0 + 12a_4 t_0^2 + 20a_5 t_0^3$$

$$q_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 + a_4 t_f^4 + a_5 t_5^5$$

$$v_f = a_1 + 2a_2 t_f + 3a_3 t_f^2 + 4a_4 t_f^3 + 5a_5 t_f^4$$

$$\alpha_f = 2a_2 + 6a_3 t_f + 12a_4 t_f^2 + 20a_5 t_f^3$$

These six equations can be combined into matrix equation as

$$\begin{bmatrix} q_0 \\ v_0 \\ \alpha_0 \\ q_f \\ v_f \\ \alpha_f \end{bmatrix} = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 & t_0^4 & t_0^5 \\ 0 & 1 & 2t_0 & 3t_0^2 & 4t_0^3 & 5t_0^4 \\ 0 & 0 & 2 & 6t_0 & 12t_0^2 & 20t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 & t_f^4 & t_f^5 \\ 0 & 1 & 2t_f & 3t_f^2 & 4t_f^3 & 5t_f^4 \\ 0 & 0 & 2 & 6t_f & 12t_f^2 & 20t_f^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$$

Trajectories: Quintic polynomials

$$\begin{bmatrix} q_0 \\ v_0 \\ \alpha_0 \\ q_f \\ v_f \\ \alpha_f \end{bmatrix} = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 & t_0^4 & t_0^5 \\ 0 & 1 & 2t_0 & 3t_0^2 & 4t_0^3 & 5t_0^4 \\ 0 & 0 & 2 & 6t_0 & 12t_0^2 & 20t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 & t_f^4 & t_f^5 \\ 0 & 1 & 2t_f & 3t_f^2 & 4t_f^3 & 5t_f^4 \\ 0 & 0 & 2 & 6t_f & 12t_f^2 & 20t_f^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$$

Suppose we want to move from initial to final points in 2s, and initial and final velocities are 0, initial and final acceleration are 0

Therefore we have $t_0 = 0s$, $t_f = 2s$, $v_0 = 0$ and $v_f = 0$, $\alpha_0 = 0$ and $\alpha_f = 0$, .

$$\Rightarrow \begin{bmatrix} q_0 \\ 0 \\ 0 \\ q_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & 2 & 4 & 8 & 16 & 32 \\ 0 & 1 & 4 & 12 & 32 & 80 \\ 0 & 0 & 2 & 12 & 48 & 160 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$$

Trajectories: Quintic polynomials

$$\Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} q_0 \\ 0 \\ 0 \\ \frac{5}{4} q_f - \frac{5}{4} q_0 \\ \frac{15}{16} q_0 - \frac{15}{16} q_f \\ \frac{3}{16} q_f - \frac{3}{16} q_f \end{bmatrix}$$

Trajectories: Quintic polynomials

Quintic trajectory, velocity and acceleration is in the form of

$$q(t) = q_0 + \frac{5}{4}(q_f - q_0)t^3 + \frac{15}{16}(q_0 - q_f)t^4 + \frac{3}{16}(q_f - q_0)t^5$$

$$\dot{q}(t) = 3 * \frac{5}{4}(q_f - q_0)t^2 + 4 * \frac{15}{16}(q_0 - q_f)t^3 + 5 * \frac{3}{16}(q_f - q_0)t^4$$

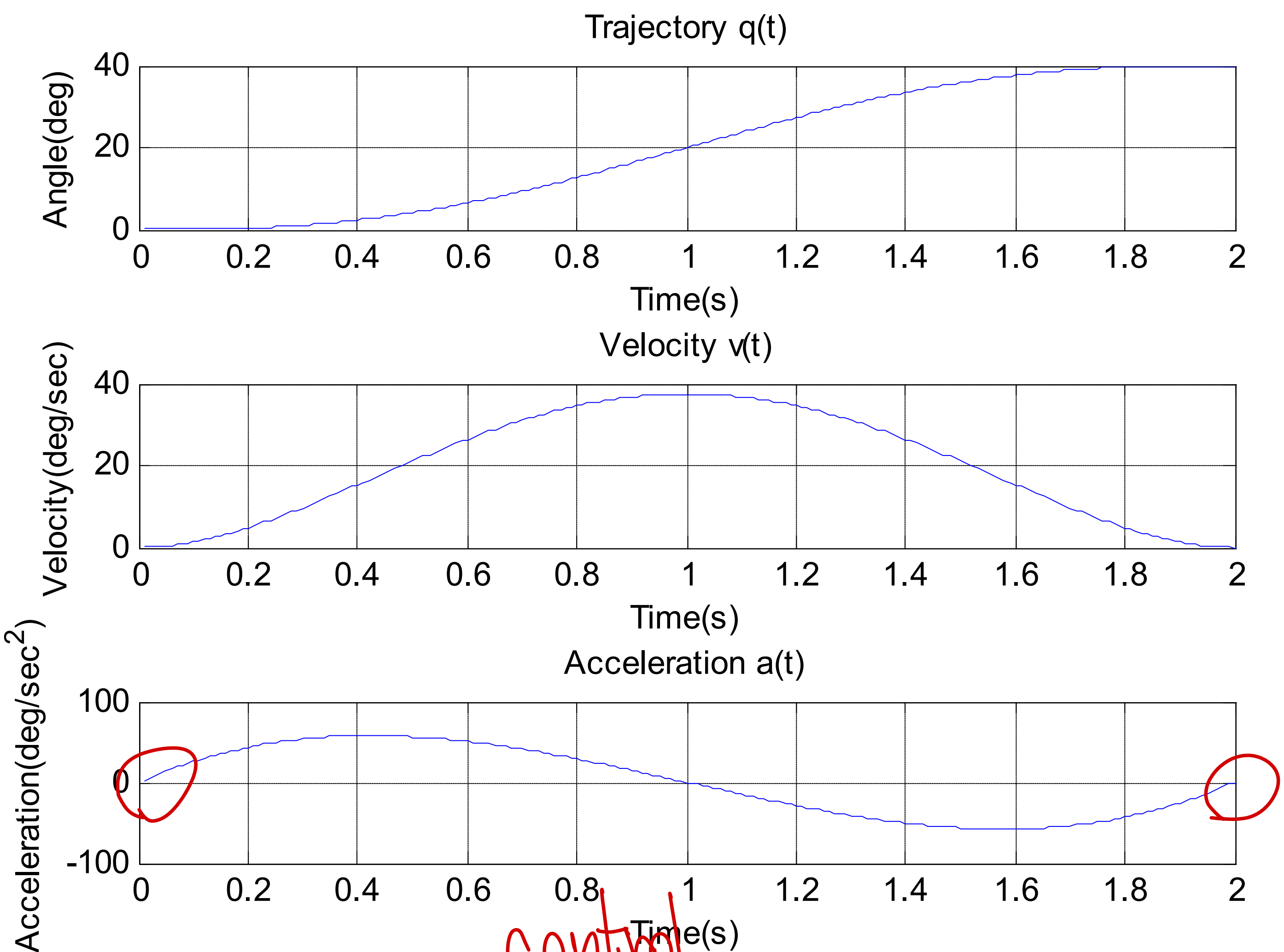
$$\ddot{q}(t) = 6 * \frac{5}{4}(q_f - q_0)t + 12 * \frac{15}{16}(q_0 - q_f)t^2 + 20 * \frac{3}{16}(q_f - q_0)t^3$$

Trajectories: Quintic polynomials

$q_0 = 0°,$

$q_f = 40°,$

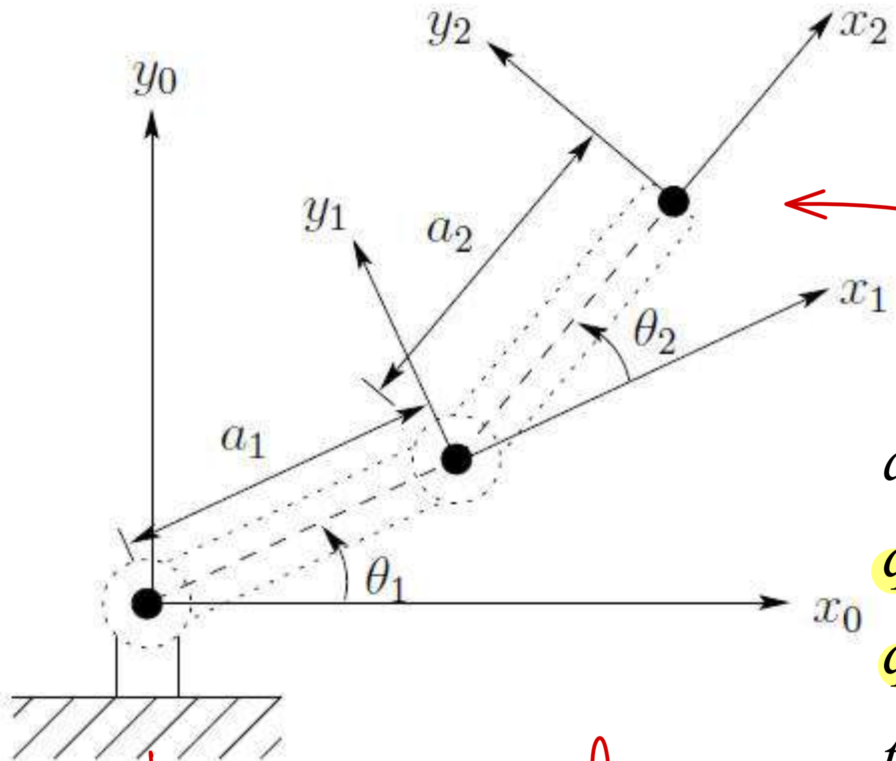
$time\, t = 0\, to\, 2s$



control

0/s² → 0/s²

Example : 2 links manipulator with MATLAB



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*

position form $\cancel{\begin{bmatrix} 3 \\ 0 \\ 2 \\ 0 \end{bmatrix}}$

$a_1 = 1, a_2 = 2$

$q_0 = [3, 0, 0],$

$q_f = [2, 2, 0],$

$time\ t = 0\ to\ 2s$

$initial\ velocity\ and\ final\ velocity = 0\ /\ s$

movement

$\theta_1 \sim \theta_f \rightarrow$ trajectory

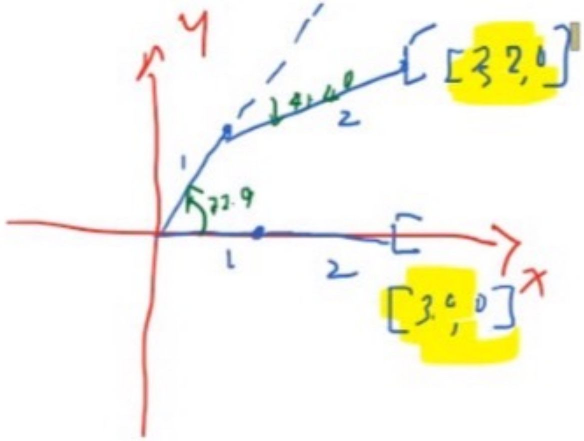
$\theta_2 \sim \theta_f \rightarrow$ trajectory

1st: desired data for θ_1 & θ_2

$\rightarrow$ ikine

Tuesday, September 14, 2021 9:47 AM

$\theta_1^0 = 0^\circ$ $\theta_2^0 = 0^\circ$



$\theta_1^f = 72.9^\circ$ $\theta_2^f = -41.4^\circ$

$$\begin{array}{ccc} y & B & A \\ \begin{bmatrix} q_0 \\ v_0 \\ q_f \\ v_f \end{bmatrix} = & \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 0 & 1 & 2t_0 & 3t_0^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix} & \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \end{array}$$

$$q_0 = a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3$$

$$v_0 = a_1 + 2a_2 t_0 + 3a_3 t_0^2$$

$$q_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3$$

$$v_f = a_1 + 2a_2 t_f + 3a_3 t_f^2$$

$q_0 =$ Joint 1 $= 54.7t^2 - 18.2t^3$

$v_0 = \dot{q}_1(t) = 109.4t - 54.6t^2$

$\dot{q}_1(t) = a_1 + 2a_2 t + 3a_3 t^2$

$\alpha_0 = \ddot{q}_1(t) = 109.4 - 109.2t$

$\ddot{q}_1(t) = 2a_2 + 6a_3 t$

Joint2 $= -31.06\,t^2 + 10.35t^3$

$\dot{q}_2(t) = - \quad t +$

$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2$

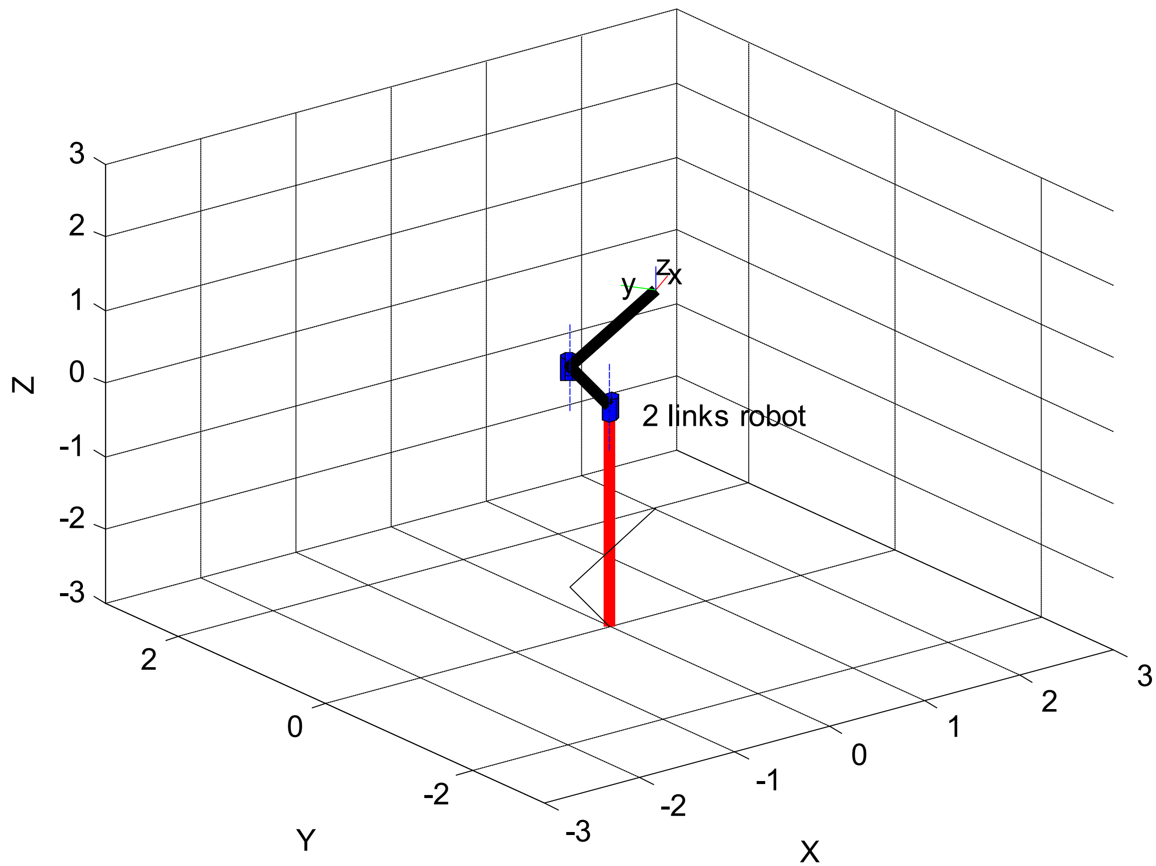
$\ddot{q}_2(t) = \quad - \quad t$

$\ddot{q}_2(t) = 2a_2 + 6a_3 t$

Example : 2 links manipulator with MATLAB

```
Q = jtraj(Q0, Q2,tq);   %interpolated position
```

$$a_1 = 1, a_2 = 2$$
$$q_0 = [3, 0, 0],$$
$$q_f = [2, 2, 0],$$
$$time\, t = 0\ to\ 2s$$



path equation → cubric → position, velocity } controlled

path equation → quantic → position, velocity, acceleration } controlled

path equation

↓

matrix ⇒ coefficient

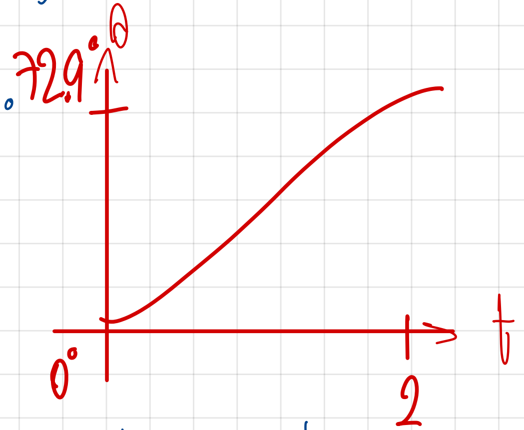
⇓

equation

Plot and get smooth curve

$\theta_1^0 = 0°$

$\theta_1^f = 72.9°$



$q(t) = 54.7t^2 - 18.2t^3$

$q(0) \rightarrow t = 0$

$q(0.5) \rightarrow t = 0.5$

$q(1) \rightarrow t = 1$

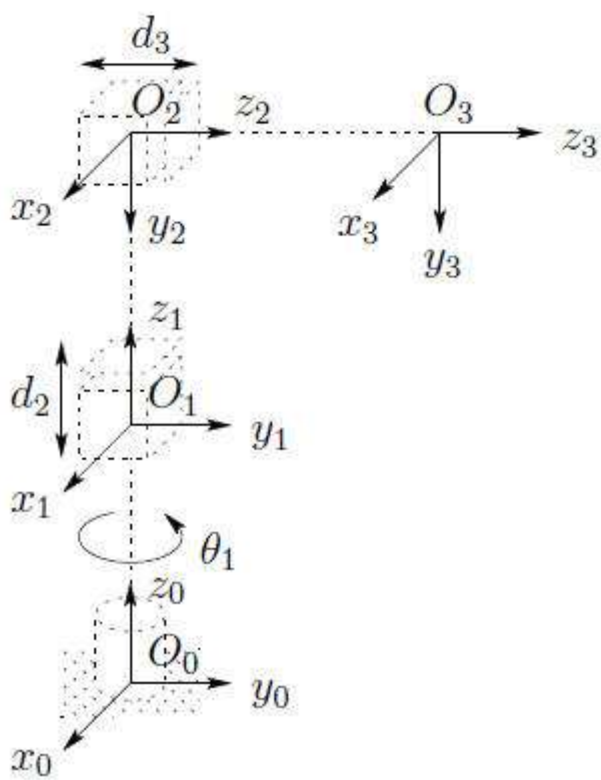
$q(1.5) \rightarrow t = 1.5$

$q(2) \rightarrow t = 2$

0
0.2
0.4
⋮
20

} 10 points

Example : RPP with MATLAB



Link	a_i	α_i	d_i	θ_i
1	0	0	0 (offset)	θ_1^*
2	0	-90°	d_2^*	0
3	0	0	d_3^*	0

$$d_1 = 0.5$$

$$q_0 = [0, 0.2, 0.5],$$

$$q_f = [1, 1.2, 0.5],$$

$$time\, t = 0\, to\, 2s$$

Example : RRP with MATLAB

$$d_1 = 0.5$$

$$q_0 = [0, 0.2, 0.5],$$

$$q_f = [1, 1.2, 0.5],$$

$$time\, t = 0\, to\, 2s$$