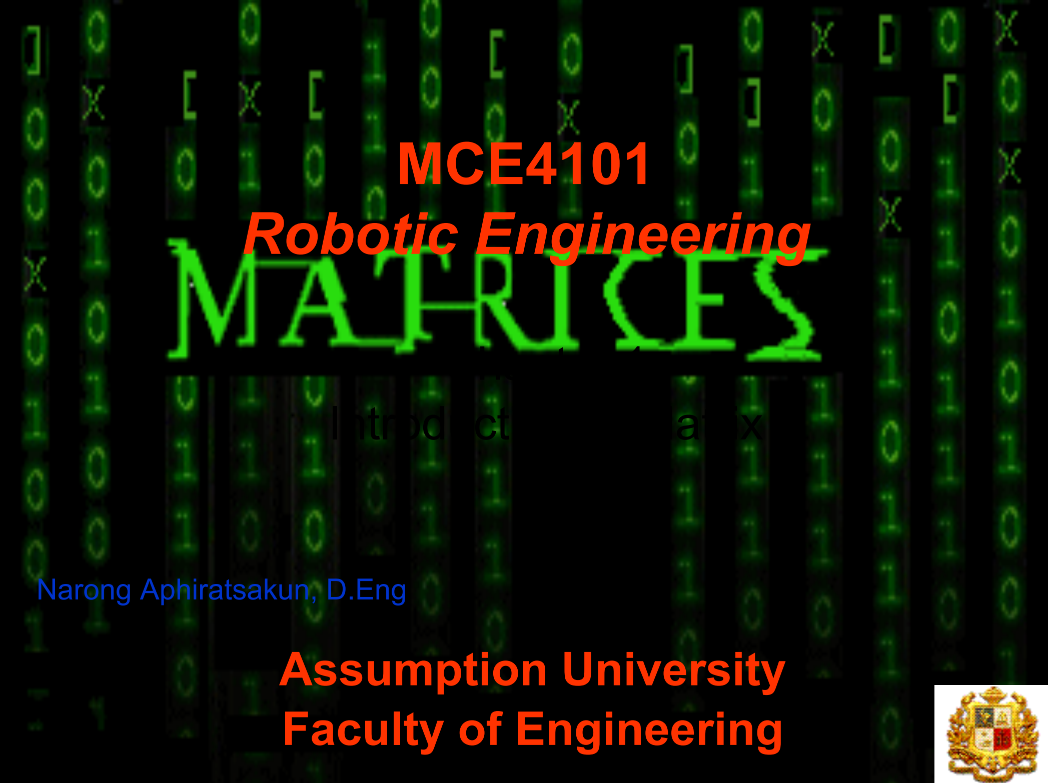

MCE4101
Robotic Engineering

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Matrix

$$\begin{array}{cc} \text{COLUMN} & \\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{[ROWXCOLUMN]} & \text{ROW} \end{array}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{[2x2]} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{[2x3]} \quad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{21} & a_{22} \end{bmatrix}_{[3x2]} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{33} \end{bmatrix}_{[3x3]}$$

Matrix: Addition

Two matrices of the same order can be added or subtracted.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{33} \end{bmatrix} + \left[\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{21} & b_{22} & b_{33} \end{bmatrix}\right] = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & a_{13} + b_{13} \\ a_{21} + b_{21} & a_{22} + b_{22} & a_{23} + b_{23} \\ a_{31} + b_{31} & a_{32} + b_{32} & a_{33} + b_{33} \end{bmatrix}$$

Matrix: Scalar Multiplication

A matrix is multiplied by scalar, every element of the matrix is multiplied by that scalar.

$$kA = k\begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} & ka_{12} \end{bmatrix}$$

Matrix: Multiplication

Two matrices can be multiplied if the number of rows in B is equal to number of column in A.

$$A\,xB = \begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}_{[1x3]} \; x \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix}_{[3x1]} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} \end{bmatrix}_{[1x1]}$$

In general $\boxed{A\,xB \neq BxA}$

Matrix: Multiplication (3 by 3)

$$\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix} \begin{pmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \\ b31 & b32 & b33 \end{pmatrix} =$$

$$\begin{pmatrix} a11.b11 + a12.b21 + a13.b31 & a11.b12 + a12.b22 + a13.b32 & a11.b13 + a12.b23 + a13.b33 \\ a21.b11 + a22.b21 + a23.b31 & a21.b12 + a22.b22 + a23.b32 & a21.b13 + a22.b23 + a23.b33 \\ a31.b11 + a32.b21 + a33.b31 & a31.b12 + a32.b22 + a33.b32 & a31.b13 + a32.b23 + a33.b33 \end{pmatrix}$$

Matrix: Multiplication (4 by 4)

If A = [a b c d;
e f g h;
i j k l;
m n o p];

B = [A B C D;
E F G H;
I J K L;
M N O P];

A*B=

[a*A+b*E+c*I+d*M, a*B+b*F+c*J+d*N, a*C+b*G+c*K+d*O, a*D+b*H+c*L+d*P
e*A+f*E+g*I+h*M, e*B+f*F+g*J+h*N, a*C+f*G+g*K+h*O, e*D+f*H+g*L+h*P
i*A+j*E+k*I+l*M, i*B+j*F+k*J+l*N, a*C+j*G+k*K+l*O, i*D+j*H+k*L+l*P
m*A+n*E+o*I+p*M, m*B+n*F+o*J+p*N, a*C+n*G+o*K+p*O, m*D+n*H+o*L+p*P]

Matrix: Transpose

The transpose of matrix A is obtained by writing the rows of A in order, as columns.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}_{[1x3]}$$

$$A^T = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}_{[3x1]}$$

Matrix: Identity (unity)

The matrix is called identity or unity if $\begin{array}{ll} a_{ij} = 0 & i \neq j \\ a_{ij} = 1 & i = j \end{array}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Note: Transpose of a unity matrix is a unity matrix (I)

Matrix: Inversion (2x2 Matrix)

Inverse matrix will exist if matrix A is square matrix and non singular.

Singular : when the determinant is zero

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A^{-1} = \frac{1}{determinant}\left[matrix\ of\ cofactors\right]^T = \frac{1}{|A|}\left[matrix\ of\ cofactors\right]^T$$

$$A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix}$$

Note: $AA^{-1} = I$

Matrix: Inversion (3x3 Matrix)

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{\begin{bmatrix} ei - fh & hc - ib & bf - ce \\ gf - di & ai - gc & dc - af \\ dh - ge & gb - ah & ae - db \end{bmatrix}}{|\mathbf{A}|}$$

$$= \frac{\begin{bmatrix} ei - fh & hc - ib & bf - ce \\ gf - di & ai - gc & dc - af \\ dh - ge & gb - ah & ae - db \end{bmatrix}}{aei + bfg + cdh - gec - hfa - idb}$$

Examples

1. Addition

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

2. Scale

$$5x \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

Examples

3A. Multiplication

$$A\,x\,B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{[1x3]} \; x \; \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}_{[3x1]} = \begin{bmatrix} 32 \end{bmatrix}_{1x1}$$

$$B\,x\,A = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}_{[3x1]} \; x \; \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{[1x3]} = \begin{bmatrix} 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \end{bmatrix}_{3x3}$$

Examples

3B. Multiplication

$$A\,x\,B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{[3x3]} \quad x \quad \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}_{[3x3]} = \begin{bmatrix} 30 & 24 & 18 \\ 84 & 69 & 54 \\ 138 & 114 & 90 \end{bmatrix}_{3x3}$$

Examples

4. Transpose

$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{[1x3]}$$

$$A^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3x1}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{[2x3]}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3x2}$$

Examples

5a. Inverse

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \det A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = -2$$

-6

4

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

Examples

5b. Inverse

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 2 \\ 5 & 4 & 2 \end{bmatrix}$$

$$\begin{array}{ccccc} & & -15 & -8 & -16 \\ \det A = \left[\begin{matrix} 1 & 2 & 3 \\ 4 & 1 & 2 \\ 5 & 4 & 2 \end{matrix}\right. & \hspace{-1em}\left.\begin{matrix} \\ \\ \\ \end{matrix}\right] & \begin{matrix} 1 \\ 4 \\ 5 \end{matrix} & \begin{matrix} 2 \\ 1 \\ 4 \end{matrix} & = 31 \\ & & 2 & 20 & 48 \end{array}$$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} +\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} & -\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} & +\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \\ -\begin{bmatrix} 4 & 2 \\ 5 & 2 \end{bmatrix} & +\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix} & -\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \\ +\begin{bmatrix} 4 & 1 \\ 5 & 4 \end{bmatrix} & -\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix} & +\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} \end{bmatrix} = \frac{1}{31}\begin{bmatrix} -6 & 8 & 1 \\ 2 & -13 & 10 \\ 11 & 6 & -7 \end{bmatrix}$$

Matrix: Equation to Matrix

Assume equations are given

$$x_1 = A_{11}x + A_{12}y$$
$$y_1 = A_{21}x + A_{22}y$$

Above equations can be view as matrix form as

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Examples

6. Calculate x1, y1

$$x_1 = 2x + 3y$$
$$y_1 = x - 2y$$

If x = 5 and y =4

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ -3 \end{bmatrix}$$

Examples with MATLAB

Row Matrix : [1;2;3]

Column Matrix : [1 2 3]

Addition : A+B

Subtraction : A-B

Scale and Multiplication : 5*A

Identity : eye(matrix size: n)

Transpose : A.'

Inverse : inv(A)

Determinant : det(A)

Dimension : size(A)

pi : 3.14……