

Assumption University (ABAC)
Classwork 1

Name: Todsavard T ID: 6114215 Date: 15/07/21

1. Obtain A^{-1} show your working steps.

if

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 1 & 4 & 3 \end{bmatrix}$$

Ans:

$$\det A = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix} = 1(-9 - 6) - 1(3 - 2) = -15$$

(Note: The handwritten calculation in the image shows $\det A = 2$, which is incorrect. The correct calculation is $\det A = -15$.)

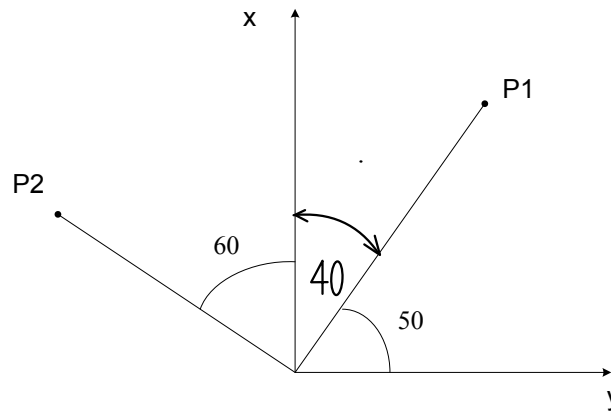
$$= \frac{1}{\det A} \begin{bmatrix} + \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} & - \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix} & + \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \\ - \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} & + \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} & - \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \\ + \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} & - \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix} & + \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \end{bmatrix} = \frac{1}{-15} \begin{bmatrix} 1 & 6 & -5 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

Assumption University (ABAC)

Classwork 2

Name...Todsavard T.....ID.....6114215.....Date: 22/6/21

1. Obtain P1 and P2 location if L = 2. X is starting axis,



Ans:

$$p_0 = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} \cos(40^\circ) & -\sin(40^\circ) & 0 \\ \sin(40^\circ) & \cos(40^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_1 = R_1^0 p_0$$

$$= \begin{bmatrix} \cos(40^\circ) & -\sin(40^\circ) & 0 \\ \sin(40^\circ) & \cos(40^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5321 \\ 1.2456 \\ 0 \end{bmatrix}$$

$$R_2^0 = \begin{bmatrix} \cos(-60^\circ) & -\sin(-60^\circ) & 0 \\ \sin(-60^\circ) & \cos(-60^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_2 = R_2^0 p_0$$

$$= \begin{bmatrix} \cos(-60^\circ) & -\sin(-60^\circ) & 0 \\ \sin(-60^\circ) & \cos(-60^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1.7321 \\ 0 \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} \cos(-100^\circ) & -\sin(-100^\circ) & 0 \\ \sin(-100^\circ) & \cos(-100^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_2 = R_2^1 p_1$$

$$= \begin{bmatrix} \cos(-100^\circ) & -\sin(-100^\circ) & 0 \\ \sin(-100^\circ) & \cos(-100^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5321 \\ 1.2456 \\ 0 \end{bmatrix}$$

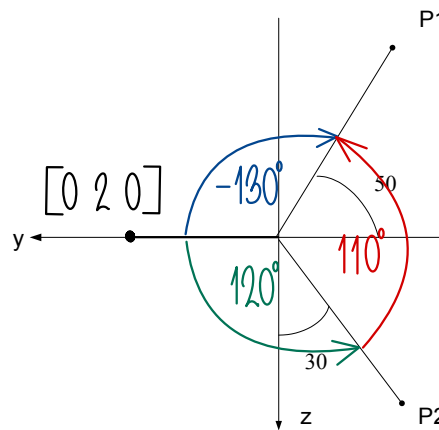
$$= \begin{bmatrix} 1 \\ -1.7321 \\ 0 \end{bmatrix}$$

Assumption University (ABAC)

Classwork 3

Name.. Todsaad T.....ID... 6114215..... Date: 24 June 2021

1. Obtain P1 and P2 locations (applying rotating matrix method, and show those steps), given $L = 2$. Y is starting axis of robot arm, and X is rotating axis.



Ans:

$$p_0 = [0 \ 2 \ 0]^T = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-130^\circ) & -\sin(-130^\circ) \\ 0 & \sin(-130^\circ) & \cos(-130^\circ) \end{bmatrix}$$

$$p_1 = R_1^0 p_0$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-130^\circ) & -\sin(-130^\circ) \\ 0 & \sin(-130^\circ) & \cos(-130^\circ) \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1.246 \\ -1.532 \end{bmatrix}$$

$$R_2^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(120^\circ) & -\sin(120^\circ) \\ 0 & \sin(120^\circ) & \cos(120^\circ) \end{bmatrix}$$

$$p_2 = R_2^0 p_0$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(120^\circ) & -\sin(120^\circ) \\ 0 & \sin(120^\circ) & \cos(120^\circ) \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1 \\ 1.732 \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(110^\circ) & -\sin(110^\circ) \\ 0 & \sin(110^\circ) & \cos(110^\circ) \end{bmatrix}$$

$$p_2 = R_2^1 p_1$$

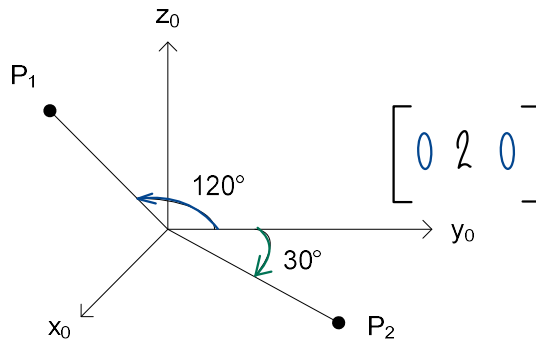
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(110^\circ) & -\sin(110^\circ) \\ 0 & \sin(110^\circ) & \cos(110^\circ) \end{bmatrix} \begin{bmatrix} 0 \\ -1.246 \\ -1.532 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1 \\ 1.732 \end{bmatrix}$$

Assumption University (ABAC)
Classwork 4

Name.....Todsavrad T.....ID.....6.114215..... Date: 29 June 2021

1. Obtain P1 and P2 locations (applying rotating matrix method, and show those steps), given $L = 2$. Y is starting axis of robot arm, and X is rotating axis.



$$p_0 = [0 \ 2 \ 0]^T = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(120^\circ) & -\sin(120^\circ) \\ 0 & \sin(120^\circ) & \cos(120^\circ) \end{bmatrix}$$

$$p^1 = R_1^0 p_0$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(120^\circ) & -\sin(120^\circ) \\ 0 & \sin(120^\circ) & \cos(120^\circ) \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1 \\ 1.732 \end{bmatrix}$$

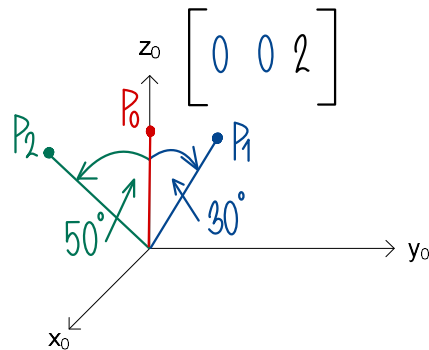
$$R_2^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-30^\circ) & -\sin(-30^\circ) \\ 0 & \sin(-30^\circ) & \cos(-30^\circ) \end{bmatrix}$$

$$p_2 = R_2^0 p_0$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-30^\circ) & -\sin(-30^\circ) \\ 0 & \sin(-30^\circ) & \cos(-30^\circ) \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1.732 \\ -1 \end{bmatrix}$$

2. P1 is 30° CW from the starting axis and P2 is 50° CCW from starting axis. Given $L = 2$ and Z is starting axis of robot arm, and X is rotating axis. Draw and locate P1 and P2 on the diagram.



$$p_0 = \begin{bmatrix} 0 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

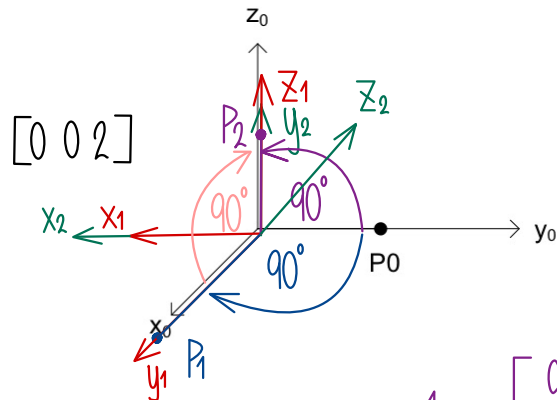
$$R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-30^\circ) & -\sin(-30^\circ) \\ 0 & \sin(-30^\circ) & \cos(-30^\circ) \end{bmatrix}$$

$$\begin{aligned} p^1 &= R_1^0 p_0 \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-30^\circ) & -\sin(-30^\circ) \\ 0 & \sin(-30^\circ) & \cos(-30^\circ) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 1 \\ 1.732 \end{bmatrix} \end{aligned}$$

$$R_2^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(50^\circ) & -\sin(50^\circ) \\ 0 & \sin(50^\circ) & \cos(50^\circ) \end{bmatrix}$$

$$\begin{aligned} p_2 &= R_2^0 p_0 \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(50^\circ) & -\sin(50^\circ) \\ 0 & \sin(50^\circ) & \cos(50^\circ) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ -1.532 \\ 1.246 \end{bmatrix} \end{aligned}$$

3. Given $L = 2$ and P_0 is on the initial position on the robot arm.
- P_1 is rotate of 90° CW on the Z-axis.
 - From Frame 1, P_2 is rotate of 90° CCW on the current X-axis.
- Draw the Frame 1 and Frame 2 on the given axis and locate the P_2 on the diagram.
- Find P_2 locations from transformation matrix (please show working steps).



$$p_0 = [0 \ 2 \ 0]^T = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} \cos(-90^\circ) & -\sin(-90^\circ) & 0 \\ \sin(-90^\circ) & \cos(-90^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{z-axis}$$

$$p_1 = R_1^0 p_0$$

$$= \begin{bmatrix} \cos(-90^\circ) & -\sin(-90^\circ) & 0 \\ \sin(-90^\circ) & \cos(-90^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} \cos(90^\circ) & 0 & -\sin(90^\circ) \\ 0 & 1 & 0 \\ \sin(90^\circ) & 0 & \cos(90^\circ) \end{bmatrix} \quad \text{y-axis}$$

$$p_2 = R_2^1 p_1$$

$$\text{ref of } p_1 = \begin{bmatrix} \cos(90^\circ) & 0 & -\sin(90^\circ) \\ 0 & 1 & 0 \\ \sin(90^\circ) & 0 & \cos(90^\circ) \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{ref of } p_0 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$p_2 = [R_1^0 \times R_2^1] p_0$$

$$= \begin{bmatrix} \cos(-90^\circ) & -\sin(-90^\circ) & 0 \\ \sin(-90^\circ) & \cos(-90^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(90^\circ) & -\sin(90^\circ) \\ 0 & \sin(90^\circ) & \cos(90^\circ) \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

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Classwork 5

Name: Todsavard TID: 6114215Date: 6/7/21

1. Don't change question reference frame.
2. Draw all the frames on the same frame
3. Pls use different colour

Perform the below transformations, where Frame 0 (original frame) is shown. Given

$$p^0 = [1, 0, 0]$$

- a. Rotate -90 degree CURRENT y-axis.
- b. Translate -3 unit CURRENT z-axis.
- c. Rotate 90 degree CURRENT x-axis.

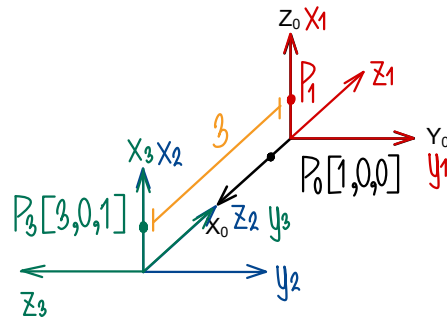
$$R_1^0 = \begin{bmatrix} \cos(-90^\circ) & 0 & -\sin(-90^\circ) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(-90^\circ) & 0 & \cos(-90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i. Find transformation matrix for the following steps, show T_{03} with working steps.

ii. Calculate the value of p^3 wrt frame 0.

iii. Draw the following frames and point out p^3 on the diagram.

$$\begin{aligned} R_1^0 &= \text{Rot}(y, -90^\circ) \\ D_2^1 &= D(z, -3) \\ R_3^2 &= \text{Rot}(x, 90^\circ) \end{aligned}$$



$$P_1^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad D_2^1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(90^\circ) & -\sin(90^\circ) & 0 \\ 0 & \sin(90^\circ) & \cos(90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$i) T_3^0 = R_1^0 D_2^1 R_3^2$$

$$\begin{aligned} &= \begin{bmatrix} \cos(-90^\circ) & 0 & -\sin(-90^\circ) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(-90^\circ) & 0 & \cos(-90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(90^\circ) & -\sin(90^\circ) & 0 \\ 0 & \sin(90^\circ) & \cos(90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$ii) P_3 = T_3^0 P^0$$

$$= \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Assumption University (ABAC)

Classwork 6

Name.....Todsavrad T.....ID.....6114215.....Date: 8/7/21

Perform the below transformations, where Frame 0 (original frame) is shown. Given $p^0 = [1, 0, 0]$.

- Rotate -90 degree FIXED y-axis.
- Translate -3 unit FIXED z-axis.
- Rotate 90 degree FIXED x-axis.

- Find transformation matrix for the following steps, show T_{03} with working steps.
- Calculate the value of p^3 wrt frame 0.
- Draw the following frames and point out p^3 on the diagram.

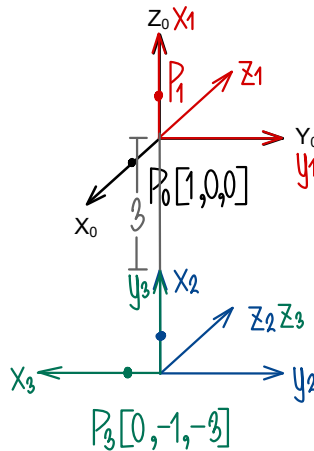
iii)

$$T_1^0 = \text{Rot}(y, -90^\circ)$$

$$T_2^1 = D(z, -3)$$

$$T_3^2 = \text{Rot}(x, 90^\circ)$$

-3



$$i) T_3^0 = T_2^1 [T_3^2 T_1^0]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(90^\circ) & -\sin(90^\circ) & 0 \\ 0 & \sin(90^\circ) & \cos(90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-90^\circ) & 0 & -\sin(-90^\circ) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(-90^\circ) & 0 & \cos(-90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ii) P_3 = T_3^0 P^0$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

Assumption University (ABAC)

Classwork 7

Name.....Todsavrad T.....ID.....6114215.....Date: 13/7/21

Perform the below transformations, where Frame 0 (original frame) is shown. Given $p^0 = [1, 0, 0]$.

- Rotate -90 degree FIXED y-axis.
- Translate -3 unit FIXED z-axis.
- Rotate 90 degree FIXED x-axis.
- Translate 3 unit FIXED y-axis.

- Find transformation matrix for the following steps, show T_{04} with working steps.
- Calculate the value of p^4 wrt frame 0.
- Draw the following frames and point out p^4 on the diagram.

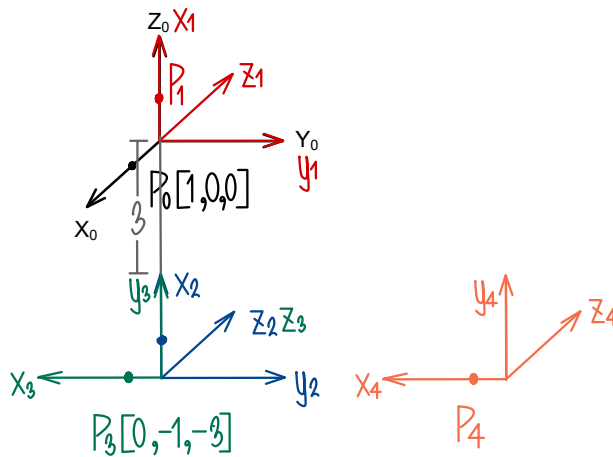
iii)

$$T_1^0 = \text{Rot}(y, -90^\circ)$$

$$T_2^1 = D(z, -3)$$

$$T_3^2 = \text{Rot}(x, 90^\circ)$$

$$T_4^3 = D(y, 3)$$



$$i) T_3^0 = T_4^3 T_3^2 T_2^1 T_1^0$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(90^\circ) & -\sin(90^\circ) & 0 \\ 0 & \sin(90^\circ) & \cos(90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-90^\circ) & 0 & -\sin(-90^\circ) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(-90^\circ) & 0 & \cos(-90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ii) P_4 = T_4^0 P^0$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -3 \\ 1 \end{bmatrix}$$