Plot by Robotics Toolbox (MATLAB)

MT411

Robotic Engineering

Chapter 4

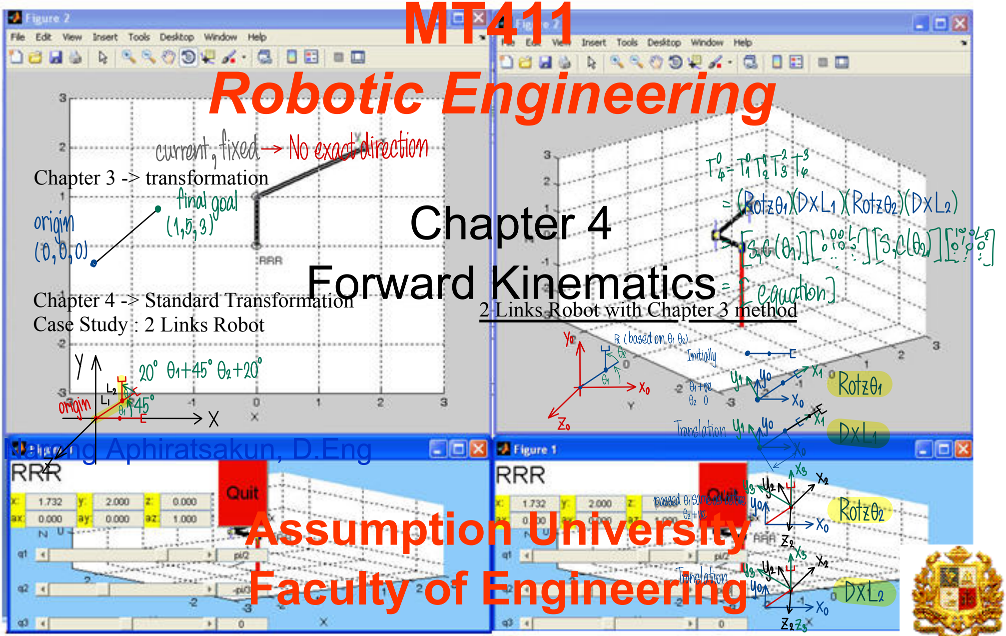
Forward Kinematics

Chapter 3 -> transformation

Chapter 4 -> Standard Transformation

Case Study : 2 Links Robot

2 Links Robot with Chapter 3 method



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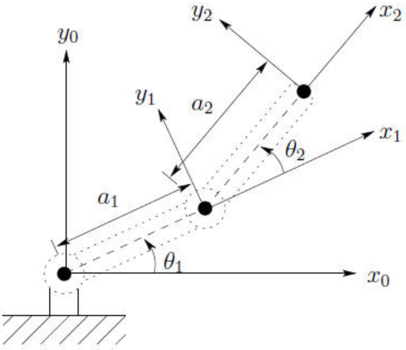
Forward Kinematics

Kinematics is a description on motion of the manipulator <u>without consideration of the forces and torques</u> which cause the motion.

Forward kinematic is to determine the position and orientation of the end effector given the values for the joint variables of the manipulator.



Kinematics Chains

Robot manipulator is composed of a set of ***links*** connected by ***joints***.

A robot manipulator with ***n*** joints will have ***n+1*** links.

Joint is number from 1 to n.

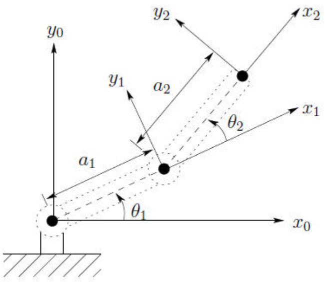
Links is number from 0 to n (0 is base).

Therefore ***joint i connects link i-1 to link i***.

When joint i is actuated, link i moves, with link 0 is fixed.

2 Links manipulator

Determine the position and orientation of the end effector in term of joint variables.



$$x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$$

$$y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)$$

the x2 and y2 wrt to frame 0 is:

$$x_2 . x_0 = \cos(\theta_1 + \theta_2)$$

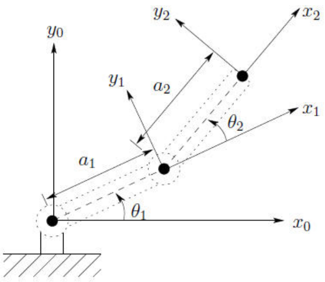
$$y_2 . x_0 = -\sin(\theta_1 + \theta_2)$$

$$x_2 . y_0 = \sin(\theta_1 + \theta_2)$$

$$y_2 . y_0 = \cos(\theta_1 + \theta_2)$$

$$R_2^0 = \begin{bmatrix} x_2 . x_0 & y_2 . x_0 \\ x_2 . y_0 & y_2 . y_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) \end{bmatrix}$$

2 Links manipulator



$$R_1^0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D_{1A}^1 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2^{1A} = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D_{2A}^2 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{2A}^0 = R_1^0 D_{1A}^1 R_2^{1A} D_{2A}^2$$

$R_{2A}^0 =$

[cos(theta1)*cos(theta2) - sin(theta1)*sin(theta2), - cos(theta1)*sin(theta2) - cos(theta2)*sin(theta1), 0, L2*(cos(theta1)*cos(theta2) - sin(theta1)*sin(theta2)) + L1*cos(theta1) ;
cos(theta1)*sin(theta2) + cos(theta2)*sin(theta1), cos(theta1)*cos(theta2) - sin(theta1)*sin(theta2), 0, L2*(cos(theta1)*sin(theta2) + cos(theta2)*sin(theta1)) + L1*sin(theta1) ;
0, 0, 1, 0;
0, 0, 0, 1]

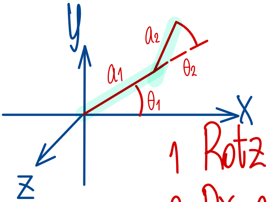
2 Links manipulator

$R^0_{2A} =$

a_1, a_2 Robot armlength

1st classical method

2nd DH Table method



1 Rotz θ_1

2 Dx, a_1

3 Rotz θ_2

4 Dx, a_2

$$T = \text{Rotz}\,\theta_1 \times \text{Dx},a_1 \times \text{Rotz}\,\theta_2 \times \text{Dx},a_2$$

The Denavit-Hartenberg (DH) Convention

It is possible to carry out forward kinematics analysis as we did for 2 links manipulator. However, the kinematic analysis of an n-link manipulator can be extremely complex and the Denavit-Hartenberg (DH) simplify the analysis. This DH is a universal language with which engineer can communicate.

$$\boxed{A_i = Rot_{z,\theta_i} \mathrm{Trans}_{z,d_i} \mathrm{Trans}_{x,a_i} Rot_{x,\alpha_i}}$$

* always make z-axis as may joints

R → d

R → θ

A = Rot(θ) D(d) D(a) Rot(α)

ai : link length

ai : link twist

di : link offset (prismatic joint)

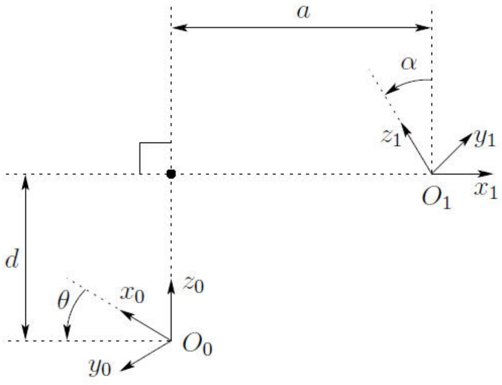
□i : joint angle (revolute joint)

*****Assign zi to be the axis of actuation for joint i+1.***

The Denavit-Hartenberg (DH) Convention

DH1: the axis x1 is perpendicular to the axis z0.

DH2: the axis x1 intersects the axis z0.



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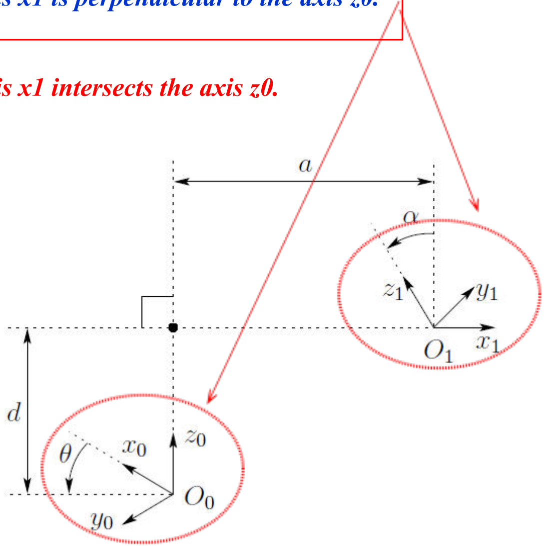

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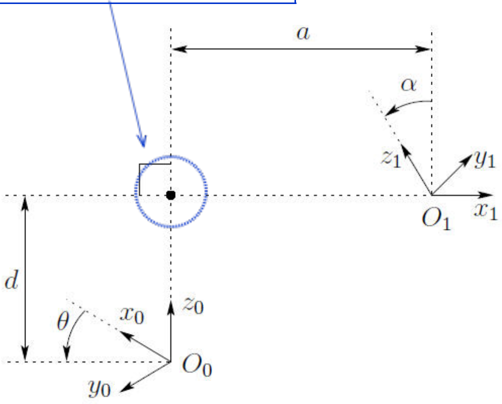


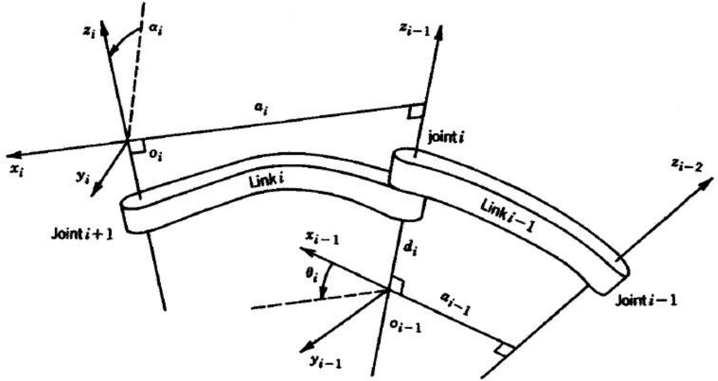
The Denavit-Hartenberg (DH) Convention

ai : distance between axes zi and zi+1, and <u>measured along the axis xi</u> .

a*i : angle between axes zi and zi+1, and <u>measured in a plane normal to xi</u> .*

di : distance from origin to the intersection of the axis xi+1 with zi, and <u>measured along the axis zi</u> .

□i : angle from xi to xi+1, and <u>measured in a plane normal to zi</u> .



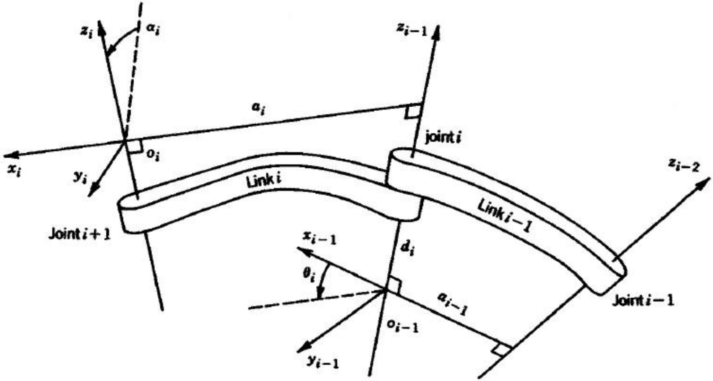
The Denavit-Hartenberg (DH) Convention

a*: *distance between axes z0 and z1, and <u>measured along the axis x1</u>.

a ***: angle between axes z0 and z1, and <u>measured in a plane normal to x1</u>.***

d : distance from origin O0 to the intersection of the axis x1 with z0, and <u>measured along the axis z0</u>.

: angle from x0 to x1, and <u>measured in a plane normal to z0</u>.



The Denavit-Hartenberg (DH) Convention

$$
\begin{aligned}
A_i &= Rot_{z,\theta_i}\text{Trans}_{z,d_i}\text{Trans}_{x,a_i}Rot_{x,\alpha_i} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

DH Examples : 2 links manipulator

link 1

a1 (distance between axes z0 and z1, and measured along the axis x1) : a1

a***1 (angle between axes z0 and z1, and measured in a plane normal to x1) : 0***

d1 (distance from origin O0 to the intersection of the axis x1 with z0, and measured along the axis z0) : 0

□1 (angle from x0 to x1, and measured in a plane normal to z0) : □1*

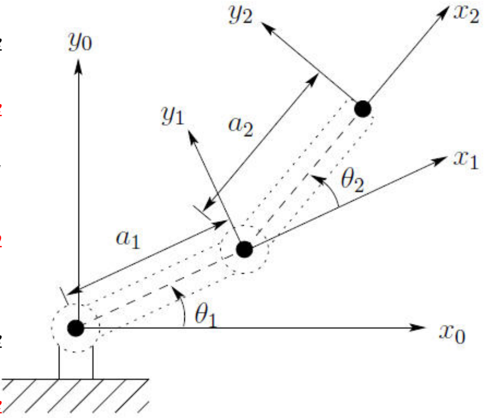
link 2

a2 (distance between axes z1 and z2, and measured along the axis x2) : a2

a***2 (angle between axes z1 and z2, and measured in a plane normal to x2) : 0***

d2 (distance from origin O1 to the intersection of the axis x2 with z1, and measured along the axis z1) : 0

□2 (angle from x1 to x2, and measured in a plane normal to z1) : □2*



DH Examples : 2 links manipulator

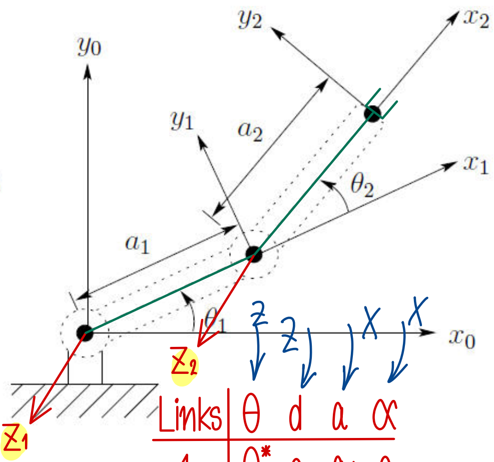
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*

$$A_i = Rot_{z,\theta_i}\,\text{Trans}_{z,d_i}\,\text{Trans}_{x,a_i}\,Rot_{x,\alpha_i}$$

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Links	θ	d	a	α
1	θ_1^*	0	a_1	0
2	θ_2^*	0	a_2	0

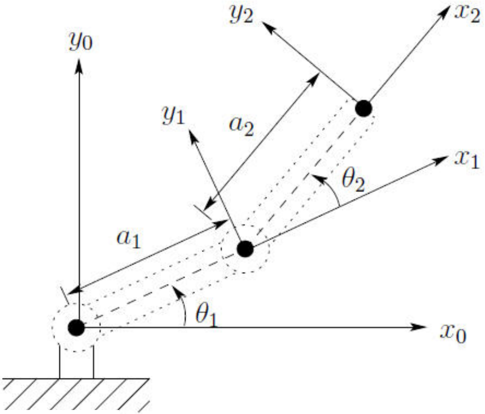
matlab function

matlab [α, a, θ, d]

toolbox follow variable

DH Examples : 2 links manipulator

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*



$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

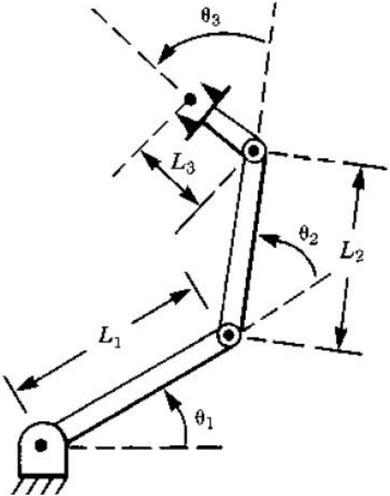
$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = A_1$$

$$T_2^0 = A_1A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DH Examples : 3 links planar RRR

Link	a_i	α_i	d_i	θ_i
1	L_1	0	0	θ_1^*
2	L_2	0	0	θ_2^*
3	L_3	0	0	θ_3^*



*: *denote variables*

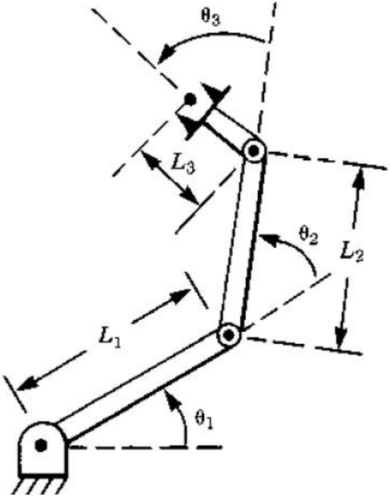
DH Examples : 3 links planar RRR

$$A_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & L_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & L_1\sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & L_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_3\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & L_3\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3$$



DH Examples : 3 links planar RRR

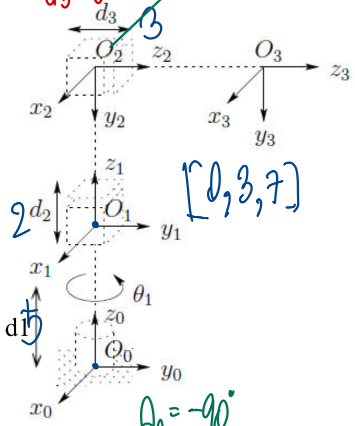
$T_3^0 =$

DH Examples : RPP

Links	θ	d	a	α
1	θ_1^*	d_1	0	0
2	0	d_2^*	0	-90
3	0	d_3^*	0	0

Pend[ ]

Link	a_i	α_i	d_i	θ_i
1	0	0	d1 (offset)	θ_1^*
2	0	-90°	d_2^*	0
3	0	0	d_3^*	0



[0,3,7]

$\theta_1 = -90°$

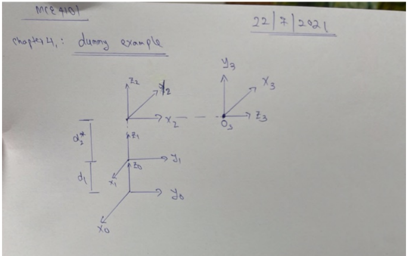
*: *denote variables*

DH Examples : RPP

$T_3^0 =$

Links	θ	d	a	α
1	θ_p^*	d_1	0	0
2	0	d_2^*	0	−90
3	0	d_3^*	0	0

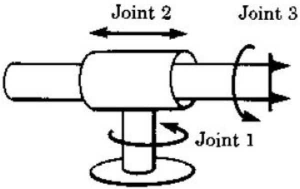
Assumption University (ABAC)
Dummy Example (22/7/21)



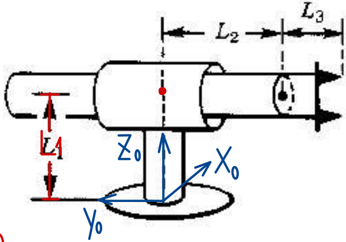
L	θ	d	a	α
1	θ_1^*	d_1	0	0
2	90°	d_2^*	0	0
(dummy) 2-3	90°	0	0	90°
3	0	d_3^*	0	0

```
A1 = link([0 0 th1 d1, 0]);
A2 = link([0 0 pi/2 d2, 1]);
A2_3 = link([pi/2 0 pi/2 0, 0])
A3 = link([0 0 0 d3, 1])
Dummy_example = robot({A1 A2 A2_3 A3});
```

DH Examples : RPR



Links	θ	d	a	α
1	θ_1^*	L_1	0	0
2	0	L_2^*	0	+90
3	θ_3^*	0	0	0



Link	a_i	α_i	d_i	θ_i
1	0	+90	L1 (offset L_1)	θ_1^*
2	0	0	L*2	0
3	0	0	0	θ_3^*

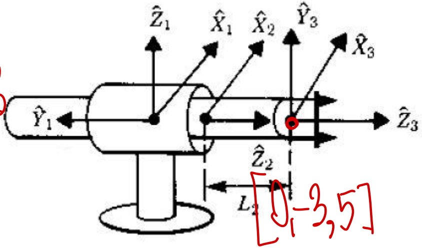
wrong

$\theta_1 = 0$

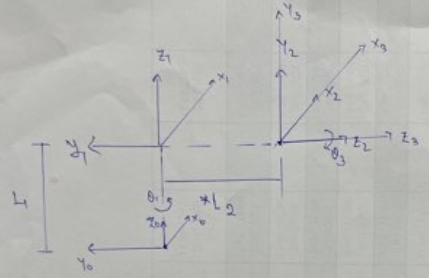
$L_1 = 5$

$L_2 = 3$

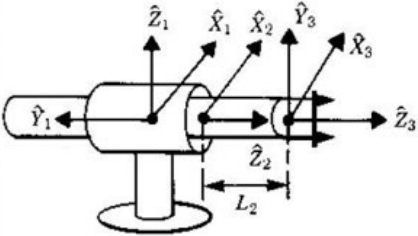
$\theta_3 = 0$



*: *denote variables*



L	θ	d	a	α
1	θ_1^*	L_1	0	0
(dummy) 1-2	0	0	0	90°
2	0	L_2^*	0	0
3	θ_3^*	0	0	0




```
A1 = link([0 0 th1 L1, 0]);
A1_2 = link([pi/2 0 0 0, 0]);
A2 = link([0 0 0 L2, 1]);
A3 = link([0 0 th3 0, 0]);
RPR = robot({A1 A1_2 A2 A3});
```

*: *denote variables*

DH Examples : RPR

$T_3^0 =$

2 links manipulator (MATLAB)

```
syms L1 L2 th1 th2;
syms r1 r2 theta1 theta2;

D2R = (pi/180);   %Rad to Deg
r1 = L1;
r2 = L2;

P0 = [0,0,0]';

th1 = theta1;   %Rotaton for pt1
th2 = theta2;   %Rotaton for pt2
```

```
A1 = [cos(th1),-sin(th1), 0 ,0;
       sin(th1), cos(th1),0,0;
       0, 0 ,1, 0;
       0, 0, 0, 1];
A2 = [1, 0, 0 ,L1;
      0, 1, 0 ,0;
      0, 0 ,1, 0;
      0, 0, 0, 1];
A3 = [cos(th2),-sin(th2), 0 ,0;
       sin(th2), cos(th2),0,0;
       0, 0 ,1, 0;
       0, 0, 0, 1];
A4 = [1, 0, 0 ,L2;
      0, 1, 0 ,0;
      0, 0 ,1, 0;
      0, 0, 0, 1];

T04 = A1*A2*A3*A4;
```

DH Examples : 2 links manipulator (MATLAB)

Manipulate by MATLAB with DH formula

T02 =
[cos(theta1)*cos(theta2) - sin(theta1)*sin(theta2), - cos(theta1)*sin(theta2) - cos(theta2)*sin(theta1), 0, L1*cos(theta1) + L2*cos(theta1)*cos(theta2) - L2*sin(theta1)*sin(theta2) ;
cos(theta1)*sin(theta2) + cos(theta2)*sin(theta1),   cos(theta1)*cos(theta2) - sin(theta1)*sin(theta2), 0, L1*sin(theta1) + L2*cos(theta1)*sin(theta2) + L2*cos(theta2)*sin(theta1) ;

0, 0, 1, 0 ;

0, 0, 0, 1]

DH Examples : 2 links manipulator (Robotics Toolbox)

Manipulate by MATLAB with DH formula by Robotics Toolbox

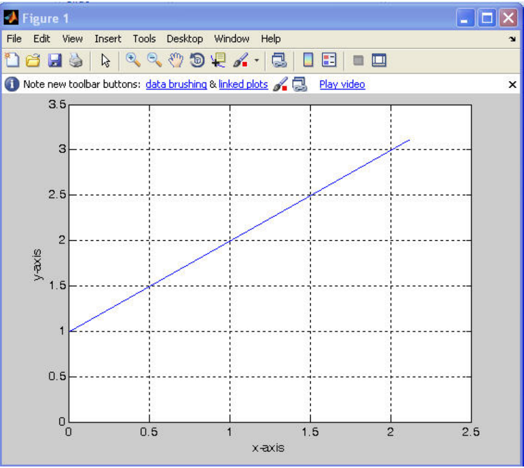
```
syms L1 L2 th1 th2;
syms r1 r2 theta1 theta2;

D2R = (pi/180);  %Rad to Deg
r1 = L1;
r2 = L2;

P0 = [0,0,0]';

th1 = theta1;  %Rotaton for pt1
th2 = theta2;  %Rotaton for pt2
```

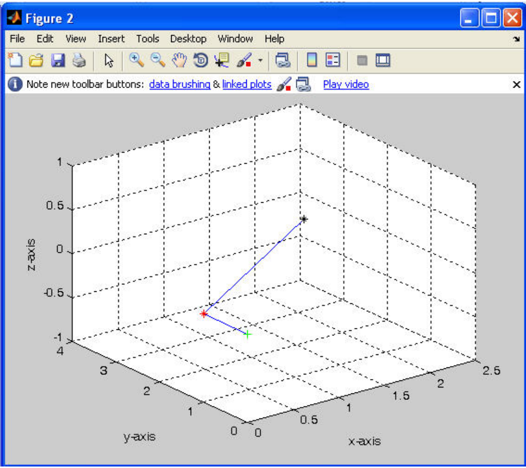
```
A1 = rotz(theta1);
A2 = transl(r1,0,0);
A3 = rotz(theta2);
A4 = transl(r2,0,0);

T04 = A1*A2*A3*A4;
```

Plot of 2 links manipulator (MATLAB)

```
r1 = 1;
r2 = 3;
P0 = [0,0,0]';

th1 = 90*D2R;  %Rotaton for pt1
th2 = -45*D2R; %Rotaton for pt2

        :
        :

figure(1)
plot(X,Y);
hold on;
plot(P0(1,1),P0(2,1),'+g');
plot(P1(1,1),P1(2,1),'*r');
grid;
xlabel('x-axis');ylabel('y-axis');

figure(2)
plot3(X,Y,Z);
hold on;
plot3(P0(1,1),P0(2,1),P0(3,1),'+g');
plot3(P1(1,1),P1(2,1),P1(3,1),'*r');
plot3(P2(1,1),P2(2,1),P2(3,1),'-*k');
grid;
xlabel('x-axis');ylabel('y-axis');zlabel('z-axis');
```

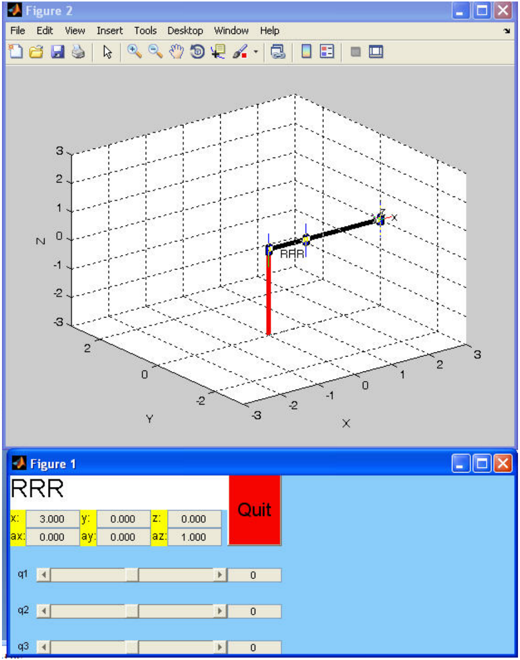
Plot of 2 links manipulator (MATLAB)



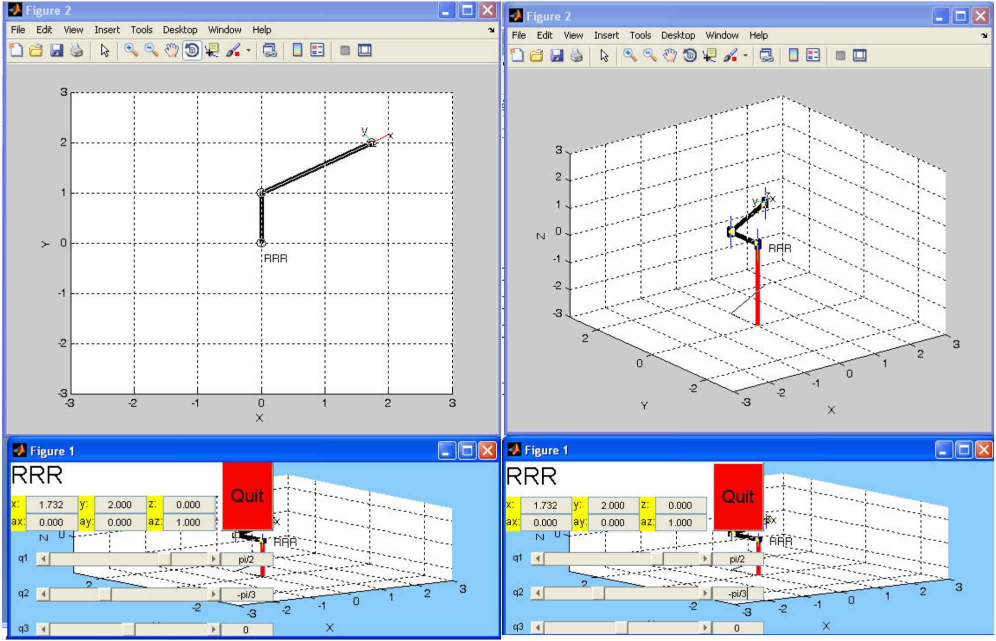


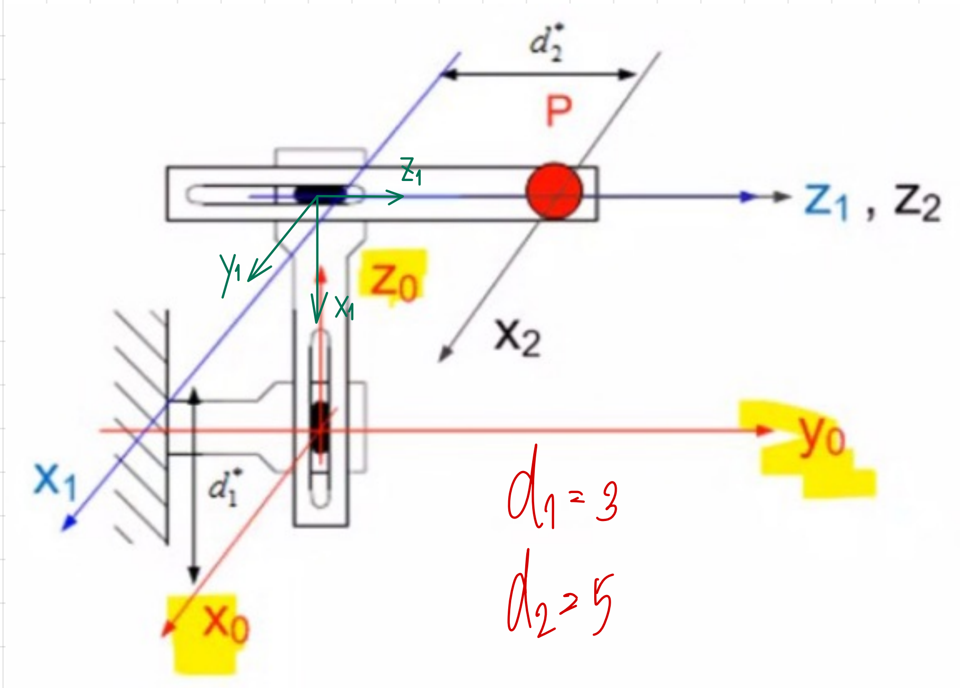
Plot by Robotics Toolbox (MATLAB)

```
clear all
close all
clc

L1 = Link([0 1 0 0]);
L2 = Link([0 2 0 0]);
L3 = Link([0 0 0 0]);

r = robot({L1 L2 L3}, 'RRR')
drivebot(r)
```



Plot by Robotics Toolbox (MATLAB)

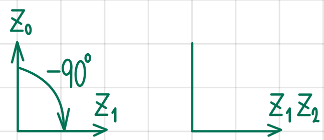




$d_1 = 3$

$d_2 = 5$

Links	θ	d	a	α
1	0	d_1^*	0	$-90°$
2	0	d_2^*	0	0

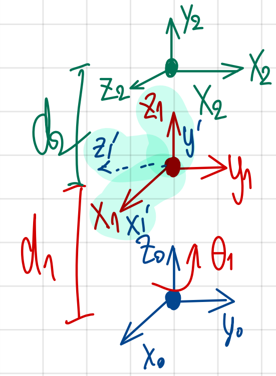


1→2 can't go in 1 step

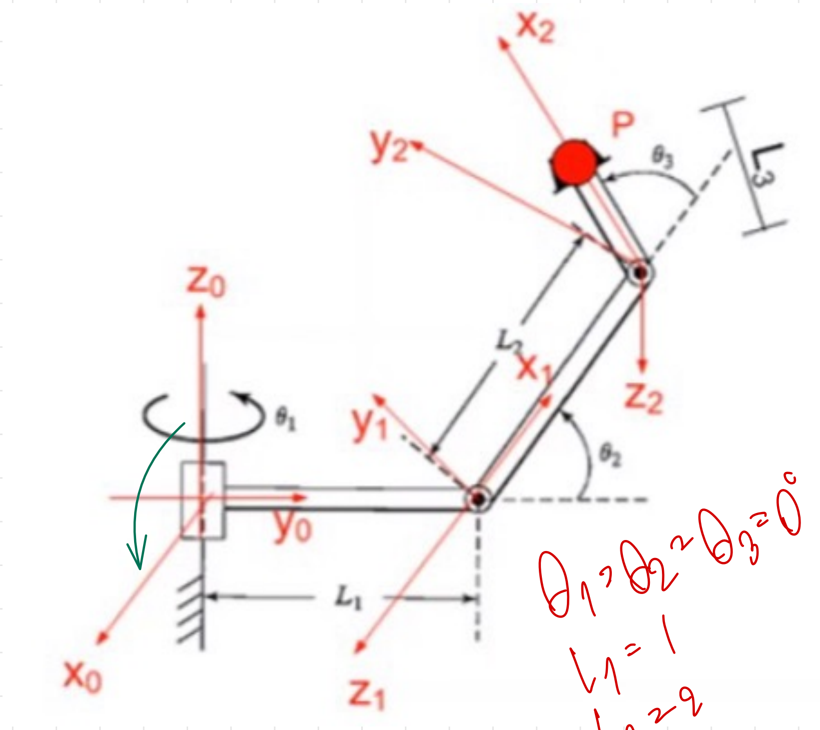
↓

add dummy link

example 3 lines



Links	θ	d	a	α
1	θ_1^*	d_1	0	0
2a	0	d_2^*	0	$+90°$
2b		0	0	



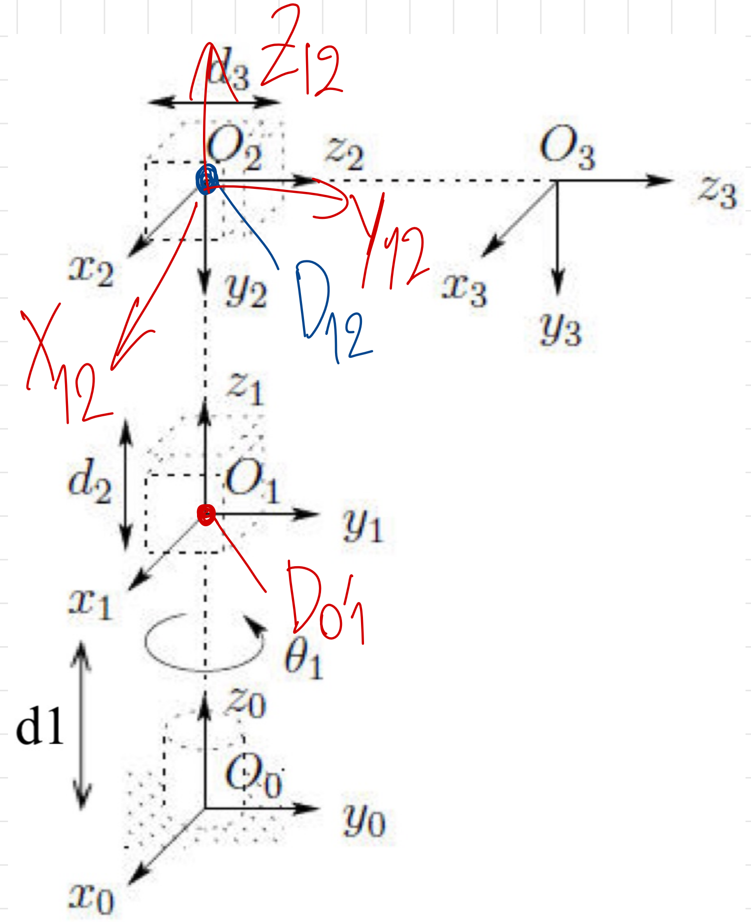
$\theta_1 = \theta_2 = \theta_3 = 0°$

$L_1 = 1$

$L_2 = 2$

$L_3 = 3$

Links	θ	d	a	α
1	θ_1^*	0	L_1	$+90°$
2	θ_2^*	0	L_2	0
3	θ_3^*	0	L_3	0



$R_{01} = rot(\theta_1^*)$

$D_{01} = transl_z(d_1)$

$D_{12} = transl_z(d_2^*)$

$R_{12} = rotx(-90^\circ)$

$D_{23} = transl_z(d_3^*)$