TEAM NOTE
CRET

Mathematics

ax + by = c

```
def abcsol(a, b, c):
    A = a
    B = b

    fa = [1, 0]
    fb = [0, 1]
    while a % b:
        fa, fb = fb, [(x - y * (a // b)) for x, y in zip(fa, fb)]
        a, b = b, a % b

    gcd = b
    if c % gcd:
        return False

    return [fb[i] * c // gcd for i in range(2)]

print(abcsol(5, 2, 7))
# (7, -14)
```

number of relative prime number

```
# phi using primes => []
def phi(n, primes):
    rpn = n
    for p in primes:
        if n % p:
            continue

        while n % p == 0:
            n //= p

        rpn //= p
        rpn *= p - 1

        if n == 1:
            break

    if n != 1:
        rpn *= n - 1
        rpn //= n
    return rpn

# phi using factorization
from math import *
def phi_f(factor):
    return prod([fact - 1 for fact in list(set(factor))])
```

FFT, NTT with precision

```
from math import *

def FFT(f, w):
    n = len(f)
    if n == 1:
        return f

    even = [f[i] for i in range(0, n, 2)]
    odd = [f[i] for i in range(1, n, 2)]

    even = FFT(even, w ** 2)
    odd = FFT(odd, w ** 2)
    wp = complex(1)

    for i in range(n//2):
        f[i] = even[i] + wp * odd[i]
        f[i + n//2] = even[i] - wp * odd[i]
        wp *= w
    return f

# A, B => index = degree
def multiple(A, B):
    n = max(len(A), len(B))
    N = 2 ** ceil(log2(2 * n))
    A += [0] * (N - len(A))
    B += [0] * (N - len(B))
    rw = complex(cos(tau / N), sin(tau / N))

    # FFT 된 A 와 B 의 inner product
    AA = FFT(A, rw)
    BB = FFT(B, rw)
    CC = [AA[i] * BB[i] for i in range(N)]

    # inner product 된 값을 다시 inverse FFT (1 / rw)
    C = FFT(CC, complex(1) / rw)
    for i in range(N):
        C[i] /= complex(N)
        C[i] = round(C[i].real)
    return C

from math import *
w = 3
mod1 = 2281701377
mod2 = 998244353
mod3 = 2130706433

def power(a, b, mod):
    ret = 1
    a %= mod
    b %= mod
    while b:
        if b & 1:
            ret = (ret * a) % mod
        a = (a * a) % mod
        b >>= 1
    return ret

def NTT(A, mod, inv=False):
    n = len(A)

    rev = [0] * n
    for i in range(n):
        rev[i] = rev[i >> 1] >> 1
        if i & 1:
            rev[i] |= n >> 1
        if i < rev[i]:
            A[i], A[rev[i]] = A[rev[i]], A[i]

    x = power(w, (mod - 1) // n, mod)
    if inv:
        x = power(x, mod - 2, mod)

    root = [1]
    for i in range(1, n):
        root.append((root[i-1] * x) % mod)

    i = 2
    while i <= n:
        step = n // i
        for j in range(0, n, i):
            for k in range(i>>1):
                u = A[j|k]
                v = (A[j|k|i >> 1] * root[step*k]) % mod
                A[j|k] = (u + v) % mod
                A[j|k|i >> 1] = (u - v) % mod
                if A[j|k|i >> 1] < 0: A[j|k|i >> 1] += mod
        i <<= 1

    if inv:
        t = power(n, mod - 2, mod)
        for i in range(n):
            A[i] = (A[i] * t) % mod
    return A

def multiple(a, b, mod):
    n = max(len(a), len(b))
    n = 2 ** ceil(log2(2 * n))
    a += [0] * (n - len(a))
    b += [0] * (n - len(b))
    D = NTT(a, mod, inv=False)
    E = NTT(b, mod, inv=False)
    return NTT([(D[i]*E[i]) % mod for i in range(n)], mod, inv=True)

import copy
input()
a = list(map(int, input().split()))
b = list(map(int, input().split()))
c = copy.copy(a)
d = copy.copy(b)
C = multiple(c, d, mod1)
c = copy.copy(a)
d = copy.copy(b)
D = multiple(c, d, mod2)
E = multiple(a, b, mod3)

answer = []
for i in range(len(C)):
    ans = 0
    ans += C[i] * mod2 * mod3 * power(mod2*mod3, mod1-2, mod1)
    ans += D[i] * mod1 * mod3 * power(mod1*mod3, mod2-2, mod2)
    ans += E[i] * mod1 * mod2 * power(mod1*mod2, mod3-2, mod3)
    ans %= mod1 * mod2 * mod3
    answer.append(ans)
```

FFT, NTT cpp

```
#include <iostream>
#include <vector>
#include <cmath>
#include <algorithm>
#include <string>
#include <iomanip>

typedef long long ll;
using namespace std;

const ll mod = 2281701377;
const ll w = 3;

ll power(ll a, ll b) {
    long long ret = 1;
    while (b) {
        if (b & 1)
            ret = (ret * a) % mod;
        a = (1LL * a * a) % mod;
        b /= 2;
    }
    return ret;
}

vector<ll> NTT(vector<ll>& A, bool inv=false) {
    int n = A.size();

    vector<ll> rev(n);
    for (int i = 0; i < n; ++i) {
        rev[i] = rev[i >> 1] >> 1;
        if (i & 1)
            rev[i] |= n >> 1;
        if (i < rev[i])
            swap(A[i], A[rev[i]]);
    }

    ll x = power(w, (mod - 1) / n);
    if (inv) {
        x = power(x, mod - 2);
    }

    vector<ll> root(n, 1);
    for (int i = 1; i < n; ++i) {
        root[i] = (root[i-1] * x) % mod;
    }

    for (int i = 2; i <= n; i <<= 1) {
        ll step = n / i;
        for (int j = 0; j < n; j += i) {
            for (int k = 0; k < (i >> 1); ++k) {
                ll u = A[j|k];
                ll v = (A[j|k|(i >> 1)] * root[step*k]) % mod;
                A[j|k] = (u + v) % mod;
                A[j|k|(i >> 1)] = (u - v) % mod;
                if (A[j|k|(i >> 1)] < 0)
```

```
                    A[j|k|(i >> 1)] += mod;
            }
        }
    }

    if (inv) {
        ll t = power(n, mod - 2);
        for (int i = 0; i < n; ++i)
            A[i] = (A[i] * t) % mod;
    }
    return A;
}

vector<ll> multiply(vector<ll>& a, vector<ll>& b) {
    int n = max(a.size(), b.size());
    n = 2 * pow(2, ceil(log2(n)));
    a.resize(n);
    b.resize(n);
    vector<ll> A = NTT(a, false);
    vector<ll> B = NTT(b, false);
    vector<ll> result(n);
    for (int i = 0; i < n; ++i){
        result[i] = (A[i] * B[i]) % mod;
    }
    return NTT(result, true);
}
```

miller rabin, polard rho

```
import math, random
def power(x, y, p):
    res = 1
    piv = x % p
    while y:
        if y & 1:
            res *= piv
            res %= p
        piv *= piv
        piv %= p
        y >>= 1
    return res

# True => 합성수이다
def miller_rabin(n, p):
    if n % p == 0:
        return True

    d = n - 1
    while 1:
        cur = power(p, d, n)
        if cur == n - 1:
            return False
        elif d & 1:
            return not (cur == 1 or cur == n - 1)
        d >>= 1

# 소수면 True
def prime_test(n):
    for i in [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41]:
        if n == i:
            return True

        if miller_rabin(n, i):
            return False
    return True

def func(c, x, n):
    return (c + ((x ** 2) % n)) % n

def rho(n, factor):
    if n == 1:
        return factor if factor else [1]

    if n % 2 == 0:
        factor.append(2)
        factor = rho(n // 2, factor)
        return factor

    if prime_test(n):
        factor.append(n)
        return factor

    a, b, c = 0, 0, 0
    g = n

    while 1:
        if g == n:
            b = random.randint(2, n - 1)
            a = b
            c = random.randint(1, 20)
        a = func(c, a, n)
        b = func(c, func(c, b, n), n)
        g = math.gcd(abs(a - b), n)
        if g != 1:
            break
    factor = rho(g, factor)
    factor = rho(n // g, factor)
    return factor
```

sum of divisors

```
def divsum(factor):
    d = {}
    for f in factor:
        if f in d:
            d[f] += 1
        else:
            d[f] = 1

    S = 1
    for f, n in d.items():
        S *= f ** (n+1) - 1
        S //= f-1
    return S
```

taylor series

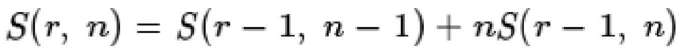
```
from decimal import Decimal
fact = [Decimal('0') for _ in range(40)]
fact[1] = Decimal('1')
for i in range(2, 40):
    fact[i] = Decimal(str(i)) * fact[i - 1]

def sin(x: Decimal):
    x %= 2 * Decimal('strinrg of phi')
    res = Decimal('0')
    value = x
    for k in range(19):
        addition = (-1 if k % 2 == 1 else 1) * (value / fact[2 * k + 1])
        res += addition
        value *= x * x
    return res
```

Combinatorics

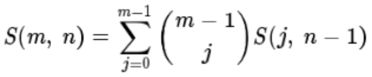
lucas theorem

```
def lucas(n, k, m):
    ans = 1
    while k != 0:
        ans = (ans * math.comb(n % m, k % m)) % m
        n //= m
        k //= m
    return ans
```

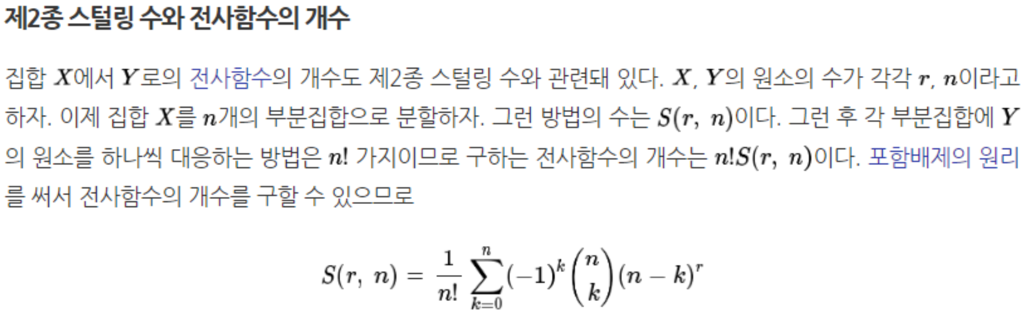
stirling number



$$S(r,\ n) = S(r-1,\ n-1) + nS(r-1,\ n)$$



$$S(m,\ n) = \sum_{j=0}^{m-1} \binom{m-1}{j} S(j,\ n-1)$$

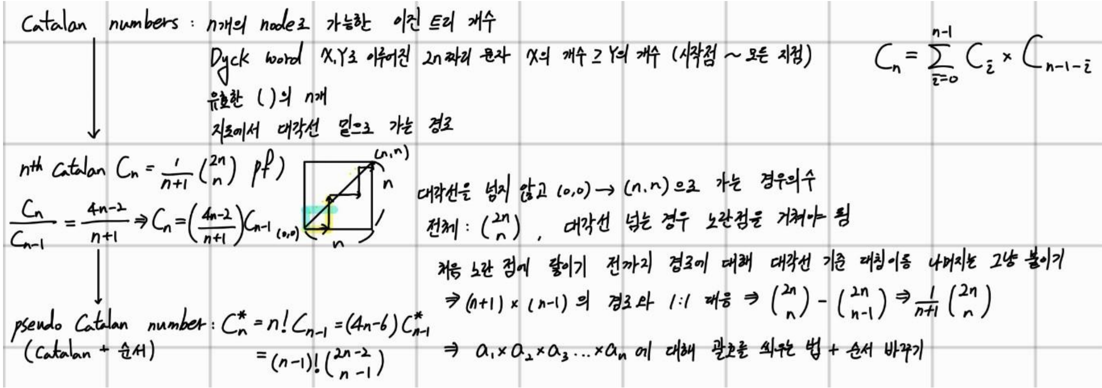


제2종 스털링 수와 전사함수의 개수

집합 X에서 Y로의 전사함수의 개수도 제2종 스털링 수와 관련돼 있다. X, Y의 원소의 수가 각각 r, n이라고 하자. 이제 집합 X를 n개의 부분집합으로 분할하자. 그런 방법의 수는 $S(r,\ n)$이다. 그런 후 각 부분집합에 Y의 원소를 하나씩 대응하는 방법은 $n!$ 가지이므로 구하는 전사함수의 개수는 $n!S(r,\ n)$이다. 포함배제의 원리를 써서 전사함수의 개수를 구할 수 있으므로

$$S(r,\ n) = \frac{1}{n!}\sum_{k=0}^{n}(-1)^k\binom{n}{k}(n-k)^r$$

catalan number



Catalan numbers : n개의 node로 가능한 이진 트리 개수

Dyck word X,Y로 이루어진 2n짜리 문자 X의 개수 ≥ Y의 개수 (시작점 ~ 모든 지점)

유효한 ( )의 n개

지점에서 대각선 밑으로 가는 경로

$C_n = \sum_{i=0}^{n-1} C_i \times C_{n-1-i}$

nth catalan $C_n = \frac{1}{n+1}\binom{2n}{n}$ pf)

$\frac{C_n}{C_{n-1}} = \frac{4n-2}{n+1} \Rightarrow C_n = \left(\frac{4n-2}{n+1}\right)C_{n-1}$

대각선을 넘지 않고 (0,0) → (n,n)으로 가는 경우의수

전체 : $\binom{2n}{n}$, 대각선 넘는 경우 노란점을 거쳐야 됨

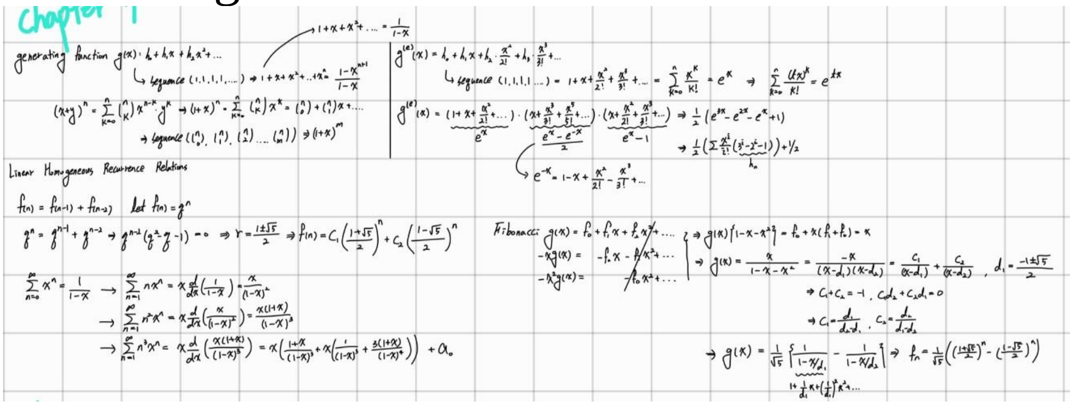
처음 노란 점에 닿이기 전까지 경로에 대해 대각선 기준 대칭이동 나머지는 그냥 붙이기

$\Rightarrow (n+1) \times (n-1)$ 의 경로와 1:1 대응 $\Rightarrow \binom{2n}{n} - \binom{2n}{n-1} \Rightarrow \frac{1}{n+1}\binom{2n}{n}$

pseudo Catalan number : $C_n^* = n!C_{n-1} = (4n-6)C_{n-1}^*$
(Catalan + 순서) $= (n-1)!\binom{2n-2}{n-1}$

$\Rightarrow a_1 \times a_2 \times a_3 \ldots \times a_n$ 에 대해 괄호를 씌우는 법 + 순서 바꾸기

Generating Function



Chapter 1

generating function $g(x) : h_0 + h_1x + h_2x^2 + \ldots$

↳ sequence $(1,1,1,1,\ldots) \Rightarrow 1 + x + x^2 + \ldots + x^n = \frac{1-x^{n+1}}{1-x}$

$\rightarrow 1 + x + x^2 + \ldots = \frac{1}{1-x}$

$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}\cdot y^k \Rightarrow (1+x)^n = \sum_{k=0}^{n}\binom{n}{k}x^k = \binom{n}{0} + \binom{n}{1}x + \ldots$

$\Rightarrow$ sequence $\left(\binom{n}{0}, \binom{n}{1}, \binom{n}{2} \ldots \binom{n}{m}\right) \Rightarrow (1+x)^m$

$g^{(e)}(x) = h_0 + h_1x + h_2\cdot\frac{x^2}{2!} + h_3\cdot\frac{x^3}{3!} + \ldots$

↳ sequence $(1,1,1,1\ldots) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{k=0}^{n}\frac{x^k}{k!} = e^x \Rightarrow \sum_{k=0}^{n}\frac{(tx)^k}{k!} = e^{tx}$

$g^{(e)}(x) = \underbrace{\left(1 + x + \frac{x^2}{2!} + \ldots\right)}_{e^x}\cdot\underbrace{\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right)}_{\frac{e^x - e^{-x}}{2}}\cdot\underbrace{\left(x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right)}_{e^x - 1} \Rightarrow \frac{1}{2}\left(e^{3x} - e^{2x} - e^x + 1\right)$

$\rightarrow \frac{1}{2}\left(\sum\frac{x^i}{i!}\underbrace{(3^i - 2^i - 1)}_{h_n}\right) + 1/2$

$\rightarrow e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$

Linear Homogeneous Recurrence Relations

$f(n) = f(n-1) + f(n-2)$ let $f(n) = q^n$

$q^n = q^{n-1} + q^{n-2} \rightarrow q^{n-2}(q^2 - q - 1) = 0 \Rightarrow r = \frac{1\pm\sqrt{5}}{2} \Rightarrow f(n) = C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n$

Fibonacci: $g(x) = f_0 + f_1x + \cancel{f_2x^2} + \ldots$
$-xg(x) = \quad -f_0x - \cancel{f_1x^2} + \ldots$
$-x^2g(x) = \quad \cancel{-f_0x^2} + \ldots$

$\Rightarrow g(x)\{1 - x - x^2\} = f_0 + x(f_1 + f_0) = x$

$\Rightarrow g(x) = \frac{x}{1-x-x^2} = \frac{-x}{(x-d_1)(x-d_2)} = \frac{C_1}{(x-d_1)} + \frac{C_2}{(x-d_2)}, \quad d_1 = \frac{-1\pm\sqrt{5}}{2}$

$\Rightarrow C_1 + C_2 = -1, \ C_1d_2 + C_2d_1 = 0$

$\Rightarrow C_1 = \frac{d_1}{d_2 - d_1}, \ C_2 = \frac{d_2}{d_1 - d_2}$

$\Rightarrow g(x) = \frac{1}{\sqrt{5}}\left\{\underbrace{\frac{1}{1 - x/d_1}}_{1 + \frac{1}{d_1}x + \left(\frac{1}{d_1}\right)^2x^2 + \ldots} - \frac{1}{1 - x/d_2}\right\} \Rightarrow f_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$

$\sum_{n=0}^{\infty}x^n = \frac{1}{1-x} \rightarrow \sum_{n=1}^{\infty}nx^n = x\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{x}{(1-x)^2}$

$\rightarrow \sum_{n=1}^{\infty}n^2x^n = x\frac{d}{dx}\left(\frac{x}{(1-x)^2}\right) = \frac{x(1+x)}{(1-x)^3}$

$\rightarrow \sum_{n=1}^{\infty}n^3x^n = x\frac{d}{dx}\left(\frac{x(1+x)}{(1-x)^3}\right) = x\left(\frac{1+x}{(1-x)^3} + x\left(\frac{1}{(1-x)^3} + \frac{3(1+x)}{(1-x)^4}\right)\right) + a_0$

Geometry

line segment

```
A = [0, 0, 0, 0]
B = [0, 0, 0, 0]

def CP(v1, v2):
    return v1[0] * v2[1] - v1[1] * v2[0]

def PTV(p1, p2):
    return [p2[0] - p1[0], p2[1] - p1[1]]

def intersection(A, B):
    # True -> intersection
    # False -> Not intersection
    VA = PTV(A[:2], A[2:])
    AtoB1 = CP(VA, PTV(A[:2], B[:2]))
    AtoB2 = CP(VA, PTV(A[:2], B[2:]))
    if AtoB1 * AtoB2 > 0:
        return False

    VB = PTV(B[:2], B[2:])
    BtoA1 = CP(VB, PTV(B[:2], A[:2]))
    BtoA2 = CP(VB, PTV(B[:2], A[2:]))
    if BtoA1 * BtoA2 > 0:
        return False

    if AtoB1 * AtoB2 == 0 or BtoA1 * BtoA2 == 0:
        if max(A[0], A[2]) < min(B[0], B[2]):
            return False
        if max(B[0], B[2]) < min(A[0], A[2]):
            return False
        if max(A[1], A[3]) < min(B[1], B[3]):
            return False
        if max(B[1], B[3]) < min(A[1], A[3]):
            return False
    return True
```

area of triangle

```
def area(p1, p2, p3):
    A = p1[0] * p2[1] + p2[0] * p1[1] + p3[0] * p1[1]
    B = p1[1] * p2[0] + p2[1] * p1[0] + p3[1] * p1[0]
    return abs(A - B)
```

convex hull

```
def ccw(p1, p2, p3):
    return (p2[0] - p1[0]) * (p3[1] - p1[1]) - (p2[1] - p1[1]) * (p3[0] - p1[0])

def convex_hull(points):
    points = sorted(points)
    lower = []
    for p in points:
        while len(lower) >= 2 and ccw(lower[-2], lower[-1], p) <= 0:
            lower.pop()
        lower.append(p)
    upper = []
    for p in reversed(points):
        while len(upper) >= 2 and ccw(upper[-2], upper[-1], p) <= 0:
            upper.pop()
        upper.append(p)

    return lower[:-1] + upper[:-1]
```

Rotating Calipers

```
def cald(A, B):
    return (A[0] - B[0]) ** 2 + (A[1] - B[1]) ** 2

def MD(t):
    P = [(A[i][0] + t*A[i][2], A[i][1] + t*A[i][3]) for i in range(N)]

    HP = CH(P)
    M = 0
    L = len(HP)
    j = 0
    for i in range(len(HP)):
        cv = (HP[i][0] - HP[(i+1)%L][0], HP[i][1] - HP[(i+1)%L][1])
        while vccw((HP[(j+1)%L][0] - HP[j%L][0], HP[(j+1)%L][1] - HP[j%L][1]), cv) > 0:
            M = max(M, cald(HP[i], HP[j%L]))
            j += 1

        M = max(M, cald(HP[i], HP[j%L]), cald(HP[i], HP[(j+1)%L]))
    return M
```

Graph

bipartite matching

```
L = []
R = []

E = [[] for _ in range(len(L))]
S = [-1 for _ in range((len(R)))]
V = [False for _ in range(len(R))]

def dfs(u):
    for v in E[u]:
        if V[v]:
            continue

        V[v] = True
        if S[v] == -1 or dfs(S[v]):
            S[v] = u
            return True
    return False

for i in range(len(L)):
    V = [False for _ in range(len(R))]
    dfs(i)
```

SCC

```
from collections import deque
N = 1
E = [[] for _ in range(N)]

F = [False for _ in range(N)]
L = [0 for _ in range(N)]
level = 0
ANS = []
S = deque([])

def scc(u):
    global level
    level += 1
    last = L[u] = level

    S.append(u)
    for v in E[u]:
        if not L[v]:
            last = min(last, scc(v))
        elif not F[v]:
            last = min(last, L[v])

    if last == L[u]:
        scc_set = []
        while S:
            p = S.pop()
            scc_set.append(p)
            F[p] = True
            if u == p:
                break
        ANS.append(scc_set)
    return last

for i in range(N):
    if not F[i]:
        scc(i)
```

SCC – CPP

```
class SCC{
public:
  int V;
  vector<vector<int>>& graph;
  int groupId = 0;
  vector<int> groupIdOf;
  vector<bool> visited;
  vector<int> stack;
  vector<int> stackIdx;
  const int MAX = 987654321;

  SCC(int V, vector<vector<int>>& graph):V(V), graph(graph){}

  vector<int> getScc(){
    groupIdOf = vector<int>(V, -1);
    stackIdx = vector<int>(V, -1);
    visited = vector<bool>(V, false);

    for (int v = 0; v < V; v++) {
      if (visited[v]) continue;
      DFS(v, stack, stackIdx);
    }
    return groupIdOf;
  }

  int DFS(int curNode, vector<int>& stack, vector<int>& stackIdx) {

    visited[curNode] = true;
    stack.push_back(curNode);
    stackIdx[curNode] = stack.size();

    int minParentIdx = MAX;

    for (auto e : graph[curNode]) {
      if (stackIdx[e] != -1 && stackIdx[e] < minParentIdx) {
        minParentIdx = stackIdx[e];
        continue;
      }

      if (visited[e]) continue;

      minParentIdx = min(DFS(e, stack, stackIdx), minParentIdx);
    }

    if (minParentIdx == stackIdx[curNode] || minParentIdx == MAX) {
      while (stack.size() > 0 && stack.back() != curNode) {
        groupIdOf[stack.back()] = groupId;
        stackIdx[stack.back()] = -1;
        stack.pop_back();
      }
      groupIdOf[stack.back()] = groupId;
      stackIdx[stack.back()] = -1;
      stack.pop_back();

      groupId += 1;

      return MAX;
    }

    return minParentIdx;
  }
};
```

Dinic

```
from collections import deque

N = 1
F = [[0 for _ in range(N)] for _ in range(N)]
L = [-1 for _ in range(N)]
P = [ 0 for _ in range(N)]
ADJ = [[] for _ in range(N)]
S, E = 0, N - 1

def bfs(S, E):
    global L
    L = [-1 for _ in range(N)]
    L[S] = 0
    queue = deque([S])
    while queue:
        u = queue.popleft()
        for v in ADJ[u]:
            if L[v] == -1 and F[u][v]:
                L[v] = L[u] + 1
                queue.append(v)
    return L[E] != -1

def dfs(u, f, E):
    global P
    if u == E:
        return f

    while P[u] <= E:
        v = P[u]
        if L[u] < L[v] and F[u][v]:
            add = dfs(v, min(f, F[u][v]), E)
            if add:
                F[u][v] -= add
                F[v][u] += add
                return add
        P[u] += 1
    return 0

MF = 0
while bfs(S, E):
    P = [0 for _ in range(N + 2)]
    while add := dfs(S, float('inf'), E):
        MF += add
```

Tree

HLD – Euler Route

```
vector<vector<int>> graph;

class hldNode{
public:
  int idx;
  int root;
  int leaf;
  int parent;
  int childEdIdx;
  int heavy;
  int level;
};
vector<hldNode> hldList;
vector<int> hldNodeQueue;

int getChildCntAndInitHeavyNode(int parent, int idx){
  int cnt = 0;

  int heavyIdx = -1;
  int heavyCnt = 0;
  for(auto e: graph[idx]){
    if(e == parent) continue;
    int curCnt = getChildCntAndInitHeavyNode(idx, e);
    cnt += curCnt;

    if(heavyCnt < curCnt){
      heavyCnt = curCnt;
      heavyIdx = e;
    }
  }
  hldList[idx].heavy = heavyIdx;
  return cnt + 1;
}
int hldInit(int parent, int cur, int root, int level){

  if(hldList[cur].heavy == -1){
    hldList[cur].idx = hldNodeQueue.size();
    hldList[cur].root = root;
    hldList[cur].leaf = cur;
    hldList[cur].parent = parent;
    hldList[cur].childEdIdx = hldList[cur].idx;
    hldList[cur].level = level;
    hldNodeQueue.push_back(cur);
    return cur;
  }

  hldList[cur].idx = hldNodeQueue.size();
  hldList[cur].root = root;
  hldNodeQueue.push_back(cur);
  hldList[cur].leaf = hldInit(cur, hldList[cur].heavy, root, level + 1);
  hldList[cur].parent = parent;

  for(auto e: graph[cur]){
    if(e == parent || e == hldList[cur].heavy) continue;
    hldInit(cur, e, e, level + 1);
  }
  hldList[cur].childEdIdx = hldNodeQueue.size() - 1;
  hldList[cur].level = level;

  return hldList[cur].leaf;
}

int lca(int a, int b){
  while(a != b){
    if(hldList[a].root == hldList[b].root){
      if(hldList[a].level < hldList[b].level)
        b = a;
      else
        a = b;
    }else{
      if(hldList[hldList[a].root].level < hldList[hldList[b].root].level)
        b = hldList[hldList[b].root].parent;
      else if(hldList[hldList[a].root].level > hldList[hldList[b].root].level)
        a = hldList[hldList[a].root].parent;
      else{
        a = hldList[a].parent;
        b = hldList[b].parent;
      }
    }
  }
  return a;
}
class segNode{
public:
  int st;
  int ed;
  ll val;
  pair<ll, ll> lazy;
};
vector<segNode> segTree;
int leafNum = 1;

void segInit(int N){
  while(leafNum < N) leafNum *= 2;
  segTree = vector<segNode>(leafNum * 2);

  for(int i = 0; i < N; i++){
    segTree[leafNum + i].st = i;
    segTree[leafNum + i].ed = i + 1;
    segTree[leafNum + i].val = 0;
    segTree[leafNum + i].lazy = {1, 0};
  }
  for(int i = N; i < leafNum; i++){
    segTree[leafNum + i].st = i;
    segTree[leafNum + i].ed = i + 1;
    segTree[leafNum + i].val = 0;
    segTree[leafNum + i].lazy = {0, 0};
  }

  for(int i = leafNum - 1; i > 0; i--){
    segTree[i].st = segTree[2 * i].st;
    segTree[i].ed = segTree[2 * i + 1].ed;
    segTree[i].val = 0;
    segTree[i].lazy = {1, 0};
  }
}

void segUpdate(int idx, int st, int ed, pair<ll, ll> op){
  if(segTree[idx].st == st && segTree[idx].ed == ed){
    segTree[idx].lazy.first *= op.first;
    segTree[idx].lazy.second *= op.first;
    segTree[idx].lazy.second += op.second;
    segTree[idx].val = segTree[idx].val * op.first + (op.second) * (ed - st);

    segTree[idx].lazy.first %= DIV;
    segTree[idx].lazy.second %= DIV;
    segTree[idx].val %= DIV;
    return;
  }

  if(segTree[idx].lazy.first != 1 || segTree[idx].lazy.second != 0){
    segUpdate(2 * idx, segTree[2 * idx].st, segTree[2 * idx].ed,
segTree[idx].lazy);
    segUpdate(2 * idx + 1, segTree[2 * idx + 1].st, segTree[2 * idx + 1].ed,
segTree[idx].lazy);
    segTree[idx].lazy = {1, 0};
  }

  if(ed <= segTree[2 * idx].ed)
    segUpdate(2 * idx, st, ed, op);
  else if(segTree[2 * idx + 1].st <= st)
    segUpdate(2 * idx + 1, st, ed, op);
  else{
    segUpdate(2 * idx, st, segTree[2 * idx].ed, op);
    segUpdate(2 * idx + 1, segTree[2 * idx + 1].st, ed, op);
  }
  segTree[idx].val = segTree[2 * idx].val + segTree[2 * idx + 1].val;
  segTree[idx].val %= DIV;
  return;
}
ll segGetVal(int idx, int st, int ed){
  if(segTree[idx].st == st && segTree[idx].ed == ed){
    return segTree[idx].val;
  }

  if(segTree[idx].lazy.first != 1 || segTree[idx].lazy.second != 0){
    segUpdate(2 * idx, segTree[2 * idx].st, segTree[2 * idx].ed,
segTree[idx].lazy);
    segUpdate(2 * idx + 1, segTree[2 * idx + 1].st, segTree[2 * idx + 1].ed,
segTree[idx].lazy);
    segTree[idx].lazy = {1, 0};
    segTree[idx].val = segTree[2 * idx].val + segTree[2 * idx + 1].val;
    segTree[idx].val %= DIV;
  }

  if(ed <= segTree[2 * idx].ed)
    return segGetVal(2 * idx, st, ed);
  if(segTree[2 * idx + 1].st <= st)
    return segGetVal(2 * idx + 1, st, ed);
  return (segGetVal(2 * idx, st, segTree[2 * idx].ed) + segGetVal(2 * idx + 1,
segTree[2 * idx + 1].st, ed)) % DIV;
}
```

String

suffix array & LCP

```
import math

def LCPSUFFIX(S, L):
    def radix_sort(rank, max_rank, rktoi, L):
        radix = [0 for _ in range(max_rank + 1)]
        new_rktoi = [0 for _ in range(L)]

        # radix 를 cummulative 로 구성
        for rk in range(L):
            radix[rank[rktoi[rk]]] += 1
        for ra in range(max_rank):
            radix[ra + 1] += radix[ra]

        # sorting 후의 ranking 위치로 i 값을 옮김
        for rk in range(L - 1, -1, -1):
            radix[rank[rktoi[rk]]] -= 1
            new_rktoi[radix[rank[rktoi[rk]]]] = rktoi[rk]

        # rank 로 sorting 된 rktoi 반환
        return new_rktoi

    def update_rank(rank1, rank2, rktoi, L):
        rank_count = 1
        new_rank = [0 for _ in range(L)]

        new_rank[rktoi[0]] = 1
        for i in range(1, L):
            if rank1[rktoi[i - 1]] != rank1[rktoi[i]] or rank2[rktoi[i - 1]] !=
rank2[rktoi[i]]:
                rank_count += 1
            new_rank[rktoi[i]] = rank_count

        # rank1, rank2 를 기준으로 rank1 의 동점자가 처리된 rank 를 반환 및 rank 갯수
반환
        return new_rank, rank_count

    rank1 = [ord(S[i]) - ord('A') + 1 for i in range(L)]
    rank2 = [0 for _ in range(L)]
    rktoi = radix_sort(rank1, max(rank1), [i for i in range(L)], L)

    rank1, rank_count = update_rank(rank1, rank2, rktoi, L)
    for i in range(math.ceil(math.log2(L))):
        for rk in range(L):
            rank2[rktoi[rk]] = rank1[rktoi[rk] + 2 ** i] if rktoi[rk] + 2 ** i < L
else 0
        rktoi = radix_sort(rank2, rank_count, rktoi, L)
        rktoi = radix_sort(rank1, rank_count, rktoi, L)

        rank1, rank_count = update_rank(rank1, rank2, rktoi, L)
        if rank_count == L:
            break

    itork = [0 for _ in range(L)]
    for i in range(L):
        itork[rktoi[i]] = i

    # LCP 배열 생성
    LCP = [0 for _ in range(L)]
    val = 0
    for i in range(L):
        if itork[i] == 0:
            continue

        uprki = rktoi[itork[i] - 1]
        while i + val < L and uprki + val < L and S[i + val] == S[uprki + val]:
            val += 1
        LCP[itork[i]] = val
        val = max(val - 1, 0)
    return LCP
```

manacher

```
import sys

# 필수 전역 변수
S = sys.stdin.readline()[:-1]   # 문자열
S = "#".join(S)               # 문자열 전처리

N = len(S)                      # 문자열 길이
r = [0 for i in range(len(S))]  # r[i] : i 번째 단어의 펠린드롬 반지름
far = -1                          # 가장 멀리 온 팰린드롬 끝부분
farMid = -1                       # 가장 멀리 온 팰린드롬 끝부분의 중심점
################################################################ 함수부

## ind 의 펠린드롬 반지름 길이 구하는 함수
def getPalinRadius(ind):
  global S, N, r, far, farMid

  curR = 0
  # curR 초기화 부분
  if ind <= far:
    curR = min(r[2 * farMid - ind], far - ind)

  # 좌우로 넓혀가며 반지름 찾는 부분
  while 0 <= ind - curR - 1 and ind + curR + 1 < N and S[ind - curR - 1] == S[ind +
curR + 1]:
    curR += 1

  r[ind] = curR

  # far 갱신
  if far < ind + curR:
    far = ind + curR
    farMid = ind

  return r[ind]

## 인덱스와 계산된 길이를 줬을 때 펠린드롬 길이를 구하는 함수
## getPalinRadius 는 전처리된 반지름을 기준으로 하기 때문에 무작정 2 곱하면 안됨
def getPalinLen(ind, r):
  if ind % 2 == 0:  # 실제 단어 일때
```

```
    return 1 + (r // 2) * 2
  else: # 더미 단어 일때
    return (r + 1) // 2 * 2
################################################################ 실행부

ans = 0
for i in range(len(S)):
  curAns = getPalinLen(i, getPalinRadius(i))
  if ans < curAns:
    ans = curAns

print(ans)
```

kmp

```
int main() {
    string T;
    getline(cin, T);
    string P;
    getline(cin, P);

    //p 배열 설정하기
    vector<int> p(P.size(), 0);

    int curCompIdx = 0;
    for (int i = 1; i < P.size(); i++) {
        while (curCompIdx > 0 && P[curCompIdx] != P[i])
            curCompIdx = p[curCompIdx - 1];

        if (P[curCompIdx] == P[i])
            p[i] = ++curCompIdx;
    }
    //찾기
    int curPatternIdx = 0;
    vector<int> idxArray;
    for (int k = 0; k < T.size(); k++) {
        if (curPatternIdx == P.size()) {
            idxArray.push_back(k - P.size());
            curPatternIdx = p[curPatternIdx - 1];
        }
        while (curPatternIdx > 0 && P[curPatternIdx] != T[k])
            curPatternIdx = p[curPatternIdx - 1];
        if (P[curPatternIdx] == T[k])
            curPatternIdx++;
        else
            curPatternIdx = 0;
    }
    if (curPatternIdx == P.size()) {
        idxArray.push_back(T.size() - P.size());
    }
    cout << idxArray.size() << "\n";
    for (auto e : idxArray)
        cout << e + 1 << " ";

    return 0;
}
```

Well-Known

2-sat

```
for i in range(N6):
    if not finished[i]:
        scc(i)

res = [0] * N3
for i in range(N3):
    if scc_num[i] == scc_num[i + N3]:
        print(-1)
        break
    if scc_num[i + N3] < scc_num[i]:
        res[i] = 1
else:
    print(res.count(1))
    for i in range(N3):
        if res[i]:
            print(i + 1, end=' ')
```

second MST

두번째 MST

```
#include <iostream>
#include <vector>
#include <algorithm>
#include <cmath>

using namespace std;

//Graph
int V, E;
vector<vector<int>> edges;

//Union-Find
vector<int> groupOf;
int getGroup(int v){
  if(v != groupOf[v]) groupOf[v] =
getGroup(groupOf[v]);
  return groupOf[v];
}
void unionGroup(int v1, int v2){
  v1 = getGroup(v1);
  v2 = getGroup(v2);

  if(v1 < v2)
    groupOf[v2] = v1;
  else
    groupOf[v1] = v2;
}
```

```
//LCA
vector<int> levelOf;
vector<vector<int>> spTable;
vector<vector<int>> spTableMax;
vector<vector<int>> spTable2ndMax;
void initSpTable(){

  for(int j = 1; j < 20; j++){
    for(int i = 0; i < V + 1; i++){
      spTable[i][j] = spTable[spTable[i][j - 1]][j
- 1];

      spTableMax[i][j] = -1;
      spTable2ndMax[i][j] = -1;

      vector<int> curVals(4);
      curVals[0] = spTableMax[i][j - 1];
      curVals[1] = spTable2ndMax[i][j - 1];
      curVals[2] = spTableMax[spTable[i][j -
1]][j - 1];
      curVals[3] = spTable2ndMax[spTable[i][j
- 1]][j - 1];
      sort(curVals.rbegin(), curVals.rend());

      spTableMax[i][j] = curVals[0];
      for(int k = 1; k < 4; k++){
        if(curVals[k] == curVals[0]) continue;
        spTable2ndMax[i][j] = curVals[k];
        break;
      }

    }
  }

}

int LCA(int v1, int v2){

  int curMaxEdge = -1;

  while(levelOf[v1] < levelOf[v2]){
    curMaxEdge = max(curMaxEdge,
spTableMax[v2][(int)(floor(log2(levelOf[v2]
- levelOf[v1])))]);
    v2 =
spTable[v2][(int)(floor(log2(levelOf[v2] -
levelOf[v1])))];
  }
  while(levelOf[v1] > levelOf[v2]){
    curMaxEdge = max(curMaxEdge, spTableMax[v1][(int)(floor(log2(levelOf[v1] - levelOf[v2])))]);
    v1 = spTable[v1][(int)(floor(log2(levelOf[v1] - levelOf[v2])))];
  }

  while(v1 != v2){

    int st = 1;
    int ed = 20;
    while(st <= ed){
      int mid = (st + ed) / 2;

      if(spTable[v1][mid] == spTable[v2][mid])
        ed = mid - 1;
      else
        st = mid + 1;
    }

    curMaxEdge = max(curMaxEdge, spTableMax[v1][ed]);
    curMaxEdge = max(curMaxEdge, spTableMax[v2][ed]);
    v1 = spTable[v1][ed];
    v2 = spTable[v2][ed];
  }

  return curMaxEdge;
}

int LCA2nd(int v1, int v2, int maxEdge){

  int cur2ndMaxEdge = -1;

  while(levelOf[v1] < levelOf[v2]){
    if(spTableMax[v2][(int)(floor(log2(levelOf[v2] - levelOf[v1])))] < maxEdge)
      cur2ndMaxEdge = max(cur2ndMaxEdge, spTableMax[v2][(int)(floor(log2(levelOf[v2] - levelOf[v1])))]);
    if(spTable2ndMax[v2][(int)(floor(log2(levelOf[v2] - levelOf[v1])))] < maxEdge)
      cur2ndMaxEdge = max(cur2ndMaxEdge, spTable2ndMax[v2][(int)(floor(log2(levelOf[v2] - levelOf[v1])))]);

    v2 = spTable[v2][(int)(floor(log2(levelOf[v2] - levelOf[v1])))];
  }

  while(levelOf[v1] > levelOf[v2]){
    if(spTableMax[v1][(int)(floor(log2(levelOf[v1] - levelOf[v2])))] < maxEdge)
      cur2ndMaxEdge = max(cur2ndMaxEdge, spTableMax[v1][(int)(floor(log2(levelOf[v1] - levelOf[v2])))]);
    if(spTable2ndMax[v1][(int)(floor(log2(levelOf[v1] - levelOf[v2])))] < maxEdge)
      cur2ndMaxEdge = max(cur2ndMaxEdge, spTable2ndMax[v1][(int)(floor(log2(levelOf[v1] - levelOf[v2])))]);

    v1 = spTable[v1][(int)(floor(log2(levelOf[v1] - levelOf[v2])))];
  }

  while(v1 != v2){

    int st = 1;
    int ed = 20;
    while(st <= ed){
      int mid = (st + ed) / 2;

      if(spTable[v1][mid] == spTable[v2][mid])
        ed = mid - 1;
      else
        st = mid + 1;
    }

    if(spTableMax[v1][ed] < maxEdge)
      cur2ndMaxEdge = max(cur2ndMaxEdge, spTableMax[v1][ed]);
    if(spTable2ndMax[v1][ed] < maxEdge)
      cur2ndMaxEdge = max(cur2ndMaxEdge, spTable2ndMax[v1][ed]);

    if(spTableMax[v2][ed] < maxEdge)
      cur2ndMaxEdge = max(cur2ndMaxEdge, spTableMax[v2][ed]);
    if(spTable2ndMax[v2][ed] < maxEdge)
      cur2ndMaxEdge = max(cur2ndMaxEdge, spTable2ndMax[v2][ed]);

    v1 = spTable[v1][ed];
    v2 = spTable[v2][ed];
  }

  return cur2ndMaxEdge;
}

//MST
```

```
int MSTWeight;
vector<bool> isMSTEdge;
vector<vector<pair<int, int>>> MST;
void getMST(){
  sort(edges.begin(), edges.end());

  for(int i = 0; i < E; i++){
    if(getGroup(edges[i][1]) ==
getGroup(edges[i][2])) continue;

    MSTWeight += edges[i][0];
    isMSTEdge[edges[i][3]] = true;
    unionGroup(edges[i][1], edges[i][2]);

    MST[edges[i][1]].push_back({edges[i][2],
edges[i][0]});
    MST[edges[i][2]].push_back({edges[i][1],
edges[i][0]});
  }
}

//인접 리스트로 된 트리를 dfs 를 통해
spTable 과 levelOf 를 초기화 하기 위함
void dfs(int cur, int prv, int level){
  levelOf[cur] = level;

  for(auto e: MST[cur]){
    if(e.first == prv) continue;
    dfs(e.first, cur, level + 1);

    spTable[e.first][0] = cur;
    spTableMax[e.first][0] = e.second;
  }
}
```

log LIS

```
import sys

A = [1, 2, 3, 4, 5]
N = len(A)
D = [A[0]] # 그 갯수인 최소값
I = [0] # 그 최소값의 index
DA = [1 for _ in range(N)] # 모든
위치에서의 최대
IA = [-1 for _ in range(N)] # 최대를
이루는 바로 이전 index

def find_index(left, right, value):
    while left != right:
        mid = (left + right) >> 1

        if D[mid] < value:
            left = mid + 1
        elif value <= D[mid]:
            right = mid
    return left

for i in range(1, N):
    index = -1

    if A[i] > D[-1]:
        index = len(D)
        D.append(A[i])
        I.append(i)
    else:
        index = find_index(0, len(D) -
1, A[i])
        if A[i] < D[index]:
            D[index] = A[i]
            I[index] = i
    DA[i] = index + 1
    IA[i] = -1 if index - 1 < 0 else
I[index - 1]

ans = []
c = I[-1]
while c != -1:
    ans.append(A[c])
    c = IA[c]
print(ans)
```

Aho-Corasik, trie

```
#include <iostream>
#include <string>
#include <vector>
#include <queue>

using namespace std;

class Node {
public:

   bool ifEnd;
   vector<Node*> children;
   Node* fail;

   Node() {
      ifEnd = false;
      for (int i = 0; i < 26; i++)
         children = vector<Node*>(26,
NULL);
      fail = NULL;
   }
};

int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    cout.tie(NULL);

    int N;
    cin >> N;

    //트라이 트리 구축
    Node* root = new Node();
    for (int i = 0; i < N; i++) {
        string str;
        cin >> str;

        Node* curNode = root;
        for (int j = 0; j < str.size(); j++) {
            if (curNode->children[str[j] - 'a'] == NULL)
                curNode->children[str[j] - 'a'] = new Node();
            curNode = curNode->children[str[j] - 'a'];
        }
        curNode->ifEnd = true;
    }

    //실패함수 매핑
    queue<Node*> Queue;

    //첫번째 레벨 node 들의 fail 은 루트로 지정해주고, Q 에 넣기
    for(int i = 0; i < 26; i++){
      if(root->children[i] == NULL) continue;

      root->children[i]->fail = root;
      Queue.push(root->children[i]);
    }

    while (!Queue.empty()) {
        Node* curNode = Queue.front();
        Queue.pop();

        //현재 노드의 자식들의 실패함수를 매핑해주기.
        for (int i = 0; i < 26; i++) {
            if (curNode->children[i] == NULL) continue;

            Node* curFail = curNode->fail;

            while (curFail != NULL && curFail->children[i] == NULL)
                curFail = curFail->fail;

            if (curFail != NULL){
                if(curFail->children[i]->ifEnd)
                  curNode->children[i]->ifEnd = true;
                curNode->children[i]->fail = curFail->children[i];
            }
            else{
              curNode->children[i]->fail = root;
            }

            Queue.push(curNode->children[i]);
        }
    }

    //탐색 수행
    int Q;
    cin >> Q;
    for(int i = 0; i < Q; i++){
      string str;
      cin >> str;

      Node* curNode = root;
      for(int j = 0; j < str.size(); j++){
        if(curNode->children[str[j] - 'a'] != NULL){
          curNode = curNode->children[str[j] - 'a'];
        }
        else{
          if(curNode->fail == NULL){
            curNode = root;
          } else{
            curNode = curNode->fail;
            j--;
          }
        }

        if(curNode->ifEnd){
          break;
        }
      }

      if(curNode->ifEnd)
        cout << "YES\n";
      else
        cout << "NO\n";
    }


    return 0;
```

```
}
```

Game-Theory

```
    if p == 'R':
        answer ^= x^y
    if p == 'B':
        answer ^= min(x,y)
    if p == 'K':
        temp = 0

        if min(x,y) % 2 == 0:
            if max(x,y) % 2 == 0:
                temp = 0
            else:
                temp = 1
        else:
            if max(x,y) % 2 == 0:
                temp = 3
            else:
                temp = 2
        answer ^= temp
    if p == 'N':
        a = (x - min(x,y) + 
min(x,y)  % 3)
        b = (y - min(x,y) + 
min(x,y)  % 3)
        c = (a+b) // 3
        answer ^= min(a,b,c)
    if p == 'P':
        answer ^= ((x+y) % 3 + 
((x//3)^(y//3)) * 3)
```

Slope Trick

```
import heapq
n = int(input())
a = list(map(int, input().split()))
a = [a[i]-i for i in range(n)]

q = []
heapq.heappush(q, -a[0])

ans = 0
R = [0 for _ in range(n)]
R[0] = a[0]

for i in range(1, n):
    heapq.heappush(q, -a[i])

    R[i] = -q[0] + i

    if a[i] < -q[0]:
        ans += -q[0] - a[i]
        heapq.heappop(q)
        heapq.heappush(q, -a[i])
for i in range(n-2, -1, -1):
    R[i] = min(R[i], R[i+1] - 1)
print(*R, sep='\n')
```

DNC Optimization

```
import sys

L, G = map(int, sys.stdin.readline().split())
C = list(map(int, sys.stdin.readline().split()))
DP = [[0 for _ in range(L + 1)] for _ in 
range(G + 1)]
CC = [0 for _ in range(L + 1)]


CC[1] = C[0]
for i in range(2, L + 1):
    CC[i] = C[i - 1] + CC[i - 1]

def dnc_dp(i, s, e, l, r):
    if s > e:
        return

    m = (s + e) // 2
    opt = l
    mini = DP[i - 1][m]
    for k in range(l, m + 1):
        if mini >= DP[i - 1][k] + (CC[m] - 
CC[k]) * (m - k):
            mini = DP[i - 1][k] + (CC[m] - CC[k]) 
* (m - k)
            opt = k
    DP[i][m] = mini

    dnc_dp(i, s, m - 1, l, opt)
    dnc_dp(i, m + 1, e, opt, r)

for i in range(1, L + 1):
    DP[1][i] = CC[i] * i

for i in range(2, G + 1):
    dnc_dp(i, 1, L, 1, L)

print(DP[G][L])
```