Maths for Economists

A Comprehensive Handbook

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Preface

This book is written as a handbook for economists rather than a note for courses like Econ Math, where materials are selected from various sources including course handouts from SUFE, lecture notes from open scources and many great books. To make it a comprehensive one, I'll try to cover almost all maths needed for studying economics, including logic, algebra, analysis, topology, probability theory and the theory of optimization. And I'll add something more, basically based on the syllabus of the course I've taken. In this way can those(especially from SUFE) who're taking courses like Mathematical analysis refer to it.

Though served as a handbook, the book will be written as intuitive as possible so that all techniques and definitions are expressed in a natural way and readers can better understand them.

The book may be updated in a monthly basis.

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First Author & Second Author

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Part I

Preliminaries

Chapter 1

Logic

To make complex ideas clear, we often separate them into statements and check whether they're right or wrong. Here we use symbolic logic. Symbolic logic is basically about statements which can be claimed to be either true or false and "operations" of statements.

We start with what is a statement. For sure, most of the sentences can be seen as a statement. But to notice, not all sentences are statements. For example, "This sentence is false." is not a statement, that is because when it is true, it is false and vice versa.

In maths, we call statements as propositions and we can categorize them into theorems, axioms, lemmas, corollaries, etc. Axioms are statements that we accept to be true without proof. Theorems are statements that can be proven to be true based on axioms and previously proven theorems. Lemmas are "helper" theorems used to prove larger theorems and corollaries are statements that follow directly from a theorem.

1.1 Negation

We use the symbol $\neg$ to express "NOT". Suppose A is a statement, then $\neg A$ is also a statement and $\neg A$ means "A is NOT true". We can use a truth table to show the relationship between A and $\neg A$.

Statement	A	$\neg A$
Truth	T	F
Value	F	T

1.2 Conjunction and Disjunction

Conjunction and Disjunction are ways to combine statements together. We use the symbol $\wedge$ to express "AND". Suppose A and B are statements, then $A \wedge B$ is also a statement and $A \wedge B$ means "A AND B are both true". Similarly, we use the symbol $\vee$ to express "OR". Suppose A and B are statements, then $A \vee B$ is also a statement and $A \vee B$ means "Either A or B is true (or both are true)".

We can use a truth table to show the relationship between A, B, $A \wedge B$ and $A \vee B$.

Statement	A	B	$A \wedge B$	$A \vee B$
	T	T	T	T
Truth	T	F	F	T
Value	F	T	F	T
	F	F	F	F

1.3 Quantifiers

1.3.1 Property

We first define the concept of property. We say $P(x)$ is a property of x (x is from a particular class) if when x is replaced with a certain object, $P(x)$ becomes a statement. For example, let $P(x)$ be "x is even", then $P(2)$ is true and $P(3)$ is false. The set $\{x : P(x)\}$ consists of all values of x such that $P(x)$ is true.

1.3.2 Forall and Exists

Now we can create another kind of statement using quantifiers. There are two quantifiers: "for all" and "there exists". The expression $\forall x \in X : P(x)$ means "for all components x in the set X, $P(x)$ is true". And the expression $\exists x \in X : P(x)$ means "there exists at least one component x in the set X, such that $P(x)$ is true".

A very important proposition is that

$$\neg[\exists x \in X : P(x)] = \forall x \in X : \neg P(x) \tag{1.1}$$

And to understand all statements, we also need $\neg(\neg A) = A, \neg(A \wedge B) = \neg A \vee \neg B, \neg(A \vee B) = \neg A \wedge \neg B$. This becomes trivial to deal with quantifiers that we only have to interchange $\vee$ and $\wedge$ and change $\forall$ to $\exists$ and vice versa when we negate a statement.

1.4 Implications

1.4.1 Implication

The implication $(A \implies B) := (\neg A) \vee B$ is false if and only if A is true and B is false. The definition is simple, but note that $A \implies B$ not necessarily mean that A causes B to be true. When A and B are false, the implication is also true, which makes it different from the familiar meaning of "imply".

We also say that A is a sufficient condition for B, and B is a necessary condition for A when we write the statement $A \implies B$.

1.4.2 Equivalence

When A is both necessary and sufficient for B, we say that A is equivalent to B, and we write $A \iff B$. Note that $A \iff B$ is equivalent to $(A \implies B) \wedge (B \implies A)$.

1.4.3 Prove by Contrapositive

With equivalence, we know that the equation in (1.1) actually means equivalent and we can make clear of the above relationships. Moreover, using the interchanging technique, we can find the counterpositive statement

$$(A \implies B) \iff (\neg B \implies \neg A)$$

This inspires us to prove a statement, we can turn to its contrapositive if difficulties arise when proving the original statement.

1.4.4 Prove by Contradiction

Consider statements $(A \implies C), (C \implies B)$, then

$$(A \implies B) \iff (A \implies C) \wedge (C \implies B)$$

Suppose B is false and assume A is true, then $(C \implies B)$ and $(A \implies C)$ are false, that is to say, we can find a statement C with the false truth value.

This leads us to the proof by contradiction method. To prove $A \implies B$, we can assume A is true and B is false, then try to find a statement with a false truth value and we prove the original statement.

Chapter 2

Set, Relation and Operation

Since all readers should be familiar with the high-school-level set theory, we only give a brief review of the basic concepts and operations of sets.

We can't define what is a set but we can give it a description. A set is all objects that satisfy a certain property. The following lists some of the basics.

Definition 2.0.1 (Subset)**.** Let A, B be sets, when we say $A \subset B$, we mean $\forall x \in A, x \in B$. Notice

$$A = B \iff (A \subset B) \wedge (B \subset A)$$

Definition 2.0.2 (Complement,Intersection and Union)**.** Let A, B be sets, we define the following operations:

> **Complement** The (relative) complement of set B in set A is defined as $A \setminus B = \{x : (x \in A) \wedge (x \notin B)\}$.
>
> **Intersection** The intersection of sets A and B is defined as $A \cap B = \{x : (x \in A) \wedge (x \in B)\}$.
>
> **Union** The union of sets A and B is defined as $A \cup B = \{x : (x \in A) \vee (x \in B)\}$.

By definition, one can easily prove:

Proposition 2.0.1. *Let A, B, C be sets, then*

> **Commutativity** $A \cap B = B \cap A$ *and* $A \cup B = B \cup A$.
>
> **Associativity** $(A \cap B) \cap C = A \cap (B \cap C)$ *and* $(A \cup B) \cup C = A \cup (B \cup C)$.
>
> **Distributivity** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ *and* $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
>
> **de Morgan's Laws** $(A \cap B)^c = A^c \cup B^c$ *and* $(A \cup B)^c = A^c \cap B^c$
>
> **Subset** $A \subset B \iff A \cap B = A \iff A \cup B = B$.

2.1 Relation and Operation

2.1.1 Relation

2.1.1.1 Equivalence

2.1.1.2 Order

2.1.1.3 Mapping

2.1.2 Operation

2.2 The Operation of Sets

2.2.1 Power Set

Definition 2.2.1 (Power Set)**.** Let A be a set, define:

$$2^A = \{X : X \subset A\}$$

2.2.2 Cartesian Product

We define an ordered pair or a n-tuple $x = (x_1, x_2, \cdots, x_n)$ and the equity means that all components are equal. Denote $x_j := pr_j(x)$ and we call it the jth projection of x.

The Cartesian product is to describe the set of ordered pairs.

Definition 2.2.2 (Cartesian Product)**.** Let A, B be sets, define:

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

and by deduction,

$$\prod_{i=1}^{n} A_i = (A_1 \times \cdots \times A_{n-1}) \times A_n$$

Specially, when $A_i = A$ for all i, we denote:

$$A^n = \prod_{i=1}^{n} A$$

To practice logic, we show how to prove the following proposition.

Proposition 2.2.1. *Let A, B be sets, then*

(1) $A \times B = \emptyset \iff (A = \emptyset) \vee (B = \emptyset)$
(2) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

2.2.3 Family of Sets

We now extend the operation of intersection and union a little bit. Consider we're intersecting a series of sets $A, B, C, \cdots$, we first rename the sets as $\{A_i\}_{i \in I}$, where I is called an index set, and intersect them according to the index i, that is

$$\bigcap_{i \in I} A_i = \{x : \forall i \in I, x \in A_i\}.$$

Similarly, for union, we have

$$\bigcup_{i \in I} A_i = \{x : \exists i \in I, x \in A_i\}.$$

Proposition 2.2.2. *Associativity, Distributivity, de Morgan's Laws*

2.3 Countability

2.3.1 Cardinal

2.3.2 Countable Set

Chapter 3

Group, Ring and Field

Part II

Vector Space and Linear Algebra

Chapter 4

Vector Space and Linear Mapping

Chapter 5

Normed Vector Space

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Chapter 6

Multilinear Form and Determinants

One good way to introduce determinant is through solving equation systems and define it by deduction. Consider solving

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

then we have

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11}a_{22} - a_{12}a_{21}}, x_2 = \frac{b_2 a_{11} - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

and denote

$$\det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

We can rewrite the solution as

$$x_1 = \frac{\det\begin{pmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{pmatrix}}{\det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}, x_2 = \frac{\det\begin{pmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{pmatrix}}{\det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}$$

Now we turn to the 3-dimensional case:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

Similarly, by elimination method, we have

$$x_1 = \frac{b_1 \det\begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} - a_{12} \det\begin{pmatrix} b_2 & a_{23} \\ b_3 & a_{33} \end{pmatrix} + a_{13} \det\begin{pmatrix} b_2 & a_{22} \\ b_3 & a_{32} \end{pmatrix}}{a_{11} \det\begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} - a_{12} \det\begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix} + a_{13} \det\begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}}$$

$$x_2 = \frac{a_{11} \det\begin{pmatrix} b_2 & a_{23} \\ b_3 & a_{33} \end{pmatrix} - b_1 \det\begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix} + a_{13} \det\begin{pmatrix} a_{21} & b_2 \\ a_{31} & b_3 \end{pmatrix}}{a_{11} \det\begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} - a_{12} \det\begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix} + a_{13} \det\begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}}$$

$$x_3 = \frac{a_{11} \det\begin{pmatrix} a_{22} & b_2 \\ a_{32} & b_3 \end{pmatrix} - a_{12} \det\begin{pmatrix} a_{21} & b_2 \\ a_{31} & b_3 \end{pmatrix} + b_1 \det\begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}}{a_{11} \det\begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} - a_{12} \det\begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix} + a_{13} \det\begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}}$$

This leads us to denote

$$\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \det\begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} - a_{12} \det\begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix} + a_{13} \det\begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$

and rewrite the solution as

$$x_1 = \frac{\det\begin{pmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{pmatrix}}{\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}, x_2 = \frac{\det\begin{pmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{pmatrix}}{\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}, x_3 = \frac{\det\begin{pmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{pmatrix}}{\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}$$

In this way can we define determinant in a deductive way and achieve the important Cramer's Law. However, this definition arises only as a notation, lacking the geometric understanding of determinants. To do this, we start from the concept of **multilinear form**.

6.1 Multilinear Form

6.1.1 Bilinear Form

Chapter 7

Eigenvalue and Eigenvector

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Part III

Metric Space and Convergence

Part IV

Topology Space and Functions

Part V

Differentiation in One Variable

Part VI

More in Convergence: Uniform Convergence

Appendix A

Notation

Appendix B

Supplementary Scripts

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