Everybody stroops

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Speeded choice response time

Speeded response times (aka reaction times)

These result from easy tasks where the participant is instructed to respond as quickly as possible. They happen on short time scales – less than a second on average. You might call them 'split-second' decisions. Real life examples are tasks such as deciding when to brake while driving, or whether to duck or jump if an object flies your way.

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Choice response times

These result from tasks in which the participant is specifically instructed to choose between several alternatives. They not only have to press a button but additionally have to choose which button to press. There may be two alternatives (two-choice response times) or more than two (multialternative response times).

Visualizing CRT data

Quantiles can be used to summarize nonstandard distributions

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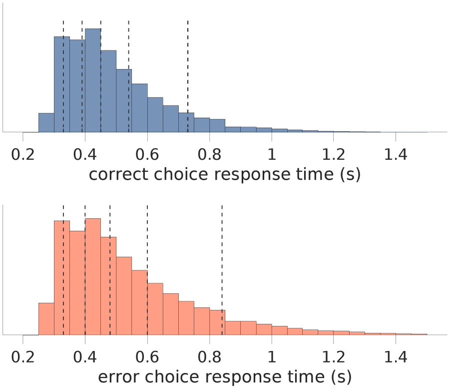
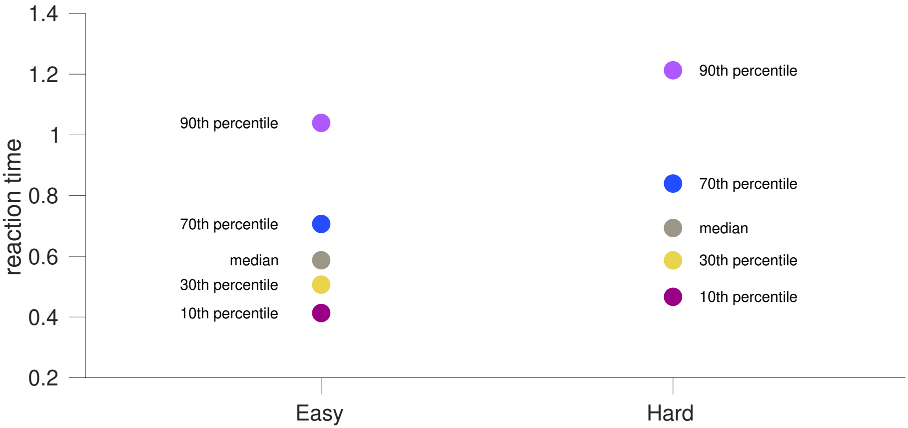


Direct comparison of quantiles



$$\Delta_n = P_n(t_2) - P_n(t_1)$$

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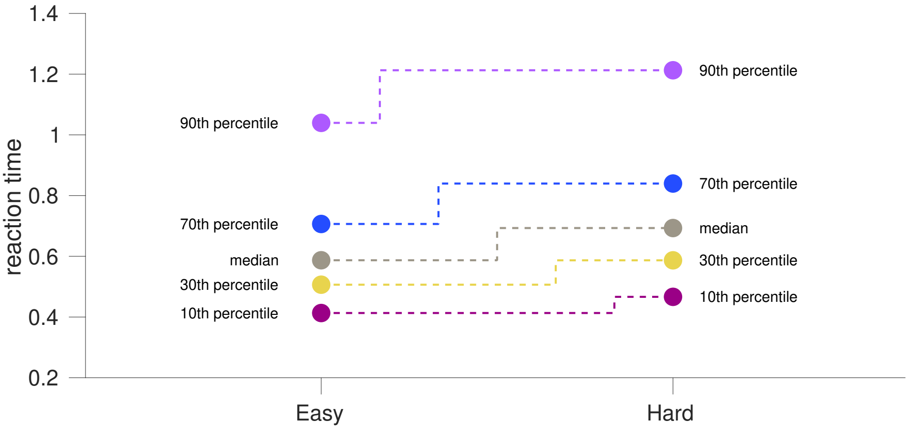

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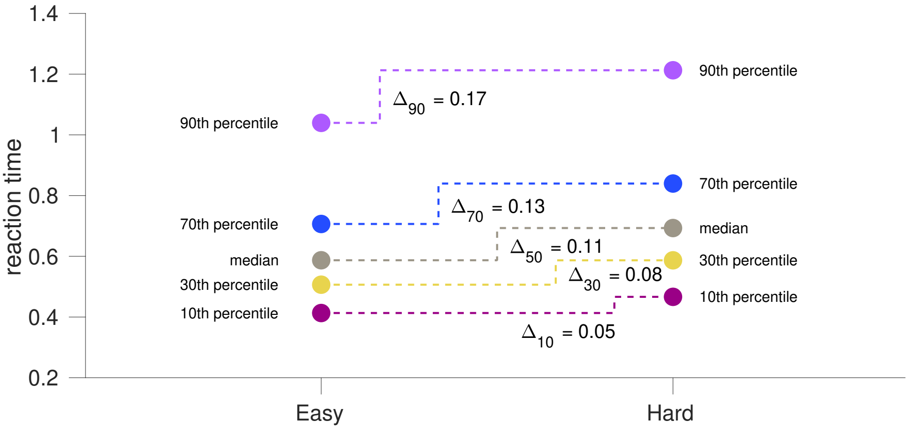

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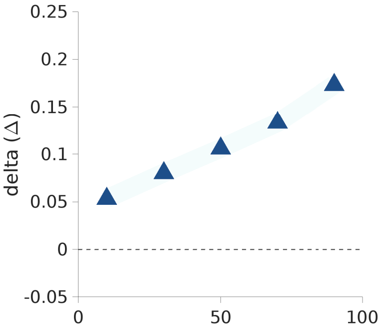
Delta plots

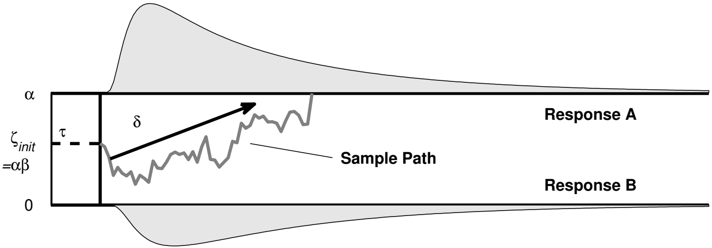
$$\Delta_{10} = 0.05$$
$$\Delta_{30} = 0.08$$
$$\Delta_{50} = 0.11$$
$$\Delta_{70} = 0.13$$
$$\Delta_{90} = 0.17$$



Delta plots can quickly reveal patterns of effects due to experimental manipulations

Diffusion models for two-choice response times

	parameter	*interpretation*
δ	drift rate	dominance $(\eta,\, d')$
α	boundary separation	caution
τ	nondecision time	time for encoding and responding
β	initial bias	a priori response bias



Diffusion model parameters in delta plots

Diffusion model parameter effects become tell-tale patterns in delta plots

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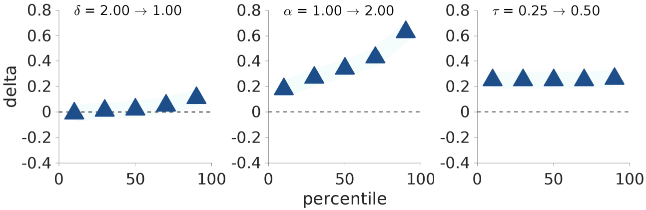

EZ diffusion

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- The likelihood of the DDM is quite complicated so maximum likelihood estimation can be slow.
- Sampling from the DDM is not so difficult but a little time consuming.

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- EZ diffusion involves two sets of equations, the forward equations give the summary statistics in terms of parameters, and the inverse equations give the parameters in terms of summary statistics.

EZ diffusion – forward

Forward EZ equations:

$$R^{\text{pred}} = \frac{1}{y+1} \tag{1}$$

$$M^{\text{pred}} = \tau + \left(\frac{\alpha}{2\delta}\right)\left(\frac{1-y}{1+y}\right), \tag{2}$$

$$V^{\text{pred}} = \left(\frac{\alpha}{2\delta^3}\right)\left\{\frac{1-2\alpha\delta y-y^2}{(y+1)^2}\right\} \tag{3}$$

with $y = \exp(-\alpha\delta)$.

EZ diffusion – inverse

Inverse EZ equations:

$$
\begin{aligned}
\delta^{\text{est}} &= \operatorname{sgn}\left(R^{\text{obs}} - \frac{1}{2}\right) \sqrt[4]{\frac{L\left({R^{\text{obs}}}^2 L - R^{\text{obs}} L + R^{\text{obs}} - \frac{1}{2}\right)}{V^{\text{obs}}}} && (4) \\
\alpha^{\text{est}} &= \frac{L}{\delta^{\text{est}}} && (5) \\
\tau^{\text{est}} &= M^{\text{obs}} - \left(\frac{\alpha^{\text{est}}}{2\delta^{\text{est}}}\right)\left[\frac{1 - \exp\left(-\delta^{\text{est}}\alpha^{\text{est}}\right)}{1 + \exp\left(-\delta^{\text{est}}\alpha^{\text{est}}\right)}\right]. && (6)
\end{aligned}
$$

with $L = \log\left(\frac{R^{\text{obs}}}{1-R^{\text{obs}}}\right)$.

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There is a solution that requires a bit of statistical manipulation,

EZ diffusion – sampling distributions

If N observations are drawn from a diffusion model whose accuracy rate is R^{pred}, then the sampling distribution of the observed number of correct trials $T^{\text{obs}} = N \times R^{\text{obs}}$ is:

$$T^{\text{obs}} \sim \text{Binomial}\left(R^{\text{pred}}, N\right). \tag{7}$$

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And the sampling distribution of the variance of the RTs follows this probability law:

$$V^{\text{obs}} \sim \text{Gamma}\left(\frac{N-1}{2}, \frac{2V^{\text{pred}}}{N-1}\right). \tag{9}$$

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Now we can once again apply our parameter decomposition techniques to cognitive model distributions:

$$\delta_{pc} = \mu + \beta \text{Condition}_c + \varepsilon_p$$

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(Except now the computation takes seconds instead of days.)

Stroop Stroop

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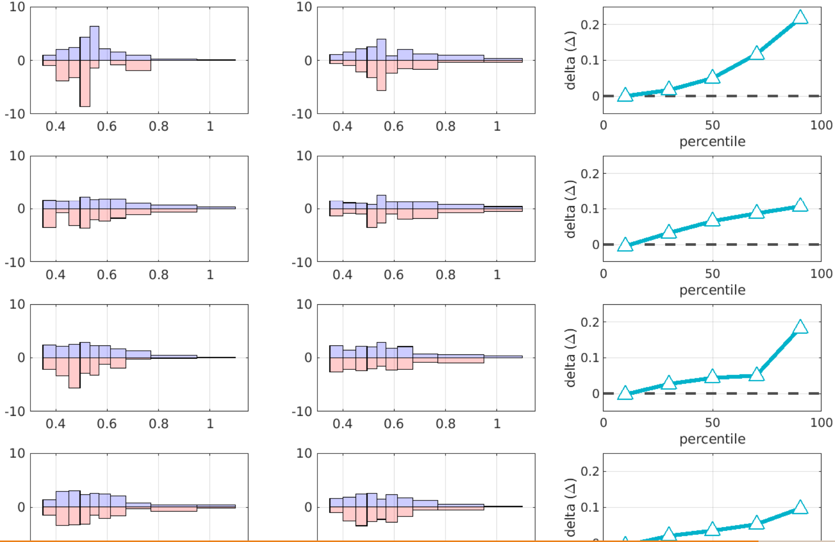
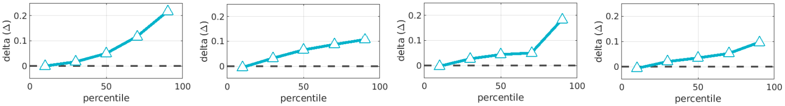
green	green
red	red
orange	orange
blue	blue

Stroop Stroop



Stroop Stroop — Ability δ

Parameter	Posterior			95% Cr. Int.		Rhat		ESS
	Mean	Median	SD	Lower	Upper	Point	Upper	
delta[1]	1.540	1.542	0.106	1.332	1.745	1.005	1.012	1419
delta[2]	0.991	0.993	0.132	0.725	1.245	1.002	1.008	2997
delta[3]	1.407	1.407	0.106	1.207	1.619	1.001	1.005	1619
delta[4]	1.004	1.005	0.097	0.809	1.196	1.002	1.009	2197
delta[5]	1.508	1.510	0.129	1.254	1.752	1.000	1.002	1927
delta[6]	1.282	1.287	0.113	1.048	1.499	1.005	1.010	2008
delta[7]	1.452	1.453	0.131	1.188	1.700	1.001	1.006	2061
delta[8]	1.228	1.228	0.109	1.011	1.437	1.002	1.004	1805



Stroop Stroop — Nondecision Time τ

	Posterior			95% Cr. Int.		Rhat		
Parameter	Mean	Median	SD	Lower	Upper	Point	Upper	ESS
tau[1]	0.178	0.179	0.029	0.117	0.232	1.004	1.015	1917
tau[2]	0.389	0.390	0.020	0.347	0.426	1.002	1.008	1888
tau[3]	0.321	0.323	0.028	0.264	0.372	1.003	1.006	1966
tau[4]	0.313	0.315	0.035	0.241	0.378	1.005	1.018	2108
tau[5]	0.316	0.317	0.020	0.274	0.352	1.004	1.015	1968
tau[6]	0.297	0.298	0.026	0.244	0.345	1.001	1.002	1990
tau[7]	0.344	0.345	0.019	0.306	0.378	1.003	1.011	2048
tau[8]	0.275	0.276	0.028	0.218	0.326	1.001	1.002	2061

Stroop Stroop — Caution α

	Posterior			95% Cr. Int.		Rhat		
Parameter	Mean	Median	SD	Lower	Upper	Point	Upper	ESS
alpha[1]	1.513	1.510	0.064	1.396	1.646	1.010	1.032	997
alpha[2]	1.039	1.037	0.033	0.980	1.107	1.002	1.008	1445
alpha[3]	1.408	1.404	0.052	1.313	1.517	1.002	1.005	1181
alpha[4]	1.461	1.458	0.047	1.377	1.562	1.007	1.023	1313
alpha[5]	1.145	1.144	0.040	1.072	1.230	1.002	1.007	1231
alpha[6]	1.301	1.299	0.044	1.221	1.393	1.002	1.008	1155
alpha[7]	1.080	1.078	0.036	1.015	1.156	1.003	1.009	1323
alpha[8]	1.333	1.331	0.045	1.251	1.423	1.001	1.003	1281

Steps of a diffusion modeling study

Suppose you are asked to determine if some experimental manipulation Q_c has an effect on information processing speed or nondecisional components, and if the effect is different between participant groups A and B.

a) Make some delta plots of the data, and visually inspect the patterns

b) Construct and run a model that has parameters for the relevant effect sizes

$$\delta_{pc} = \mu + \beta_1 Q_c + \beta_2 B_p + \beta_3 Q_c B_p + \varepsilon_p$$

c) Thoroughly evaluate the convergence of the model with tables and figures

d) Characterize the posterior distributions

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