Identifiability

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Recap: What is Non-Identifiability?

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Structural Non-Identifiability: Inherent in model equations.

Consequences:

- Poor MCMC convergence ($\hat{R} \gg 1$, low ESS).
- Unreliable posterior distributions (drifting, odd shapes).
- Strong correlations between confounded parameters.
- Misleading inferences.

1

Example: Hierarchical SDT Model

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Hierarchical Parameter Models (Non-Identified Version):

$$
\begin{aligned}
d'_{ip} &= \alpha_p + \gamma_i \\
\gamma_i &= \zeta_0 + \zeta_1 \cdot \text{predictor}_i && \text{(Condition effect on } d') \\
\alpha_p &\sim \text{Normal}(\mu_\alpha, \sigma_\alpha^2) && \text{(Person sensitivity intercept)} \\
c_p &\sim \text{Normal}(\mu_c, \sigma_c^2) && \text{(Person criterion)}
\end{aligned}
$$

Spotting the Non-Identifiability

The core issue lies in the definition of sensitivity d'_{ip}:

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d'_{ip} &= \alpha_p + \gamma_i \\
&= \alpha_p + (\zeta_0 + \zeta_1 \cdot \mathsf{predictor}_i) \\
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Let $\tilde{\alpha}_p = \alpha_p - C$ and $\tilde{\zeta}_0 = \zeta_0 + C$.

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This is structural non-identifiability.

Fitting the non-identified model is expected to show:

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- **Sampling Difficulties**: Potential warnings (divergences, etc.).

Any or all of these can occur.

4

PyMC Code (Non-Identified SDT)

Key parameter definitions causing the issue:

```
1 # Uninformative priors for Sensitivity (d') parameters
2 mu_alpha = pm.Normal("mu_alpha", mu=0.0, sigma=1.0e9) # << Part 1
3 sigma_alpha = pm.HalfNormal("sigma_alpha", sigma=1.0e9)
4 zeta0 = pm.Normal("zeta0", mu=0.0, sigma=1.0e9)         # << Part 2
5 zeta1 = pm.Normal("zeta1", mu=0.0, sigma=1.0e9)
6
7 # Person sensitivity intercepts (drawn from mu_alpha)
8 alpha_p = pm.Normal("alpha_p", ...)
9
10 # Calculate d' using alpha_p and gamma_i
11 d_ip = pm.Deterministic("d_ip", alpha_p[...] + gamma_i) # << Part 3
12
13 # ... SDT calculations ...
```

Possible Diagnosis 1: Trace Plots

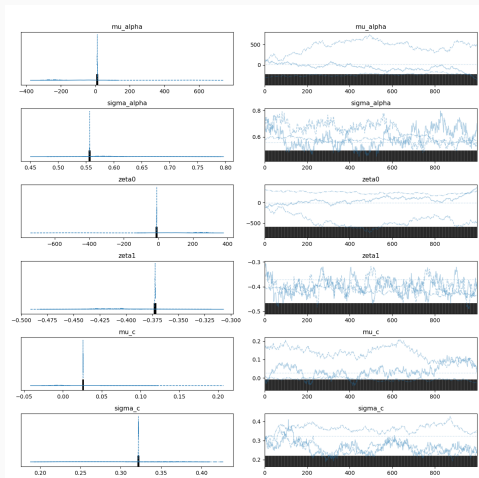Use 'arviz.plot_trace()' focusing on 'mu_alpha' and 'zeta0'.

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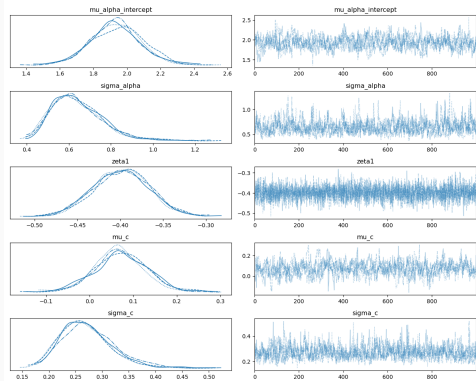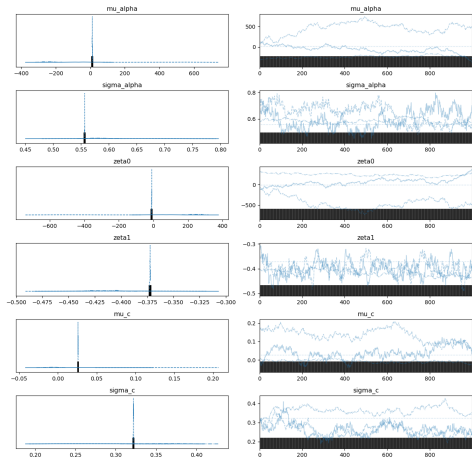Use 'arviz.plot_trace()' focusing on 'mu_alpha' and 'zeta0'.

Look For:

- Poor mixing for 'mu_alpha', 'zeta0'.

- Chains exploring different levels.

- Compare to well-behaved traces (e.g., 'mu_c').

- Corroborate with high $\hat{R}$ in summary.

For comparison, here are trace plots of the fixed (identified) model:

MCMC Output Table

```
    --- Non-Identified SDT Model Summary ---
                   mean      sd     hdi_3%   hdi_97%  mcse_mean  mcse_sd  ess_bulk  ess_tail  r_hat
mu_alpha          40.779  268.443 -304.985  570.827    128.271   70.199       5.0      17.0   2.70
zeta0            -38.809  268.503 -569.032  307.068    128.298   70.209       5.0      17.0   2.70
zeta1             -0.397    0.028   -0.446   -0.350      0.008    0.001      15.0     180.0   1.22
mu_c               0.048    0.058   -0.020    0.173      0.027    0.014       5.0      26.0   2.35
Sampling 4 chains for 1_500 tune and 1_000 draw iterations (6_000 + 4_000 draws total) took 379 seconds.
There were 1000 divergences after tuning. Increase 'target_accept' or reparameterize.


    --- Identified SDT Model Summary ---
                      mean     sd   hdi_3%  hdi_97%  mcse_mean  mcse_sd  ess_bulk  ess_tail  r_hat
mu_alpha_intercept   1.936  0.149    1.659    2.221      0.008    0.004     380.0     598.0   1.01
zeta1               -0.398  0.033   -0.463   -0.341      0.001    0.000    3558.0    3080.0   1.00
mu_c                 0.072  0.062   -0.046    0.186      0.003    0.002     426.0     735.0   1.01
Sampling 4 chains for 1_500 tune and 1_000 draw iterations (6_000 + 4_000 draws total) took 12 seconds.
```
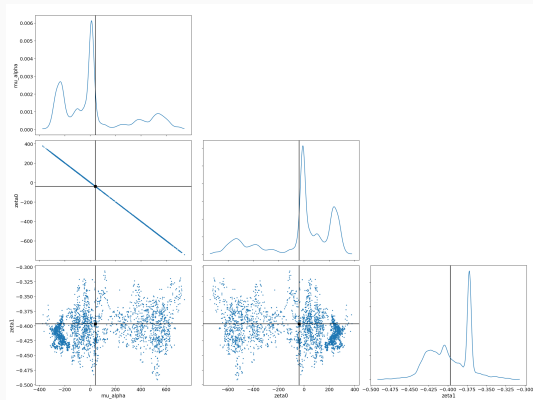
Possible Diagnosis 2: Pair Plots

Use 'arviz.plot_pair()' for sensitivity parameters 'mu_alpha', 'zeta0'.

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Look For:

- Strong linear correlation "ridge."

- Specifically, a negative correlation between 'mu_alpha' and 'zeta0'.

- Poorly conditioned marginal densities.

Fixing the Non-Identifiability

Common strategies:

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a) Reparameterization (Preferred):

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- Use contrasts relative to a baseline condition.

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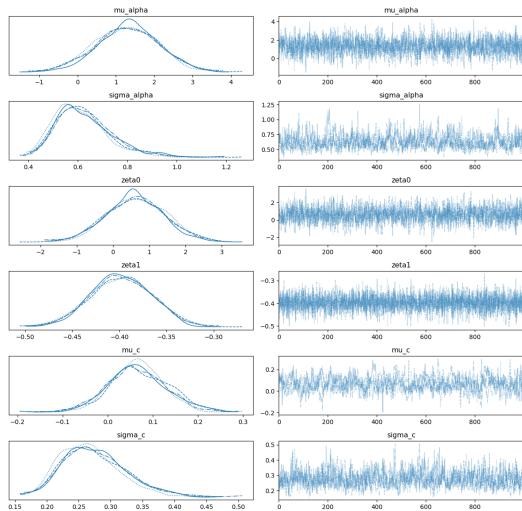Let's see what informative priors can do...

Using Informative Priors

Setting $\mu_\alpha \sim N(0, 1.5)$ gives converging chains:

```
    --- Semi-Identified SDT Model Summary ---
        mean    sd  hdi_3%  hdi_97%  mcse_mean  mcse_sd  ess_bulk  ess_tail  r_hat
mu_alpha  1.301  0.827  -0.310   2.807     0.018    0.013    2023.0    2365.0   1.00
zeta0     0.629  0.825  -0.924   2.183     0.018    0.013    2002.0    2339.0   1.00
zeta1    -0.398  0.033  -0.459  -0.338     0.001    0.001    3915.0    2744.0   1.00
mu_c      0.067  0.064  -0.051   0.190     0.003    0.002     480.0     720.0   1.01
```
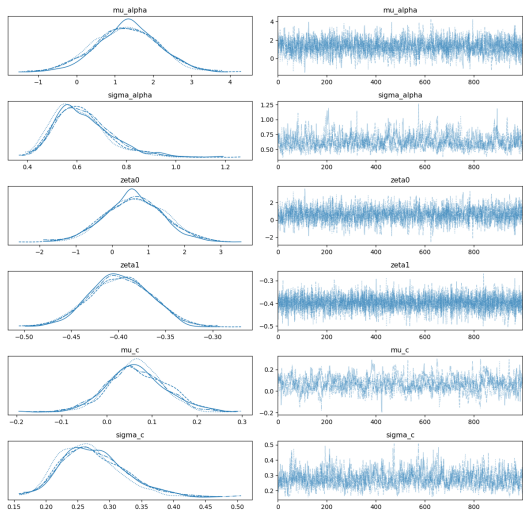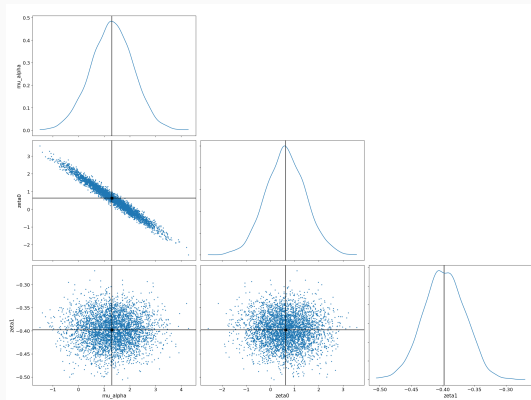
Even reasonable trace plots:

Using Informative Priors

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But the trade-off pops out of the pair plot:

Applying the Real Fix: Remove ζ_0

Interpretation changes: μ_α becomes 'mu_alpha_int', average d' when predictor=0.

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Interpretation changes: μ_α becomes 'mu_alpha_int', average d' when predictor=0.

```python
1  # Priors for Sensitivity (d') parameters
2  mu_alpha_int = pm.Normal("mu_alpha_int", ...) # Was mu_alpha
3  sigma_alpha = pm.HalfNormal("sigma_alpha", ...)
4  # zeta0 = pm.Normal("zeta0", ...) # <<< REMOVED
5  zeta1 = pm.Normal("zeta1", ...)
6
7  # Person sensitivity intercepts (drawn from mu_alpha_intercept)
8  alpha_p = pm.Normal("alpha_p", ...)
9
10 # Condition Effects on Sensitivity (NO intercept)
11 gamma_i = pm.Deterministic("gamma_i", zeta1 * cond_pred_data)
12
13 # SDT likelihood calculations are structurally identical
14 # ...
```

Results After Fixing
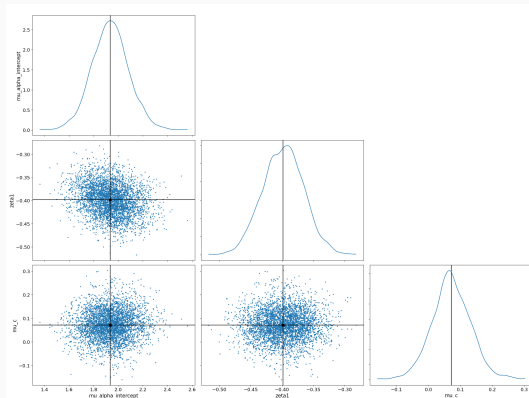
The identified model should exhibit good MCMC behavior.

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Expected Results:

- Good convergence ($\hat{R} \approx 1$).

- High Effective Sample Sizes (ESS).

- Well-mixed, stable trace plots.

Now there should also be no problematic correlations involving 'mu_alpha_int'.

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Always check for potential identifiability issues before interpreting model parameters. A model with convergence issues will generally not yield reliable inferences!

15

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