Executive Summary

In this report, `mtcars` data set will be analyzed, to explore the relationship between a set of variables and Miles Per Gallon(MPG). The data was extracted from the 1974 *Motor Trend* US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32 automobiles (1973–74 models). Regression models and exploratory data analyses are used to mainly explore how **automatic** (am = 0) and **manual** (am = 1) transmissions features affect the **MPG** feature. T-test shows that the performance difference between cars with automatic and manual transmission. And it is about 7 MPG more for cars with manual transmission than those with automatic transmission. Then, several linear regression models are fitted and one with highest Adjusted R-squared value is selected. So, given that weight and 1/4 mile time are held constant, manual transmitted cars are 14.079 + (-4.141)*weight more MPG (miles per gallon) on average better than automatic transmitted cars. Thus, cars that are lighter in weight with a manual transmission and cars that are heavier in weight with an automatic transmission will have higher MPG values.

Exploratory Data Analysis

First, loading data set `mtcars` and change some variables from `numeric` class to `factor` class.

```
library(ggplot2)
data(mtcars)
mtcars[1:3, ] # Sample Data
```

```
##                mpg cyl disp  hp drat    wt  qsec vs am gear carb
## Mazda RX4     21.0   6  160 110 3.90 2.620 16.46  0  1    4    4
## Mazda RX4 Wag 21.0   6  160 110 3.90 2.875 17.02  0  1    4    4
## Datsun 710    22.8   4  108  93 3.85 2.320 18.61  1  1    4    1
```

```
dim(mtcars)
```

```
## [1] 32 11
```

```
mtcars$cyl <- as.factor(mtcars$cyl)
mtcars$vs <- as.factor(mtcars$vs)
mtcars$am <- factor(mtcars$am)
mtcars$gear <- factor(mtcars$gear)
mtcars$carb <- factor(mtcars$carb)
attach(mtcars)
```

```
## The following object is masked from package:ggplot2:
##
##     mpg
```

Next, some basic exploratory data analyses is done. Please refer to the **Appendix: Figures** section for the plots. According to the box plot, it can be seen that manual transmission yields higher values of MPG in general. And as for the pair graph, it can be seen that some higher correlations between variables like “wt”, “disp”, “cyl” and “hp”.

Inference

At this step, null hypothesis is made as the MPG of the automatic and manual transmissions are from the same population (assuming the MPG has a normal distribution). Two sample T-tests are used to show it.

```
result <- t.test(mpg ~ am)
result$p.value
```

```
## [1] 0.001373638
```

```r
result$estimate
```

```
## mean in group 0 mean in group 1
##        17.14737        24.39231
```

Since the p-value is 0.00137, null hypothesis is rejected. So, the automatic and manual transmissions are from different populations. And the mean for MPG of manual transmitted cars is about 7 more than that of automatic transmitted cars.

Regression Analysis

First, the full model is being fit as the following.

```r
fullModel <- lm(mpg ~ ., data=mtcars)
summary(fullModel) # results hidden
```

This model has the Residual standard error as 2.833 on 15 degrees of freedom. And the Adjusted R-squared value is 0.779, which means that the model can explain about 78% of the variance of the MPG variable. However, none of the coefficients are significant at 0.05 significant level.

Then, backward selection is used to select some statistically significant variables.

```r
stepModel <- step(fullModel, k=log(nrow(mtcars)))
summary(stepModel) # results hidden
```

This model is "mpg ~ wt + qsec + am". It has the Residual standard error as 2.459 on 28 degrees of freedom. And the Adjusted R-squared value is 0.8336, which means that the model can explain about 83% of the variance of the MPG variable. All of the coefficients are significant at 0.05 significant level.

Please refer to the **Appendix: Figures** section for the plots again. According to the scatter plot, it indicates that there appear to be an interaction term between "wt" variable and "am" variable, since automatic cars tend to weigh heavier than manual cars. Thereby, following model including the interaction term is generated:

```r
amIntWtModel<-lm(mpg ~ wt + qsec + am + wt:am, data=mtcars)
summary(amIntWtModel) # results hidden
```

This model has the Residual standard error as 2.084 on 27 degrees of freedom. And the Adjusted R-squared value is 0.8804, which means that the model can explain about 88% of the variance of the MPG variable. All of the coefficients are significant at 0.05 significant level, which is pretty good.

Next, the simple model is fitted with MPG as the outcome variable and Transmission as the predictor variable.

```r
amModel<-lm(mpg ~ am, data=mtcars)
summary(amModel) # results hidden
```

It shows that on average, a car has 17.147 mpg with automatic transmission, and if it is manual transmission, 7.245 mpg is increased. This model has the Residual standard error as 4.902 on 30 degrees of freedom. And the Adjusted R-squared value is 0.3385, which means that the model can explain about 34% of the variance of the MPG variable. The low Adjusted R-squared value also indicates that other variables should be added to the model.

Finally, the final model is selected.

```r
anova(amModel, stepModel, fullModel, amIntWtModel)
confint(amIntWtModel) # results hidden
```

Model with the highest Adjusted R-squared value is selected, "mpg ~ wt + qsec + am + wt:am".

```r
summary(amIntWtModel)$coef
```

```
##               Estimate Std. Error   t value     Pr(>|t|)
## (Intercept)  9.723053  5.8990407  1.648243 0.1108925394
## wt          -2.936531  0.6660253 -4.409038 0.0001488947
## qsec         1.016974  0.2520152  4.035366 0.0004030165
## am1         14.079428  3.4352512  4.098515 0.0003408693
## wt:am1      -4.141376  1.1968119 -3.460340 0.0018085763
```

Thus, the result shows that when "wt" (weight lb/1000) and "qsec" (1/4 mile time) remain constant, cars with manual transmission add 14.079 + (-4.141)*wt more MPG (miles per gallon) on average than cars with automatic transmission. That is, a manual transmitted car that weighs 2000 lbs have 5.797 more MPG than an automatic transmitted car that has both the same weight and 1/4 mile time.
Residual Analysis and Diagnostics
Please refer to the **Appendix: Figures** section for the plots. According to the residual plots, the following underlying assumptions can be varified:
1. The Residuals vs. Fitted plot shows no consistent pattern, supporting the accuracy of the independence assumption.
2. The Normal Q-Q plot indicates that the residuals are normally distributed because the points lie closely to the line.
3. The Scale-Location plot confirms the constant variance assumption, as the points are randomly distributed.
4. The Residuals vs. Leverage argues that no outliers are present, as all values fall well within the 0.5 bands.

As for the Dfbetas, the measure of how much an observation has effected the estimate of a regression coefficient, this is the following result:

```r
sum((abs(dfbetas(amIntWtModel)))>1)
```

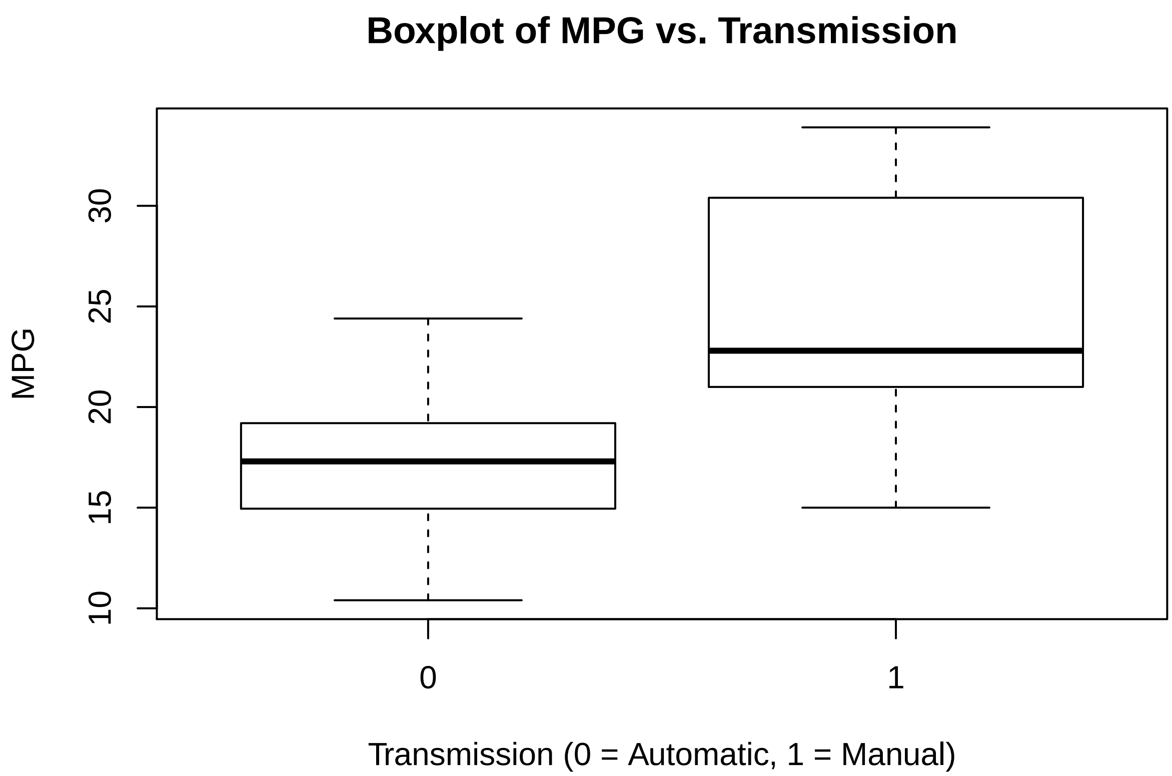
```
## [1] 0
```

Therefore, the above analyses meet all basic assumptions of linear regression and well answer the questions.

Appendix: Figures

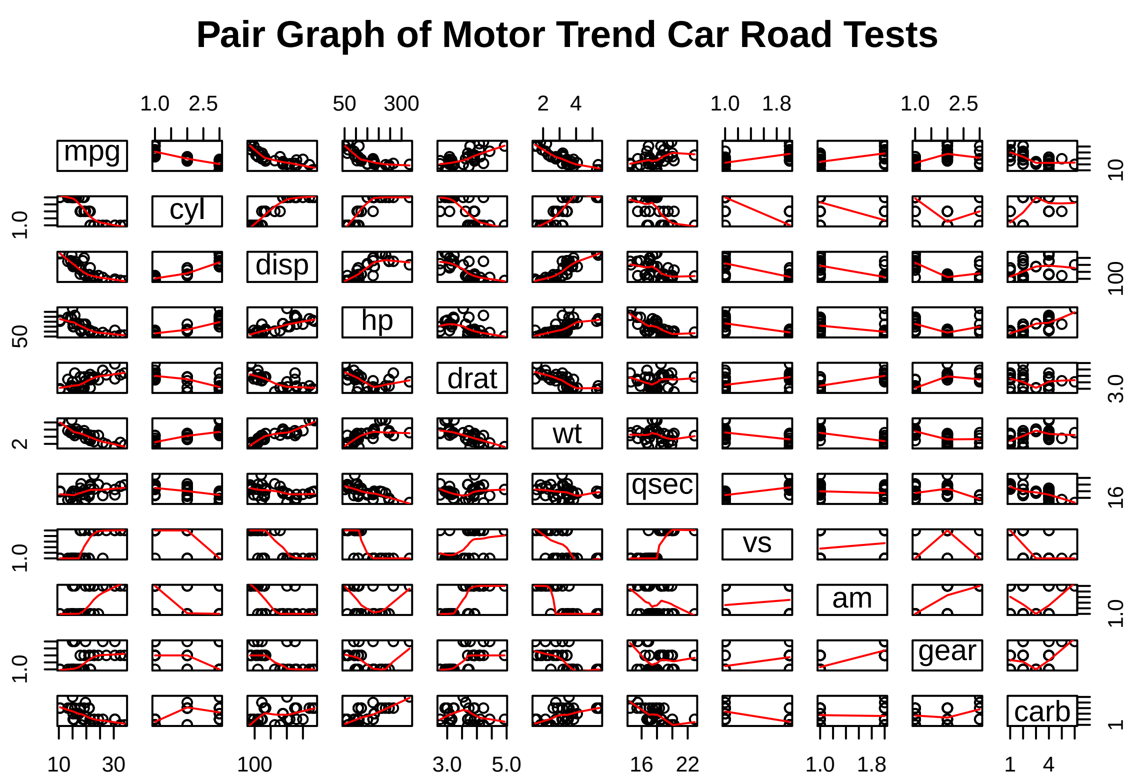
1. Boxplot of MPG vs. Transmission

```r
boxplot(mpg ~ am, xlab="Transmission (0 = Automatic, 1 = Manual)", ylab="MPG",
        main="Boxplot of MPG vs. Transmission")
```



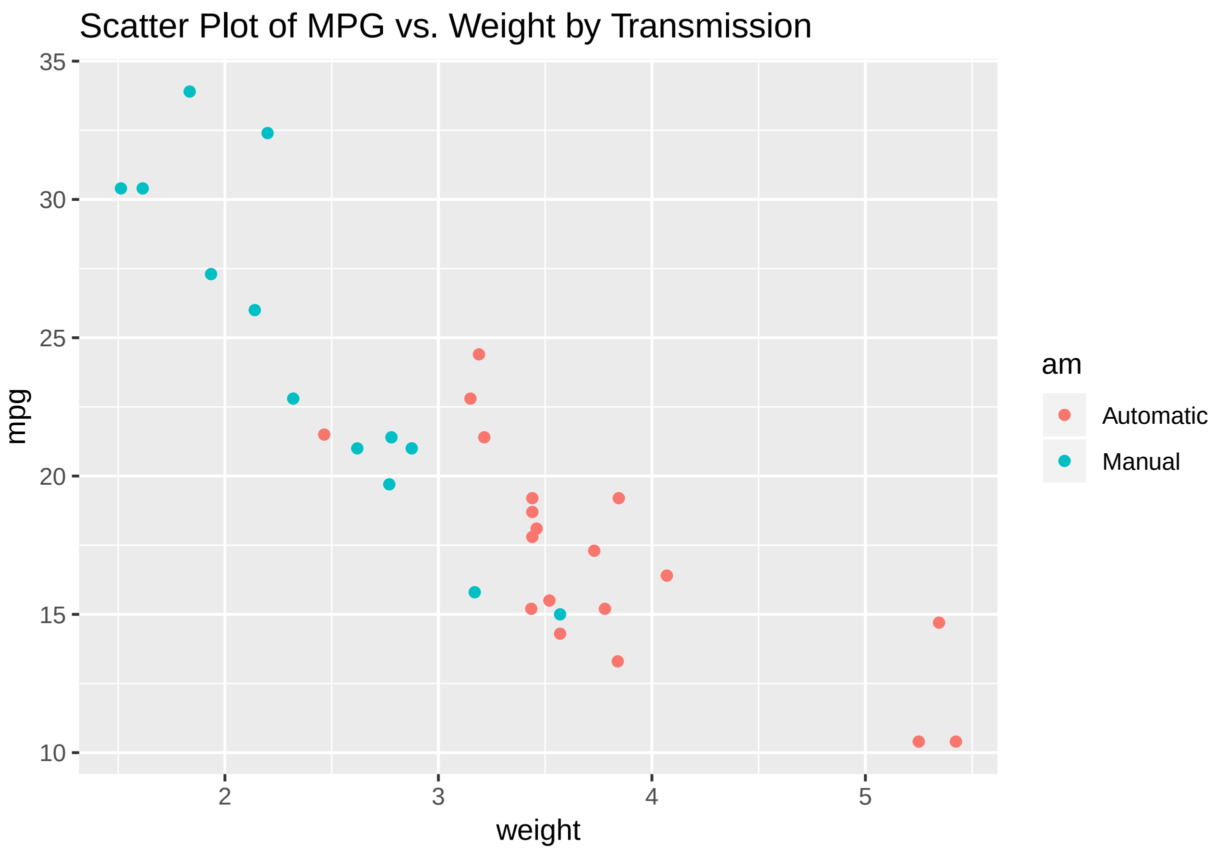
2. Pair Graph of Motor Trend Car Road Tests

```
pairs(mtcars, panel=panel.smooth, main="Pair Graph of Motor Trend Car Road Tests")
```



3. Scatter Plot of MPG vs. Weight by Transmission

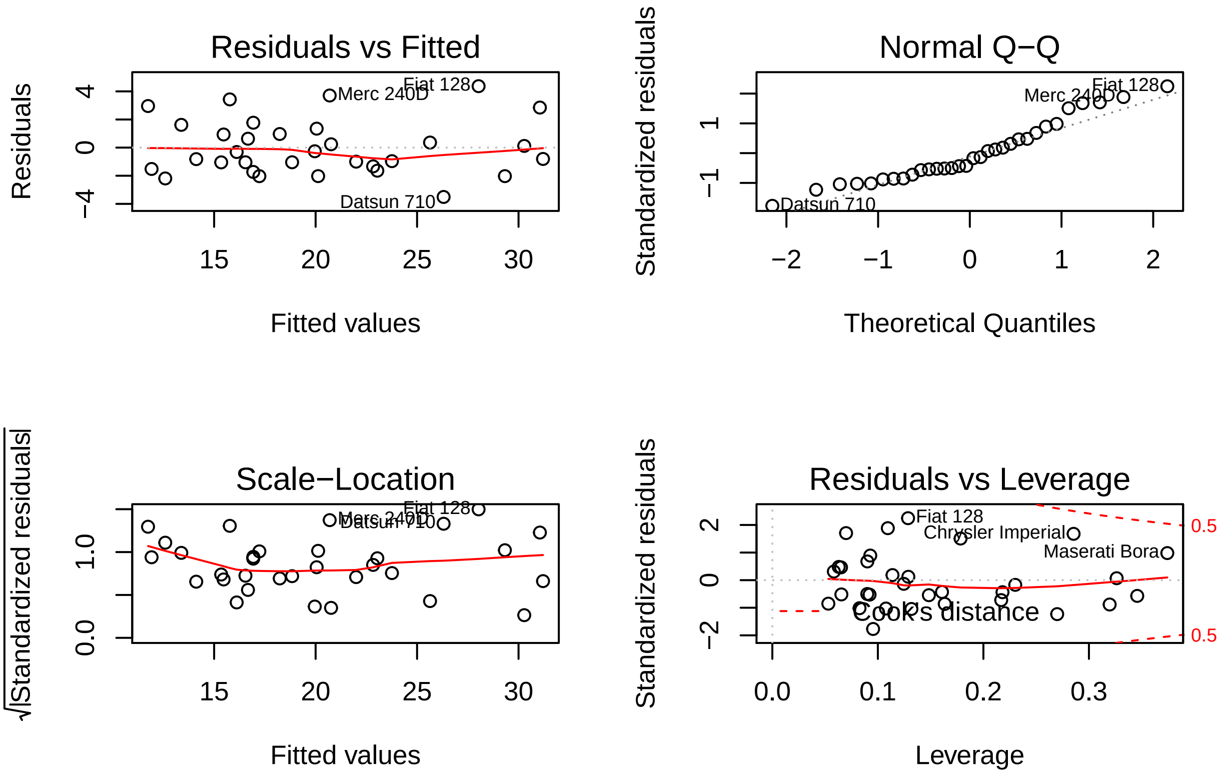
```r
ggplot(mtcars, aes(x=wt, y=mpg, group=am, color=am, height=3, width=3)) + geom_point() +
scale_colour_discrete(labels=c("Automatic", "Manual")) +
xlab("weight") + ggtitle("Scatter Plot of MPG vs. Weight by Transmission")
```



4.

Residual Plots

```r
par(mfrow = c(2, 2))
plot(amIntWtModel)
```


Residuals vs Fitted
Residuals
Fitted values
Merc 240D
Fiat 128
Datsun 710
Normal Q−Q
Standardized residuals
Theoretical Quantiles
Scale−Location
√|Standardized residuals|
Residuals vs Leverage
Chrysler Imperial
Maserati Bora
Cook's distance
Leverage
0.5