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New consensus measure for group decision-making based on Spearman's correlation coefficient for reciprocal fuzzy preference relations

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ABSTRACT

In group decision-making, reaching to a level of agreement about the decision between the group members is important. Indeed, it is considered as the main challenge to be solved in any group decision-making problem. Typically, similarity and distance measures are used to measure the degree of similarities of preference intensities in group decision-making. In this paper, we propose a new consensus measure based on Spearman's correlation coefficient for group decision-makers with reciprocal preference relations. We rely on ranked preference intensities of a decision-maker and find its correlation with respect to other decision-makers in the group. The novelty of this work relies on considering the coherence of decision-maker preference degree ranks as a whole rather than in a pairwise manner. This work does not rely directly on similarity/distance functions, as do most of the consensus models in the literature, but rather adopts the idea of measuring the monotonic degree among them. In addition, based on this model, we propose a feedback mechanism to act as a mediator to guide the group into the consensus solution. We illustrate the new model by presenting a numerical example. Moreover, the model results show robustness when validated on several problems.

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1. Introduction

In group decision-making, reaching a level of agreement about the decision between the group members is important, even if each member has different goals or objectives about the alternatives. In fact, reaching consensus along with aggregation function and ranking method are considered as the main open-ended research problems in group decision-making [1]. The consensus is the main goal in group decision-making problems, since obtaining an acceptable solution by the group is important.

Therefore, it is very important to measure the consensus degree between the individuals in the group to find the degree of agreement among them. The consensus in group decision-making can be interpreted in three ways [2]. It could mean full agreement or unanimous decision by the group members or reach consensus by a moderator who facilitates the process of agreement, or it could mean attaining a consent in which some individuals might not completely agree but are willing to go with the opinion of the group.

Generally, two processes are employed in group decision-making: consensus and selection. The selection process could be applied without adopting a consensus process by applying the preference relations provided by the decision-makers [3]. However, this could lead to a solution that might not be accepted by some of the decision-makers since it does not reflect their preferences [4,5]. Therefore, they might reject the solution. Thus, it is important to reach a consensus before applying the selection process [6].

Generally, a consensus is an interactive and iterative process that is composed of a number of rounds where decision-makers in each round are asked to revise their preferences to reach a consensus level by a facilitator or moderator. The facilitator's role is to gather all the information from the experts and apply some consensus measures to check if the group has reached a state of agreement or not. Therefore, the main step in the consensus process is to measure the consensus degrees of the experts. In fact, it is hard and inconvenient to reach full and unanimous agreement among all the experts in the group, besides, a full agreement is not always necessary in practice. A soft consensus has been developed, which does not require a full agreement among the experts, relies mainly on consensus measure [7]. In the field of decision-making theory, mostly similarity/distance measures are used to measure the consensus among the individuals in the group. If the similarity

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degree is greater than or equal to a pre-defined threshold, then the group has reached a consensus state. However, as mentioned in a number of papers such as Pérez et al. [8], Cabrerizo et al. [9], and Herrera-Viedma et al. [2], developing a new consensus measure is beneficial to strengthen the field. A study done by Chiclana et al. [10] compares five different similarity/distance measures of consensus in group decision-making, namely, Manhattan, Euclidean, Cosine, Dice, and Jaccard. They found that different similarity/distance measures could generate significantly different results. Moreover, the chosen measure could affect the speed of convergence to consensus. Furthermore, sometimes similarity/distance functions do not correctly reflect the agreement among the experts. For example, if a decision-maker provides his/her preference degrees on the alternatives by shifting (increasing/decreasing) other decision-maker preference degrees by 0.05, then the similarity/distance function will show that both decision-makers are not fully in agreement even though both prefer the same alternatives but with different intensities.

Recently González-Arteaga et al. [11] proposed a new consensus measure based on Pearson correlation for reciprocal preference relations. They introduced a correlation consensus degree to measure the agreement degree between a pair of decision-makers and group consensus degree to measure the group consensus. Their method inherits the advantages of similarity/distance measures. Moreover, it does not require that the two reciprocal preference relations be coincided to show full agreement between the pair of decision-makers. However, the consensus measure does not preserve the information since the correlation consensus degree measure does not maintain the correlation properties. In addition to that, the method does not have a feedback mechanism to guide the group towards a consensus solution.

In this paper, we introduce a new consensus measure based on Spearman's correlation to measure monotonic relations rather than linear relations between decision-makers. Also, we propose a feedback mechanism, which acts as a mediator, to help bring the decision–makers who are far in their preferences from the group closer to the consensus state. The new consensus measure calculates the rank correlation consensus between each pair of experts. The new measure values range from 1 when the two experts are in full agreement in their preference rankings to a value of −1 when there is total disagreement. The closer the rank correlation is to 1, the more positive correlation between decision-makers' ranked preferences, which means that the preference degree ranks are in the same direction. Conversely, the closer the rank correlation to −1, the more negative correlation between the decision-makers' ranked preferences, which means the preference degree ranks are in the opposite direction.

The new consensus measure will be developed to use with reciprocal preference relations. The advantage of this measure over the similarity/distance functions is that it measures the ranks of preference degrees instead of intensities of preference degrees between a pair of decision-makers. Thus, if both decision-makers have likewise ranked preference degrees despite the values given to the pair of alternatives (preference degrees), then the new measure shows that both decision-makers are fully in consensus unlike similarity/distance measures which calculates consensus based on the values of the intensities. Moreover, the new measure deals with the data qualitatively, while, Pearson correlation measure handles them quantitatively.

The rest of the paper is organized as follows: We present a brief preliminary knowledge on preference relations in section 2. Next, we provide a literature review on consensus in preference relations in section 3. Following this, we present the proposed methodology in section 4. Subsequently, we illustrate the proposed methods by providing examples in section 5. Additionally, we validate the proposed methods in section 6. Finally, we finish with the conclusions and future works in section 7.

2. Preliminary knowledge

Definition 1 [12]: A preference relation R is a binary relation defined on the set X that is characterized by a function $\mu_p : X \times X \to D$, where D is the domain of representation of preference degrees provided by the decision-maker.

Definition 2 [13]: A fuzzy preference relation P on a finite set of alternatives X is represented by a matrix $P = \left(p_{ij}\right)_{n\times n} \subset X \times X$ with:

$$p_{ij} \in [0,1],\ p_{ij} + p_{ji} = 1,\ p_{ii} = 0.5 \quad \forall i,j = 1,\ldots,n.$$

$$P = \left(p_{ij}\right)_{nxn} = \begin{array}{c c} & \begin{array}{cccc} A_1 & A_2 & \cdots & A_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array} & \begin{bmatrix} 0.5 & p_{12} & \cdots & p_{1n} \\ p_{21} & 0.5 & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & 0.5 \end{bmatrix} \end{array}$$

when $p_{ij} > 0.5$ indicates that the expert prefers alternative x_i over alternative x_j; $p_{ij} < 0.5$ indicates that the expert prefers alternative x_j over alternative x_i; $p_{ij} = 0.5$ indicates that the expert is indifference between x_i and x_j, thus, $p_{ii} = 0.5$.

Furthermore, the fuzzy preference relation $P = (p_{ij})_{n \times n}$ is additive consistent if and only if the following additive transitivity is satisfied [14,12,15,16]

$$p_{ij} + p_{jk} = p_{ik} + 0.5 \ \forall ijk = 1, 2, \ldots, n.$$

Definition 3: Let $V^k = \left(p^k_{12}, p^k_{13}, \ldots, p^k_{1n}, p^k_{23}, \ldots, p^k_{(n-1)n}\right)$ be a vector of preference degrees or intensities of decision-maker k of a reciprocal preference relation $P^k = \left(p^k_{ij}\right)_{n \times n}$, such that V^k represents the upper triangular relation with $\frac{n^2-n}{2}$ elements.

This vector represents the upper triangular relation of P^kprovided by decision-maker$k$. We could also define another vector for the lower triangular relation in the same manner. However, that is not necessary since the reciprocal rule guarantees the same results if we apply any of the vectors. Then, we rank the elements of the vector by using true rank scores to get Ranked Vector.

Definition 4: For every preference degree in V^k, there is a true rank score such that the preference degrees are given a score based on their intensity degree such that the largest preference degree is assigned a rank score of 1 and the smallest assigned a rank score of $\frac{n^2-n}{2}$,

$$\begin{aligned} RV^k &= \left(\left(p^k_{12}, o^k_{12}\right), \left(p^k_{13}, o^k_{13}\right), \ldots, \left(p^k_{(n-1)n}, o^k_{(n-1)n}\right)\right) \\ &= \left(o^k_{12}, o^k_{13}, \ldots, o^k_{(n-1)n}\right) \end{aligned}$$

where o^k_{ij} represents the true rank $\left(e.g: 1, 2, \ldots, \frac{n^2-n}{2}\right)$ of p^k_{ij} among the elements of vector V^k.

Moreover, the Ranked Vector of the upper triangular relation (RV) and the Ranked Vector of the lower triangular relation (RV_*) of P have the following property:

$$o_{ij} + c_{ji} = \frac{n^2 - n + 2}{2} \tag{1}$$

where o_{ij} is the true rank on (RV) and o_{ji} is the true rank on (RV_*).

2.1. Spearman's rank correlation coefficient

The Spearman's rank correlation is a special case of Pearson correlation coefficient (Chen, Smithson & Popovich)[17]. Spearman's correlation measures the degree of monotonic relation between vector X and vector Y, while Pearson correlation measures only the linear relationship [18–20]. Pearson correlation treats real data in a quantitative way, whereas Spearman's correlation treats them to some extent in a qualitative way [19].

Spearman's correlation has the following properties [18]:

(1) Symmetry: $\rho(X, Y) = \rho(Y, X)$.
(2) Normalization: $\rho(X, Y) \in [-1, 1]$.
(3) Comonotonic: $\rho(X, Y) = 1 \Leftrightarrow X, Y$.
(4) Countermonotonic: $\rho(X, Y) = -1 \Leftrightarrow X, Y$.
(5) For X strictly monotonic: $\rho(X, Y) = \begin{cases} \rho(X, Y) & X \textit{ increasing} \\ -\rho(X, Y) & X \textit{ decreasing} \end{cases}$
(6) Uncorrelated (independent): $\rho(X, Y) = 0 \Leftrightarrow X, Y$.

The Spearman's correlation is a simplification of the Pearson correlation coefficient on the rank. The Pearson correlation coefficient on the rank is defined as follows:

Definition 5: Given a sample of two n – dimensional vectors $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$, then for each vector, the variables are given true rank scores such that the largest variable is assigned a score of 1 and the smallest is assigned a score of $\frac{n^2-n}{2}$. Thus, having RV of the vectors x and y, $\left\{\left(o^x_1, o^y_1\right), \ldots, \left(o^x_{\frac{n^2-n}{2}}, o^y_{\frac{n^2-n}{2}}\right)\right\}$, the *Pearson correlation coefficient on the rank* is computed as

$$cor(\boldsymbol{x}, \boldsymbol{y}) = \rho^p = \frac{\sum_i^m \left(o^x_i - \bar{o}^x\right)\left(o^y_i - \bar{o}^y\right)}{\sqrt{\sum_i^m \left(o^x_i - \bar{o}^x\right)^2}\sqrt{\sum_i^m \left(o^y_i - \bar{o}^y\right)^2}} \tag{2}$$

where $\bar{o}^x = \frac{1}{m}\sum_i^m o^x_i$ and $\bar{o}^y = \frac{1}{m}\sum_i^m o^y_i$ are the arithmetic means of the true ranked scores, o^x_i and o^y_i are the true ranked of x_i and y_i, respectively.

In our case, *rank correlation consensus(rcc)*measure based on the *Pearson correlation coefficient on the rank of preference relation* is given by:

$$rcc(x,y)=\rho^{p}=\frac{\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{x}-\bar{o}^{x}\right)\left(o_{ij}^{y}-\bar{o}^{y}\right)}{\sqrt{\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{x}-\bar{o}^{x}\right)^{2}}\sqrt{\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{y}-\bar{o}^{y}\right)^{2}}} \tag{2'}$$

where $\bar{o}^{x}=\frac{2}{n^2-n}\sum_{i}^{\frac{n^2-n}{2}}o_{ij}^{x}$ and $\bar{o}^{y}=\frac{2}{n^2-n}\sum_{i}^{\frac{n^2-n}{2}}o_{ij}^{y}$ are the arithmetic means of the true ranked scores, o_{ij}^{x}, and o_{ij}^{y} are the true ranked of RV^{x} and RV^{y}, respectively.

In the presence of tied ranks within a vector, the Pearson correlation coefficient formula is applied (Chen & Popovich, 2002). However, when there are no tied ranks, Spearman's rank correlation coefficient formula is used. To calculate Spearman's rank correlation coefficient, the *Pearson correlation coefficient* is simplified to (Chen, Smithson & Popovich; Kendall & Gibbons)[17,21]:

$$cor(\boldsymbol{x},\boldsymbol{y})=\rho^{s}=1-\frac{6\sum_{i=1}^{m}d_i^2}{m^3-m} \tag{3}$$

where d_i is the difference of the ranking of the two vectors and m is the number of elements or variables of the vector.

Proposition 1: To calculate the *rank correlation consensus*(*rcc*)measure based on Spearman's rank correlation coefficient with no tied ranks for (RV^k) and (RV^h), the following formula is equivalent to (3);

$$rcc^{kh}=\rho^{s}=1-\frac{48\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{k}-o_{ij}^{h}\right)^{2}}{\left(n^3(1-n)^2-4n\right)(1-n)} \tag{4}$$

where o_{ij}^{k} and o_{ij}^{h} are the true ranked of V^k and V^h respectively and n is the number of alternatives.

Proof: from the Ranked Vector (RV^k) and (RV^h), the number of variables is $\frac{n^2-n}{2}$. Thus, $m=\frac{n^2-n}{2}$ by substituting this and $d_i=d_{ij}=o_{ij}^{k}-o_{ij}^{h}$ into (3) we get;

$$\begin{aligned}cor\left(RV^k,RV^h\right)=\rho&=1-\frac{6\sum_{i=1}^{m}d_i^2}{m^3-m}\\&=1-\frac{6\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{k}-o_{ij}^{h}\right)^{2}}{\left(\frac{n^2-n}{2}\right)^{3}-\left(\frac{n^2-n}{2}\right)}\\&=1-\frac{6\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{k}-o_{ij}^{h}\right)^{2}}{\frac{(n^2-n)^3}{8}-\frac{n^2-n}{2}}\\&=1-\frac{6\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{k}-o_{ij}^{h}\right)^{2}}{\frac{(n^2-n)^3-4(n^2-n)}{8}}\\&=1-\frac{48\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{k}-o_{ij}^{h}\right)^{2}}{(n^2-n)^3-4(n^2-n)}\\&=1-\frac{48\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{k}-o_{ij}^{h}\right)^{2}}{\left[(n^2-n)^2-4\right](n^2-n)}\\&=1-\frac{48\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{k}-o_{ij}^{h}\right)^{2}}{\left[n^2(n-1)^2-4\right](n-1)n}\\&=1-\frac{48\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{k}-o_{ij}^{h}\right)^{2}}{\left[n^3(n-1)^2-4n\right](n-1)}\end{aligned}$$

Note: The tied numbers are handled by midrank method. The midrank method is based on averaging the ranks that these tied numbers possess (Kendall & Gibbons, 1990). For instance, if the fifth and sixth numbers are tied, then each is assigned number $5\frac{1}{2}$, and if the third to the seventh are tied, each is assigned the number $\frac{(3+4+5+6+7)}{5}=5$ and thus the next assigned rank number is 8 if it's not tied with others and so on.

Definition 6: The general rule of the midrank method is when there are z elements of tied ranks in V at l^{th} rank position, then the assigned number of each of these is $l+[(z-1)\cdot 0.5]$.

The *rank correlation consensus* (*rcc*) measure has a value range from –1 to 1. The value 1 means that both decision-makers' ranked preferences are the same (both are positively rank correlated). Whereas –1 means that the two decision-makers' ranked preferences are opposite (both are negatively rank correlated). When (*rcc*) equals zero, there is no correlation between the two decision-makers' ranked preferences; both are independent. Ideally, the closer (*rcc*) to 1, the closer the decision-makers' ranked preferences are to the consensus and vice versa.

The rank correlation consensus measure could be mapped to the domain $[0,1]$ to have the rank correlation consensus degree (*rccd*), as in the following definition;

Definition 7: The rank correlation consensus degree $(rccd)$ is a function $f : rcc \rightarrow [0, 1], f(rcc) = rccd = 0.5 \times (1 + rcc)$.

Thus, $rccd$ is interpreted as follows: when $rccd = 0.5$ then the two relations are rank independent where no rank correlation exists between the ranked preferences. For $ccd < 0.5$, the ranks of preference relations are negatively correlated (moving in the opposite direction), whereas for$rccd > 0.5$, the ranked preferences are moving in the same direction.

3. Literature review

Recently Fei et al. [22] introduced vector valued similarity measure for intuitionistic fuzzy sets based on OWA operators to overcome the shortcomings of single-valued measures. Their method was based on a two-dimensional vector consisting of similarity and uncertainty measures, where the uncertainty measure was used to evaluate or measure the performance of the similarity measure. Ureña et al. [23] proposed an open framework source executed in R for fuzzy group decision-making. One of its features is the ability to perform a number of consensus rounds with visualization tools to detect decision-makers who are far from the group and those who are inconsistent in your opinions. Sun and Ma [24] proposed an approach for a consensus measure on linguistic preference relations. They used consensus measure based on the dominance degree to measure the consensus between group preference relation and individuals' preference relations. Cabrerizo et al. [25] introduced a consensus model by relying on granular format on fuzzy preference relation. Their model utilizes the flexibility in opinion from decision-makers that granular fuzzy preference relation offers to increase the level of consensus within the group. They used the level of granularity to increase the consensus within the group and to increment the consistency of individual decision-makers. The model is based on similarity measure uses under population-based stochastic optimization (PSO) technique. Zhang and Dong [26] proposed an interactive consensus reaching process based on optimization consensus rules to increase consensus of individuals and minimize the number of adjustments of adjusted preference values. Guha and Chakraborty [27] introduced an iterative fuzzy multi-attribute group decision-making technique to reach consensus by using a fuzzy similarity measure to measure consensus degree. In addition, their method considers the degrees of confidence of experts' opinions in the procedure. Pérez et al. [28] introduced a mobile prototype for group decision-making. Their model utilizes selection and consensus processes, in addition to dealing with dynamic decision-making, where the set of alternatives could change during the decision-making process. The prototype provides on-time updates to the group along with recommendations to decision-makers who need to modify their preferences to increase consensus level. The decision-makers can use any of four preference representation formats, namely, fuzzy preference relation, multiplicative preference relation, preference ordering and utility values. Mata et al. [29] proposed an adaptive consensus model for group decision-making in multigranular fuzzy linguistic relation. The model implies an adaptive strategy to increase the rate of convergence to a consensus solution. The model distinguishes between three consensus levels: very low, low and medium. Thus, the number of suggestions and preferences needs to be modified depends on the level of consensus. As the level increases, fewer modifications are needed and thus less consensus rounds. Xu [30] proposed an automatic approach for reaching consensus in multi-attribute group decision-making. His approach was based on numerical settings, where each individual constructs a decision matrix. Then, these matrices are aggregated into one group decision matrix. The method calculates the similarity measure between each individual matrix and the group decision matrix to determine the degree of consensus. Moreover, he introduced a convergent iterative algorithm for individual matrices to reach the consensus. Herrera-Viedma et al. [31] introduced an automated consensus model for group decision-making in multigranular fuzzy linguistic preference relations. The model uses two consensus criteria: consensus degree to measure the group consensus and proximity measures to find and detect those decision-makers who are far from the rest of the group in their opinion. The model provides recommendations to decision-makers to increase their consensus contribution and thus the group consensus level. Herrera-Viedma et al. [32] proposed a consensus model suitable for four different preference structures. Their model uses two consensus criteria: a consensus measure for measuring the degree of consensus between the experts and a proximity measure to measure the difference between the preferences of individuals and the group preference relation.

Consensus and consistency measures have been used in the literature lately to guide the consensus process. For example, Herrera-Viedma et al. (2007a) proposed a consensus model based on consensus and consistency measures. They used two consensus measures: consensus degrees, to find the agreement of all experts, and proximity degrees, to find the agreement between the individuals and the group preference. Recently, Cabrerizo et al. [36] proposed a consensus model for group decision-making in an unbalanced fuzzy linguistic setting using consistency and consensus measures. They used three different levels of consensus degrees: consensus degree on pairs of alternatives, consensus degree on alternatives, and consensus degree on the relation, in addition to proximity measures.

4. A new consensus measure based on Spearman's rank correlation coefficient

We propose a new measurement of consensus based on Spearman's rank correlation. This new method utilizes the advantages of rank correlation coefficient among decision-makers' ranked preferences to measure the rank correlation consensus between each pair of experts. The new measurement is suitable for reciprocal preference relations.

The reason for choosing Spearman's correlation over Pearson correlation is that we are interested in general monotonic relations rather than linear relations. Moreover, Spearman's correlation is a kind of qualitative measure whereas Pearson correlation is a quantitative one. In addition, the proposed feedback mechanism relies on the decision-makers' input with respect to others and gives suggestions based on the rank of preference degrees rather than the valuations of other decision-makers. Thus, if the preference intensities among decision-makers vary, it does not matter as long as all the decision-makers have the same or almost the same rank preference degrees. This simply means that all prefer the same alternative but with different degrees. Therefore, trying to bring the preference intensities closer is not our main concern here. Making preference intensities almost the same could face resistance from decision-makers especially if their inputs are far from the original ones. Consequently, focusing on the preference ranks could make decision-makers more willing to accept changes, since this procedure, in general, asks them to rearrange their inputs in such a way that brings them closer to the rest of the decision-makers.

4.1. Rank similarity degree

From Ranked Vector (RV) we could also find the rank similarity degree (rsd) between any pair of decision-makers' preference ranks. This degree shows how far the two preference rankings of a pair of decision-makers are from each other. The rank similarity degree value at 0 means there is no similarity at all and that one of the preference ranks is ranked first and the other is ranked last, while 1 means that both alternatives are ranked at the same position by both decision-makers.

Proposition 2: Given two Ranked Vector RV^k and RV^h, for a pair of decision-makers k and $h \in T = 1, \ldots t$, the rank similarity degree $\left(rsd_{ij}^{kh}\right)$ between each pair of preference ranks of the two decision-makers is given as follows:

$$rsd_{ij}^{kh} = 1 - \left| \frac{2\left(o_{ij}^k - o_{ij}^h\right)}{n(n-1) - 2} \right| \tag{5}$$

This formula has been derived by normalizing the difference of rank to the maximum possible difference in scores as follows: the maximum score a Ranked Vector (RV) could have is $\frac{n^2-n}{2}$ and the minimum is 1. Thus, the difference between the maximum score and the minimum is $\frac{n^2-n-2}{2}$. By dividing the difference of rank by this difference, we get $\frac{2\left(o_{ij}^k - o_{ij}^h\right)}{n(n-1)-2}$. Then, we take the absolute value of this normalization formula to prevent negative difference. Thus, the rank similarity degree is 1 minus the absolute value of the normalization formula.

4.2. Rank correlation consensus algorithm

The proposed consensus measure is based on rank correlation coefficient. This measure uses Spearman's rank correlation coefficient to measure rank correlation consensus coefficient for reciprocal preference relation $P^k = \left(p_{ij}^k\right)_{n \times n}$. To measure rank correlation consensus, the following steps are applied:

(1) For each P^k, $k \in T = \{1, 2, \ldots, t\}$ establish a preference vector $V^k = \left(p_{12}^k, p_{13}^k, \ldots, p_{1n}^k, p_{23}^k, \ldots, p_{(n-1)n}^k\right)$ from the preference relation $\left(P^k\right)$, which is provided by the decision-maker$k$. This vector represents the preference intensities in the upper triangular relation of $\left(P^k\right)$.

(2) Give a true rank (e.g. $1, 2, \ldots, \frac{n^2-n}{2}$) for each element in V^k such that the largest preference degree is allotted score 1 and the smallest is allotted score $\frac{n^2-n}{2}$, then a ranked vector $\left(RV^k\right)$ of $\frac{n^2-n}{2}$ elements can be established as follows:

$$RV^k = \left(o_{12}^k, o_{13}^k, \ldots, o_{(n-1)n}^k\right)$$

where o_{ij}^k represents the true rank of p_{ij}^k with respect to the preference degrees of vector V^k.

(3) For every pair of decision-makers, k and $h \in T$, calculate rank similarity degree $\left(rsd_{ij}^{kh}\right)$ on the vectors RV^k and RV^h:

$$rsd_{ij}^{kh} = 1 - \left| \frac{2\left(o_{ij}^{k} - o_{ij}^{h}\right)}{n(n-1)-2} \right|$$

(4) For every pair of decision-makers k and $h \in T$, calculate the rank correlation consensus coefficient $\left(rcc^{kh} = rcc^{hk}\right)$ using (4) if there is no tied rank or (2′) if the ties exist with adopting the midrank method. The rank correlation consensus, $rcc \in [-1, 1]$, where −1 means strong negative rank correlation between decision-maker k and h, 0 no rank correlation, and 1 strong positive rank correlation. Ideally, the closer rcc is to 1 the better it is. This can be transformed into rank correlation consensus degree by:

$$rccd^{kh} = \frac{1}{2}\left(1 + rcc^{kh}\right) \in [0, 1] \tag{6}$$

(5) Calculate the rank correlation consensus for each decision-maker $e \in T$

$$rcc^{k} = \frac{\sum_{h=1,h\neq k}^{t} rcc^{kh}}{t-1} \tag{7}$$

(6) Calculate experts' rank correlation consensus

$$rcc^{T} = \frac{\sum_{k=1}^{t} rcc^{k}}{t} \tag{8}$$

(7) Calculate collective relation's rank correlation consensus

$$rcc^{c} = \frac{\sum_{k=1}^{t} rcc^{ck}}{t} \tag{9}$$

By applying the rank correlation consensus measure, five types of measures are obtained:

(1) Between individuals' rank correlation consensus rcc^{kh}, which shows the similarity degree of the preference ranks between decision-maker k and h.
(2) Individual rank correlation consensus rcc^{k}, which represents the similarity degree of the preference ranks of decision-maker k to other decision-makers.
(3) Among decision-makers' rank correlation consensus rcc^{T}, which represents the similarity degree of the preferences ranks among them.
(4) Between individuals and collective rank correlation consensus rcc^{ck}, which represents the degree of similarity of the preferences ranks between the collective relation$c$ and the decision-maker k.
(5) Collective relation's rank correlation consensus rcc^{c}, which represents the similarity degree of the preference ranks between individuals and collective preference relation.

Note: Rank correlation consensus degree is not equivalent to consensus degree obtained by similarity/distance functions. In general, rank correlation consensus should be the one to rely on to attain the consensus level.

4.3. Feedback mechanism

The purpose of this mechanism is to help and guide decision-makers to improve their consensus level. The proposed feedback mechanism uses consensus results to help individual experts with low consensus to improve their evaluations and thus their consensus level with regards to other decision-makers. This method uses other decision-makers' assessments, which usually have the best consensus in the group, to generate suggestions to the individuals who have fewer contributions to the consensus state. This feedback mechanism relies heavily on the rank similarity degrees between the experts.

Theorem 1: Let rcc^{kh} be the rank correlation consensus between preference relation of decision-maker$k$ and preference relation of decision-maker h, then increasing rsd_{ij}^{kh} leads to increase rcc^{kh}.

Proof: From $rsd_{ij}^{kh} = 1 - \left|\frac{2\left(o_{ij}^{k} - o_{ij}^{h}\right)}{n(n-1)-2}\right|$, we get $\left|o_{ij}^{k} - o_{ij}^{h}\right| = \left(1 - rsd_{ij}^{kh}\right) \cdot \frac{(n(n-1)-2)}{2}$.

When no ties in ranks,

$$rcc^{kh} = 1 - \frac{48\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{k} - o_{ij}^{h}\right)^{2}}{\left[n^{3}(n-1)^{2} - 4n\right](n-1)}$$

$$= 1 - \frac{48\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(\left(1 - rsd_{ij}^{kh}\right) \cdot \frac{(n(n-1)-2)}{2}\right)^{2}}{\left[n^{3}(n-1)^{2} - 4n\right](n-1)}$$

$$= 1 - \frac{12[(n(n-1)-2)]^{2}}{\left[n^{3}(n-1)^{2} - 4n\right](n-1)} \cdot \sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(1 - rsd_{ij}^{kh}\right)^{2}$$

When there are ties in ranks,

$$rcc^{kh} < 1 - \frac{12[(n(n-1)-2)]^{2}}{\left[n^{3}(n-1)^{2} - 4n\right](n-1)} \cdot \sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(1 - rsd_{ij}^{kh}\right)^{2}$$

Therefore,

$$rcc^{kh} \le 1 - \frac{12[(n(n-1)-2)]^2}{\left[n^3(n-1)^2 - 4n\right](n-1)} \cdot \sum_{i=1}^{n-1} \sum_{j=2, i<j}^{n} \left(1 - rsd_{ij}^{kh}\right)^2$$

Moreover, the rank similarity degree has the following properties:

(1) $rsd_{ij}^{kh} = rsd_{ij}^{hk}$

Proof: It is obvious from the absolute value in(5).

(2) $rsd_{ij}^{kh} = rsd_{ji}^{kh}$

Proof: By substituting $o_{ji} = \frac{n^2-n+2}{2} - o_{ij}$, from(1), into $rsd_{ji}^{kh} = 1 - \left|\frac{2\left(o_{ji}^k - o_{ji}^h\right)}{n(n-1)-2}\right|$, we get

$$\begin{aligned} rsd_{ji}^{kh} &= 1 - \left|\frac{2\left(o_{ji}^k - o_{ji}^h\right)}{n(n-1)-2}\right| \\ &= 1 - \left|\frac{2\left(\frac{n^2-n+2}{2} - o_{ij}^k - \frac{n^2-n+2}{2} + o_{ij}^h\right)}{n(n-1)-2}\right| \\ &= 1 - \left|\frac{2\left(o_{ij}^h - o_{ij}^k\right)}{n(n-1)-2}\right| = rsd_{ij}^{hk} = rsd_{ij}^{kh} \end{aligned}$$

The feedback mechanism can be conducted in two ways depending on the main concern: A) if the rank correlation consensus between decision-makers is the priority. Or B) if the rank correlation consensus of the collective relation is important. Figure 1 shows the flowchart of the consensus process.

A. *Feedback mechanism for rank correlation consensus between decision-makers*:

(1) Select the decision-maker who has the lowest rank correlation consensus $\left(rcc^k\right)$ for him/her to review their judgments.
(2) Once the decision-maker is selected (k), look at his/her rank similarity degrees $\left(rsd_{ij}^{kh}\right)$ with respect to other decision-makers. Find the lowest $\sum_{h=1, h \ne k}^{t} rsd_{ij}^{kh}$ to identify the element of the preference vector to modify and then find which decision-maker has the lowest rank similarity degree with respect to the identified vector element, $\min_{\forall h \in T}\{rsd_{ij}^{kh}\}$. For instance, if we have the following rsd:

$rsd_{13}^{31} = 0.4$, $rsd_{13}^{32} = 0.2$, $rsd_{13}^{34} = 0.6$, and
$rsd_{12}^{31} = 0.9$, $rsd_{12}^{32} = 0.6$, $rsd_{12}^{34} = 0.3$,
then the decision-maker modifies p_{13}^{3} since $\sum_{h=1, h \ne k}^{4} rsd_{13}^{3h} = 1.2 < \sum_{h=1, h \ne k}^{4} rsd_{12}^{3h} = 1.8$,

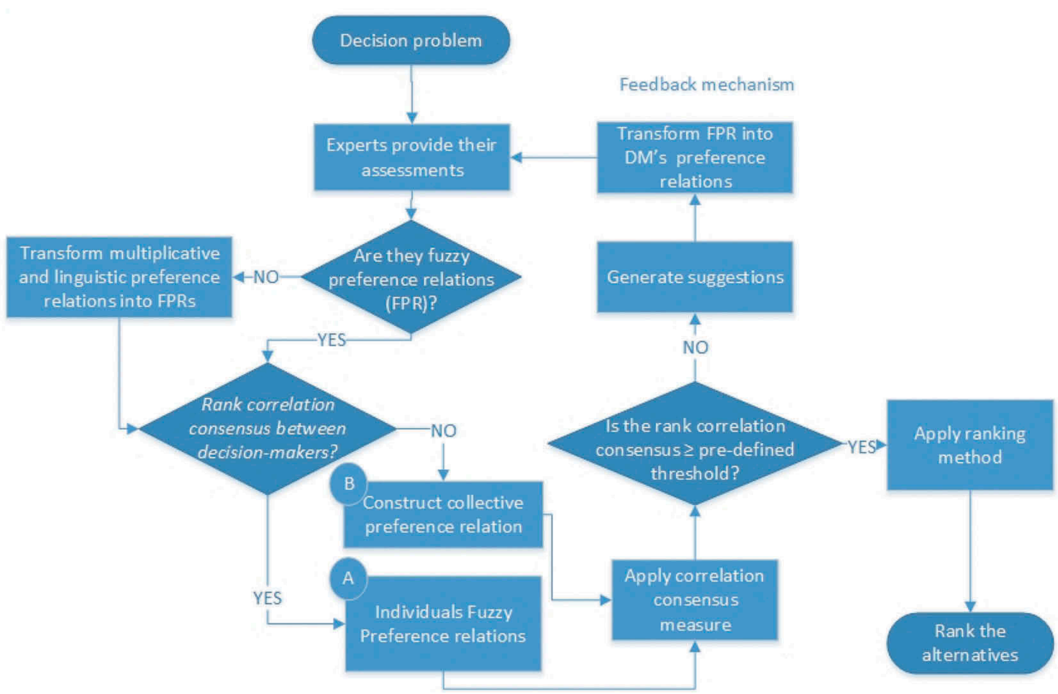

Figure 1. Consensus process.

with respect to o_{13}^2 position since $\min_{\forall h \in T}\{rsd_{13}^{kh}\} = rsd_{13}^{32}$.

(3) Once the preference degree that needs to be changed is known $\left(p_{ij}^k\right)$, make the modification based on the position of the ranked element of decision-maker h who has the lowest rank similarity degree as following:

➢ If $o_{ij}^k > o_{ij}^h \Rightarrow p_{ij}^k \in \left[p^k\left\{o_{ij}^h\right\}, p^k\left\{o_{ij}^h - 1\right\}\right]$;

➢ If $o_{ij}^k < o_{ij}^h \Rightarrow p_{ij}^k \in \left[p^k\left\{o_{ij}^h + 1\right\}, p^k\left\{o_{ij}^h\right\}\right]$

where $p^k\left\{o_{ij}^h\right\}$ is the preference degree value of k at the rank position o_{ij}^h. Moreover,

- If $p^k\left\{o_{ij}^h - 1\right\} = p^k\{0\} \Rightarrow p^k = 1$
- If $p^k\left\{o_{ij}^h + 1\right\} = p^k\{\frac{n^2-n}{2} + 1\} \Rightarrow p^k = 0$
- If $\left\{o_{ij}^h\right\}$ does not exist exactly in k then find where it lies such that p_{ij}^k should fall in the rank between $\left\{o_{ij}^h\right\}$ and $\left\{o_{ij}^h + 1\right\}/\left\{o_{ij}^h - 1\right\}$. For example, if $p^k\left\{o_{ij}^h\right\} = p^k\{1.5\}$ but there is no rank position at 1.5 in k then we could approximate it to $p^k\{1\}$ or $p^k\{2\}$ depending on the other rank position $\left\{o_{ij}^h + 1\right\}/\left\{o_{ij}^h - 1\right\}$.

(4) After the adjustment, recalculate the rank correlation consensus and repeat steps 1–3.

(5) The process is finished when $rcc^T \geq \alpha$ and/or $rcc^{kh} \geq \beta$, where α and β are the agreeable consensus level between the experts and among the pair of experts, respectively.

This feedback mechanism is built to improve rank consensus level of the experts without relying on the collective preference relation. Once the consensus level is attained then collective preference relation could be constructed.

B. *Feedback mechanism for rank correlation consensus on the collective preference relation*:

We could apply the feedback mechanism between the experts and the collective preference relation by modifying step 2 and 5 into:

Step 2′. Identify the lowest rank similarity degree $\left(rsd_{ij}^{ck}\right)$, that has the farthest rank between the rank position o_{ij}^c and o_{ij}^k.

Step 5′. The process is finished when $rcc^c \geq \alpha$ and/or $rcc^{ck} \geq \beta$, where α and β are the agreeable consensus level of the collective relation and between the experts and the collective relation, respectively.

Thus, in this case, we only need to know which preference degree to modify, since k is known from the rank correlation consensus $\left(\min_{\forall k \in T}\{rcc^{ck}\}\right)$ between RV^k and ranked collective vector RV^c.

5. Numerical examples

Suppose four decision-makers provide their assessments on four alternatives using fuzzy preference relations as following:

$$P^1 = \begin{pmatrix} 0.50 & 0.38 & 0.20 & 0.28 \\ 0.62 & 0.50 & 0.32 & 0.40 \\ 0.80 & 0.68 & 0.50 & 0.58 \\ 0.72 & 0.6 & 0.42 & 0.50 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.50 & 0.38 & 0.25 & 0.33 \\ 0.62 & 0.50 & 0.37 & 0.45 \\ 0.75 & 0.63 & 0.50 & 0.58 \\ 0.67 & 0.55 & 0.42 & 0.50 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.50 & 0.75 & 0.55 & 0.41 \\ 0.25 & 0.50 & 0.30 & 0.16 \\ 0.45 & 0.70 & 0.50 & 0.36 \\ 0.59 & 0.84 & 0.64 & 0.50 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.50 & 0.40 & 0.30 & 0.60 \\ 0.60 & 0.50 & 0.40 & 0.70 \\ 0.70 & 0.60 & 0.50 & 0.80 \\ 0.40 & 0.30 & 0.20 & 0.50 \end{pmatrix}$$

A.*Consensus Measure*

To find the rank correlation consensus among these experts (preferences), we apply the rank correlation consensus coefficient measure as mentioned above without relying on the collective relation.

(1) $V^1 = (0.38, 0.20, 0.28, 0.32, 0.40, 0.58)$,
$V^2 = (0.38, 0.25, 0.33, 0.37, 0.45, 0.58)$
$V^3 = (0.75, 0.55, 0.41, 0.30, 0.16, 0.36)$,
$V^4 = (0.40, 0.30, 0.60, 0.40, 0.70, 0.80)$

(2) $RV^1 = (3, 6, 5, 4, 2, 1)$, $RV^2 = (3, 6, 5, 4, 2, 1)$
$RV^3 = (1, 2, 3, 5, 6, 4)$,
$RV^4 = (4.5, 6, 3, 4.5, 2, 1)$

Table 1. Rank similarity degrees between decision-makers.

	Rank Similarity degrees					
	p^1/p^2	p^1/p^3	p^1/p^4	p^2/p^3	p^2/p^4	p^3/p^4
rsd12	1	0.6	0.7	0.6	0.7	0.3
rsd15	1	0.2	1	0.2	1	0.2
rsd14	1	0.6	0.6	0.6	0.6	1
rsd23	1	0.8	0.9	0.8	0.9	0.9
rsd24	1	0.2	1	0.2	1	0.2
rsd34	1	0.4	1	0.4	1	0.4

(3) **The rank similarity degrees are summarized in** Table 1.

1 means that both decision-makers rank the associated preference degree in the same position. If the rank similarity degree is less than 1, more differentiation exists on the rank between both experts.

(4) From step 2, we see that all of the decision-makers have no tied ranks except for decision-maker 4. Thus, we apply (7) to find the rank correlation consensus coefficient for rcc^{12}, rcc^{13} and rcc^{23}. Eq (5′) is used to find the rank correlation coefficient for rcc^{14}, rcc^{24} and rcc^{34}. For example,

$$rcc^{13} = 1 - \frac{48\sum_{i=1}^{n-1}\sum_{j=2,i<j}^{n}\left(o_{ij}^{k} - o_{ij}^{h}\right)^2}{\left(n^3(1-n)^2 - 4n\right)(1-n)}$$

$$= 1 - \frac{48\sum_{i=1}^{3}\sum_{j=2,i<j}^{4}\left(o_{ij}^{1} - o_{ij}^{3}\right)^2}{\left(4^3(1-4)^2 - 4\times 4\right)(1-4)}$$

$$= 1 - \frac{48\sum_{i=1}^{3}\sum_{j=2,i<j}^{4}\left(o_{ij}^{1} - o_{ij}^{3}\right)^2}{1680}$$

$$= 1 - \frac{48\left[(3-1)^2 + (6-2)^2 + (5-3)^2 + (4-5)^2 + (2-6)^2 + (1-4)^2\right]}{1680}$$

$$= 1 - \frac{48[50]}{1680} = -0.429$$

Table 2 shows all the rcc^{kh}, rcc^{k} and rcc^{T}:

From Table 2, we see that all the decision-makers have a good positive rank correlation with each other except for decision-maker 3, which has the lowest rank correlation consensus $(rcc^1 = rcc^2 > rcc^4 > rcc^3)$. Moreover, all the rank correlation consensus with decision-maker 3 are negative. In fact, if we rank each decision-maker preference on the alternatives by using $A_i = \frac{2}{n^2}\sum_{j=1}^{n} P_{ij}^{c}$, we would see that decision-makers 1 and 2 favor A_3 over the others and have the same ranking order $A_3 \succ A_4 \succ A_2 \succ A_1$, which is reflected by $rcc^{12} = rcc^{21} = 1$. Decision-maker 4 favors A_3 and has a ranking order $A_3 \succ A_2 \succ A_1 \succ A_4$ while decision-maker 3 favors A_4 over the others ($A_4 \succ A_1 \succ A_3 \succ A_2$). Thus, to improve the consensus level among decision-makers, decision-maker 3 has to revise his/her assessments.

B. *Feedback mechanism*

1. We see that decision-maker 3 has the lowest $rcc^3 = -0.469$. Thus, he/she is selected to revise his/her assessments.
2. Table 3 shows all rank similarity degrees of all decision-makers with respect to decision-maker 3:

Notice that $\min_{\forall h \in t}\left\{\sum_{\substack{h=1\\ h\neq 3}}^{4} rsd_{ij}^{3h}\right\} = rsd_{13}^{3h} = rsd_{24}^{3h}$ $= 0.6$, thus we pick any of them. For $\min_{\forall h \in t}\{rsd_{24}^{3h}\} \Rightarrow rsd_{24}^{31} = rsd_{24}^{32} = rsd_{24}^{34} = 0.2$ for all decision-makers. That means the rank position for the decision-makers are the same, $o_{24}^{1} = o_{24}^{2} = o_{24}^{4} = 2$.

3. **Thus,** p_{24}^{3} **is the one to modify,** $p_{24}^{3}\{o_{24}^{3}\} = p_{24}^{3}\{6\} \Rightarrow p_{24}^{3}\{2\}$. **In this case:** $o_{24}^{3} > o_{24}^{1}$

$$\Rightarrow p_{24}^{3} \in \left[p^3\{2\}, p^3\{1\}\right]$$

$$\Rightarrow p_{24}^{3} \in [0.55, 0.75]$$

Suppose that the decision-maker is willing to change his/her assessment for this preference degree from $p_{24}^{3} = 0.16$ to $p_{24}^{3'} = 0.65$.

Table 2. Rank correlation consensus between DMs.

	rcckh			
	e1	e2	e3	e4
e1	1	1	-0.429	0.812
e2	1	1	−0.429	0.812
e3	−0.429	−0.429	1	−0.551
e4	0.812	0.812	−0.551	1
rcck	0.461	0.461	−0.469	0.358
rccT		0.203		

Table 3. Rank similarity degrees between DM 3 and other DMs.

	Rank Similarity degrees			
	p^1/p^3	p^2/p^3	p^3/p^4	Sum
rsd12	0.6	0.6	0.3	1.5
rsd13	0.2	0.2	0.2	0.6
rsd14	0.6	0.6	1	2.2
rsd23	0.8	0.8	0.9	2.5
rsd24	0.2	0.2	0.2	0.6
rsd34	0.4	0.4	0.4	1.2

Table 4. The new rank correlation consensuses.

	rcckh			
	e1	e2	e3*	e4
e1	1	1	0.886	0.812
e2	1	1	0.886	0.812
e3*	0.886	0.886	1	0.754
e4	0.812	0.812	0.754	1
rcck	0.899	0.899	0.842	0.792
rccT	0.858			

This changes the result on new rcc^{kh}: $rcc^{1'} = 0.613, rcc^{2'} = 0.613, rcc^{3'} = -0.039, rcc^{4'} = 0.483$ and $rcc^{T'} = 0.418$.

4. Again $rcc^{3'}$ is the lowest one. The same steps are repeated. This time p^3_{34} is the one to modify $p^3_{34}\{5\} \Rightarrow p^3_{34}\{1\}$. In this case; $o^3_{24} > o^1_{24}$

$$\Rightarrow p^3_{34} \in \left[p^3\{1\}, p^3\{0\}\right]$$

$$\Rightarrow p^3_{34} \in [0.75, 1]$$

Suppose that the decision-maker decides to change it to $p^3_{34} = 0.8$. This results in a newrcc^{kh}: $rcc^{1'} = 0.842, rcc^{2'} = 0.842, rcc^{3'} = 0.640, rcc^{4'} = 0.705$ and $rcc^{T'} = 0.757$.

5. If these results satisfy the condition of the consensus level, then stop and construct the collective relation. However, if the consensus level is at 0.8, then carry on the process. Still decision-maker 3 has the lowest rank correlation consensus. This time $\min_{\forall h \in t}\left\{\sum_{\substack{h=1 \\ h \neq 3}}^{4} rsd^{3h}_{ij}\right\} = rsd^{3h}_{13} = 1.8$, and $\min_{\forall h \in t}\left\{rsd^{3h}_{13}\right\} \Rightarrow rsd^{31}_{13} = rsd^{32}_{13} = rsd^{34}_{13} = 0.6$. Thus, p^3_{13} is the one to modify $p^3_{13}\{4\} \Rightarrow p^3_{34}\{6\}$. In this case; $o^3_{13} < o^1_{13}$

$$\Rightarrow p^3_{13} \in \left[p^3\{7\}, p^3\{6\}\right]$$

$$\Rightarrow p^3_{13} \in [0, 0.3]$$

Suppose that the decision-maker decides to change it to $p^3_{13} = 0.29$. This results in a new rcc^{kh}: $rcc^{1'} = 0.899, rcc^{2'} = 0.899, rcc^{3'} = 0.842, rcc^{4'} = 0.792$ and $rcc^{T'} = 0.858$, which attains the consensus level. Therefore, the feedback mechanism is finished and the new P^{3*} is:

$$P^{3*} = \begin{pmatrix} 0.50 & 0.75 & 0.29 & 0.41 \\ 0.25 & 0.50 & 0.30 & 0.65 \\ 0.71 & 0.70 & 0.50 & 0.8 \\ 0.59 & 0.35 & 0.2 & 0.50 \end{pmatrix}$$

With this update or revision, decision-maker 3 would prefer A_3 over the others and ranks his alternatives as $A_3 \succ A_1 \succ A_2 \succ A_4$.

The new rank correlation consensus for decision-maker 3 with the other decision-makers, each decision-maker's rank correlation consensus and experts' rank correlation consensus are shown in Table 4.

Notice the improvement in the rank correlation consensus for the three types of correlations. They all become strongly positive correlated and more importantly, the expert's rank correlation consensus has increased.

If the collective preference relation is constructed using a weighted averaging operator with equal weights for the experts, the following preference relation is obtained:

$$P^c = \begin{pmatrix} 0.50 & 0.48 & 0.26 & 0.40 \\ 0.52 & 0.50 & 0.35 & 0.55 \\ 0.74 & 0.65 & 0.50 & 0.69 \\ 0.60 & 0.45 & 0.31 & 0.50 \end{pmatrix}$$

By using$A_i = \frac{2}{n^2}\sum_{j=1}^{n} P^c_{ij}$, which is equivalent to the sum normalization method, we get$A_3 \succ A_2 \succ A_4 \succ A_1$.

6. Validation

We conducted some tests on the proposed method using some examples available in the literature. Our method shows almost the same conclusions to the other researchers' conclusions. However, we should mention that some of these methods are not based on reciprocal preference relations. In addition, the aggregation operators and selection methods are different, which play a significant role in the final results. Table 5 summarizes these findings.

We point out that the examples of Zhang et al. [33] and Wu and Xu [34] are based on reciprocal relations while the others are not. Herrera-Viedma et al. [32] started the problem with reciprocal preference relations; however, the aggregation operator does not maintain this property. It can be seen from Table 5 that the proposed method has similar results to the problems with reciprocal relations. Also, it performs great on problems that are not based on reciprocal relations. We should mention that for the non-reciprocal problems in Table 5, the upper triangular relation of the problems was considered. The results are almost the same except for minor ranking orders for Pérez et al.'s [35] and Herrera-Viedma et al.'s [32] problems, which could be linked to the level of consensus to be achieved. Also, the effect of using different selection methods and aggregation operators in finding the solution should not be ignored.

Table 5. Proposed method vs some available methods.

	Authors		
	Zhang et al. [33]	Pérez et al. [35]	Wu and Xu [34]
Consensus model based	Similarity	Similarity+expert weights	Distance
Aggregation Operator	Weighted Averaging	Not Given	Weighted Averaging
Group Members	4	4	5
No. of Alternatives	4	4	6
Selection Method	QGDD	Not Given	Sum Normalization
Alternatives Ranking	$A_2 \succ A_1 \succ A_3 \succ A_4$	$A_1 \succ A_2 \succ A_3 \succ A_4$	$A_3 \succ A_2 \succ A_1 \succ A_4 \succ A_6 \succ$ A5
Porpose Method Ranking	$rcc^c \approx 0.9$, $A_2 \succ A_1 \succ A_3 \succ A_4$	$rcc^c = 0.92$, $A_1 \succ A_2 \succ A_4 \succ A_3$	$rcc^c \approx 0.93$, $A_3 \succ A_2 \succ A_1 \succ A_4 \succ A_6 \succ$ A5

	Authors	
	Herrera-Viedma et al, [2007a)	[32]
Consensus model based	Similarity+Consistency	ordinal/dissimilarity
Aggregation Operator	IOWA	S-OWA OR-LIKE
Group Members	4	8
No. of Alternatives	4	6
Selection Method	Not Available	QGDD
Alternatives Ranking	$A_2 \succ A_1 \succ A_4 \succ A_3$*	$A_2 \succ A_3 \succ A_1 \succ A_5 \succ A_4 \succ A_6$
Porpose Method Ranking	$rcc^c \approx 0.9$, $A_2 \succ A_1 \succ A_4 \succ A_3$	$rcc^c \approx 0.77$, $A_3 \succ A_2 \succ A_1 \succ A_5 \succ A_4 \succ A_6$

QGDD-quantifier guided domonance degree, IOWA-induced ordered weighted averaging, * the ranking was generated by using weighted averaging method with equal weights using sum normalization by us

7. Conclusions and future work

In this paper, we presented a consensus model based on Spearman's correlation. The proposed model does not rely directly on similarity/distance measure rather than on ranks of preference degrees on reciprocal preference relations. The novelty of this work lies in considering the coherence of decision-maker preference degrees ranks as a whole in comparison with the rest of the group members. In addition, a feedback mechanism is proposed to play the role of the mediator to provide suggestions to the decision-makers who are not close to rank correlation consensus of the group members. The model was tested on several problems and the results proved the validity of the model. Two examples dealing with different information types are illustrated.

In future works, we would like to compare this model with other consensus models to see how they are related in terms of consensus level, in addition to the speed of convergence of the consensus solution. Also, we would like to work on Pearson correlation with a feedback mechanism and compare it to the proposed work in terms of consensus level and solution obtained.

Disclosure statement

No potential conflict of interest was reported by the authors.

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