Computer Vision Homework 2

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1. Fitting Lines through Points [30 pts]

1.1. Least-Squares Line Fitting [10 pts]

The line model $\hat{\mathbf{a}}^T\hat{\mathbf{x}} = 0$, can also be written as $\mathbf{y} = \tilde{\mathbf{a}}^T\tilde{\mathbf{x}} = \tilde{\mathbf{x}}^T\tilde{\mathbf{a}}$, where $\mathbf{x} = [\tilde{\mathbf{x}}; y]$. $\mathbf{y}$ ($\mathbb{R}^1$) is the label data.

The optimization problem of the line fitting is the following, we want to minimize the Euclidean distance between the label y and the output $\tilde{\mathbf{x}}^T\tilde{\mathbf{a}}$: $\text{argmin}_{\tilde{\mathbf{a}}} \sum_{i=1}^{N} \|\mathbf{y_i} - \tilde{\mathbf{x}}^T\tilde{\mathbf{a}}$. It can also be written in the format:

$$\text{argmin}_{\tilde{\mathbf{a}}} \|\mathbf{X}\tilde{\mathbf{a}} - \mathbf{Y}\|^2 \tag{1}$$

where $\mathbf{Y}(\mathbb{R}^{N\times 1})$ and $\mathbf{X}(\mathbb{R}^{N\times(n+1)})$ are the samples. N is the number of samples.

The solution of Equation 2 is :

$$\tilde{\mathbf{a}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T \tag{2}$$

Then $\hat{\mathbf{a}}$ can be obtained by inserting -1 before the last row of $\tilde{\mathbf{a}}$.

Result. The result of $pts1$ is: $[2.0084, 0.9027]$. The line model is $y = 2.0084x + 0.9027$. The plot is shown in Figure 1.

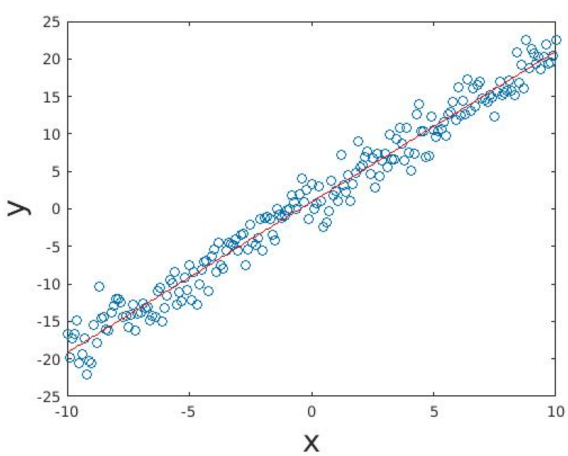

Figure 1: **Line Fitting of** $pts1$.

1.2. LS Line Fitting with Outliers [5 pts]

The result of $pts2$ is: $[1.1247, 1.2933]$. The line model is $y = 1.1247x + 1.2933$. The plot is shown in Figure 2.

Discuss difference. It is clear that the predicted line under the noisy data $pts2$ rotates towards to the outliers a little compared to the predicted line using data $pts1$. The reason is that the Equation 2 counts every points the same. Therefore to minimize the error, the line will rotate towards the outliers a little bit.

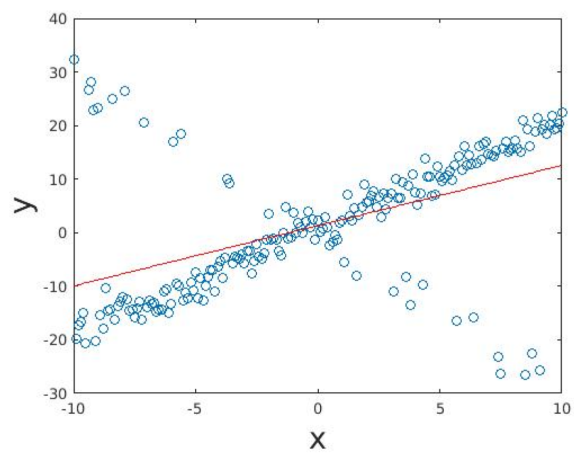

Figure 2: **Line Fitting of** $pts2$.

1.3. Robust Line Fitting with RANSAC [15 pts]

Distance equation. Line is $\mathbf{a}^T\mathbf{x} + a_0 = 0$. The point is $\mathbf{y}$, denoted as P. The $|PQ|$ is the distance between point P and the plane p. The point Q in at the plane, its vector is $\mathbf{y}_Q$. $\mathbf{a}^T\mathbf{y}_Q + a_0 = 0$. The normal vector of the plane is a.

$$d(\mathbf{y}|p(\mathbf{a}, a_0)) = |\vec{PQ}| = \frac{|\mathbf{a}^T\vec{PQ}|}{\|\mathbf{a}\|_2} = \frac{|\mathbf{a}^T\mathbf{y} - \mathbf{a}^T\mathbf{y}_Q|}{\|\mathbf{a}\|_2} = \frac{|\mathbf{a}^T\mathbf{y} + a_0|}{\|\mathbf{a}\|_2}$$

RANSAC line fitting algorithm. RANSAC line fitting algorithm is shown in Algorithm 1. The hyper parameters we use in the experiment is: the maximum iterations

$N = 1000$, the distance threshold is $\sigma = 3$, the number of samples per iteration is $m = 2$. $m = 2$ is the minimal samples for the reason that at least two points are needed to determine a 2D line.

Algorithm 1: Robust Line Fitting with RANSAC

Given samples of input data $\mathbf{x}(\mathbb{R}^n)$ and label $\mathbf{y}(\mathbb{R}^1)$. Append zeros in the last column of $\mathbf{x}$ and obtain X $(\mathbb{R}^{n+1})$.

Build a line fitting model, $y = \mathbf{X}^T\mathbf{a}$. The maximum iterations is N, the distance threshold is σ, the number of samples per iteration is m.

while *not terminated* **do**

1. randomly pick m samples X_i, y_i;
2. solve the parameter of the line model by $\mathbf{a} = (\mathbf{X}_i^T\mathbf{X}_i)^{-1}\mathbf{X}_i^T$;
3. calculate the distance $d(\mathbf{x}|p(\mathbf{a}))$;
4. count inlier points with distance $d < \sigma$.

5) repeat steps 1) to 4) for N times. Choose $\mathbf{a}$ with the most inliers.

Use the inlier points to recompute the $\mathbf{a} = (\mathbf{X}_{inlier}^T\mathbf{X}_{inlier})^{-1}\mathbf{X}_{inlier}^T$. Fit the line use the final parameter $\mathbf{a}$.

Result. Figure 3 shows the result of using RANSAC. The obtained line model is $y = 1.9873x + 1.5814$. This figure clearly show that RANSANC is more robust to the existence of noise then simply using the least-square line fitting.

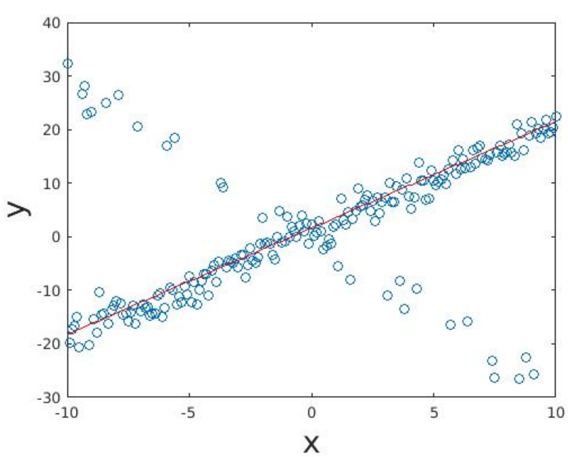

Figure 3: **Line Fitting of** $pts2$.

2. Image Alignment and Stitching [70 pts]

2.1. SIFT points detection

Figure 4 shows the SIFT points of im01 and im02. Points are matched according to Lowe's distance ratio rule. Given two sets of SIFT features D_1 and D_2 from image 1 and image 2, the Lowe's distance ratio rule will discard the incorrect matches whose distance between two features is not significant small. A descriptor d_1 will only be matched to d_2 only if the distance $d(d_1, d_2)$ multiplied by a threshold is not greater than the distance of d_1 to all other descriptors among D_2.

2.2. Calculate homographies using RANSAC and DLT

Robust alignment method. From Figure 4, one can see that using the Lowe's distance ratio rule still result in many incorrect matches. RANSAC can be further used to robust align images. The robust alignment method is the following: 1) sample four random potential matches; 2) compute $\mathbf{H}$ using normalized DLT; 3) project points of image 2 using $\mathbf{H}$ to image 1 for each potentially matching pair; 4) count inlier points with projected distance < a fixed threshold. 5) repeat steps 1) to 4) for N times. Choose $\mathbf{H}_{max}$ with the most inliers. One can also recompute the final $\mathbf{H}$ by the matches of image 1 and image 2 when use the $\mathbf{H}_{max}$.

Estimated homographies.

$$\mathbf{H}_{21} = \begin{bmatrix} -0.4684 & 0.0041 & -232.6984 \\ 0.0500 & -0.5620 & -19.8571 \\ 0.0002 & 0.0000 & -0.6026 \end{bmatrix}$$

$$\mathbf{H}_{32} = \begin{bmatrix} -0.4927 & 0.0196 & -234.0505 \\ 0.0407 & -0.5793 & -27.6116 \\ 0.0002 & 0.0000 & -0.6263 \end{bmatrix}$$

$$\mathbf{H}_{43} = \begin{bmatrix} -0.5055 & 0.0086 & -185.6992 \\ 0.0329 & -0.5731 & -1.8272 \\ 0.0001 & -0.0000 & -0.6071 \end{bmatrix}$$

The transformed images using $\mathbf{H}_{21}, \mathbf{H}_{32}, \mathbf{H}_{43}$ showed in the second row of Figure 6.

2.3. Image stitching

Image transformation. To transform images im02, im03, im04 to im01 coordinates, we calculate the $\mathbf{H}_{21}, \mathbf{H}_{31}, \mathbf{H}_{41}$ at first. They can be calculated simply by $\mathbf{H}_{31} = \mathbf{H}_{21} \times \mathbf{H}_{32}$ and $\mathbf{H}_{41} = \mathbf{H}_{31} \times \mathbf{H}_{43}$. The transformed images of im02, im03, im04 are shown in the third row of Figure 6.

Panoramic image. Figure 7 shows the final panoramic image of im01, im02, im03 and im04. Pixels in the panoramic image that are not covered are set to black.

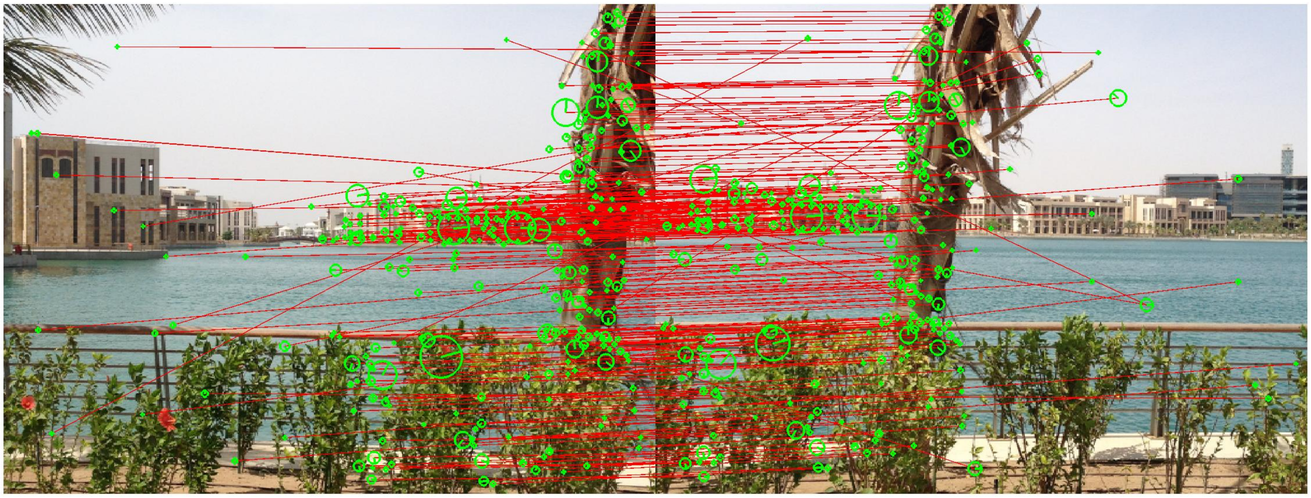

Figure 4: **Match image pair (im01, im02)**. Their point matches are plotted using red lines.

Figure 5: **Match image pair (im01, im02) after RANSAC**. Their point matches are plotted using red lines.

Figure 6: **Transformed images using the estimated homographies**. First row shows the original images. Second row shows the projected images of im02, im03, im04 using homographies $\mathbf{H}_{21}, \mathbf{H}_{32}, \mathbf{H}_{43}$. Third row shows the projected images of im02, im03, im04 using homographies $\mathbf{H}_{21}, \mathbf{H}_{31}, \mathbf{H}_{41}$. are plotted using colored lines.

Figure 7: **Panoramic image**. Pixels in the panoramic image that are not covered are set to black.