Nonequilibrium DMFT

1 Green's function

The Kadanoff contour-ordered Green's function is defined as

$$G(t,t') = -i\langle T_C c(t) c^\dagger(t')\rangle. \tag{1.1}$$

The Green's function has 9 components that

$$G(t,t') = \begin{pmatrix} G^{11} & G^{12} & G^{13} \\ G^{21} & G^{22} & G^{23} \\ G^{31} & G^{32} & G^{33} \end{pmatrix}, \tag{1.2}$$

where the 1st branch is the '+' branch, 2nd is the '-' branch, and 3rd is the imaginary branch.

2 Lattice Green's function

Consider the lattice Green's function

$$G_{ij}(t,t') = -i\langle T_C c_i(t) c_j^\dagger(t')\rangle, \tag{2.1}$$

its Dyson equation is

$$G_{ij}(t,t') = G_{ij}^{(0)}(t,t') + \sum_{kl} \int \mathrm{d}t_1 \mathrm{d}t_2 G_{ik}^{(0)}(t,t_1)\Sigma_{kl}(t_1,t_2) G_{lj}(t_2,t'). \tag{2.2}$$

In momentum space, we have

$$G_k(t,t') = G_k^{(0)}(t,t') + \int \mathrm{d}t_1 \mathrm{d}t_2 G_k^{(0)}(t,t_1)\Sigma_k(t_1,t_2) G_k(t_2,t'). \tag{2.3}$$

Employing the approximation that $\Sigma_k(t_1,t_2) \approx \Sigma(t_1,t_2)$, we have

$$G_k(t,t') = G_k^{(0)}(t,t') + \int \mathrm{d}t_1 \mathrm{d}t_2 G_k^{(0)}(t,t_1)\Sigma(t_1,t_2) G_k(t_2,t'). \tag{2.4}$$

Once $\Sigma(t,t')$ is known, in principle $G_k(t,t')$ can be obtained via this equation.

3 Impurity Green's function

When interaction is off, we have

$$G_0(t,t') = g(t,t') + \int \mathrm{d}t_1 \mathrm{d}t_2\, g(t,t_1)\Delta(t_1,t_2) G_0(t_2,t'), \tag{3.1}$$

where $g(t,t')$ is the impurity Green's function without the interaction and bath.

With the interaction, we have the final Green's function as

$$G(t,t') = G_0(t,t') + \int \mathrm{d}t_1 \mathrm{d}t_2 G_0(t,t_1)\Sigma(t_1,t_2) G(t_2,t'). \tag{3.2}$$

Inverting this equation gives us $\Sigma(t_1,t_2)$ in principle.