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A New Resource Allocation Mechanism for Security of Mobile Edge Computing System

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ABSTRACT Mobile-Edge Computing (MEC) is a new computing paradigm that provides a capillary distribution of cloud computing capabilities to the network edge. In this paper, we studied the security defense problem in MEC network environment. One big challenge is how to efficiently allocate resources to deploy Mobile-Edge Computing-Intrusion Detection Systems (MEC-IDS) in this system, since all the MEC hosts are composed of resource-constrained network devices. To tackle this challenge, a new resource allocation mechanism based on deterministic differential equation model is proposed and investigated. Existence, uniqueness and stability of the positive solution of this model are obtained by using Lyapuonv stability theory. Furthermore, we extended our study to MEC network environment with stochastic perturbation and established a new stochastic differential equation model. We proved the existence, uniqueness, persistence and oscillatory of the positive solution of this model and quantitatively analyzed the relationship between oscillation and intensity of stochastic perturbation. Numerical simulations are carried out to illustrate the effectiveness of the main results.

INDEX TERMS Mobile-edge computing, resource allocation, intrusion detection systems, stability theory, stochastic perturbation.

I. INTRODUCTION

In recent years, cloud computing as a mainstream paradigm of computing has attracted a lot of attention from both academia and industry. However, the cloud computing paradigm is unable to meet certain requirements, such as low latency and jitter, context awareness, mobility supporting [1]. In order to meet the specific requirements of practical application, various novel computing paradigms have emerged, such as MEC that promises a dramatic reduction in latency and mobile energy consumption [2]. MEC is an emergent architecture where cloud computing services are extended to the edge of mobile network devices, which resides between cloud layer and mobile devices layer. Since the MEC deploys computing services near edge devices, it can improve user experience by providing high bandwidth. According to the white paper published by European Telecommunications Standards Institute (ETSI), mobile edge computing can be characterized on-premises, proximity, lower latency, location awareness and network context information [3]. The infrastructure is derived as a three-layer hierarchy including cloud layer, MEC layer and mobile devices layer [4], which is depicted in Figure 1.

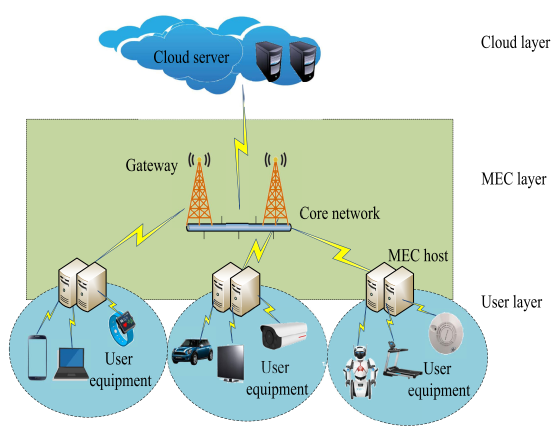

FIGURE 1. Three-layer MEC architecture diagram [4].

Edge devices exchange information with each other and may bypass the central systems security mechanism since

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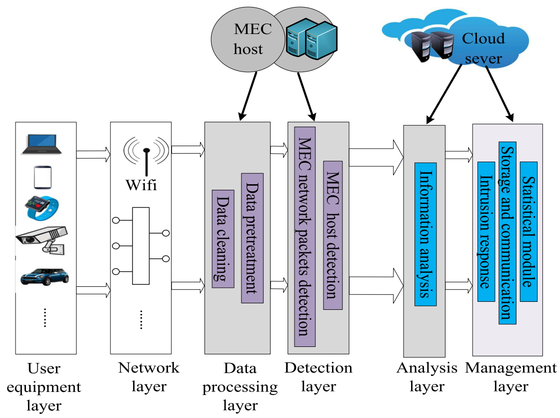

FIGURE 2. **MEC-IDS architecture diagram [9].**

the deployment of MEC at the network edge, which may be vulnerable to various attacks, such as Denial of Service (DoS) [5]. Therefore, it is of paramount importance to develop IDS for MEC to quickly detect intrusion and security risks in the network [6], [7]. The research of this paper is based on An *et al.* [8], [9] proposed a six-layer MEC-IDS architecture for MEC environment, as illustrated in Figure 2. Users in the user experience layer connect the MEC network with different protocols of the network layer. The MEC host is mainly responsible for data processing and intrusion detecting. Correspondingly, the detection layer and data processing layer are deployed on MEC host. Since the cloud server needs to monitor and manage the security of the MEC host, the cloud server is responsible for the main work of the analysis layer and management layer. MEC-IDS is a security technology that collaborates cloud and MEC host. Among them, the MEC host needs to perform tasks such as data preprocessing, intrusion detecting and security log storing.

However, MEC host typically consist of edge servers (or network devices) with limited resources, resulting in limited computing power, memory, and storage capacity. And most MEC host are battery-powered, such as hand-held devices, wearables, and wireless sensor units [10]. Therefore, the biggest challenge in deploying IDS on MEC host is resource allocation. Resources in MEC usually include computing resources, storage resources, bandwidth resources and energy consumption resources. In this paper, the resources we emphasized are specific computing resources. It is a contradiction that limited resources should not only provide services for users, but also perform security detection tasks. The more resources provided to MEC-IDS, the less services provided to users by MEC host will be affected. Conversely, the fewer resources available to MEC-IDS, the less secure the system is. Whether this contradiction can be solved harmoniously is directly related to the security and practical application of MEC network environment. Therefore, how to efficiently allocate resources among MEC host so that the equipment can not only provide users with stable quality services, but also ensure security in MEC network environment will be the difficulty of this paper.

In this paper, we aim to develop a new resource allocation mechanism in MEC environment to ensure the safe and stable operation of the system. Based on the mathematical modeling idea of differential equation, we describe the relationship between MEC-IDS resources and user resources using a deterministic differential equation model. And we use the important mathematical concept of stability to quantitatively study the working state of MEC network environment. Under the mechanism that we set up, there is a globally asymptotically stable equilibrium, which represents that the system can work safely and steadily. In other words, this mechanism not only effectively completes the resource allocation, but also guarantees the security and stability of the MEC network environment.

In fact, there are some inevitable stochastic perturbations that affect the stability of the network system, for example, the online or offline of a large number of users who do not have the expected value on a MEC host, the unconventional behavior of some users and the destruction of large natural environment to network link and so on. Therefore, in order to make the resource allocation mechanism more suitable for the practical MEC network environment, stochastic differential equation is a good tool. A large number of scholars have established and studied stochastic models and achieved meaningful results [11], [12]. The standard Brownian motion is used to describe the stochastic perturbation quantitatively and the deterministic model is extended to a stochastic model. We have studied the dynamics properties of the stochastic model comprehensively and obtained the persistence of the solution. As it were, the deterministic model is the basis of the resource allocation mechanism, and the extended stochastic model is the focus of this paper. Some numerical simulations are carried out to verify and extend our theoretical result. The main contributions of this paper are summarized as follows.

- Based on the architecture of MEC-IDS, a secure resource allocation mechanism is designed and relevant theoretical analysis is completed. For the first time, we use the idea of differential equation modeling to solve the problem of resource allocation in communication networks. In addition, the classical Lyapunov stability theory is applied to prove the model theoretically, and the mathematical concept of stability is taken as important metrics to judge the working state of the system. We proved that the stability of the positive solution of this model, which means that our resource allocation mechanism is feasible.
- Furthermore, we study the impact of some uncertainties in the MEC environment on our resource allocation mechanism. Stochastic differential equations are used to quantitatively analyze MEC networks with stochastic perturbation. We use standard Brownian motion to simulate the random disturbance and apply the theory of stochastic differential equation to obtain the existence, uniqueness and persistence of the positive

solution. Finally, we obtain the correlation between the oscillatory of positive solution and the intensity stochastic perturbation. This theoretical result will guide us to effectively prevent and control some stochastic factors in the MEC network to ensure the stability of the network. The rationality and usability of the proposed method are extended.

- The resource allocation mechanism we proposed in this paper, is strictly proved based on mathematical theory. This method can be extended to solve resource allocation and other problems in communication networks. The idea of differential equation modeling and Lyapunov stability theory provides a new way to solve the problem of communication network.

The paper is organized as follows. In Section 2, we mainly reviews the research work related to the security and resource allocation of MEC networks under resource constraints. In Section 3, the deterministic model of resource allocation mechanism is established and relevant theoretical proof has been completed. We then propose the stochastic model based on deterministic model and study its dynamic behavior in Section 4. Some numerical simulations are also given to show the effectiveness of the established allocation mechanism in Section 5. Concluding remarks are given in Section 6.

II. RELATED WORK

MEC is an emerging architecture where cloud computing services are extended to the edge of networks leveraging mobile base stations to offer an improved user experience in terms of expected throughout, latency, scalability and automation [4]. MEC represents the key technologies and architectural concepts that will evolve towards 5G as it helps drive the shift of mobile broadband networks to the programmable world [13]–[15]. In a new computing paradigm, network security issues are the first to be considered. It is necessary to carry out relevant research of MEC, especially in the aspect of security defense. Many literatures studied security problems in different architectural elements of MEC and presented relevant security mechanisms for MEC, for example, identification and authentication [16]–[19], network security mechanisms [20], [21], virtualization security mechanisms [22], [23], data security [24], [25], data computation security [26], [27].

In particular, Roman *et al.* [1] analyzed the threats and challenges existing in the current marginal paradigm (such as fog computing, MEC and mobile cloud computing) from an overall perspective, all of which need to be further studied and solved by scholars. Because of physical proximity, the survivability characteristics of MEC are closer to its associated mobile devices than to the distant cloud [20]. Therefore, MEC can serve as a proxy for the cloud and perform its critical services if there is temporary disconnection between MEC and the cloud server or during failures, such as security authentication and detection.

Gobbo *et al.* [28] described a new attack that threatens the security of mobile networks allowing an unauthenticated malicious mobile device to inject traffic in the mobile operator's infrastructure. This also exists in the network security problem faced by MEC, and this threat will lead to significant service degradation up to a full-fledged Denial of Service (DoS) attack. Vassilakis *et al.* [29] investigated security challenges in virtualized MEC environments, and presented the most prominent threats and vulnerabilities against a broad range of targets. Wei *et al.* [30] proposed MobiShare that a sharing system for privacy protection of mobile network. In MobiShare, the user's location privacy is protected even if either of the entities colludes with malicious users. An *et al.* [9] presented a new lightweight IDS deployed on fog nodes/MEC hosts to defend against cyber attacks.

While addressing the issues of MEC security, we should first consider that MEC is resource-constrained. Difference from MEC, the cloud has rich resources and can deploy various protection and security mechanisms. Therefore, how to allocate resources effectively in MEC will be a subject worthy of study. Karachontzitis *et al.* [31] solved the problem of single resource allocation for down link with eavesdropping by using a mixed integer nonlinear program. The user's secrecy rate is sacrificed in order to optimize maximum and minimum resource allocation principles in this program. Lin *et al.* [6], [32] proposed a general IDS model for edge computing, which realized efficient and fair allocation of network resources in IDS considering the resource limitation of edge nodes.

Tran and Pompili [33] studied an optimization problem that is a joint task offloading and resource allocation in a multi-cell MEC network that assists mobile users in executing computation intensive tasks via task offloading. The underlying optimization problem was formulated as a Mixed-Integer Non-linear Program (MINLP), instead of getting the optimal solution, and reduced to suboptimal solution [34], [35]. Hou and Gupta [36] studied the problem of achieving weighted proportional fairness for resource allocation in self-organized wireless systems. Ghodsi *et al.* [37] proposed a new algorithms for multiple resources allocation and the algorithm achieved the principle of fair sharing of resources. Sardellitti *et al.* [38] studied joint optimization problem of radio and computational resources for multicell MEC, the global optimal solution for single user and the local optimal solution for multiple users are obtained, respectively.

The resource allocation methods mentioned in the previous literature can be divided into single resource allocation methods and multiple resource allocation methods. The disadvantage of these methods without the stability of schemes proposed by quantitative research. Stability is the ability of a scheme or algorithm to keep its metering characteristics constant over time. It is also one of the most important indicators to measure an algorithm, a scheme or even a physical system. For a stable system, even if there is some external disturbance within the allowed range, it is still able to maintain some inherent performance characteristics.

III. ESTABLISHMENT OF RESOURCE ALLOCATION MODEL IN MEC

To achieve a larger application scenario, MEC first needs to address problems. Although the resources of mobile edge nodes are significantly limited compared to cloud servers, MEC-IDS must be executed while providing services to users to ensure security. Specifically, MEC is mainly responsible for intrusion detection, data preprocessing and other tasks. This is bound to reduce the total number of users the node can provide services, but it is also essential to ensure system security. On the contrary, as the number of service users increases, a greater proportion of resources need to be devoted to MEC-IDS execution. The specific form is or MEC-IDS will lease part of the user resources for defense. At the same time, there is a natural law that restricts competition within any resource-consuming service relationship system. That is, limited resources can only provide a limited number of users, and exceeding this threshold will result in abnormal service or deterioration of service quality. According to these general rules, we will establish a MEC resource allocation mechanism based on the mathematical model.

A. MODEL FORMULATION AND PRELIMINARIES

In the real network environment, every MEC host has resource sharing or crowding relationship with other nodes. Firstly, we use physical isolation and one MEC host is taken as the research object. Let $U(t)$ be the total computation resources consumption of a MEC host at time t, and r denotes the arrival rate of user access MEC. Then total computing resources consumption can be described by the famous Malthus equation

$$\frac{\mathrm{d}U(t)}{\mathrm{d}t} = rU(t). \tag{1}$$

We calculate the solution of Malthus equation (1)

$$U(t) = U(t_0)\exp[r(t-t_0)],$$

where $U(t_0)$ denotes computing resources of MEC at time t_0. This is obviously unreasonable because $U(t)$ exponetially increase over time, and $\lim_{t\to\infty} U(t) = \infty$. The biggest disadvantage of Malthus equation is that it ignores the limitation of resources. In fact, any network device is a dissipative system, and the increase of resources will be limited by the network, bandwidth and other hardware devices. Therefore, we improved the program to be $\bar{r} = r(1 - U/K)$, where K denotes the maximum capacity of computation resources. Thus, the classical Logistic equation is obtained as follows

$$\frac{\mathrm{d}U(t)}{\mathrm{d}t} = rU(t)\left(1 - \frac{U(t)}{K}\right). \tag{2}$$

Then, we obtain the explicit solution of equation above for any $t \geq t_0$ as follows

$$U(t) = \frac{KU_0}{K\exp[-r(t-t_0)] + U_0\{1-\exp[-r(t-t_0)]\}}.$$

The long-term behavior of $U(t)$ is easy to get from this expression $\lim_{t\to\infty} U(t) = K$.

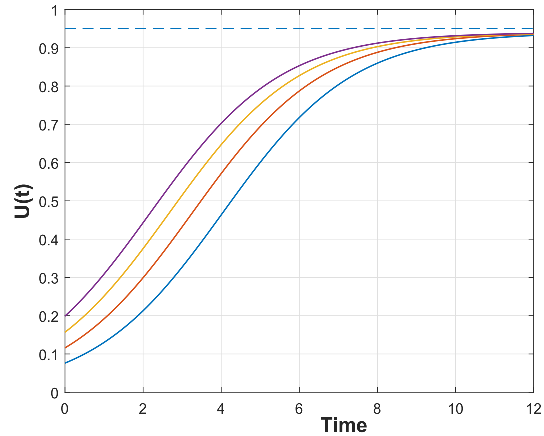

FIGURE 3. MEC-IDS architecture diagram.

As shown in the Figure 3, we selected four groups of different initial values. Under the same arrival rate r, $U(t)$ would show an "S" growth, but it would always be capped at resource limit K. That is, $U(t)$ will not increase to K. However, it is impossible for any network device to devote all its resources to providing computing services for users. In order to ensure the security of MEC network, a part of computation resources is allocated to deploy MEC-IDS, which denotes $P(t)$. As MEC provides more computation services to users, MEC-IDS needs to rent more resources from $U(t)$ to secure the network. It can be said that $P(t)$ is an indispensable computing resource in the MEC network. Therefore, we assume that computation resources of MEC-IDS are allocated by MEC host and rent from $U(t)$. Firstly, we do not consider the resources rented from $U(t)$, then $P(t)$ can be isolated to study the following differential equation description

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = P(t)\left[b - aP(t)\right], \tag{3}$$

where $b > 0$ represents the rate of deploy MEC-IDS inherent allocator, and $a > 0$ represents the rate of internal constraint of the MEC-IDS. For all $t \geq t_0$, we calculate and get the solution of the equation (3)

$$P(t) = \frac{bP_0/a}{b\exp[-b(t-t_0)]/a + P_0\{1-\exp[-b(t-t_0)]\}}.$$

The long-term behavior of $P(t)$ is easy to get from this expression $\lim_{t\to\infty} P(t) = b/a$.

Based on the above analysis, we use the method of differential equation to set up an effective resource allocation mechanism. Since MEC-IDS needs to rent computing resources of $U(t)$, we assume that $U(t)$ reduces by $a_{12}U(t)P(t)$ in unit time, which is converted into effective MEC-IDS computing resources by $a_{21}U(t)P(t)$ (where $a_{12} > 0$ and $a_{21} > 0$ are two proportionality coefficient and satisfy $a_{12} > a_{21}$). Combining equation (2) equation (3) with the conversion relationship between $U(t)$ and $P(t)$, the resource allocation mechanism of MEC-IDS architecture proposed above can be described by differential equations with Logistic growth as follows

$$\begin{cases} \dfrac{\mathrm{d}U(t)}{\mathrm{d}t} = b_1 U(t)\left(1 - \dfrac{U(t)+P(t)}{K}\right) - a_{12}U(t)P(t), \\ \dfrac{\mathrm{d}P(t)}{\mathrm{d}t} = P(t)\left[b_2 + a_{21}U(t) - a_{22}P(t)\right], \end{cases} \tag{4}$$

where $U(t)$ and $P(t)$ stand for the computation resources of MEC host for users and MEC-IDS at time t, respectively; b_1 and b_2 stand for the inputting rate of computation resources for users and MEC-IDS, respectively; $U(t)$ reduces by $a_{12}U(t)P(t)$ in unit time because $P(t)$ needs to rent its resources and the effective resource converted to MEC-IDS is $a_{21}U(t)P(t)$. a_{22} stands for the internal constraints of the MEC-IDS and K represents the maximum capacity of resources that can be provided by resources of mobile edge nodes. Based on practical significance, all parameters in the model (4) are bounded positive constants.

B. SOLUTIONS OF MODEL

By computation, it is clear that the model (4) admits one zero equilibrium $(0, 0)$ and two trivial equilibrium $(0, b_2/a_{22})$ and $(K, 0)$. If $Kb_1a_{22} - b_2(b_1 + Ka_{12}) > 1$, then model (4) admits an unique positive equilibrium $\mathbf{E}^*(U^*, P^*)$, where U^* and P^* as follows

$$U^* = \frac{Kb_1a_{22} - b_2(b_1 + Ka_{12})}{b_1a_{22} + a_{21}(b_1 + Ka_{12})},$$
$$P^* = \frac{b_1b_2 + Kb_1a_{21}}{b_1a_{22} + a_{21}(b_1 + Ka_{12})}.$$

Therefore, a basic threshold for model (4) is $R_0 = Kb_1a_{22} - b_2(b_1 + Ka_{12})$. And $R_0 > 1$ is the threshold condition for the existence of a positive equilibrium of the model (4). The equilibrium is a constant solution to the differential equation. In the physical model, we usually require that the model has a positive equilibrium. For example, one zero equilibrium $(0, 0)$ and two trivial equilibriums of the model (4) are meaningless, which is only a mathematical solution that satisfies the equation under extreme conditions. The positive equilibrium means that there is a solution to the physical problem of the model (4). Therefore, in this paper, we always assume that $R_0 > 1$ and focus on the positive equilibrium $\mathbf{E}^*(U^*, P^*)$ of the model (4). In many physical applications, we are more concerned with the stability of the positive equilibrium. Stability is the ability of a physical system to maintain its normal operation condition constant over time. This is an important indicator to judge whether a system can maintain a constant state of continuous operation. In our proposed MEC-IDS network architecture, it is also very important to study the stability of the equilibrium of resource allocation mechanism model (4). This is also the judgment basis to test the feasibility and rationality of our model.

Obviously, model (4) satisfies Lipschitz condition. Then for any given initial value $X(0) = (U(0), P(0)) \in \mathbb{R}^2_+$, there is a unique solution $X(t) = (U(t), P(t)) \in \mathbb{R}^2_+$ of model (4). Furthermore, we can get the non-trivial solution of model (4) with initial value $X(0) = (U(0), P(0)) \in \mathbb{R}^2_+$, which has the following form

$$U(t) = \frac{\exp\left\{b_1t - \left(a_{12} + \frac{b_1}{K}\right)\int_0^t P(s)\,\mathrm{d}s\right\}}{\frac{1}{U(0)} + \frac{b_1}{K}\int_0^t \exp\left\{b_1\tau - \frac{Ka_{12}+b_1}{K}\int_0^s P(\tau)\mathrm{d}\tau\right\}\mathrm{d}s},$$
$$P(t) = \frac{\exp\left\{b_2t + a_{21}\int_0^t U(s)\,\mathrm{d}s\right\}}{\frac{1}{P(0)} + a_{22}\int_0^t \exp\left\{b_2\tau + a_{21}\int_0^s U(\tau)\,\mathrm{d}\tau\right\}\,\mathrm{d}s}.$$

From the form of the solution, it is obvious that the model (4) has a unique positive solution. That is to say, the proposed resource allocation scheme is feasible. Furthermore, we need to study the stability of the model (4) for illustrating the robustness of the resource allocation scheme in MEC.

C. STABILITY ANALYSIS

Next, we will focus on the stability of the positive equilibrium $\mathbf{E}^*(U^*, P^*)$. The meaning of the stable equilibrium point is that the non-trivial solution of model (4) for any given initial value $X(0) = (U(0), P(0)) \in \mathbb{R}^2_+$ will converge to a fixed equilibrium point. The following theorem mainly states that the positive equilibrium of the model (4) is stable and sufficient conditions for its stability are obtained.

Theorem 1: The positive equilibrium $\mathbf{E}^*(U^*, P^*)$ of model (4) is globally asymptotically stable, if the following condition are satisfies

$$\frac{a_{21}}{2} < \min\left\{\frac{b_1}{K}, a_{22}\right\}, \quad 4A_1C_1 - B_1^2 < 0,$$

and $4A_2C_1 - B_2^2 < 0$, where

$$\begin{aligned} &A_1 = a_{21}/2 - b_1/K, \quad B_1 = b_1 + b_1/KU^* - a_{21}P^*, \\ &C_1 = -b_1U^*, \quad A_2 = a_{21}/2 - a_{22}, \\ &B_2 = b_2 + (b/K + a_{12})U^* + a_{22}P^*, \quad C_2 = -b_2P^*. \end{aligned} \tag{5}$$

Stability is of great physical importance to a physics system, and it is also an important index to measure the rationality of a mathematical model. Theorem 1 shows that our model (4) is reasonable and can maintain a stable working state. This means that the resource allocation scheme proposed by us has good robustness. The proof of the theorem is given in the appendix, and the verifiability of the condition will be analyzed in detail in the numerical simulation.

However, the model (4) is only suitable for relatively stable network environment. The actual MEC network environment has a lot of stochastic perturbations, and these perturbations will greatly affect the stability of the resource allocation model. In fact, any network environment is stochastic rather than deterministic. In order to get closer to the real MEC network environment, in the next section, we extend the deterministic model (4) into a stochastic model one to carry out the research.

IV. EXTENSION OF MODEL TO WITH STOCHASTIC PERTURBATION

In MEC network architecture, the user layer is usually composed of mobile phones, computers, cars, smart watches, cameras and other devices which are heterogeneous and highly mobile. The stability of MEC network will be affected due to the heterogeneity and strong mobility of these devices. In addition, there are many unpredictable random factors that will have a great impact on the MEC network, such as having a group device on-line or off-line and the unconventional behavior of some users can have a significant impact on the MEC network. In this section, we will use stochastic differential equations to describe the effects of stochastic perturbations on the resource allocation model of MEC networks.

A. MODEL FORMULATION AND PRELIMINARIES

Taking into account the effect of stochastic perturbation environment as mentioned above, we incorporate stochastic perturbation in main parameters of model (4). Throughout this paper, we let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). We assume that perturbation in the environment shall manifest themselves mainly as perturbation in the parameters: $b_1 \rightarrow b_1 + \sigma_1 dB_1(t)$, $b_2 \rightarrow b_2 + \sigma_2 dB_2(t)$, where $B_i(t)$ $(i = 1, 2)$ are mutually independent one-dimensional standard Brownian motions defined on a complete probability space with $B_1(0) = B_2(0) = 0$, $\sigma_1 > 0$ and $\sigma_2 > 0$ which denotes the intensities of stochastic perturbations. Hence a stochastic version corresponding to model (4) takes the following form

$$\begin{cases} dU(t) = f_1(t)\,dt + \sigma_1 U(t)\,dB_1(t), \\ dP(t) = f_2(t)\,dt + \sigma_2 P(t)\,dB_2(t), \end{cases} \tag{6}$$

where

$$f_1(t) = U(t)\left[b_1\left(1 - \frac{U(t)+P(t)}{K}\right) - a_{12}P(t)\right]$$

and

$$f_2(t) = P(t)\left[b_2 + a_{21}U(t) - a_{22}P(t)\right].$$

B. EXISTENCE AND UNIQUENESS OF POSITIVE SOLUTIONS

Since the model (6) is based on a real physical problem, the first concerning thing is whether the positive solution is global existence and uniqueness. Generally, in order for a stochastic differential equation to have a unique global solution for any given initial value, the coefficients of the equation are generally required to satisfy the linear growth condition and local Lipschitz condition (see [39]). However, the coefficients of model (6) do not satisfy the linear growth condition, though they are locally Lipschitz continuous, so the solution of model (6) may explode at a finite time (see [40]). In the following theorem, we will strictly prove it the solution of model (6) is positive and global.

Theorem 2: For any given initial value $(U(0), P(0)) \in \mathbb{R}^2_+$, there is a unique positive solution $(U(t), P(t))$ of model (6) for all $t \geq 0$ and the solution will remain in $\mathbb{R}^2_+$ with probability one. And we can directly calculate the explicit solution of model (6) with the initial value $(U(0), P(0)) \in \mathbb{R}^2_+$ as follows

$$U(t) = \frac{\exp\{\varphi(t) + \sigma_1 B_1(t)\}}{\frac{1}{U(0)} + \frac{b_1}{K}\int_0^t \exp\{\varphi(s) + \sigma_1 B_1(s)\}\,ds}$$

and

$$P(t) = \frac{\exp\{\psi(t) + \sigma_2 B_2(t)\}}{\frac{1}{P(0)} + a_{22}\int_0^t \exp\{\psi(s) + \sigma_2 B_2(s)\}\,ds},$$

where

$$\varphi(t) = \left[b_1 - \frac{\sigma_1^2}{2}\right]t - \left(a_{12} + \frac{b_1}{K}\right)\int_0^t P(s)\,ds$$

and

$$\psi(t) = \left[b_2 - \frac{\sigma_2^2}{2}\right]t + a_{21}\int_0^t U(s)\,ds.$$

Remark 1: We not only obtained the existence and uniqueness of positive solutions of model (6), but also get the explicit solution from Theorem 2. It is obvious from the explicit solution that the intensity (σ_1 and σ_2) of the stochastic perturbation has a direct influence on the solution.

For the deterministic model (4), there is globally asymptotically stable positive equilibrium (U^*, P^*) when the conditions of Theorem 1 are satisfied. That is, MEC host can provide users with high-quality resource services and reasonable deployment of MEC-IDS to ensure the security of the system under the resouce allocation mechanism we have established, which implies that the system can work safely and stably. However, in the real world, stochastic perturbations are ubiquitous. Whether the solution of the stochastic model (6) is stable or not, next, we will study in detail.

C. STABILITY ANALYSIS

Since there is no equilibrium in the stochastic model (6), we mainly analyze whether the solution of stochastic model (6) has an upper bound and a lower bound in the mean sense. With the positive equilibrium of the deterministic model (4) as the reference, the solution of the stochastic model will oscillate in an interval. In this paper, We define this interval as the stability interval of the solution of the stochastic model (6). Next, we focus on whether there is a stable interval for the stochastic model (6). If there is a stable interval, it can be said that the solution of the stochastic model (6) is stable or persistent. From the point of view of stochastic stochastic control theory, the stable interval indicates that the deterministic model (4) has certain anti-perturbation function. That is, we find a stochastic control condition so that the resource allocation mechanism proposed in Section 3 can still provide services continuously and stably, even if there are some unpredictable stochastic perturbations factors, the system can still work stably.

For the sake of narrative convenience, let us introduce the following preliminary knowledge. First, we introduce a mathematical symbol: if $f(t)$ is an integrable function on $[0, \infty)$, define $\langle f \rangle_t = (\int_0^t f(s)\,\mathrm{d}s)/t$, $t > 0$. And then we present a number of lemmas and definitions in order to later theoretical analysis.

First, a general stochastic differential equation is introduced as follows

$$\mathrm{d}x(t) = f(t)\,\mathrm{d}t + g(t)\,\mathrm{d}B(t). \tag{7}$$

Definition 1 ([41]): The solution $x(t)$ of stochastic differential (7) is said to be persistent in the mean if

$$0 < m \leq \liminf_{t\to\infty} \langle x(s) \rangle_t \leq \limsup_{t\to\infty} \langle x(s) \rangle_t \leq M < \infty \ a.s., \tag{8}$$

where m and M are two positive constants.

Remark 2: Considering the actual background of MEC, in this paper, we called $U(t)$ stable in the interval $[m, M]$ if $U(t)$ satisfied the condition (8), and defined $[m, M]$ is a stable interval of $U(t)$. Moreover, if $[m_1, M_1]$ and $[m_2, M_2]$ are stable interval for $U(t)$ and $P(t)$, respectively, defined $[m_1, M_1] \times [m_2, M_2]$ is a stable interval of solution of stochastic model (6).

Lemma 1 (See Theorem 6.4 in [39]): Let $x(t)$ be a d-dimensional Itô process on $t \geq 0$ with the stochastic differential (7), where $f \in \mathcal{L}^1(\mathbb{R}_+; \mathbb{R}^d)$ and $g \in \mathcal{L}^2(\mathbb{R}_+; \mathbb{R}^{d\times m})$. Let $V \in C^{2,1}(\mathbb{R}^d \times \mathbb{R}_+; \mathbb{R})$, then $V(x(t), t)$ is again an Itô process with the stochastic differential given by

$$\mathrm{d}V(t) = \mathbf{L}(x, t)\,\mathrm{d}t + V_x(x, t)g(t)\,\mathrm{d}B(t), \quad a.s. \tag{9}$$

where

$$\mathbf{L}V(x, t) = V_t(x, t) + V_x(x, t)f(t) + \frac{1}{2}tr(g^T(t)V_{xx}(x, t)g(t)),$$

and $\mathcal{L}^p(\mathbb{R}_+; \mathbb{R}^d)$ denote that the family of $\mathbb{R}^d$-valued $\mathcal{F}_t$-adapted processes $\{X(t)\}_{a\leq t\leq b}$ and $\int_a^b |X(t)|^p\,\mathrm{d}t < \infty$ a.s.

Lemma 2 (See Lemma 4 in [42]): Suppose that $x(t) \in C([0, \infty) \times \Omega, \mathbb{R}_+)$ is a stochastic process,

(i) if there exist two positive constants T and λ_0 such that for all $t \geq T$

$$\ln x(t) \leq \lambda t - \lambda_0 \int_0^t x(s)\,\mathrm{d}s + \sum_{i=1}^n \beta_i B_i(t),$$

where β_i is a constant $(1 \leq i \leq n)$, then

$$\begin{cases} \limsup\limits_{t\to\infty} \langle x(s) \rangle_t \leq \dfrac{\lambda}{\lambda_0} & a.s., \ if \ \lambda \geq 0, \\ \lim\limits_{t\to\infty} x(t) = 0 & a.s., \ if \ \lambda < 0; \end{cases}$$

(ii) if there exist three constants $T > 0$, $\lambda \geq 0$ and $\lambda_0 > 0$ such that for all $t \geq T$

$$\ln x(t) \geq \lambda t - \lambda_0 \int_0^t x(s)\,\mathrm{d}s + \sum_{i=1}^n \beta_i B_i(t),$$

where β_i is a constant $(1 \leq i \leq n)$, then $\liminf_{t\to\infty} \langle x(s) \rangle_t \geq \lambda/\lambda_0$ *a.s.*

For the stochastic control method model (6), we have the following results.

Theorem 3: Let $(U(t), P(t))$ be the solution of model (6) with initial value $(U(0), P(0)) \in \mathbb{R}_+^2$, if $b_1 - \sigma_1^2/2 > 0$, $b_2 - \sigma_2^2/2 > 0$, and $(b_1 - \sigma_1^2/2) - \lambda\,(b_1/K + a_{12}) > 0$, where

$$\lambda = \frac{(b_2 - \sigma_2^2/2) + a_{21}\frac{b_1 - \sigma_1^2/2}{b_1/K}}{a_{22}},$$

then $U(t)$ and $P(t)$ are persistent in the mean. That is

$$0 < m_1 \leq \liminf_{t\to\infty} \langle U(s) \rangle_t \leq \limsup_{t\to\infty} \langle U(s) \rangle_t \leq M_1 \leq K,$$
$$0 < m_2 \leq \liminf_{t\to\infty} \langle P(s) \rangle_t \leq \limsup_{t\to\infty} \langle P(s) \rangle_t \leq \lambda \leq K.$$

where

$$m_1 = \frac{(b_1 - \sigma_1^2/2) - \lambda\,(b_1/K + a_{12})}{b_1/K},$$
$$M_1 = \frac{b_1 - \sigma_1^2/2}{b_1/K} \quad \text{and } m_2 = \frac{b_2 - \sigma_2^2/2}{a_{22}}.$$

Remark 3: We can get that following results from the conclusion of the Theorem 3

(i) $[m_1, M_1] \times [m_2, \lambda]$ is a stable interval of solution of stochastic model (6);

(ii) if $b_1 < \sigma_1^2/2$, then $\lim_{t\to\infty} U(t) = 0$;

(iii) if $b_1 - \sigma_1^2/2 < 0$ and $b_2 - \sigma_2^2/2 < 0$, then $\lim_{t\to\infty} U(t) = \lim_{t\to\infty} P(t) = 0$.

The results (ii) and (iii) mean that when the intensity of stochastic perturbation exceeds a certain threshold, the system will not be able to provide users with any service. It will guide us to increase the service capacity (b_1 and b_2) of the system or reduce the intensities (σ_1 and σ_2) of external stochastic perturbation. In fact, according to our observations (b_1 and b_2) is much larger than (σ_1 and σ_2) although intensities of stochastic perturbation have great influence on the solution. In other words, the conditions of the theorem are easy to satisfy.

From the Theorem 3 we have obtained the persistence in the sense of the solution of the stochastic model (6), which means that the solution will oscillate within a stable interval. At the same time, we also notice that an upper bound and a lower bound of the solution are related to the intensity of the stochastic perturbation. It is necessary to quantitatively describe the relationship between the oscillation amplitude of the solution and the intensity of stochastic perturbation, effectively guiding our prevention and control, and ensuring the normal operation of the MEC network environment. We will elaborate on this in the next Theorem.

Theorem 4: Assume that $k_1 > 0$, $k_2 > 0$ are satisfied, then for the any given initial conditions $(U(0), P(0)) \in \mathbb{R}_+^2$, the solution $(U(t), P(t))$ of model (6) satisfies

$$\limsup_{t\to\infty} \frac{1}{t}\mathbf{E}\int_0^t [k_1(U - U^*)^2 + k_2(P - P^*)^2]\,\mathrm{d}t \leq \varpi,$$

TABLE 1. Parameters of model (4).

K	b_1	a_{12}	b_2	a_{21}	a_{22}
1	0.6	0.002	0.003	0.002	0.04

TABLE 2. Threshold conditions.

A_1	B_1	C_1	A_2	B_2	C_2
-0.5990	1.1281	-0.5283	-0.0390	0.5379	$-3.5709e-04$

TABLE 3. Threshold conditions.

$4A_1C_1 - B_1^2$	$4A_2C_2 - B_2^2$
-0.0067	-0.2892

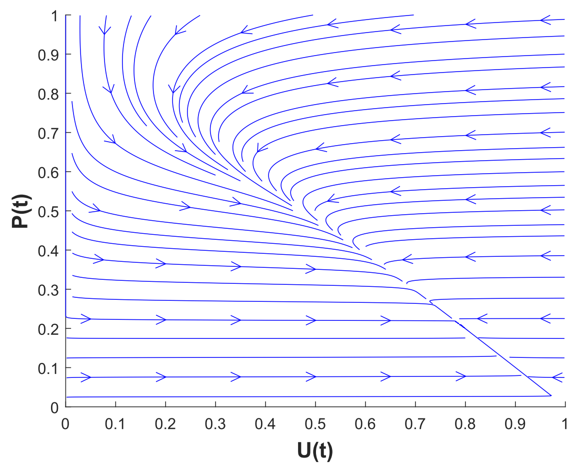

FIGURE 4. Trajectory diagram figure of model (4).

where $k_1 = b_1/K - a_{21} - \sigma_1^2$, $k_2 = b_1/K + a_{12} - a_{22} - \sigma_2^2$ and $\varpi = \sigma_1^2(U^*)^2 + \sigma_2^2(P^*)^2$.

Remark 4: Note that the oscillation amplitude $\varpi = \sigma_1^2(U^*)^2 + \sigma_2^2(P^*)^2$ of solution of stochastic model (6), which is a linear positive correlation function of σ_1^2 and σ_2^2. That means that ϖ is positively correlated with σ_1^2 and σ_2^2. And we take the positive equilibrium of the deterministic model as reference, calculate the oscillation amplitude and get the accurate calculation formula.

V. NUMERICAL SIMULATIONS

To show the effectiveness of our proposed theoretical results, in this section, we perform numerical simulations with the help of software from MATLAB soft. As an example for model (4), we chose the parameters as Table 1. Different from the method in [43], our numerical example is not placed in a particular MEC network scenario. By analyzing the relationship between the two resources, a more general mathematical rule is obtained. According to the calculation, we get $R_0 = Kb_1a_{22} - b_2(b_1 + Ka_{12}) > 1$ and the model (4) admits a positive equilibrium $E^*(0.8806, 0.1190)$. Farther, it is easy to verifies that the conditions of Theorem 1 are satisfied, see Table 2 and Table 3. Therefore, the positive equilibrium E^* of model (4) is globally asymptotically stable, which is shown in trajectory diagram Figure 4 and planar graph Figure 5.

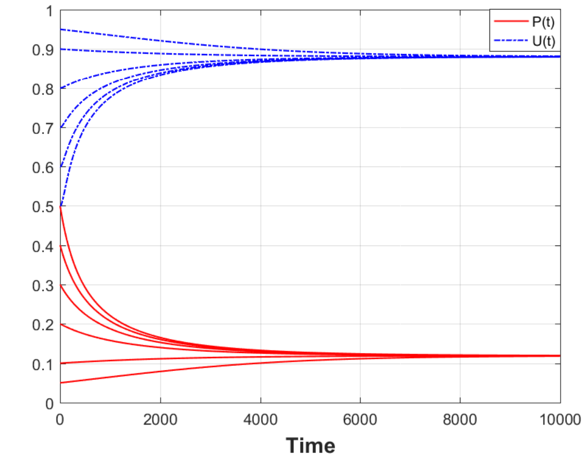

FIGURE 5. The solution of the model (4) with six different initial value conditions, where blue line represent $U(t)$ and red line represent $P(t)$.

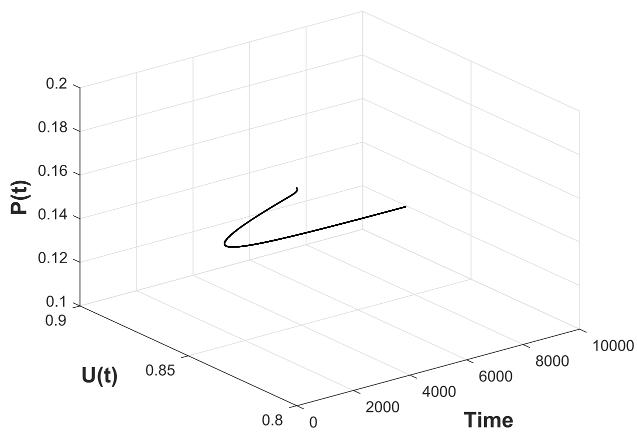

FIGURE 6. Spatial graph of deterministic model (4) with respect to the change in time.

The plots in Figure 4 show that trajectories of the model (4) will converge to a fixed point in the plane. Specifically, this fixed point is the positive equilibrium $E^*(0.8806, 0.1190)$ of model (4). The trends of $U(t)$ and $P(t)$ over time in three-dimensional space are depicted as shown in Figure 6. It can be seen that we have established the resource allocation mechanism for EMC network environment when the model parameters are fixed as shown in Table 1. This mechanism solves the resource allocation problem in the case of resource-constrained of MEC, and the method is stable. Notice that the sum of U^* and P^* is less than total resource K, because there is unnecessary consumption of network resources, which also is in line with the actual situation in the network.

The network environment is inevitably subject to a variety of unpredictable stochastic perturbation, especially the user devices such as MEC with strong mobility. In this paper, we use stochastic differential model (6) to describe this phenomenon. We know from Theorem 2, for initial value $(U(0), P(0)) = (0.8, 0.2) \in \mathbb{R}_+^2$, there is a unique positive solution $(U(t), P(t))$ of stochastic model (6) as show

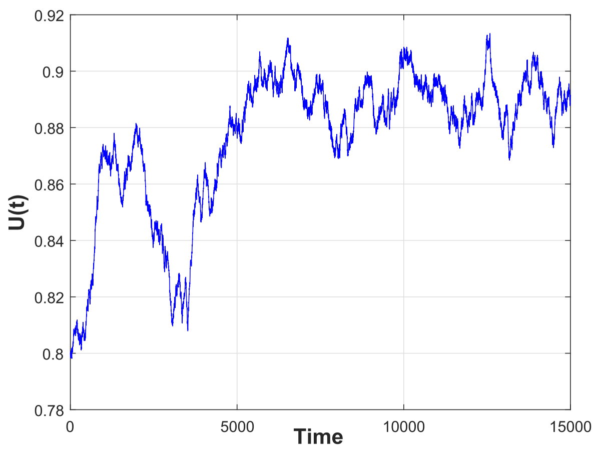

FIGURE 7. $U(t)$: The solution of stochastic model (6).

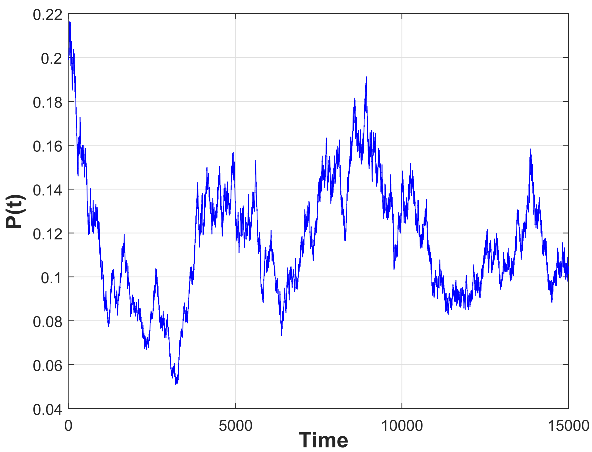

FIGURE 8. $P(t)$: The solution of stochastic model (6).

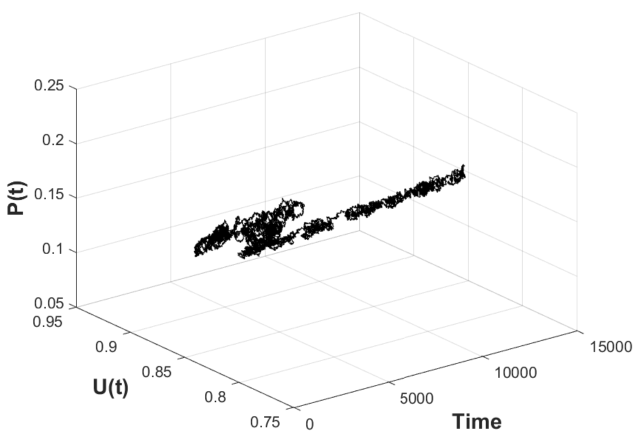

FIGURE 9. Spatial graph of stochastic model (6) with respect to the change in time.

in Figure 7 and Figure 8. Figure 9 depicts the trend of the solution of stochastic model (6) over time in three-dimensional space. Through the comparison of six pictures, it is not difficult to see that the stochastic model (6) has more complex

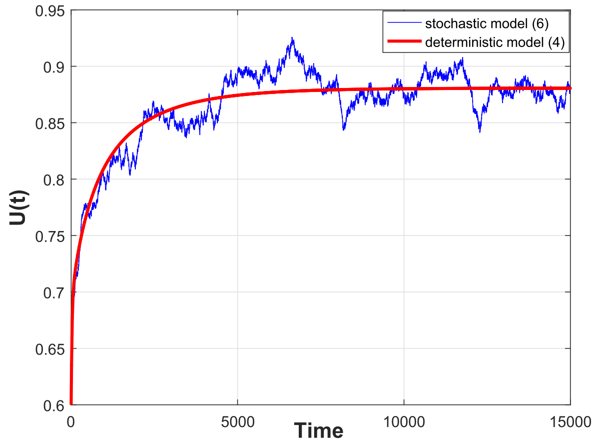

FIGURE 10. $U(t)$: Where red line represent the solution of deterministic model (4), blue line represent the solution of stochastic model (6).

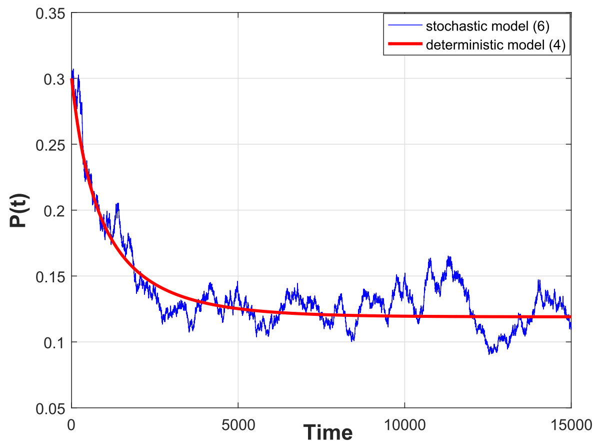

FIGURE 11. $P(t)$: Where red line represent the solution of deterministic model (4), blue line represent the solution of stochastic model (4).

dynamic behaviors than the deterministic model (4). Next we will conduct more qualitative analysis of stochastic model (6) in detail.

The basic parameters are fixed as shown in Table 1, and the intensities of stochastic perturbation are $\sigma_1 = 0.02$ and $\sigma_2 = 0.009$. Easy to calculate that $b_1 - \sigma_1^2/2 = 0.5996 > 0$, $b_2 - \sigma_2^2/2 = 0.0029 > 0$ and $\lambda = 0.1229 > 0$. Therefore, in view of Theorem 3 we get that $U(t)$ and $P(t)$ are persistent in the sense. In order to more obviously compare the dynamic differences between the stochastic model (6) and the deterministic model (4), we plot the solutions of the two models in the one figure. The plots in Figure 10 and Figure 11 indicate that solution of stochastic model (6) oscillates around the positive equilibrium of deterministic model (4). And we also notice that, the solution of stochastic model (4) exist a maximum oscillation interval when the solution of the deterministic model (4) converges to the positive equilibrium. That is, in the mean of time average, the solution of stochastic model (6) satisfies $U(t) \in [m_1, M]$ and $P(t) \in [m_2, \lambda]$. The correctness of the conclusion of the theorem refpersistence is illustrated by Figure 10 and Figure 11.

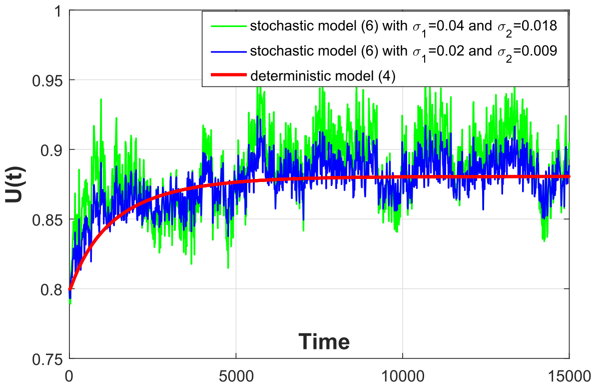

FIGURE 12. U(t): Where red line represent the solution of deterministic model (4), blue line represent the solution of stochastic (6) with large stochastic perturbation, green line represent the solution of stochastic (6) with small stochastic perturbation.

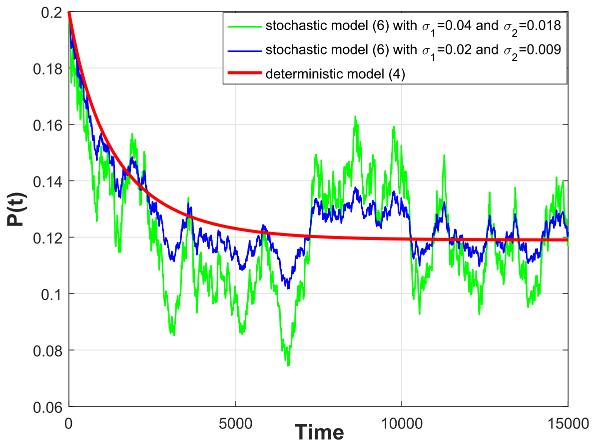

FIGURE 13. P(t): Where red line represent the solution of deterministic model (4), blue line represent the solution of stochastic model (6) with large stochastic perturbation, green line represent the solution of stochastic model (6) with small stochastic perturbation.

Furthermore, by the Theorem 4, we concluded that the greater the intensity of the stochastic perturbation, the more violent the oscillation of the solution. On the contrary, the smaller the stochastic perturbation are, the closer the solution to model (4) will be to the positive equilibrium point of deterministic model (4), which are shown in Figure 12 and Figure 13. This theory will have important guiding significance to the practical work. In the real MEC network environment, stochastic perturbation should be controlled as far as possible, so that the system can provide users with higher quality services.

VI. CONCLUSIONS

In this paper, we mainly solved the resource allocation problem of deploying MEC-IDS in MEC environment. We proposed a new allocation mechanism by means of mathematical modeling. An important conclusion that the stability of model (4) are achieved. The physical significance of the stability of the positive equilibrium in the actual MEC network environment is also gotten. Considering that the network environment has many random factors, we proposed and studied the stochastic model (6). In fact, deterministic model (4) is a special state of stochastic model (6) when the intensities of the stochastic perturbation $\sigma_1 = \sigma_2 = 0$. Therefore, it is more versatile to study the properties of the stochastic model (4) based on the deterministic model (4). We proved the persistance of global positive solution of stochastic model (6), which means the solution will oscillate in a stable interval $[m_1, M] \times [m_2, \lambda]$. That is, even if there are some stochastic perturbations, the proposed resource allocation mechanism can still ensure the MEC network environment to work stably. Finally, we got an interesting result that there is a positive correlation between the amplitude of the solution stochastic model (4) and the intensity of the stochastic perturbation by comparing with the deterministic model (4). That is to say, when the stochastic perturbation is large, it is adverse to the stability of the system. This theory will promote our technical progress in controlling stochastic perturbation in MEC network environment. Numerical simulation verifies the correctness and feasibility of the theoretical results. In addition, the method we proposed is strictly based on mathematical proof and can be applied to a wide range of resource allocation problems.

APPENDIX

Before proving Theorem 1 we first need to prove the following Lemma.

Lemma 3: For all $x > 0$ and $x^* > 0$, the following inequality holds

$$f(x) = x - x^* - x^* \ln \frac{x}{x^*} \geq 0.$$

Proof 1: In view of Lagrange Mean Value Theorem, there is a constant $\xi \in [\min\{x, x^*\}, \max\{x, x^*\}]$, such that

$$f(x) = f(x^*) + (x - x^*)f'(\xi) = (x - x^*)\left(1 - \frac{x^*}{x}\right).$$

Further, we can get

$$\begin{cases} f(x) > 0, & if\ x < x^*; \\ f(x) = 0, & if\ x = x^*; \\ f(x) > 0, & if\ x > x^*. \end{cases}$$

This completes the proof of Lemma 3.

Proof of Theorem 1:

Proof 2: Define a Lyapunoov function as follow

$$V(U(t), P(t)) = U - U^* - U^* \ln\left(\frac{U}{U^*}\right) + P - P^* - P^* \ln\left(\frac{P}{P^*}\right).$$

$V(U(t), P(t)) \geq 0$ can be obtained from the Lemma 3. Furthermore, we can compute

$$\dot{V}(t) = \left(1 - \frac{U^*}{U}\right)\left[b_1 U\left(1 - \frac{U+P}{K}\right) - a_{12}UP\right] + \left(1 - \frac{P^*}{P}\right)P\left[b_2 + a_{21}U - a_{22}P\right]$$

$$
\begin{aligned}
&= (U-U^*)[b_1 - b_1/KU - (b_1/K + a_{12})P] \\
&\quad + \left(P-P^*\right)[b_2 + a_{21}U - a_{22}P] \\
&= b_1U - b_1/KU^2 - (b_1/K + a_{12})UP - b_1U^* \\
&\quad + b_1/KUU^* + (b_1/K + a_{12})PU^* + b_2P \\
&\quad + a_{21}UP - a_{22}P^2 - b_2P^* - a_{21}P^*U + a_{22}P^*P.
\end{aligned}
$$

In view of the fact $a_{21}PU \leq a_{21}/2(P^2+U^2)$, we get

$$
\begin{aligned}
\dot{V}(t) \leq\; & (a_{21}/2 - b_1/k)U^2 + (b_1 + b_1/KU^* - a_{21}P^*)U \\
& + [b_2 + (b_1/K + a_{12})U^* + a_{22}P^*]P \\
& + (a_{21}/2 - a_{22})P^2 - b_1U^* - b_2P^*.
\end{aligned}
$$

Note that the condition of theorem $a_{21}/2 < \min\{b_1/K, a_{22}\}$, $4A_1C_1 - B_1^2 < 0$ and $4A_1C_1 - B_1^2 < 0$, which guarantee that

$$\dot{V}(t) < 0.$$

Therefore, by the Lyapunov theory we can obtain that the $\mathbf{E}^*(U^*, P^*)$ of model (4) is globally asymptotically stable. The proof is complete.

Proof of Theorem 2:

Proof 3: Since the coefficient of model (6) are locally Lipschitz continuous, then for any given initial value $(U(0), P(0)) \in \mathbb{R}_+^2$, there is a unique local solution $(U(t), P(t))$ on $t \in [0, \tau_e)$, where τ_e is the explosion time ([40]). To show this solution is global, we only need to show that $\tau_e = \infty$.

For $t \in [0, \tau_e)$, the first equation for the model (6), let $x = 1/U$. Application formula (9) we get

$$
\mathrm{d}x = \left[\frac{b_1}{K} + (\sigma_1^2 - b_1)x + \left(\frac{b_1}{K} + a_{12}\right)xP\right]\mathrm{d}t - \sigma_1 U\,\mathrm{d}B_1(t). \qquad (10)
$$

It is easy to see that this is a non-homogeneous random differential equation with respect to the variable x. The corresponding homogeneous stochastic differential equation as follows

$$
\mathrm{d}x = \left[(\sigma_1^2 - b_1)x + \left(\frac{b_1}{K} + a_{12}\right)xP\right]\mathrm{d}t - \sigma_1 x\,\mathrm{d}B_1(t).
$$

Let $y = \ln x$ and using the formula (9)

$$
\mathrm{d}y = \mathrm{d}\ln x = \left(\frac{1}{2}\sigma_1^2 - b_1\right) + \left(\frac{b_1}{k} + a_{12}\right)P\,\mathrm{d}t - \sigma_1\,\mathrm{d}B_1(t).
$$

Integrating above equality frow 0 to t and in view of $y = \ln x$ we have

$$
x(t) = x_0 \exp\{\left[\frac{\sigma_1^2}{2} - b_1\right]t - \sigma_1 B_1(t) + \left(a_{12} + \frac{b_1}{K}\right)\int_0^t P(s)\,\mathrm{d}s\}.
$$

Using the variation-of-constants formal, the explicit solution of is non-homogeneous stochastic differential equation given by

$$
U(t) = \frac{\exp\{\varphi(t) + \sigma_1 B_1(t)\}}{\frac{1}{U(0)} + \frac{b_1}{K}\int_0^t \exp\{\varphi(s) + \sigma_1 B_1(s)\}\,\mathrm{d}s}.
$$

where

$$
\varphi(t) = \left[b_1 - \frac{\sigma_1^2}{2}\right]t - \left(a_{12} + \frac{b_1}{K}\right)\int_0^t P(s)\,\mathrm{d}s
$$

Apply the same method as above, we get that for $t \in [0, \tau_e)$

$$
P(t) = \frac{\exp\{\psi(t) + \sigma_2 B_2(t)\}}{\frac{1}{P(0)} + a_{22}\int_0^t \exp\{\psi(s) + \sigma_2 B_2(s)\}\,\mathrm{d}s},
$$

where

$$
\psi(t) = \left[b_2 - \frac{\sigma_2^2}{2}\right]t + a_{21}\int_0^t U(s)\,\mathrm{d}s.
$$

Consider the following comparison equations

$$
\begin{cases}
\mathrm{d}\overline{U}(t) = b_1\overline{U}(t)\left(1 - \dfrac{\overline{U}(t)}{K}\right)\mathrm{d}t + \sigma_1\overline{U}(t)\,\mathrm{d}B_1(t), \\
\mathrm{d}\overline{P}(t) = \overline{P}(t)\left[b_2 + a_{21}\overline{U}(t) - a_{22}\overline{P}(t)\right]\mathrm{d}t \\
\qquad + \sigma_2\overline{P}(t)\,\mathrm{d}B_2(t),
\end{cases} \qquad (11)
$$

with initial value $U(0) = \overline{U}(0)$ and $P(0) = \overline{P}(0)$. Then we get the explicit solution of model (11) as follows

$$
\overline{U}(t) = \frac{\exp\left\{\left[Kb_1 - \frac{\sigma_1^2}{2}\right]t + \sigma_1 B_1(t)\right\}}{\frac{1}{U(0)} + b_1\int_0^t \exp\left\{\left[Kb_1 - \frac{\sigma_1^2}{2}\right]s + \sigma_1 B_1(s)\right\}\mathrm{d}s},
$$

and

$$
\overline{P}(t) = \frac{\exp\{\chi(t) + \sigma_2 B_2(t)\}}{\frac{1}{P(0)} + a_{22}\int_0^t \exp\{\chi(s) + \sigma_2 B_2(s)\}\,\mathrm{d}s},
$$

where

$$
\chi(t) = \left[b_2 - \frac{\sigma_2^2}{2}\right]t + a_{21}\int_0^t \overline{U}(s)\,\mathrm{d}s.
$$

By the comparison theorem, we have $U(t) \leq \overline{U}(t)$ and $P(t) \leq \overline{P}(t)$. Since all the parameters of model (11) are positive bounded constant, then $\overline{U}(t)$ and $\overline{P}(t)$ will not explode in finite time. Thus, $U(t)$ and $P(t)$ will not also explode in finite time. That is, $\tau_e = \infty$. This is complete proof.

Proof of Theorem 3:

Proof 4: Making use of Itô's formula, we drive from model (6) that

$$
\begin{aligned}
\mathrm{d}\ln U(t) &= g_1(t)\,\mathrm{d}t + \sigma_1\,\mathrm{d}B_1(t), \\
\mathrm{d}\ln P(t) &= g_2(t)\,\mathrm{d}t + \sigma_2\,\mathrm{d}B_2(t),
\end{aligned}
$$

where

$$
g_1(t) = \left(b_1 - \frac{\sigma_1^2}{2}\right) - \frac{b_1}{K}U(t) - \left(\frac{b_1}{K} + a_{12}\right)P(t),
$$

and

$$
g_2(t) = \left(b_2 - \frac{\sigma_2^2}{2}\right) + a_{21}U(t) - a_{22}P(t).
$$

Integrating from 0 to t on both sides of the above equations, we get

$$\ln U(t) = \ln U(0) + \int_0^t g_1(t)\,\mathrm{d}s + \sigma_1 B_1(t),$$
$$\ln P(t) = \ln P(0) + \int_0^t g_2(t)\,\mathrm{d}s + \sigma_2 B_2(t). \tag{12}$$

Therefore,

$$\ln U(t) \le \ln U(0) + \left(b_1 - \frac{\sigma_1^2}{2}\right)t - \frac{b_1}{K}\int_0^t U(s)\mathrm{d}s + \sigma_1 B_1(t).$$

From the large number for martingales (see [39]), we have $\lim_{t\to\infty}[\sigma_1 B(t)]/t = 0$. Then by Lemma 2, we can get

$$\limsup_{t\to\infty} \frac{1}{t}\int_0^t U(s)\,\mathrm{d}s \le \frac{b_1 - \sigma_1^2/2}{b_1/K},$$

where we used the fact $\lim_{t\to\infty} \ln U(0)/t = 0$. Similarly, we can obtain that

$$\ln P(t) \ge \ln P(0) + \left(b_2 - \frac{\sigma_2^2}{2}\right)t - a_{22}\int_0^t P(s)\mathrm{d}s + \sigma_2 B_2(t).$$

By the same method as above, it can be easily shown that

$$\liminf_{t\to\infty} \frac{1}{t}\int_0^t P(s)\,\mathrm{d}s \ge \frac{b_2 - \sigma_2^2/2}{a_{22}}.$$

Note that $\limsup_{t\to\infty}\langle U(s)\rangle_t \le \frac{b_1-\sigma_1^2/2}{b_1/K}$, for arbitrary $0 < \varepsilon_1 < 1$, there is a positive constant $T_1 = T_1(\varepsilon_1)$ such that for all $t > T_1$, $\langle U(s)\rangle_t \le \frac{b_1-\sigma_1^2/2}{b_1/K} + \varepsilon_1$. In view of (12) we have

$$\ln P(t) \le \left(b_2 - \frac{\sigma_2^2}{2}\right)t + a_{21}\left(\frac{b_1 - \sigma_1^2/2}{b_1/K} + \varepsilon_1\right)t$$
$$+ \ln P(0) - a_{22}\int_0^t P(s)\,\mathrm{d}s + \sigma_2 B_2(t).$$

By virtue of Lemma 2, we get

$$\limsup_{t\to\infty} \frac{1}{t}\int_0^t P(s)\,\mathrm{d}s \le \frac{\left(b_2 - \frac{\sigma_2^2}{2}\right) + a_{21}\left(\frac{b_1-\sigma_1^2/2}{b_1/K} + \varepsilon_1\right)}{a_{22}}.$$

According to the arbitrariness of ε_1, one can obtain that

$$\limsup_{t\to\infty}\langle P(s)\rangle_t \le \frac{(b_2 - \sigma_2^2/2) + a_{21}\frac{b_1-\sigma_1^2/2}{b_1/K}}{a_{22}} \equiv \lambda. \tag{13}$$

Then by (13), we can see that for arbitrary $0 < \varepsilon_2 < \frac{b_1-\sigma_1^2 2}{b_1/K+a_{12}} - \lambda$, there is a positive constant $T_2 = T_2(\varepsilon_2)$ such that for all $t > T_2$, $\langle P(s)\rangle_t \le \lambda + \varepsilon_2$. Therefore, for all $t > T_2$, we get

$$\ln U(t) \ge \ln U(0) + \left(b_1 - \frac{\sigma_1^2}{2}\right)t - \frac{b_1}{K}\int_0^t U(s)\,\mathrm{d}s$$
$$- \left(\frac{b_1}{K} + a_{12}\right)(\lambda + \varepsilon_2)t + \sigma_1 B(t).$$

By the same method as above, it can be easily shown that

$$\liminf_{t\to\infty}\langle U(s)\rangle \ge \frac{(b_1 - \sigma_1^2/2) - \lambda\,(b_1/K + a_{12})}{b_1/K}.$$

The proof is complete.

Proof of Theorem 4:

Proof 5: Consider the Lyapunov function as follow

$$V(U,P) = \frac{1}{2}(U - U^* + P - P^*)^2.$$

Then along the trajectories of model (6) by using Itô's formula, the total differential of function $V(U,P)$ can be obtained as

$$\begin{aligned}\mathrm{d}V(U,P) = &\left(U - U^* + P - P^*,\quad U - U^* + P - P^*\right)\\ &\cdot\begin{pmatrix} b_1\left(1 - \frac{U(t)+P(t)}{K}\right) - a_{12}P(t)\\ b_2 + a_{21}U(t) - a_{22}P(t)\end{pmatrix}\mathrm{d}t\\ &+\frac{1}{2}tr\left(\begin{pmatrix}\sigma_1 U & 0\\ 0 & \sigma_2 P\end{pmatrix}^T\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}\sigma_1 U & 0\\ 0 & \sigma_2 P\end{pmatrix}\right)\mathrm{d}t\\ &+\left(U - U^* + P - P^*,\quad U - U^* + P - P^*\right)\\ &\cdot\begin{pmatrix}\sigma_1 U & 0\\ 0 & \sigma_2 P\end{pmatrix}\begin{pmatrix}\mathrm{d}B_1(t)\\ \mathrm{d}B_2(t)\end{pmatrix},\end{aligned} \tag{14}$$

and we denote $\mathrm{d}V(U,P) = \mathbf{L}V(U,P)\,\mathrm{d}t + V_x(x,t)g(t)\,\mathrm{d}B(t)$. Note that $\mathbf{E}^*(U^*,P^*)$ is point positive equilibrium of model (4), which implies that

$$b_1\left(1 - \frac{U^*+P^*}{K}\right) - a_{12}P^* = 0,$$
$$b_2 + a_{21}U^* - a_{22}P^* = 0. \tag{15}$$

Substituting (15) into (14) gives that

$$\begin{aligned}\mathbf{L}V(U,P) &= (U - U^* + P - P^*)[-\left(\frac{b_1}{K} - a_{21}\right)(U - U^*)\\ &\quad - \left(\frac{b_1}{K} + a_{12} - a_{22}\right)(P - P^*)]\\ &\quad + \frac{1}{2}\sigma_1^2U^2 + \frac{1}{2}\sigma_2^2P^2\\ &= -\left(\frac{b_1}{K} - a_{21}\right)(U - U^*)^2 + \frac{1}{2}\sigma_1^2U^2\\ &\quad - \left(\frac{b_1}{K} + a_{12} - a_{22}\right)(P - P^*)^2 + \frac{1}{2}\sigma_2^2P^2.\end{aligned}$$

In view of elementary inequality $0 \le (a - 2b)^2$ for any $(a,b) \in \mathbb{R}_+^2$, we derive that

$$\frac{1}{2}\sigma_1^2U^2 + \frac{1}{2}\sigma_2^2P^2 \le \sigma_1^2(U^*)^2 + \sigma_1^2(U - U^*)^2$$
$$+ \sigma_2^2(P^*)^2 + \sigma_2^2(P - P^*)^2. \tag{16}$$

Therefore, it follows form $\mathbf{L}V(U,P)$ that

$$\begin{aligned}\mathbf{L}V(U,P) \le &-\left(\frac{b_1}{K} - a_{21} - \sigma_1^2\right)(U - U^*)^2\\ &- \left(\frac{b_1}{K} + a_{12} - a_{22} - \sigma_2^2\right)(P - P^*)^2\\ &+ \sigma_1^2(U^*)^2 + \sigma_2^2(P^*)^2.\end{aligned}$$

Set $k_1 = b_1/K - a_{21} - \sigma_1^2$, $k_2 = b_1/K + a_{12} - a_{22} - \sigma_2^2$ and $\varpi = \sigma_1^2(U^*)^2 + \sigma_2^2(P^*)^2$, we see that

$$\begin{aligned} \mathrm{d}V(U,P) \le {} & [-k_1(U-U^*)^2 - k_2(P-P^*)^2 + \varpi]\,\mathrm{d}t \\ & + (U-U^*+P-P^*)\,\mathrm{d}B_1(t) \\ & + (U-U^*+P-P^*)\,\mathrm{d}B_2(t). \end{aligned}$$

Integrating above inequality frow 0 to t and taking expectation on both side yields that

$$\begin{aligned} & \mathbf{E}[V(U(t),P(t))] - V(U(0),P(0)) \\ & \quad \le -\mathbf{E}\int_0^t [k_1(U-U^*)^2 + k_2(P-P^*)^2]\,\mathrm{d}t + \varpi t. \end{aligned}$$

In view of the $V(U(t),P(t)) \ge 0$ for all $t \in \mathbb{R}_+$, we can then show that

$$\limsup_{t\to\infty} \frac{1}{t}\mathbf{E}\int_0^t [k_1(U-U^*)^2 + k_2(P-P^*)^2]\,\mathrm{d}t \le \varpi.$$

The proof is therefore complete.

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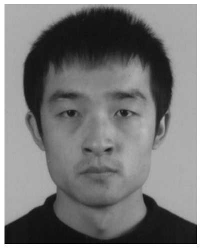

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