## Appendix to "Market Intraday Momentum"

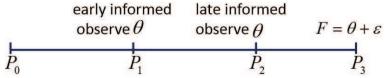
This appendix provides a simple late-informed trading model and other results not included in the paper. Below, we briefly describe the contents of the appendix.

- Appendix A: The late-informed trading model discussed in footnote 10.
- Table A.1: Predictive regression results of S&P 500 futures and index discussed in footnote 3 of the paper.
- Table A.2: Alternative time intervals.
- Table A.3: Impact of volume and liquidity on the predictive regression results.
- Table A.4: Impact of volatility or volume on market timing performance.
- Table A.5: Univariate predictive regressions using other half-hour returns.
- Table A.6: Predictive regression results using bid-bid, ask-ask, and midquote-midquote returns discussed in footnote 5.
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## A A Late-Informed Trading Model

This model is based on Hirshleifer, Subrahmanyam, and Titman (1994) and Cespa and Vives (2015). The purpose is to show that, under certain standard theoretical conditions, the intraday momentum can arise from trading by late-informed investors.<sup>1</sup> Consider a three period, t = 0, 1, 2, 3, stock market in which a single asset with liquidation value F at time 3 and a riskless asset are traded by early- and late-informed investors, and also by liquidity traders. The liquidation value is assumed as

$$F = \bar{F} + \theta + \varepsilon, \ \theta \sim N(0, \sigma_{\theta}^2), \ \varepsilon \sim N(0, \sigma_{\varepsilon}^2).$$



The mass of the early informed traders who observe  $\theta$  at time 1 is denoted by M, while the mass of late-informed traders who observe  $\theta$  at time 2 is denoted by N-M. Both groups of traders have CARA utility over terminal wealth with a common risk-aversion coefficient  $\gamma$ . Each informed trader has an endowment of  $B_0$  units of the riskless asset. The market is competitive and there is a risk-neutral market maker. Noise is introduced in the form of liquidity demand shocks for the risky asset,  $z_1$  and  $z_2$ , which arrive at times 1 and 2, and follow an AR(1) process:

$$z_1 = u_1,$$
  
 $z_2 = \beta z_1 + \sqrt{1 - \beta^2} u_2,$ 

where  $u_1$  and  $u_2$  are normally distributed with mean zero and variance  $\sigma_u^2$ , and are independent of each other, and of  $\theta$  and  $\varepsilon$ .

<sup>&</sup>lt;sup>1</sup>We are extremely grateful to Dashan Huang for his enormous help on this model.

Terminal wealths for early- and late-informed traders are

$$W^{E} = x_{2}(\bar{F} + \theta + \varepsilon) - x_{1}P_{1} - (x_{2} - x_{1})P_{2} + B_{0},$$
  

$$W^{L} = y_{2}(\bar{F} + \theta + \varepsilon) - y_{1}P_{1} - (y_{2} - y_{1})P_{2} + B_{0}.$$

At t=2, both early- and late-informed traders observe  $\theta$ , and their demands are

$$x_2(\theta, P_2) = y_2(\theta, P_2) = \frac{\bar{F} + \theta - P_2}{\gamma \sigma_{\varepsilon}^2}.$$

At t=1, only early-informed traders observe  $\theta$ , and the terminal wealth is

$$W^{E} = \frac{\bar{F} + \theta - P_{2}}{\gamma \sigma_{\varepsilon}^{2}} (\bar{F} + \theta + \varepsilon) - \frac{\bar{F} + \theta - P_{2}}{\gamma \sigma_{\varepsilon}^{2}} P_{2} - x_{1} (P_{1} - P_{2}) + B_{0}$$
$$= \frac{(\bar{F} + \theta - P_{2})^{2}}{\gamma \sigma_{\varepsilon}^{2}} + \frac{(\bar{F} + \theta - P_{2})\varepsilon}{\gamma \sigma_{\varepsilon}^{2}} - x_{1} (P_{1} - P_{2}) + B_{0}.$$

Their expected utility at time 2 is

$$E[-e^{-\gamma W^{E}}|t=2] = -\exp\left\{-\gamma \left[B_{0} - x_{1}(P_{1} - P_{2}) + \frac{(\bar{F} + \theta - P_{2})^{2}}{2\gamma\sigma_{\varepsilon}^{2}}\right]\right\}.$$

Then, assuming  $P_2 \sim N(\bar{P}_2, \sigma_{P_2}^2)$ , we have

$$E[-e^{-\gamma W^E}|t=1] = E_1\left[-\exp\left\{-\gamma[B_0 - x_1(P_1 - P_2) + \frac{(\bar{F} + \theta - P_2)^2}{2\gamma\sigma_{\varepsilon}^2}]\right\}\right].$$

Hirshleifer, Subrahmanyam, and Titman (1994) show that

$$x_1(\theta, P_1) = \frac{\bar{P}_2 - P_1}{\gamma} \left( \frac{1}{\sigma_{P_2}^2} + \frac{1}{\sigma_{\varepsilon}^2} \right) + \frac{\bar{F} + \theta - \bar{P}_2}{\gamma \sigma_{\varepsilon}^2}. \tag{1}$$

Also, they show that, in equilibrium, the late-informed traders do not trade at time 1 and the demand is

$$y_1(P_1) = 0.$$

Suppose the linear equilibrium prices at times t = 1 and t = 2 are:

$$P_2 = \bar{F} + a\theta + bz_1 + cz_2, \tag{2}$$

$$P_1 = \bar{F} + e\theta + fz_1. \tag{3}$$

Then the equilibrium prices at t = 0 and t = 3 are

$$P_0 = \bar{F},$$

$$P_3 = \bar{F} + \theta + \varepsilon.$$

Since early-informed traders can observe  $\theta$  at time 1, they can back out  $z_1$  from  $P_1$ , and so

$$\bar{P}_2 = \mathrm{E}(P_2|\theta, z_1) = \bar{F} + a\theta + (b + c\beta)z_1, \tag{4}$$

$$\sigma_{P_2}^2 = \operatorname{Var}(P_2|\theta, z_1) = c^2(1 - \beta^2)\sigma_u^2.$$
 (5)

Note that  $c\beta$  in (4) is a new term that does not appear in Hirshleifer, Subrahmanyam, and Titman (1994) since they assume that the liquidity demands are independent of each other.

Replacing  $\bar{P}_2$  and  $\sigma^2_{P_2}$  in (1) with (4) and (5) gives

$$x_{1} = \theta \frac{a\sigma_{\varepsilon}^{2} + c^{2}(1 - \beta^{2})\sigma_{u}^{2}}{\gamma c^{2}(1 - \beta^{2})\sigma_{\varepsilon}^{2}\sigma_{u}^{2}} + z_{1} \frac{b + c\beta}{\gamma c^{2}(1 - \beta^{2})\sigma_{u}^{2}} + \frac{(\bar{F} - P_{1})(\sigma_{\varepsilon}^{2} + c^{2}(1 - \beta^{2})\sigma_{u}^{2})}{\gamma c^{2}(1 - \beta^{2})\sigma_{\varepsilon}^{2}\sigma_{u}^{2}}.$$
 (6)

Let

$$\tau_{1} = \frac{M(a\sigma_{\varepsilon}^{2} + c^{2}(1 - \beta^{2})\sigma_{u}^{2})}{\gamma c^{2}(1 - \beta^{2})\sigma_{\varepsilon}^{2}\sigma_{u}^{2}}\theta + \frac{M(b + c\beta) + \gamma c^{2}(1 - \beta^{2})\sigma_{u}^{2}}{\gamma c^{2}(1 - \beta^{2})\sigma_{u}^{2}}z_{1}$$

$$= \frac{M(a\sigma_{\varepsilon}^{2} + c^{2}(1 - \beta^{2})\sigma_{u}^{2})}{\gamma c^{2}(1 - \beta^{2})\sigma_{\varepsilon}^{2}\sigma_{u}^{2}}(\theta + kz_{1}),$$

where

$$k = \frac{M(b + c\beta)\sigma_{\varepsilon}^2 + \gamma c^2(1 - \beta^2)\sigma_{\varepsilon}^2\sigma_u^2}{M(a\sigma_{\varepsilon}^2 + c^2(1 - \beta^2)\sigma_u^2)}.$$

Since  $y_1(P_1) = 0$ , the total demand at time 1 is

$$D_1(P_1) = Mx_1 + z_1 = \frac{M(\bar{F} - P_1)(\sigma_{\varepsilon}^2 + c^2(1 - \beta^2)\sigma_u^2)}{\gamma c^2(1 - \beta^2)\sigma_{\varepsilon}^2\sigma_u^2} + \tau_1.$$

To the risk-neutral market maker, the price must satisfy

$$P_{1} = \mathrm{E}[\bar{F} + \theta + \varepsilon | \tau_{1}] = \bar{F} + \frac{\mathrm{Cov}(\theta, \tau_{1})}{\mathrm{Var}(\tau_{1})} \tau_{1}$$

$$= \bar{F} + \frac{\sigma_{\theta}^{2}}{\sigma_{\theta}^{2} + k^{2} \sigma_{u}^{2}} (\theta + kz_{1}), \qquad (7)$$

$$= \bar{F} + e\theta + fz_{1}, \qquad (8)$$

where

$$e = \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + k^2 \sigma_{\pi}^2}, \qquad f = ke. \tag{9}$$

At t = 2, the total demand is

$$D_2(P_2) = \frac{N}{\gamma \sigma_{\varepsilon}^2} (\bar{F} + \theta - P_2) + z_1 + z_2 = \frac{N}{\gamma \sigma_{\varepsilon}^2} (\bar{F} - P_2) + \tau_2,$$

where

$$\tau_2 = \frac{N}{\gamma \sigma_{\varepsilon}^2} \theta + z_1 + z_2 = \frac{N}{\gamma \sigma_{\varepsilon}^2} \theta + (1+\beta)z_1 + \sqrt{1-\beta^2}u_2.$$

The market maker observes  $\tau_1$  and  $\tau_2$  and sets the price as

$$P_2 = \mathrm{E}[\bar{F} + \theta + \varepsilon | \tau_1, \tau_2] = \bar{F} + \mathrm{E}[\theta | \tau_1, \tau_2].$$

Let 
$$A = \frac{M(a\sigma_{\varepsilon}^2 + c^2(1-\beta^2)\sigma_u^2)}{\gamma c^2(1-\beta^2)\sigma_{\varepsilon}^2\sigma_u^2}$$
 and  $v = (v_1, v_2)'$ . Then

$$\tau_1 = Av_1, \qquad \tau_2 = \frac{N}{\gamma \varepsilon_{\varepsilon}^2} v_2,$$

and hence

$$v_1 \sim N\left(0, \ \sigma_{\theta}^2 + k^2 \sigma_u^2\right),$$
  
 $v_2 \sim N\left(0, \ \sigma_{\theta}^2 + \frac{\gamma^2 \sigma_{\varepsilon}^4}{N^2}(2+2\beta)\sigma_u^2\right).$ 

It follows that

$$\operatorname{Var}(v) = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} = \begin{pmatrix} \sigma_\theta^2 + k^2 \sigma_u^2 & \sigma_\theta^2 + \frac{k\gamma(1+\beta)}{N} \sigma_\varepsilon^2 \sigma_u^2 \\ \sigma_\theta^2 + \frac{k\gamma(1+\beta)}{N} \sigma_\varepsilon^2 \sigma_u^2 & \sigma_\theta^2 + \frac{\gamma^2 \sigma_\varepsilon^4}{N^2} (2+2\beta) \sigma_u^2 \end{pmatrix}.$$

Armed with the above, we can compute explicitly

$$P_{2} = \bar{F} + \text{Cov}(\theta, v)' \text{Var}(v)^{-1} v$$

$$= \bar{F} + \left(\sigma_{\theta}^{2}, \sigma_{\theta}^{2}\right) \times \frac{1}{\sigma_{1}^{2} \sigma_{2}^{2} - \sigma_{12}^{2}} \begin{pmatrix} \sigma_{2}^{2} & -\sigma_{12} \\ -\sigma_{12} & \sigma_{1}^{2} \end{pmatrix} \times \begin{pmatrix} \theta + kz_{1} \\ \theta + \frac{\gamma \sigma_{\varepsilon}^{2}}{N} z_{1} + \frac{\gamma \sigma_{\varepsilon}^{2}}{N} z_{2} \end{pmatrix}$$

$$= \bar{F} + \frac{\sigma_{\theta}^{2}}{\sigma_{1}^{2} \sigma_{2}^{2} - \sigma_{12}^{2}} \left(\sigma_{2}^{2} - \sigma_{12}, -\sigma_{12} + \sigma_{1}^{2}\right) \begin{pmatrix} \theta + kz_{1} \\ \theta + \frac{\gamma \sigma_{\varepsilon}^{2}}{N} z_{1} + \frac{\gamma \sigma_{\varepsilon}^{2}}{N} z_{2} \end{pmatrix}$$

$$= \bar{F} + \frac{\sigma_{\theta}^{2}}{\sigma_{1}^{2} \sigma_{2}^{2} - \sigma_{12}^{2}} \left[ \left(\sigma_{2}^{2} - 2\sigma_{12} + \sigma_{1}^{2}\right) \theta + \left[\left(\sigma_{2}^{2} - \sigma_{12}\right) k + \left(\sigma_{1}^{2} - \sigma_{12}\right) \frac{\gamma \sigma_{\varepsilon}^{2}}{N} \right] z_{1} + \left(\sigma_{1}^{2} - \sigma_{12}\right) \frac{\gamma \sigma_{\varepsilon}^{2}}{N} z_{2} \right]$$

Let 
$$D = \sigma_{\theta}^2 \left[ kN - (1+\beta)\gamma \sigma_{\varepsilon}^2 \right]^2 + (1-\beta^2)\gamma^2 \sigma_{\varepsilon}^4 \left[ \sigma_{\theta}^2 + k^2 \sigma_{u}^2 \right]$$
. Then,  $\sigma_{1}^2 \sigma_{2}^2 - \sigma_{12}^2 = \frac{\sigma_{u}^2}{N^2} D$ .

Equating coefficients from Eq. (10) to Eq. (2), we obtain

$$a = \frac{\sigma_{\theta}^{2} \left[ (kN - (1+\beta)\gamma \sigma_{\varepsilon}^{2})^{2} + (1-\beta^{2})\gamma^{2}\sigma_{\varepsilon}^{4} \right]}{D},$$

$$b = \frac{k\gamma \sigma_{\varepsilon}^{2} \sigma_{\theta}^{2} \left[ \gamma \sigma_{\varepsilon}^{2} - \beta(kN - \gamma \sigma_{\varepsilon}^{2}) \right]}{D}, \quad \frac{k\gamma \sigma_{\varepsilon}^{2} \sigma_{\theta}^{2} \left[ kN - (1+\beta)\gamma \sigma_{\varepsilon}^{2} \right]}{D}.$$

Therefore

$$Cov(R_3, R_1) = Cov((1-a)\theta + \varepsilon - bz_1 - cz_2, e\theta + fz_1)$$
$$= (1-a)e\sigma_{\theta}^2 - (b+c\beta)f\sigma_{\eta}^2.$$

This is positive for small enough  $\sigma_u$  under standard assumptions. The intuition is that the late-informed traders will play a bigger role as long as the liquidity shocks are not too strong.

Table A.1: Predictability using S&P 500 futures and S&P 500 index

This table reports the results of regressing the last half-hour return,  $r_{13}$ , on the first half-hour return,  $r_{1}$ , and the twelfth half-hour return,  $r_{12}$ , of the day. The first half-hour return  $r_{1}$  is calculated from the closing of the previous trading day at 4:00pm to 10:00am Eastern Time. Panel A reports the results using front-month S&P 500 futures contracts while Panel B reports the results using the index itself. The returns are annualized and in percentage, and the regression coefficients are scaled by 100. Newey and West (1987) robust t-statistics are in parentheses, and significance at the 1%, 5%, or 10% level is denoted by \*\*\*, \*\* or \*, respectively. The sample period is from February 1, 1993, through December 31, 2013.

| Predictor      | $r_1$            | $r_{12}$             | $r_1$ and $r_{12}$   | $r_1$                    | $r_{12}$           | $r_1$ and $r_{12}$ |  |  |
|----------------|------------------|----------------------|----------------------|--------------------------|--------------------|--------------------|--|--|
|                | S&               | Panel A<br>zP 500 Fu |                      | Panel B<br>S&P 500 Index |                    |                    |  |  |
| Intercept      | -0.26<br>(-0.41) | -0.24<br>(-0.39)     | -0.37<br>(-0.60)     | 0.68<br>(1.14)           | 0.65<br>(1.09)     | 0.55<br>(0.92)     |  |  |
| $\beta_{r_1}$  | 7.38*** $(4.77)$ |                      | 7.23*** $(4.73)$     | 8.74***<br>(5.81)        |                    | 8.57***<br>(5.84)  |  |  |
| $eta_{r_{12}}$ |                  | 12.75***<br>(2.59)   | $12.37^{***} (2.59)$ |                          | 17.40***<br>(3.14) | 16.99***<br>(3.23) |  |  |
| $R^2$ (%)      | 1.8              | 1.2                  | 3.0                  | 2.6                      | 2.2                | 4.8                |  |  |

**Table A.2:** Alternative Time Intervals

We separate the time intervals into overnight (4:00pm-10:00am), morning (10:00am-11:30am), noon (11:30am-1:00pm), early afternoon (1:00pm-3:00pm), and the 12th half-hour (3:00pm-3:30pm). This table reports the predictive power of these time intervals on the last half-hour return. The returns are annualized and in percentage, and the regression coefficients are scaled by 100. Newey and West (1987) robust t-statistics are in parentheses, and significance at the 1%, 5%, or 10% level is denoted by \*\*\*, \*\* or \*, respectively. The sample period for each ETF is from its inception date to December 31, 2013, excluding days with fewer than 100 trades (500 for SPY).

|                       | SPY               | QQQ               | XLF               | IWM                | DIA               | EEM                | FXI                | EFA              | VWO                | IYR                | TLT               |
|-----------------------|-------------------|-------------------|-------------------|--------------------|-------------------|--------------------|--------------------|------------------|--------------------|--------------------|-------------------|
| Intercept             | -0.62<br>(-1.04)  | -1.03<br>(-1.18)  | 1.09<br>(1.14)    | 0.41 $(0.47)$      | -0.28<br>(-0.47)  | -0.67<br>(-0.72)   | -0.74<br>(-0.64)   | 0.51<br>(0.83)   | 0.62<br>(0.60)     | 4.67***<br>(3.55)  | 0.54**<br>(2.03)  |
| $r_{4:00pm-10:00am}$  | 6.85***<br>(4.46) | 6.08***<br>(4.10) | 8.90***<br>(4.46) | 8.89***<br>(6.30)  | 5.92***<br>(3.68) | 8.86***<br>(5.16)  | 8.20***<br>(5.26)  | 5.69*** $(4.37)$ | 7.14***<br>(4.15)  | 12.07***<br>(3.42) | 3.04***<br>(4.66) |
| $r_{10:00am-11:30am}$ | 5.54**<br>(2.33)  | 4.31** $(2.33)$   | 5.16*<br>(1.66)   | 7.02*** $(3.23)$   | 4.82*<br>(1.85)   | 5.23 $(1.39)$      | 5.25 $(1.22)$      | 2.85 $(0.95)$    | 5.09 $(1.36)$      | 2.56 $(0.63)$      | 2.69**<br>(2.31)  |
| $r_{11:30am-1:00pm}$  | 2.70 $(0.77)$     | 4.07 $(1.54)$     | 8.93**<br>(2.20)  | 5.96 $(1.52)$      | 2.23 $(0.57)$     | -0.81<br>(-0.15)   | -1.32<br>(-0.21)   | -1.01<br>(-0.22) | 0.13 $(0.02)$      | 5.82 (0.93)        | -0.46<br>(-0.33)  |
| $r_{1:00pm-3:00pm}$   | -0.63<br>(-0.22)  | 3.57 $(1.62)$     | 2.36 $(0.67)$     | 2.02 $(0.54)$      | -0.18<br>(-0.05)  | 4.54 $(1.08)$      | 3.99 $(0.75)$      | 2.37 $(0.52)$    | 1.14 $(0.27)$      | 2.70 $(0.54)$      | $0.50 \\ (0.33)$  |
| $r_{12}$              | 11.44**<br>(2.56) | 9.91***<br>(2.59) | 10.68*<br>(1.84)  | 18.84***<br>(3.92) | 12.18**<br>(2.26) | 28.08***<br>(4.27) | 22.75***<br>(3.07) | 13.25*<br>(1.91) | 20.32***<br>(3.03) | 36.39***<br>(3.88) | -3.74<br>(-1.10)  |
| $R^2 \ (\%)$          | 3.3               | 3.2               | 5.8               | 6.0                | 2.7               | 12.8               | 9.9                | 5.2              | 8.8                | 11.2               | 2.1               |

Table A.3: Impact of Volume and Liquidity

This table reports the predictive regressions on days with different last half-hour liquidity and volume. The last half-hour liquidity is measured by the Amihud measure, computed daily as the ratio of the absolute stock return to the dollar trading volume from open to 3:30pm to capture the illiquidity before the last half-hour. The returns are annualized and in percentage, and the regression coefficients are scaled by 100. Newey and West (1987) robust t-statistics are in parentheses, and significance at the 1%, 5%, or 10% level is denoted by \*\*\*, \*\* or \*, respectively. The sample period is from February 1, 1993, through December 31, 2013.

|                  | Low Volume       | High Volume         | Low Volume                    | High Volume       |  |  |
|------------------|------------------|---------------------|-------------------------------|-------------------|--|--|
|                  |                  | nel A<br>hud Period | Panel B<br>High Amihud Period |                   |  |  |
| Intercept        | -1.09<br>(-1.38) | 0.57 $(0.42)$       | -1.31<br>(-1.53)              | -0.77<br>(-0.47)  |  |  |
| $eta_{r_1}$      | 3.35 $(1.41)$    | 6.59***<br>(2.95)   | 5.41***<br>(2.72)             | 7.88***<br>(2.87) |  |  |
| $\beta_{r_{12}}$ | 8.3 (1.51)       | 14.88<br>(1.70)     | 9.07**<br>(2.16)              | 10.95 $(1.43)$    |  |  |
| $R^2 \ (\%)$     | 0.8              | 3.3                 | 1.6                           | 3.2               |  |  |

Table A.4: Impact of Volatility or Volume on Out-of-Sample Timing Performance

This table reports the impact of the first half-hour volatility (Panel A) or trading volume (Panel B) on the economic value of timing the last half-hour market return, using the first half-hour return  $(r_1)$ , or the first half-hour return and the twelfth half-hour return  $(r_1)$  and  $(r_1)$ . The timing strategy is described in Table 6. We report the timing performance for three different levels of the first half-hour volatility or volume. For each strategy, we report the average return  $(Avg\ Ret)$ , standard deviation  $(Std\ Dev)$ , Sharpe ratio (SRatio), and skewness. The returns are annualized and in percentage. Newey and West (1987) robust t-statistics are in parentheses, and significance at the 1%, 5%, or 10% level is denoted by \*\*\*, \*\* or \*, respectively. The sample period is from February 1, 1993, through December 31, 2013.

|                     |                        | Panel A    | : Volati  | lity            | Panel B: Volume |                     |            |        |          |          |  |
|---------------------|------------------------|------------|-----------|-----------------|-----------------|---------------------|------------|--------|----------|----------|--|
| Timing              | Avg Ret(%)             | Std Dev(%) | SRatio    | Skewness        | Kurtosis        | Avg Ret(%)          | Std Dev(%) | SRatio | Skewness | Kurtosis |  |
|                     |                        | Low V      | olatilit  |                 | Low             | Volume              | 9          |        |          |          |  |
| Always Long         | -2.04<br>(-1.62)       | 2.95       | -0.69     | -0.51           | 2.48            | -4.03**<br>(-2.37)  | 3.98       | -1.01  | -0.78    | 6.08     |  |
| $\eta(r_1)$         | 0.54 $(0.43)$          | 2.95       | 0.18      | -0.29           | 2.57            | 1.67 $(0.98)$       | 3.98       | 0.42   | -0.54    | 6.30     |  |
| $\eta(r_1, r_{12})$ | 0.97 $(1.17)$          | 1.93       | 0.50      | 0.12            | 5.87            | 2.10**<br>(1.93)    | 2.53       | 0.83   | 1.08     | 13.25    |  |
|                     |                        | Medium     | Volati    | $\mathbf{lity}$ |                 | Medium Volume       |            |        |          |          |  |
| Always Long         | -2.36<br>(-1.13)       | 4.89       | -0.48     | -0.25           | 2.83            | 1.96<br>(0.92)      | 5.01       | 0.39   | -0.02    | 3.94     |  |
| $\eta(r_1)$         | $4.75^{**}$ $(2.27)$   | 4.89       | 0.97      | -0.14           | 2.91            | 6.46***<br>(3.03)   | 5.00       | 1.29   | 0.09     | 3.95     |  |
| $\eta(r_1,r_{12})$  | $3.78^{***}$ (2.69)    | 3.28       | 1.15      | 0.79            | 9.07            | $3.35^{**}$ (2.24)  | 3.50       | 0.96   | 0.74     | 14.09    |  |
|                     |                        | High V     | Volatilit | y               |                 | High Volume         |            |        |          |          |  |
| Always Long         | 1.05<br>(0.27)         | 9.10       | 0.12      | -0.42           | 8.64            | -1.29<br>(-0.35)    | 8.63       | -0.15  | -0.44    | 10.84    |  |
| $\eta(r_1)$         | $14.73^{***}$ $(3.80)$ | 9.06       | 1.63      | 0.76            | 8.50            | 11.87***<br>(3.23)  | 8.60       | 1.38   | 0.96     | 10.68    |  |
| $\eta(r_1, r_{12})$ | 8.42***<br>(2.91)      | 6.77       | 1.24      | 1.44            | 17.62           | $7.73^{***}$ (2.80) | 6.45       | 1.20   | 1.63     | 21.00    |  |

This table reports the simple regression results of regressing the last half-hour return  $(r_{13})$  on one of the first 12 half-hour returns for SPY and 10 most heavily traded index ETFs.  $r_k$  denotes the kth half-hour return of the day, where k = 1, 2, ..., 12. The returns are annualized and in percentage, and the coefficients are scaled by 100. Newey and West (1987) robust t-statistics are in parentheses and significance at the 1%, 5%, or 10% level is denoted by \*\*\*, \*\* or \*, respectively. The sample period for each ETF is from its inception date to December 31, 2013, excluding days with fewer than 100 trades (500 for SPY).

| ETFs | $r_1$              | $r_2$           | $r_3$            | $r_4$          | $r_5$              | $r_6$             | $r_7$               | $r_8$            | $r_9$            | $r_{10}$          | $r_{11}$          | $r_{12}$           |
|------|--------------------|-----------------|------------------|----------------|--------------------|-------------------|---------------------|------------------|------------------|-------------------|-------------------|--------------------|
| SPY  | 6.51***<br>(4.16)  | 6.15*<br>(1.70) | 6.22<br>(1.32)   | 2.84 $(0.65)$  | 10.11**<br>(2.15)  | 7.32<br>(1.05)    | -11.06*<br>(-1.76)  | -2.74<br>(-0.47) | -4.46<br>(-0.70) | 3.26<br>(0.60)    | 1.92<br>(0.39)    | 11.94***<br>(2.62) |
| QQQ  | 6.08***<br>(4.06)  | 1.63 $(0.58)$   | 8.17**<br>(2.17) | 4.14 $(1.07)$  | 7.18*<br>(1.83)    | 4.33 $(0.83)$     | -2.31<br>(-0.46)    | 3.34 $(0.83)$    | -3.64<br>(-0.77) | 5.90 $(1.38)$     | 9.95**<br>(2.47)  | 10.43***<br>(2.69) |
| XLF  | 9.43***<br>(4.82)  | 5.46 $(1.24)$   | 5.59 $(1.01)$    | 6.59 $(1.28)$  | 11.92**<br>(2.16)  | 16.08**<br>(2.07) | -4.15<br>(-0.62)    | 2.11 $(0.37)$    | -6.04<br>(-0.85) | 2.60 $(0.42)$     | 13.72**<br>(2.20) | 12.92**<br>(2.20)  |
| IWM  | 8.74***<br>(5.99)  | 6.68*<br>(1.93) | 9.90**<br>(2.18) | 4.24 $(0.97)$  | 14.43***<br>(3.27) | 10.59 $(1.49)$    | -9.33<br>(-1.20)    | 1.20 $(0.15)$    | -4.93<br>(-0.71) | 6.26 $(0.97)$     | 5.61 (0.99)       | 19.19***<br>(3.93) |
| DIA  | 5.91***<br>(3.66)  | 6.64*<br>(1.65) | 4.38 $(0.88)$    | 2.32 $(0.51)$  | 11.42**<br>(2.22)  | 6.29 $(0.93)$     | -13.71*<br>(-1.91)  | -5.51<br>(-0.88) | -0.96<br>(-0.13) | 3.54 $(0.58)$     | 2.55 $(0.48)$     | 12.43**<br>(2.27)  |
| EEM  | 10.86***<br>(5.77) | 6.13 $(0.92)$   | 8.59 $(0.98)$    | 3.23 $(0.42)$  | 9.36<br>(1.16)     | $9.08 \\ (0.85)$  | -24.35**<br>(-2.08) | 13.58 $(1.04)$   | 5.93 $(0.50)$    | 6.41 $(0.66)$     | 15.59*<br>(1.80)  | 31.59***<br>(4.27) |
| FXI  | 9.45***<br>(5.86)  | 8.26<br>(1.21)  | 4.03 $(0.45)$    | 4.64 $(0.56)$  | 7.47 $(0.77)$      | 7.61 $(0.65)$     | -25.41**<br>(-2.04) | 16.00 $(1.34)$   | 9.20<br>(0.77)   | 1.21 $(0.10)$     | 11.22 $(1.12)$    | 25.17***<br>(3.09) |
| EFA  | 5.94***<br>(4.27)  | 3.28 $(0.72)$   | 6.16 $(0.96)$    | 2.83 $(0.61)$  | 6.67 $(1.36)$      | 6.92 $(0.79)$     | -19.52**<br>(-2.05) | 1.74 $(0.22)$    | -6.02<br>(-0.63) | 8.51<br>(1.09)    | 6.92 $(0.92)$     | 14.38**<br>(2.00)  |
| VWO  | 8.07***<br>(4.32)  | 5.11 $(0.79)$   | 6.94 $(0.86)$    | 5.65 $(0.78)$  | 11.85<br>(1.63)    | 6.43 $(0.62)$     | -20.75**<br>(-2.01) | 11.67 $(1.03)$   | -2.59<br>(-0.22) | 3.07 $(0.35)$     | 7.90 $(1.00)$     | 21.28***<br>(2.96) |
| IYR  | 15.74***<br>(4.03) | 0.29 $(0.04)$   | 3.70 $(0.40)$    | 7.88 $(0.91)$  | 11.10<br>(1.09)    | 26.46**<br>(2.32) | -15.36<br>(-1.33)   | -3.76<br>(-0.33) | -9.02<br>(-0.84) | 21.14**<br>(2.12) | 12.29 $(1.26)$    | 39.44***<br>(4.13) |
| TLT  | 3.00***<br>(4.86)  | 1.94<br>(1.27)  | 4.74**<br>(2.06) | 2.06<br>(0.90) | -1.04<br>(-0.41)   | 1.14<br>(0.44)    | -0.10<br>(-0.04)    | 1.01<br>(0.53)   | 0.59<br>(0.19)   | 2.22<br>(1.02)    | 0.19<br>(0.04)    | -3.99<br>(-1.17)   |

Table A.6: Predictability of the Last Half-Hour Returns: Market Microstructure Impact

This table reports the results of regressing the last half-hour return  $(r_{13})$  on the first half-hour return  $(r_{1})$  and the twelfth half-hour return  $(r_{12})$  of the day using different return measures for SPY. Panels A, B, C and D report results using returns calculated from transaction prices, bid prices, ask prices, and mid-quote prices, respectively. The returns are annualized and in percentage, and the coefficients are scaled by 100. Newey and West (1987) robust t-statistics are in parentheses and significance at the 1%, 5%, or 10% level is denoted by \*\*\*, \*\* or \*, respectively. The sample period is from Februry 1, 1993 to December 31, 2013, excluding days with fewer than 500 trades.

| Predictor        | $r_1$             | $r_{12}$           | $r_1$ and $r_{12}$       | $r_1$              | $r_{12}$               | $r_1$ and $r_{12}$   | $r_1$                      | $r_{12}$         | $r_1$ and $r_{12}$ |  |
|------------------|-------------------|--------------------|--------------------------|--------------------|------------------------|----------------------|----------------------------|------------------|--------------------|--|
|                  | Who               | ole Sample         | Period                   | Financial          | Crisis (1              | 2/2007-6/2009)       | Excluding Financial Crisis |                  |                    |  |
|                  |                   |                    | Pan                      | el A: Tran         | saction                | Returns              |                            |                  |                    |  |
| Intercept        | -1.46<br>(-0.95)  | -1.35<br>(-0.89)   | -1.69<br>(-1.11)         | 1.94<br>(0.20)     | -3.06<br>(-0.31)       | 0.87<br>(0.09)       | -1.35<br>(-1.00)           | -1.12<br>(-0.84) | -1.46<br>(-1.08)   |  |
| $eta_{r_1}$      | 6.51***<br>(4.16) |                    | $6.42^{***}$ $(4.20)$    | 14.49***<br>(3.14) |                        | 14.05**** $(3.14)$   | 3.82***<br>(3.04)          |                  | 3.80*** $(3.05)$   |  |
| $\beta_{r_{12}}$ |                   | 11.94***<br>(2.62) | 11.72***<br>(2.62)       |                    | 21.25*<br>(1.91)       | 20.35*<br>(1.95)     |                            | 6.40*<br>(1.82)  | 6.33*<br>(1.80)    |  |
| $R^2$ (%)        | 1.5               | 1.1                | 7.8<br>Returns           | 0.6                | 0.3                    | 1.0                  |                            |                  |                    |  |
| Intercept        | -1.28<br>(-0.83)  | -1.22<br>(-0.80)   | -1.52<br>(-1.00)         | 2.91<br>(0.29)     | -2.44<br>(-0.25)       | 1.74<br>(0.18)       | -1.25<br>(-0.93)           | -1.04<br>(-0.79) | -1.36<br>(-1.02)   |  |
| $\beta_{r_1}$    | 6.58***<br>(4.13) |                    | 6.46*** $(4.15)$         | 14.82*** $(3.17)$  |                        | 14.35**** $(3.19)$   | 3.84*** $(2.97)$           |                  | 3.80*** $(2.95)$   |  |
| $\beta_{r_{12}}$ |                   | 13.49***<br>(2.88) | 13.22***<br>(2.88)       |                    | 22.68**<br>(2.02)      | 21.76**<br>(2.08)    |                            | 7.82**<br>(2.15) | 7.71**<br>(2.12)   |  |
| $R^2 \ (\%)$     | 1.5               | 1.4                | 2.9<br><b>Pan</b>        | 4.9<br>el C: Ask-  | 3.8<br><b>To-Ask</b>   | 8.4<br>Returns       | 0.6                        | 0.5              | 1.1                |  |
| Intercept        | -0.36<br>(-0.24)  | -0.35<br>(-0.23)   | -0.62<br>(-0.41)         | 3.36<br>(0.34)     | -2.04<br>(-0.21)       | 2.12<br>(0.22)       | -0.29<br>(-0.22)           | -0.12<br>(-0.09) | -0.41<br>(-0.31)   |  |
| $\beta_{r_1}$    | 6.52*** $(4.05)$  |                    | 6.40***<br>(4.06)        | 14.74***<br>(3.16) |                        | 14.29***<br>(3.17)   | 3.75***<br>(2.84)          |                  | 3.71***<br>(2.83)  |  |
| $\beta_{r_{12}}$ |                   | 13.18***<br>(2.80) | 12.89***<br>(2.80)       |                    | 22.41**<br>(2.00)      | 21.52**<br>(2.05)    |                            | 7.47** (2.03)    | 7.33**<br>(1.99)   |  |
| $R^2 \ (\%)$     | 1.5               | 1.3                | 2.8<br><b>Panel D:</b> 1 | 4.8<br>Midquote-   | 3.7<br><b>To-Mid</b> o | 8.3<br>Juote Returns | 0.6                        | 0.4              | 1.0                |  |
| Intercept        | -0.82<br>(-0.54)  | -0.79<br>(-0.52)   | -1.08<br>(-0.71)         | 3.13 $(0.32)$      | -2.24<br>(-0.23)       | 1.93 $(0.20)$        | -0.77<br>(-0.58)           | -0.59<br>(-0.45) | -0.9<br>(-0.67)    |  |
| $\beta_{r_1}$    | 6.57***<br>(4.10) |                    | 6.44***<br>(4.11)        | 14.78***<br>(3.17) |                        | 14.32***<br>(3.18)   | 3.82*** $(2.92)$           |                  | 3.77*** $(2.90)$   |  |
| $\beta_{r_{12}}$ |                   | 13.65***<br>(2.90) | 13.37***<br>(2.90)       |                    | 22.55**<br>(2.01)      | 21.64**<br>(2.07)    |                            | 8.10**<br>(2.21) | 7.98**<br>(2.18)   |  |
| $R^2 \ (\%)$     | 1.5               | 1.4                | 2.9                      | 4.9                | 3.8                    | 8.3                  | 0.6                        | 0.5              | 1.1                |  |

Table A.7: Robustness of Out-of-Sample Mean-Variance Portfolio Performance

This table reports the out-of-sample performance of different combinations of the relative risk aversion coefficient,  $\gamma$ , and portfolio weight restrictions,  $\psi_i, i = 1, \dots, 4$ . The recursive regression uses both the first half-hour return and the twelfth half-hour return as described in Table 7. We report the average return  $(Avg\ Ret)$ , standard deviation  $(Std\ Dev)$ , Sharpe ratio (SRatio), skewness, kurtosis, and the certainty equivalent gain of return (CER) as defined in Table 7. The returns are annualized and in percentage. Newey and West (1987) robust t-statistics are in parentheses, and significance at the 1%, 5%, or 10% level is denoted by \*\*\*, \*\* or \*, respectively. The sample period is from February 1, 1993, through December 31, 2013, excluding days with fewer than 500 trades.

| Weight Restriction           | Avg Ret(%)             | Std Dev(%)        | SRatio | Skewness | Kurtosis | CER(%) |  |  |  |  |  |
|------------------------------|------------------------|-------------------|--------|----------|----------|--------|--|--|--|--|--|
| Panel A: $\gamma = 5$        |                        |                   |        |          |          |        |  |  |  |  |  |
| $\psi_2: 0 \le w \le 1.0$    | 3.22***<br>(3.08)      | 3.90              | 0.82   | 0.37     | 75.40    | 3.2    |  |  |  |  |  |
| $\psi_3: -1.0 \le w \le 1.0$ | $7.35^{***}$ $(4.70)$  | 5.84              | 1.26   | 0.60     | 21.15    | 6.61   |  |  |  |  |  |
| $\psi_4: -1.0 \le w \le 2.0$ | $10.33^{***}$ $(4.47)$ | 8.65              | 1.19   | 0.62     | 47.86    | 9.55   |  |  |  |  |  |
|                              |                        | Panel B: $\gamma$ | =2     |          |          |        |  |  |  |  |  |
| $\psi_1: -0.5 \le w \le 1.5$ | 7.16***<br>(4.20)      | 6.37              | 1.12   | 0.17     | 54.88    | 6.61   |  |  |  |  |  |
| $\psi_2: 0 \le w \le 1.0$    | 3.32***<br>(3.10)      | 4.00              | 0.83   | 0.22     | 70.30    | 3.28   |  |  |  |  |  |
| $\psi_3: -1.0 \le w \le 1.0$ | $7.70^{***}$ $(4.78)$  | 6.02              | 1.28   | 0.55     | 19.28    | 6.77   |  |  |  |  |  |
| $\psi_4: -1.0 \le w \le 2.0$ | 10.85***<br>(4.47)     | 9.08              | 1.20   | 0.22     | 42.58    | 9.81   |  |  |  |  |  |
|                              |                        | Panel C: $\gamma$ | = 10   |          |          |        |  |  |  |  |  |
| $\psi_1: -0.5 \le w \le 1.5$ | 6.48***<br>(4.15)      | 5.84              | 1.11   | 0.72     | 71.26    | 6.09   |  |  |  |  |  |
| $\psi_2: 0 \le w \le 1.0$    | 3.10***<br>(3.09)      | 3.74              | 0.83   | 0.82     | 84.77    | 3.09   |  |  |  |  |  |
| $\psi_3: -1.0 \le w \le 1.0$ | $7.08^{***}$ $(4.72)$  | 5.61              | 1.26   | 0.83     | 24.28    | 6.73   |  |  |  |  |  |
| $\psi_4: -1.0 \le w \le 2.0$ | 9.69***<br>(4.44)      | 8.16              | 1.19   | 0.80     | 59.49    | 9.33   |  |  |  |  |  |