Internet Appendix:

“Optimal Portfolio Choice with Estimation Risk: No Risk-free Asset Case”

October 2018

This internet appendix provides additional results to support the findings in the paper. Figures IA.1 to IA.6 are similar to Figures 1 to 6 in the paper, but with parameters calibrated using the Fama-French 25 size and book-to-market ranked portfolios over the period of 1927/01–2017/12. Figures IA.7 to IA.10 examine the theoretical out-of-sample performance (expected out-of-sample utility and Sharpe ratio) of various portfolios under the i.i.d. normality assumption with $\gamma = 5$. Figures IA.11 to IA.18 present the theoretical out-of-sample performance of various portfolio rules under two alternative distributional assumptions based on 100,000 simulations. Figures IA.11 to IA.14 are based on the multivariate t-distribution with five degrees of freedom, and Figures IA.15 to IA.18 are based on an empirical distribution obtained from the block bootstrap procedure proposed in Politis and Romano (1994) with the expected length of the block set to 10 months. Tables IA.1 to IA.4 report empirical results similar to Tables 1 to 4 in the paper, but with $\gamma = 5$.

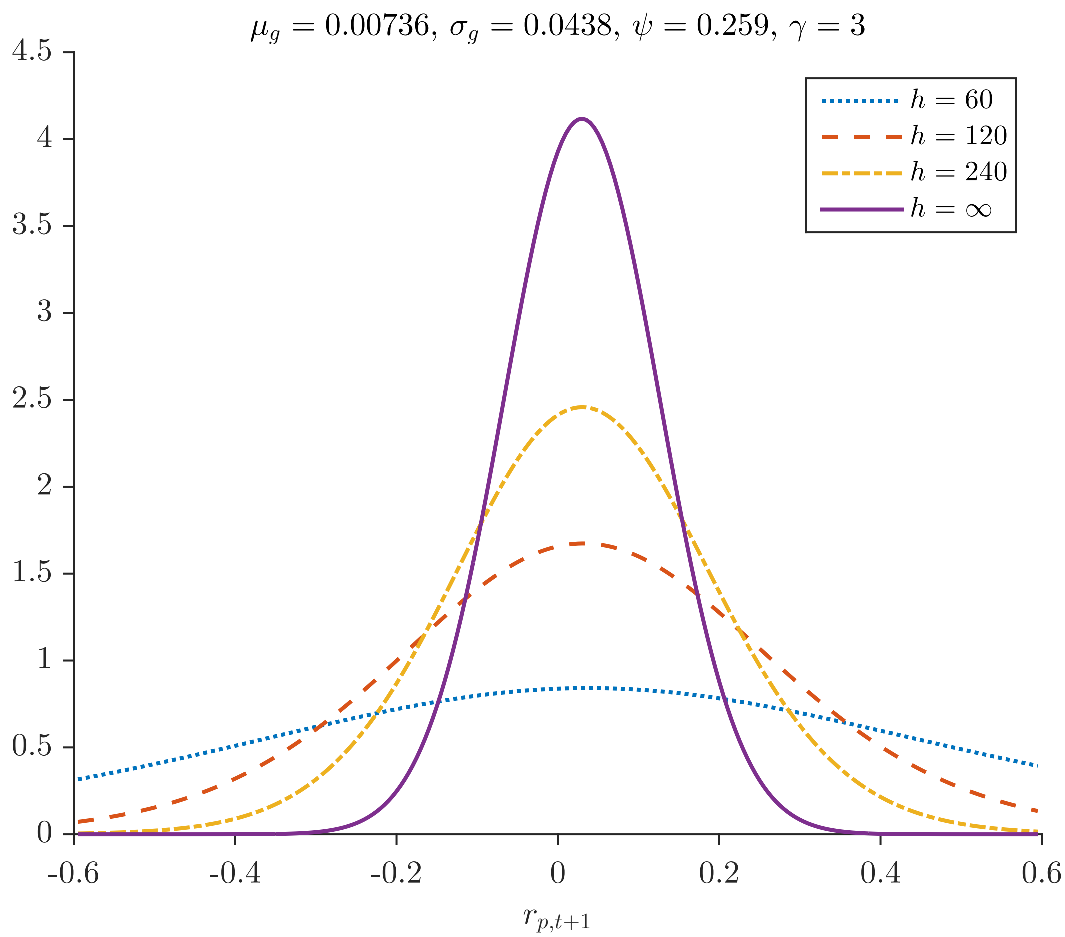


Figure IA.1: **Unconditional distribution of out-of-sample return of the plug-in rule with 25 risky assets**
This figure plots the unconditional distribution of $r_{p,t+1}$ for $h = 60$, 120, and 240 months, with parameters estimated using excess monthly returns of the Fama-French 5×5 size and book-to-market ranked portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to three ($\gamma = 3$). For comparison, the return distribution of the true optimal portfolio (i.e., $h = \infty$) is also reported.

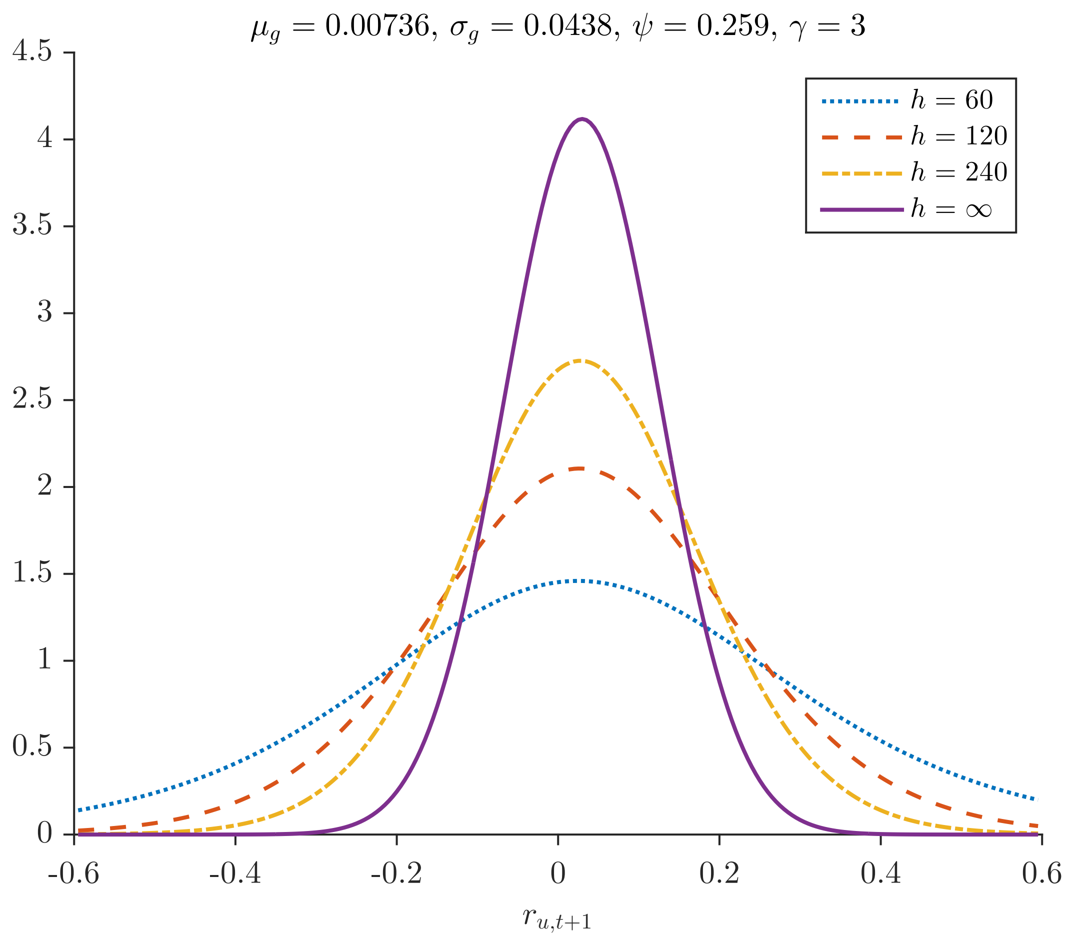


Figure IA.2: **Unconditional distribution of out-of-sample return of the unbiased rule with 25 risky assets**
This figure plots the unconditional distribution of $r_{u,t+1}$ for $h = 60$, $h = 120$, and 240 months, with parameters estimated using excess monthly returns of the Fama-French 5×5 size and book-to-market ranked portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to three ($\gamma = 3$). For comparison, the return distribution of the true optimal portfolio (i.e., $h = \infty$) is also reported.

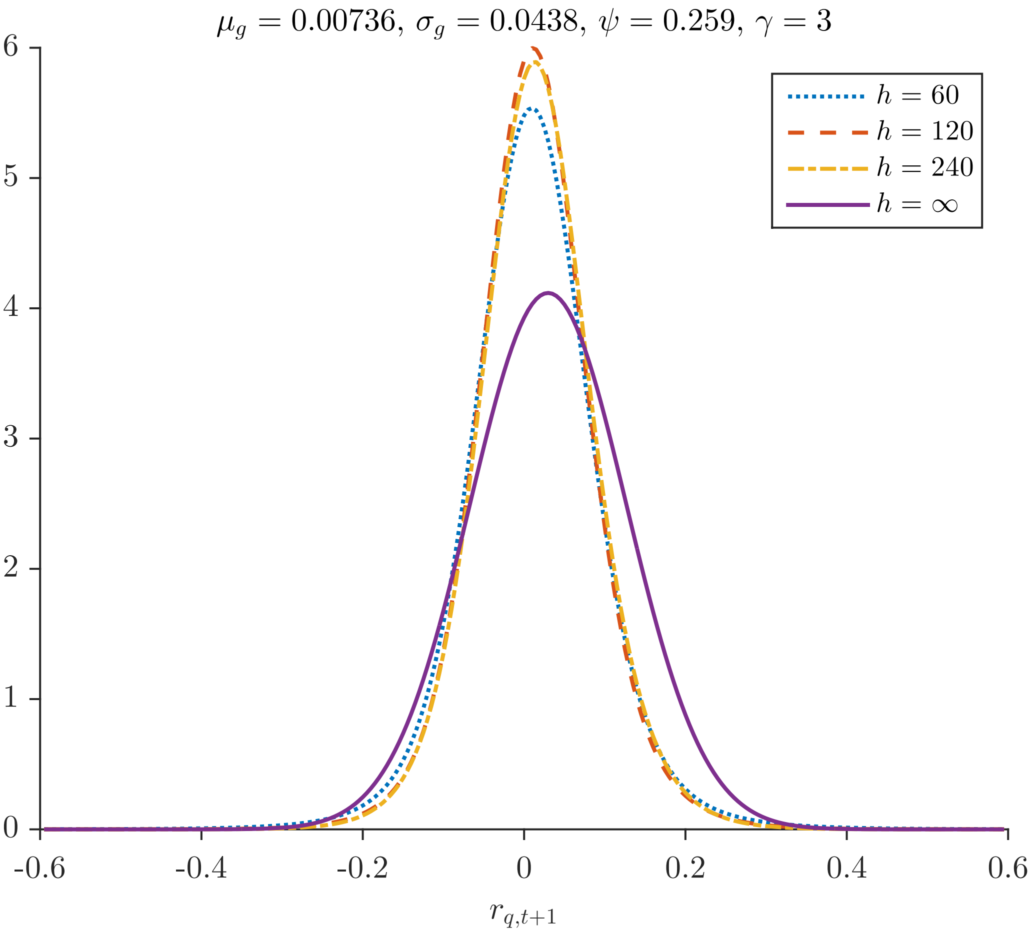


Figure IA.3: **Unconditional distribution of out-of-sample return of the implementable optimal two-fund rule with 25 risky assets**
This plots the unconditional distribution of $r_{q,t+1}$ for $h = 60$, 120, and 240 months, with parameters estimated using excess monthly returns of the Fama-French 5×5 size and book-to-market ranked portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to three ($\gamma = 3$). For comparison, the return distribution of the true optimal portfolio (i.e., $h = \infty$) is also reported.

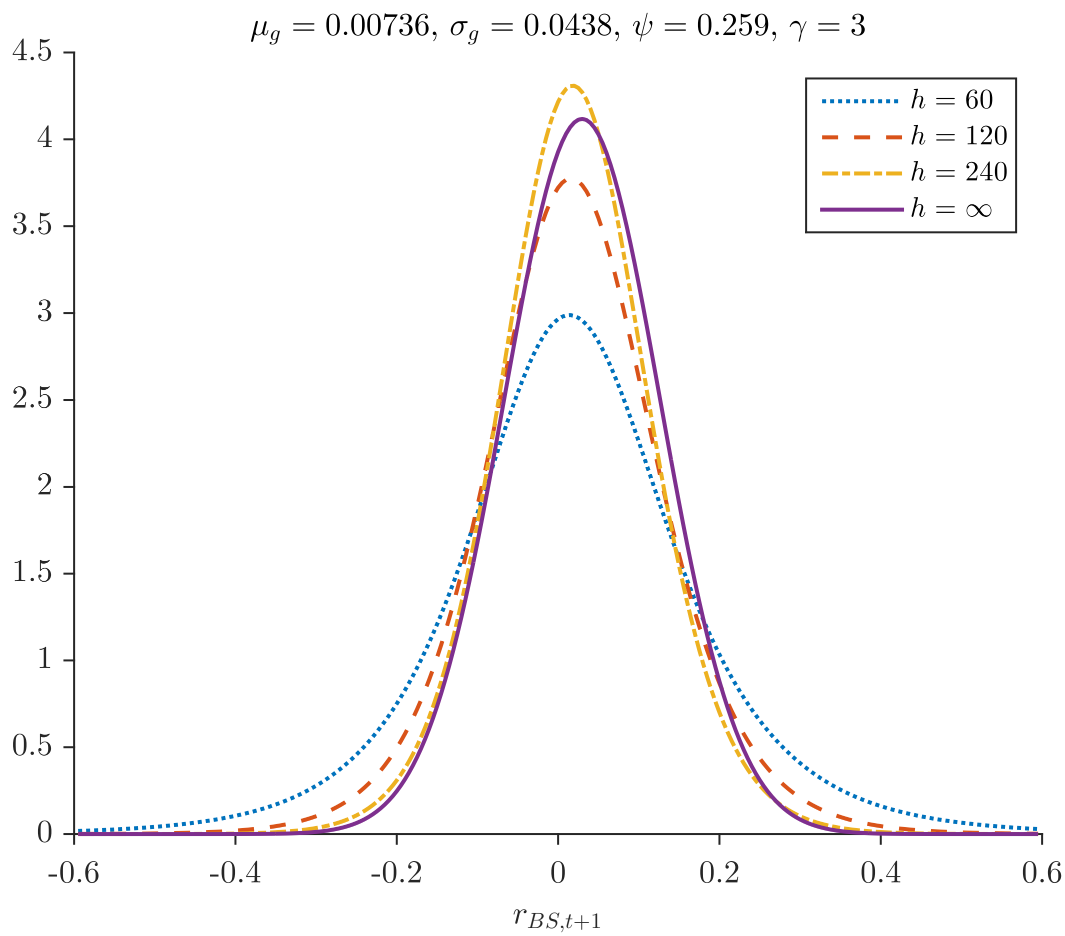
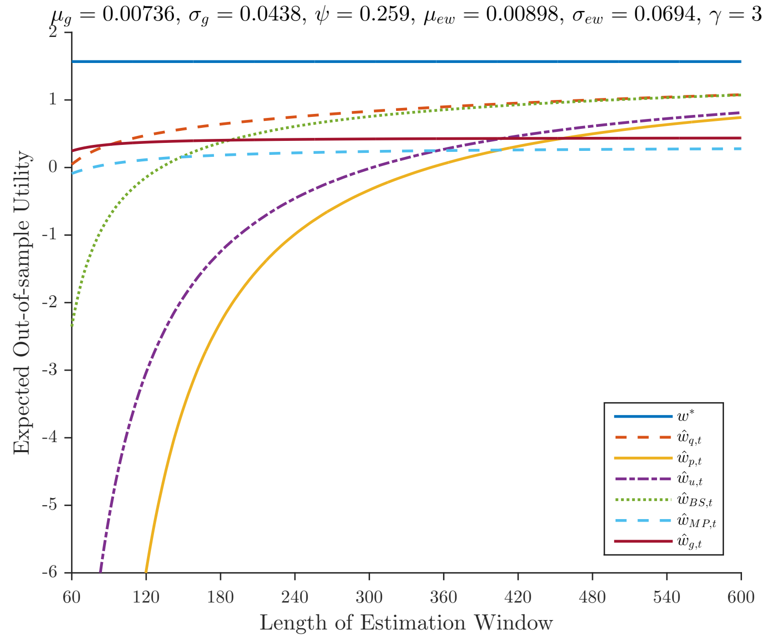


Figure IA.4: **Unconditional distribution of out-of-sample return of the optimal portfolio rule using shrinkage estimators with 25 risky assets**

This figure plots the unconditional distribution of $r_{BS,t+1}$ for $h = 60$, 120, and 240 months, with parameters estimated using excess monthly returns of the Fama-French 5×5 size and book-to-market ranked portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to three ($\gamma = 3$). For comparison, the return distribution of the true optimal portfolio (i.e., $h = \infty$) is also reported.



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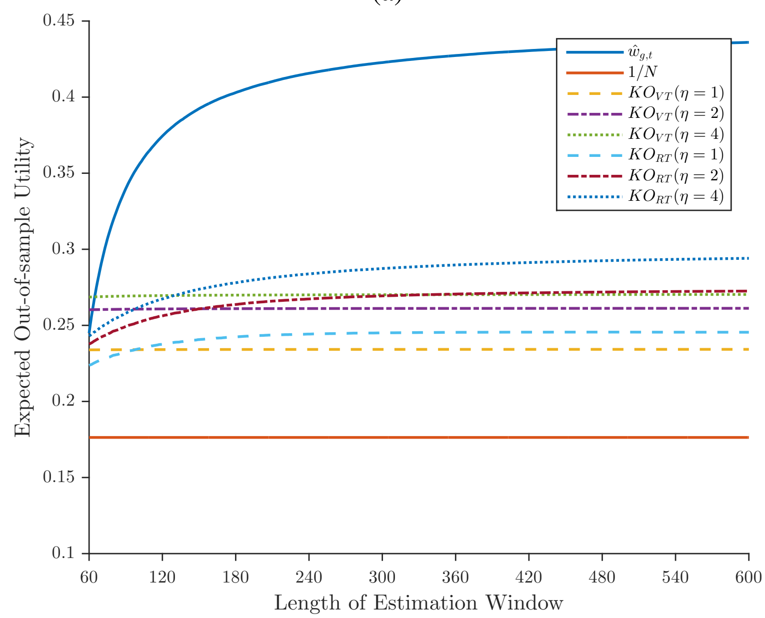
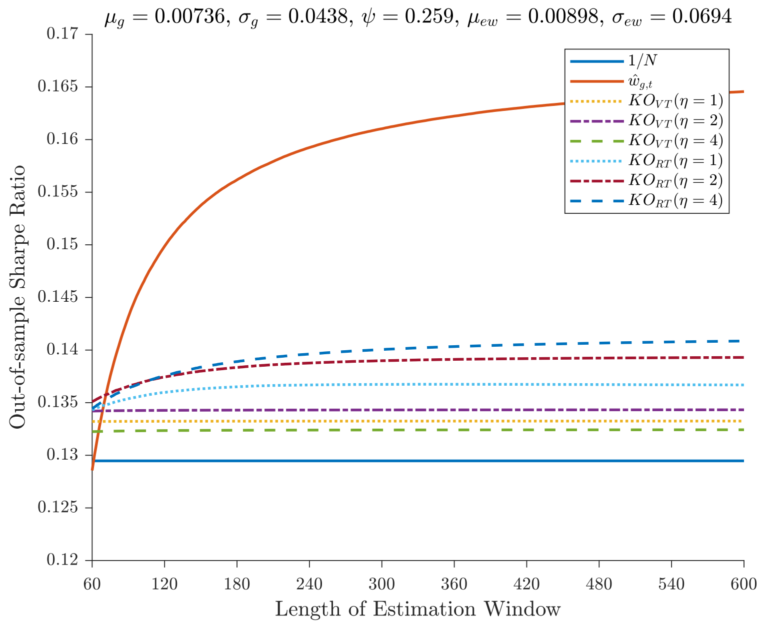


(b)

Figure IA.5: **Expected out-of-sample utility of portfolio rules with 25 risky assets ($\gamma = 3$)** Panel (a) plots the expected out-of-sample utility (in percentage points) of various optimal portfolios rules, w^*, $\hat{w}_{p,t}$, $\hat{w}_{u,t}$, $\hat{w}_{q,t}$, $\hat{w}_{BS,t}$, $\hat{w}_{MP,t}$, and $\hat{w}_{g,t}$, as a function of the length of the estimation window (h), with parameters estimated using excess monthly returns of the Fama-French 25 size and book-to-market ranked portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to $\gamma = 3$. Panel (b) plots similar results of the $1/N$ rule, the two timing strategies (KO_{VT} and KO_{RT}) with $\eta = 1$, 2, and 4. For comparison, the result of $\hat{w}_{g,t}$ is also included in Panel (b).



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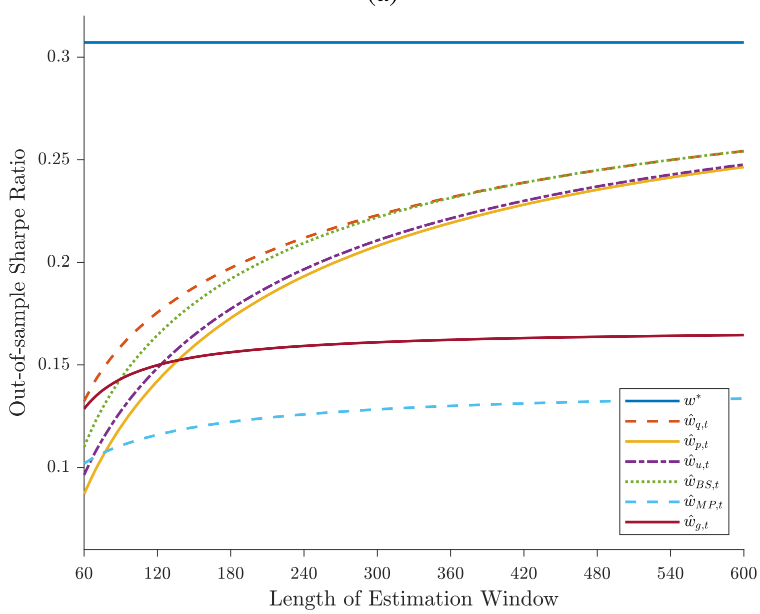
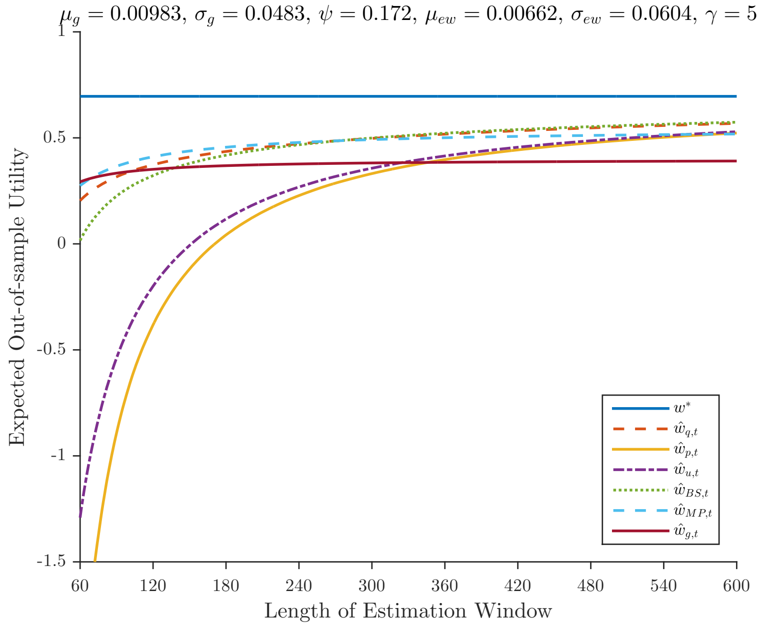


(b)

Figure IA.6: **Out-of-sample Sharpe ratio with 25 risky assets ($\gamma = 3$)**
This figure plots the out-of-sample Sharpe ratio of various portfolio rules as a function of the length of the estimation window (h), with parameters estimated using excess monthly returns of the Fama-French 25 size and book-to-market ranked portfolios over the period 1927/1–2017/12. Panel (a) presents the result for the $1/N$ rule, KO_{VT}, KO_{RT}, and $\hat{w}_{g,t}$. Panel (b) reports the results for w^*, $\hat{w}_{p,t}$, $\hat{w}_{u,t}$, $\hat{w}_{q,t}$, $\hat{w}_{BS,t}$, and $\hat{w}_{MP,t}$, with $\gamma = 3$. For comparison, the result for $\hat{w}_{g,t}$ is included in Panel (b).



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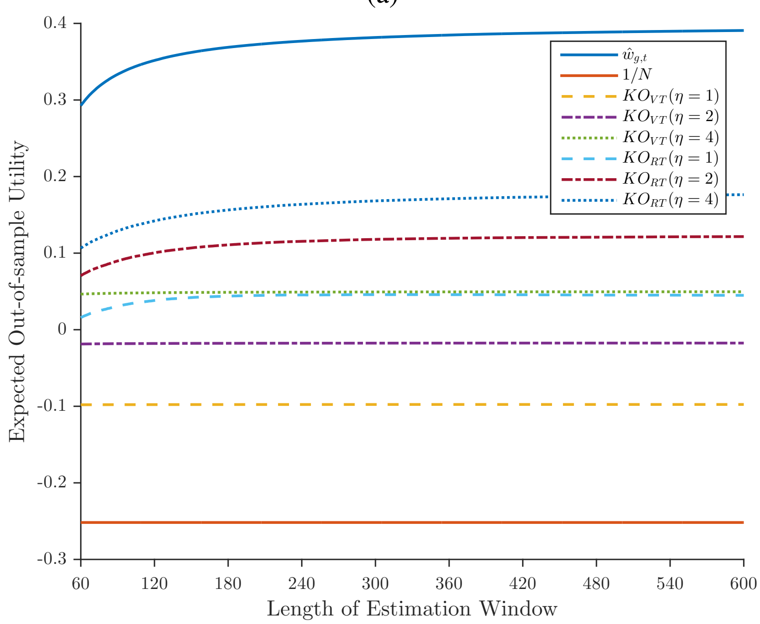
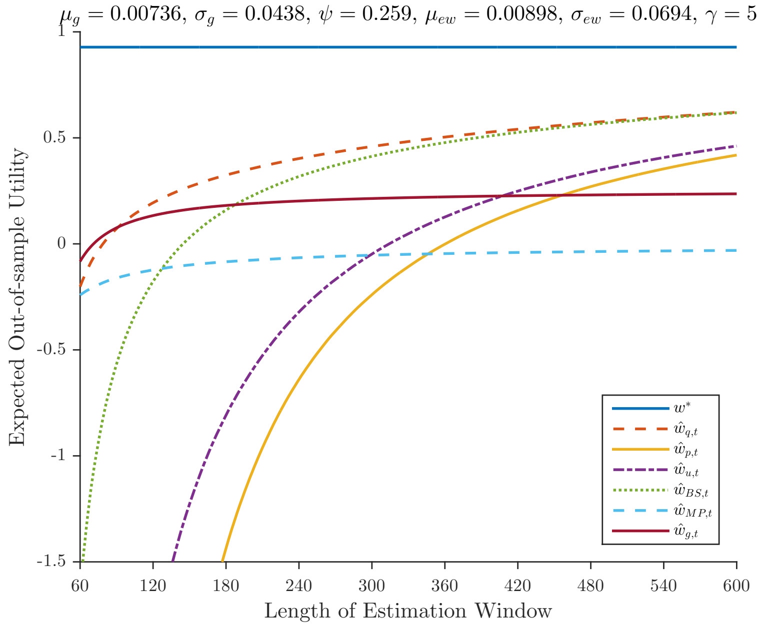


(b)

Figure IA.7: **Expected out-of-sample utility of portfolio rules with 10 risky assets ($\gamma = 5$)** Panel (a) plots the expected out-of-sample utility (in percentage points) of various optimal portfolios rules, w^*, $\hat{w}_{p,t}$, $\hat{w}_{u,t}$, $\hat{w}_{q,t}$, $\hat{w}_{BS,t}$, $\hat{w}_{MP,t}$, and $\hat{w}_{g,t}$, as a function of the length of the estimation window (h), with parameters estimated using excess monthly returns of the 10 momentum portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to $\gamma = 5$. Panel (b) plots similar results of the $1/N$ rule, the two timing strategies (KO_{VT} and KO_{RT}) with $\eta = 1$, 2, and 4. For comparison, the result of $\hat{w}_{g,t}$ is also included in Panel (b).



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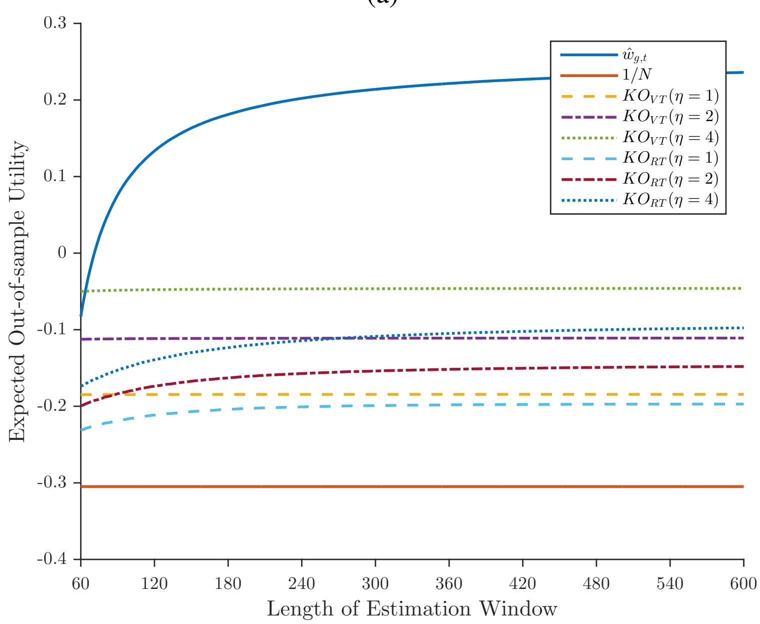
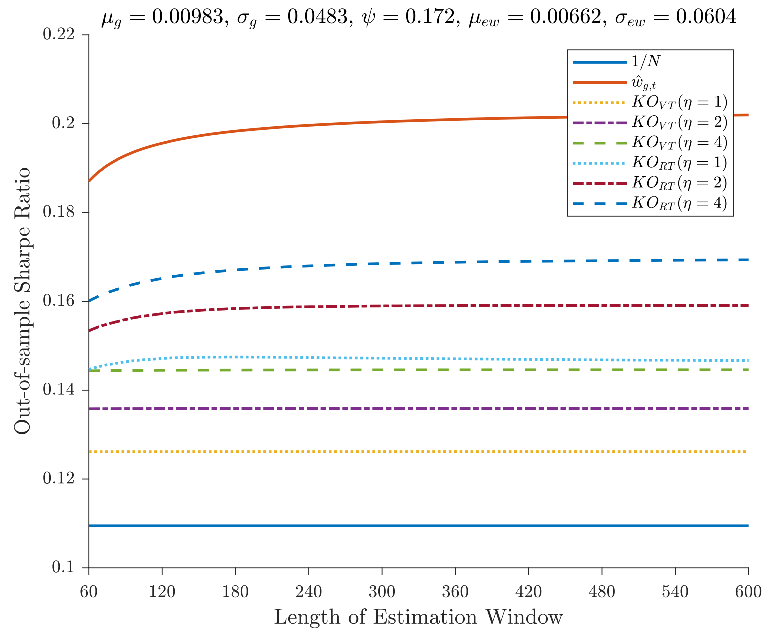


(b)

Figure IA.8: **Expected out-of-sample utility of portfolio rules with 25 risky assets ($\gamma = 5$)** Panel (a) plots the expected out-of-sample utility (in percentage points) of various optimal portfolios rules, w^*, $\hat{w}_{p,t}$, $\hat{w}_{u,t}$, $\hat{w}_{q,t}$, $\hat{w}_{BS,t}$, $\hat{w}_{MP,t}$, and $\hat{w}_{g,t}$, as a function of the length of the estimation window (h), with parameters estimated using excess monthly returns of the Fama-French 25 size and book-to-market ranked portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to $\gamma = 5$. Panel (b) plots similar results of the $1/N$ rule, the two timing strategies (KO_{VT} and KO_{RT}) with $\eta = 1$, 2, and 4. For comparison, the result of $\hat{w}_{g,t}$ is also included in Panel (b).



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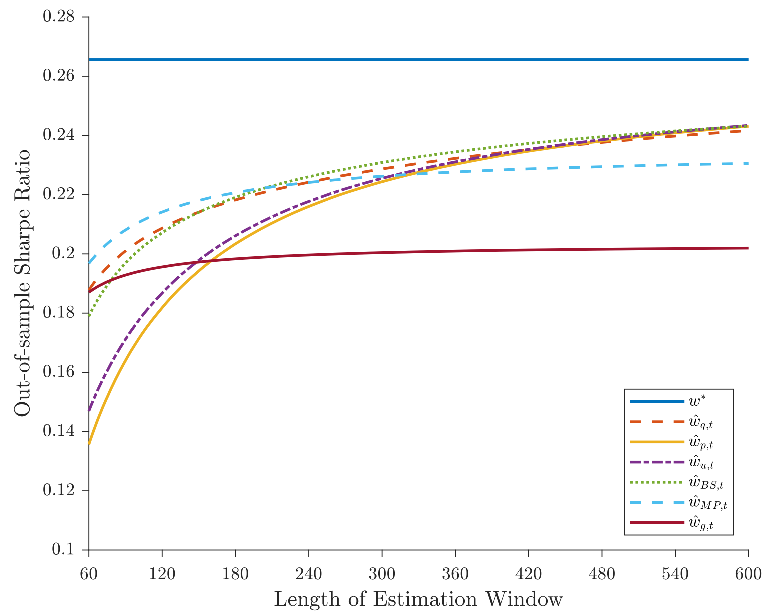
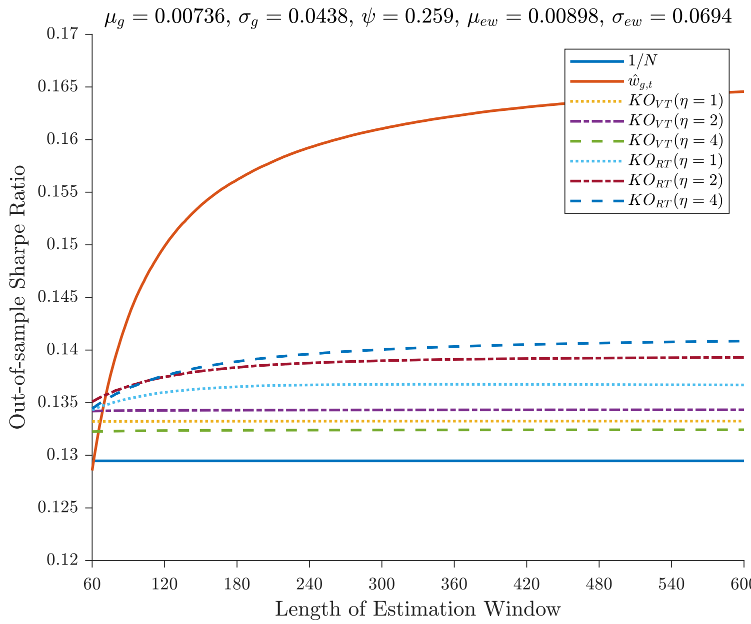


(b)

Figure IA.9: **Out-of-sample Sharpe ratio with 10 risky assets ($\gamma = 5$)**
This figure plots the out-of-sample Sharpe ratio of various portfolio rules as a function of the length of the estimation window (h), with parameters estimated using excess monthly returns of the 10 momentum portfolios over the period 1927/1–2017/12. Panel (a) presents the results for the $1/N$ rule, KO_{VT}, KO_{RT}, and $\hat{w}_{g,t}$. Panel (b) reports the results for w^*, $\hat{w}_{p,t}$, $\hat{w}_{u,t}$, $\hat{w}_{q,t}$, $\hat{w}_{BS,t}$, and $\hat{w}_{MP,t}$, with $\gamma = 5$. For comparison, the result for $\hat{w}_{g,t}$ is included in Panel (b).



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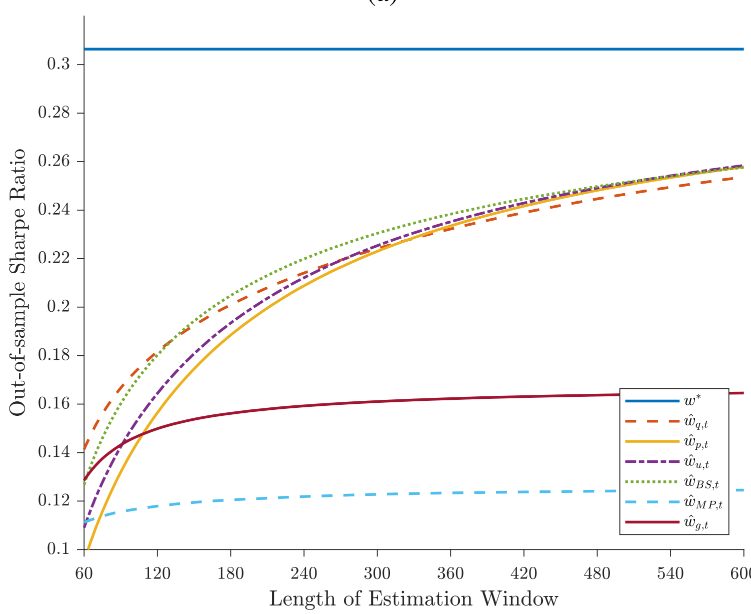
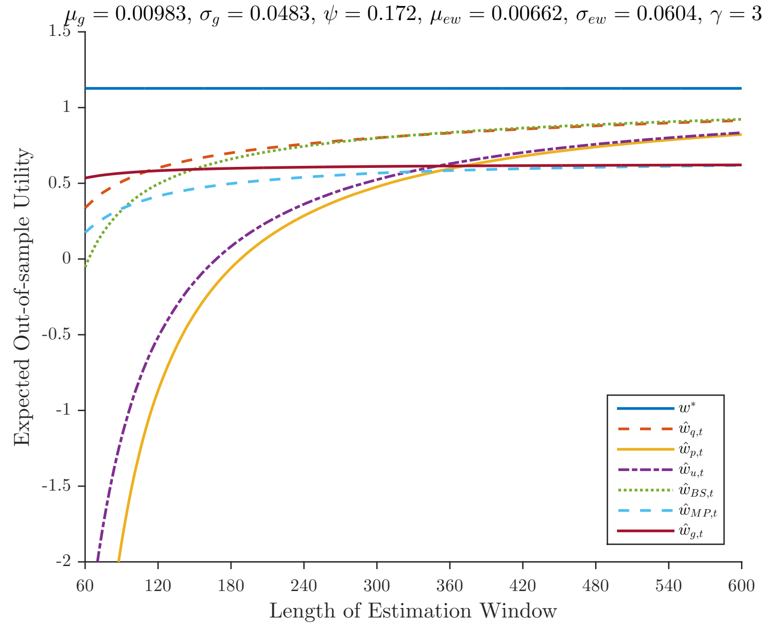


(b)

Figure IA.10: **Out-of-sample Sharpe ratio with 25 risky assets** ($\gamma = 5$)

This figure plots the out-of-sample Sharpe ratio of various portfolio rules as a function of the length of the estimation window (h), with parameters estimated using excess monthly returns of the Fama-French 25 size and book-to-market ranked portfolios over the period 1927/1–2017/12. Panel (a) presents the results for the $1/N$ rule, KO_{VT}, KO_{RT}, and $\hat{w}_{g,t}$. Panel (b) reports the results for w^*, $\hat{w}_{p,t}$, $\hat{w}_{u,t}$, $\hat{w}_{q,t}$, $\hat{w}_{BS,t}$, and $\hat{w}_{MP,t}$, with $\gamma = 5$. For comparison, the result for $\hat{w}_{g,t}$ is included in Panel (b).



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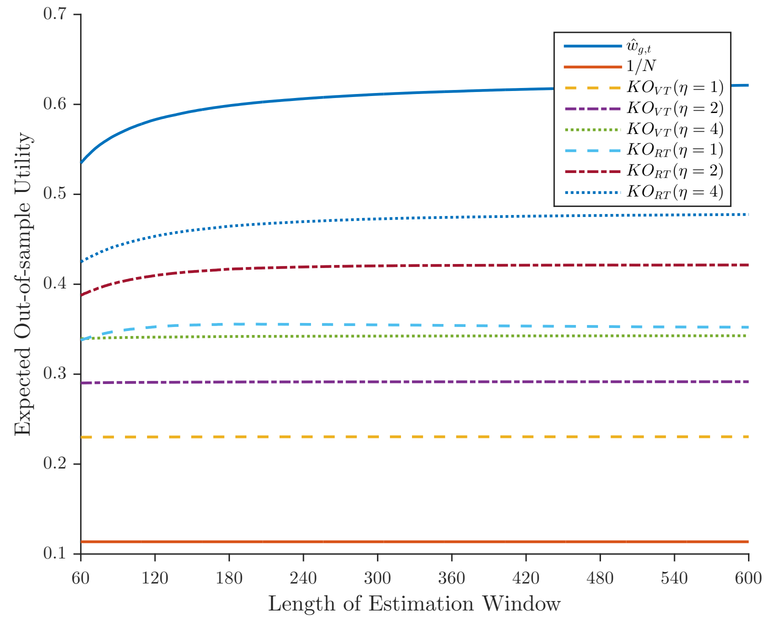
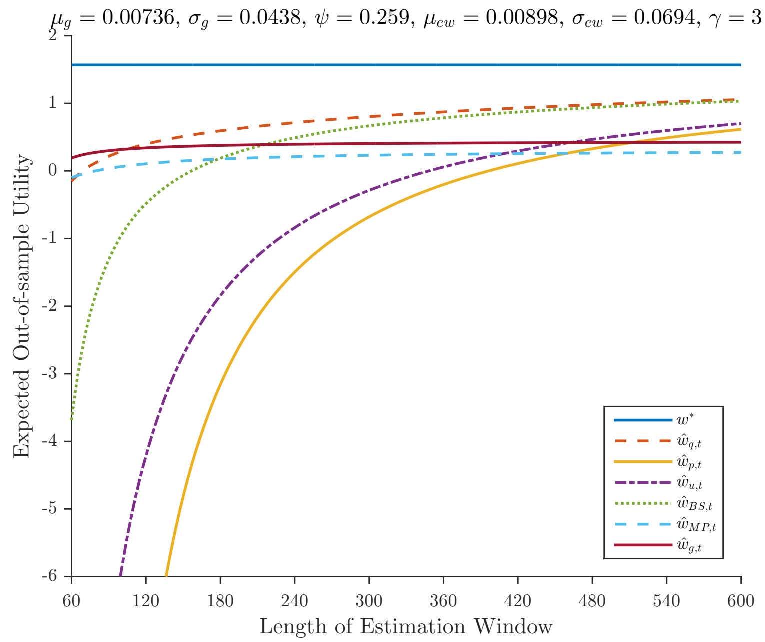


(b)

Figure IA.11: **Expected out-of-sample utility of portfolio rules with multivariate t distribution (10 risky assets, $\gamma = 3$)**
This figure plots the expected out-of-sample utility (in percentage points) of various portfolio rules as a function of the length of the estimation window (h). The returns of the risky assets are assumed to follow a multivariate t distribution with five degrees of freedom. The mean and covariance matrix are estimated using excess monthly returns of the 10 momentum portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to $\gamma = 3$.



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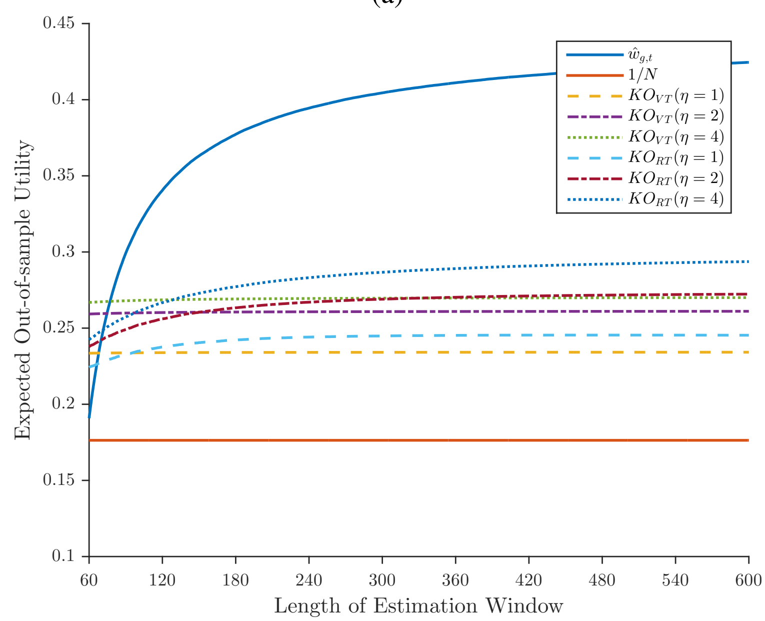
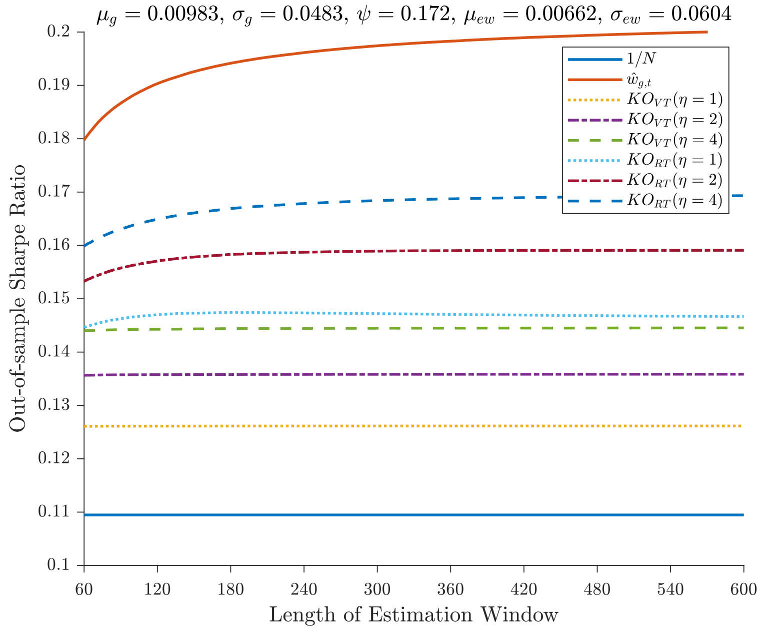
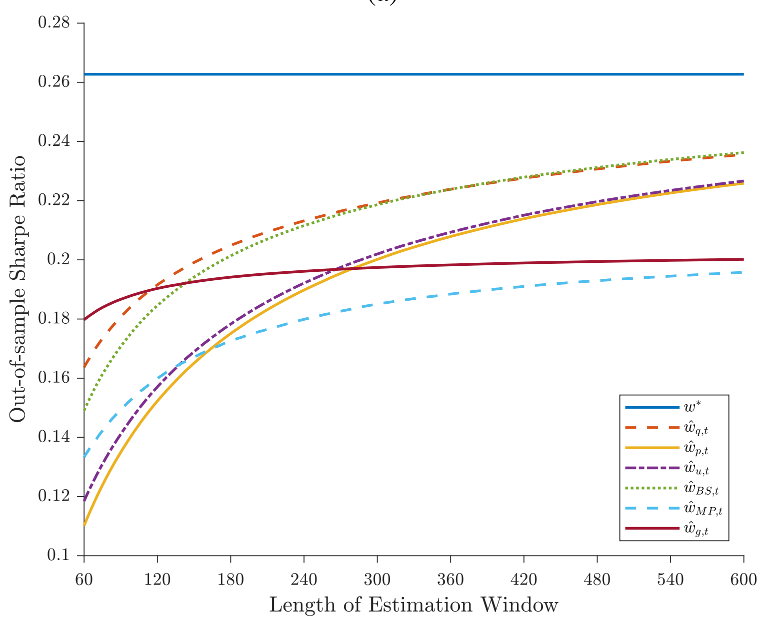


(b)

Figure IA.12: **Expected out-of-sample utility of portfolio rules with multivariate t distribution (25 risky assets, $\gamma = 3$)**
This figure plots the expected out-of-sample utility (in percentage points) of various portfolio rules as a function of the length of the estimation window (h). The returns of the risky assets are assumed to follow a multivariate t distribution with five degrees of freedom. The mean and covariance matrix are estimated using excess monthly returns of the Fama-French 25 size and book-to-market ranked portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to $\gamma = 3$.



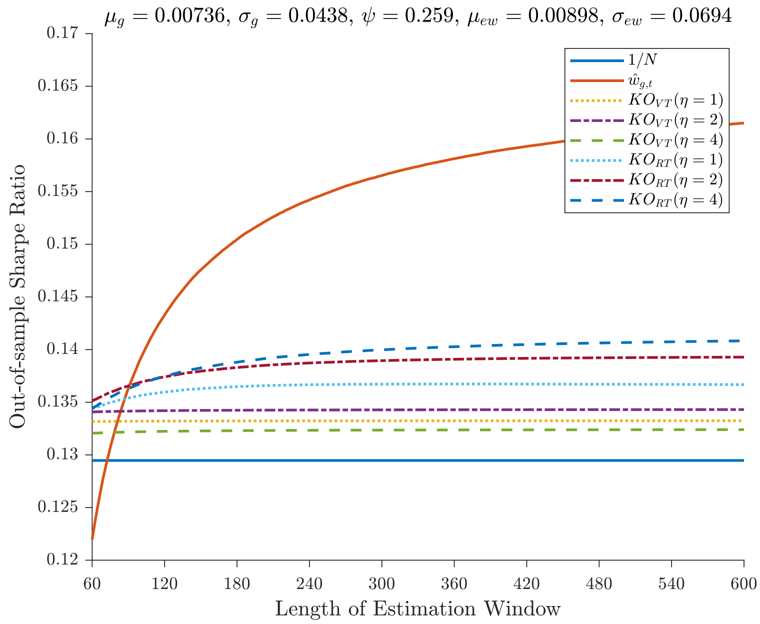
(a)



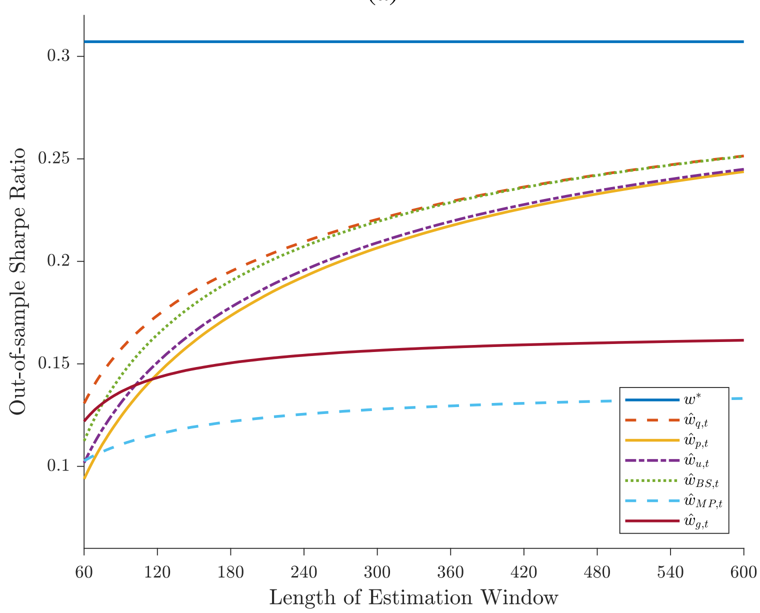
(b)

Figure IA.13: **Out-of-sample Sharpe ratio of portfolio rules with multivariate t distribution (10 risky assets, $\gamma = 3$)**

This figure plots the out-of-sample Sharpe ratio of various portfolio rules as a function of the length of the estimation window (h). The returns of the risky assets are assumed to follow a multivariate t distribution with five degrees of freedom. The mean and covariance matrix are estimated using excess monthly returns of the 10 momentum portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to $\gamma = 3$.

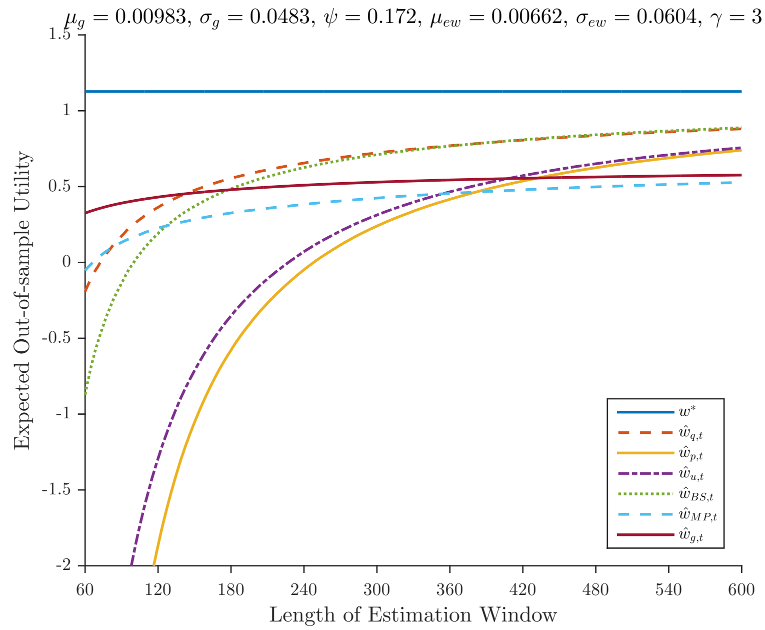


(a)



(b)

Figure IA.14: **Out-of-sample Sharpe ratio of portfolio rules with multivariate t distribution (25 risky assets, $\gamma = 3$)**
This figure plots the out-of-sample Sharpe ratio of various portfolio rules as a function of the length of the estimation window (h). The returns of the risky assets are assumed to follow a multivariate t distribution with five degrees of freedom. The mean and covariance matrix are estimated using excess monthly returns of the Fama-French 25 size and book-to-market ranked portfolios over the period of 1927/1–2017/12. The risk aversion coefficient is set to $\gamma = 3$.



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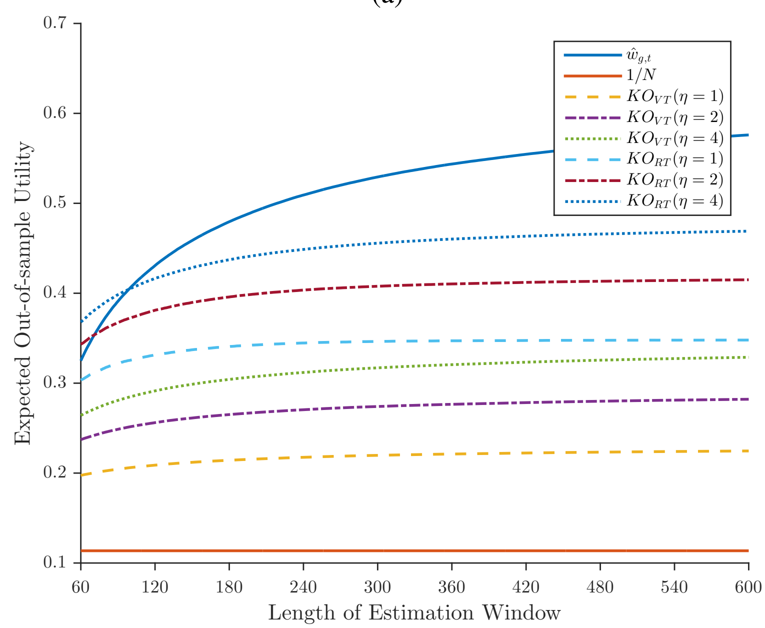
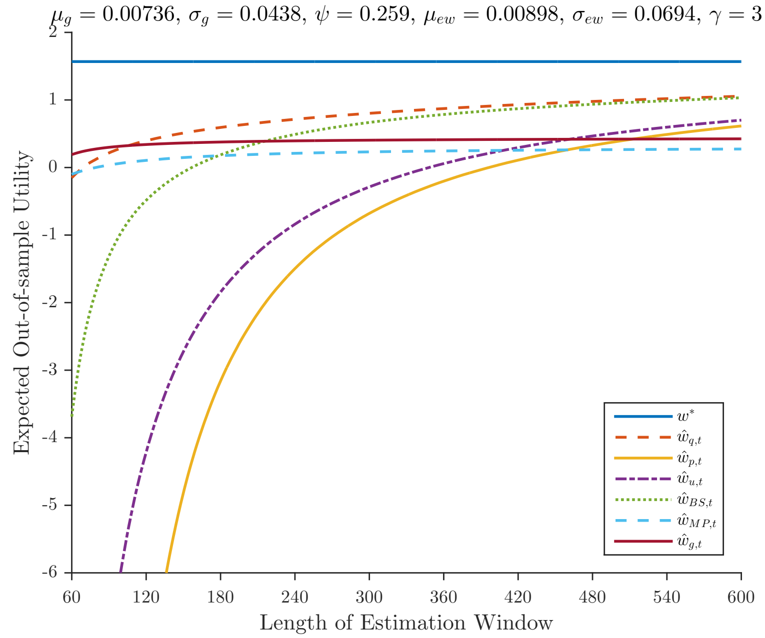
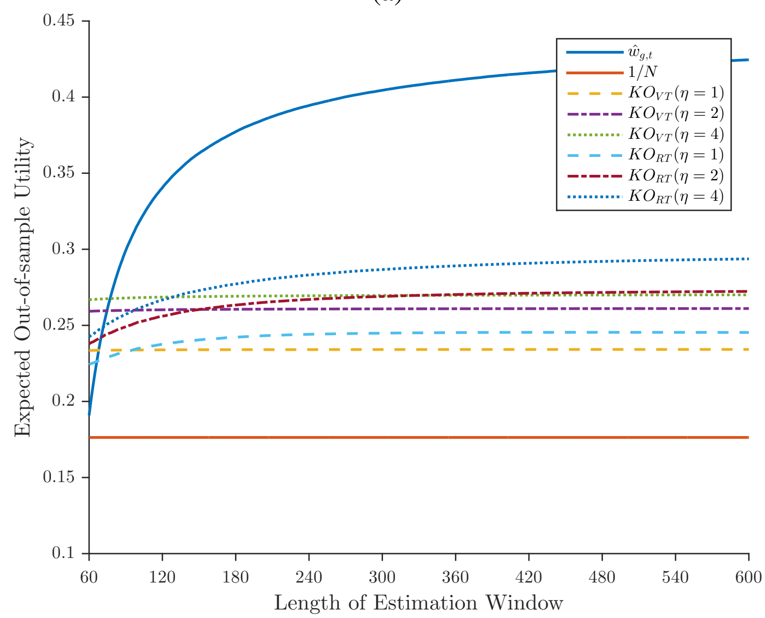


(b)

Figure IA.15: **Expected out-of-sample utility of portfolio rules with block bootstrap (10 risky assets, $\gamma = 3$)**
This figure plots the expected out-of-sample utility (in percentage points) of various portfolio rules as a function of the length of the estimation window (h). The returns of the risky assets are resampled from the excess monthly returns of the 10 momentum portfolios over the period of 1927/1–2017/12 using the block bootstrap procedure of Politis and Romano (1994) with the expected length of the block set to 10 months. The risk aversion coefficient is set to $\gamma = 3$.



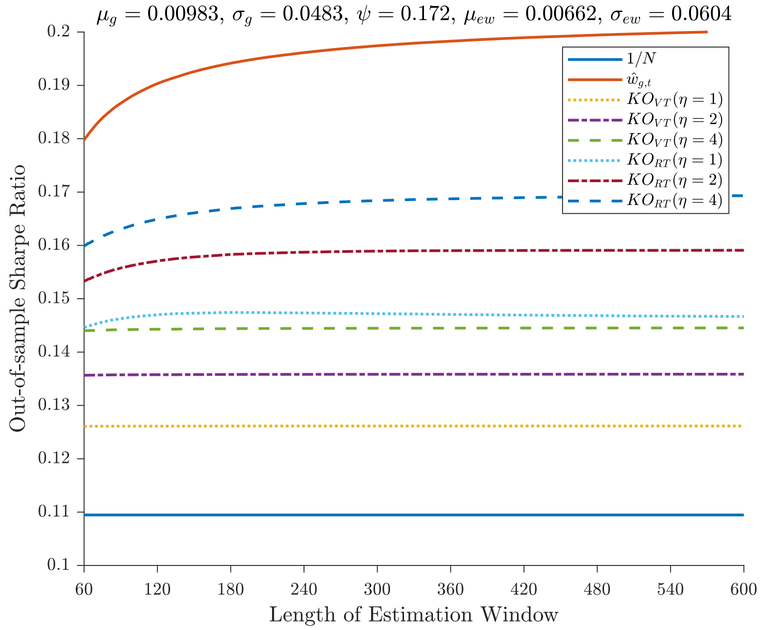
(a)



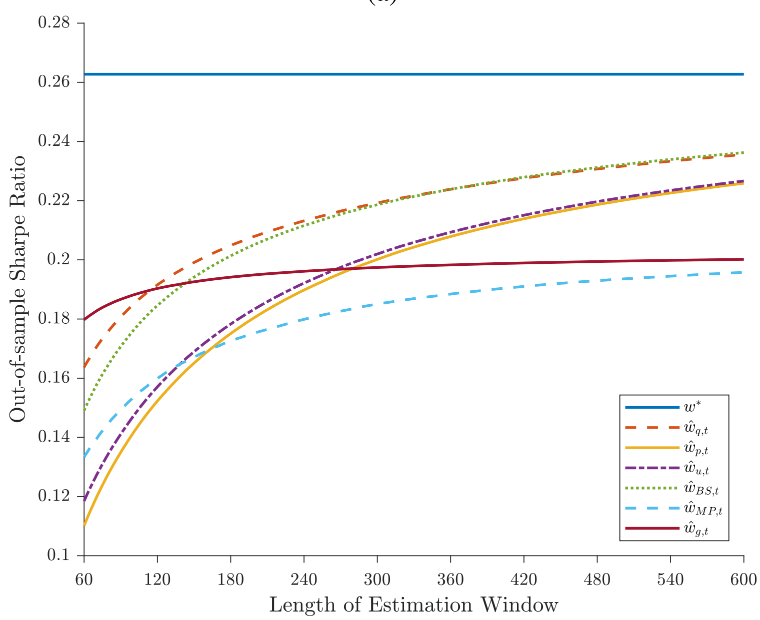
(b)

Figure IA.16: **Expected out-of-sample utility of portfolio rules with block bootstrap (25 risky assets, $\gamma = 3$)**

This figure plots the expected out-of-sample utility (in percentage points) of various portfolio rules as a function of the length of the estimation window (h). The returns of the risky assets are resampled from the excess monthly returns of the Fama-French 25 size and book-to-market ranked portfolios over the period of 1927/1–2017/12 using the block bootstrap procedure of Politis and Romano (1994) with the expected length of the block set to 10 months. The risk aversion coefficient is set to $\gamma = 3$.



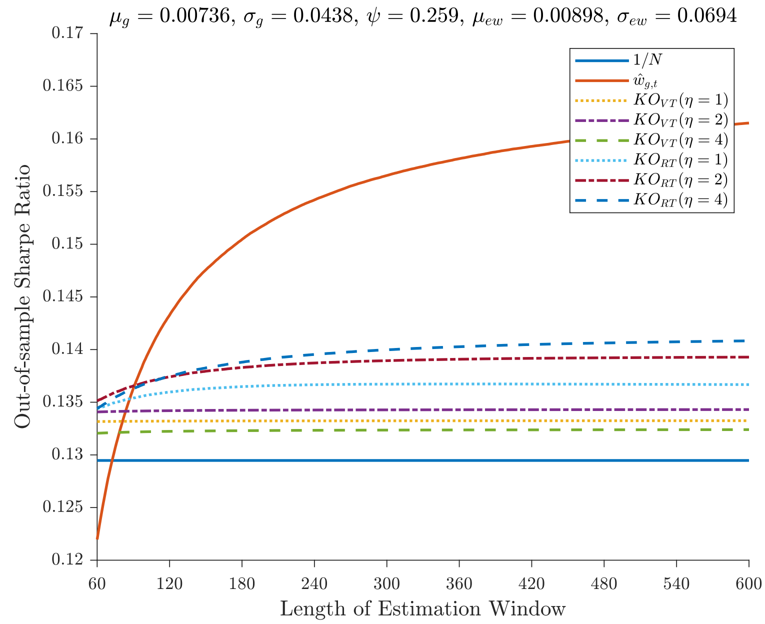
(a)



(b)

Figure IA.17: **Out-of-sample Sharpe ratio of portfolio rules with block bootstrap (10 risky assets, $\gamma = 3$)**

This figure plots the out-of-sample Sharpe ratio of various portfolio rules as a function of the length of the estimation window (h). The returns of the risky assets are resampled from the excess monthly returns of the 10 momentum portfolios over the period of 1927/1–2017/12 using the block bootstrap procedure of Politis and Romano (1994) with the expected length of the block set to 10 months. The risk aversion coefficient is set to $\gamma = 3$.



(a)

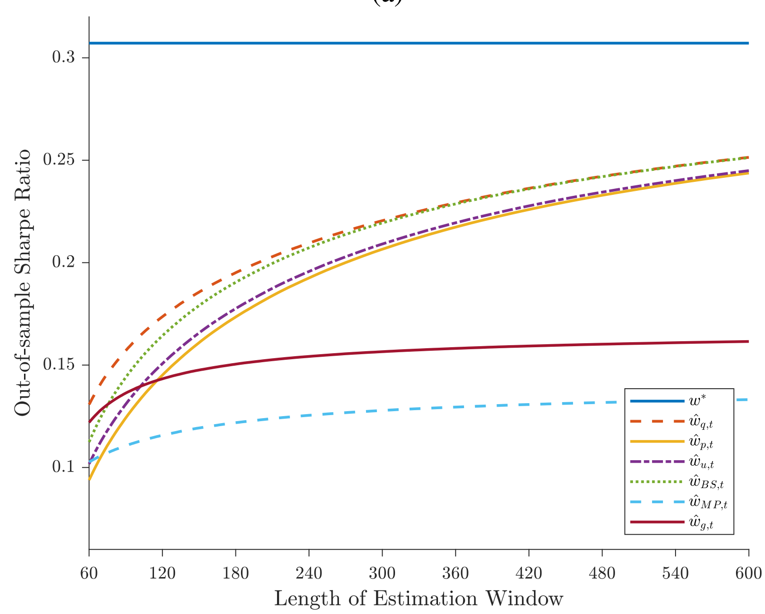


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Figure IA.18: **Out-of-sample Sharpe ratio of portfolio rules with block bootstrap (25 risky assets, $\gamma = 3$)**
This figure plots the out-of-sample Sharpe ratio of various portfolio rules as a function of the length of the estimation window (h). The returns of the risky assets are resampled from the excess monthly returns of the Fama-French 25 size and book-to-market ranked portfolios over the period of 1927/1–2017/12 using the block bootstrap procedure of Politis and Romano (1994) with the expected length of the block set to 10 months. The risk aversion coefficient is set to $\gamma = 3$.

Table IA.1: CER comparison: $h = 120$ and $\gamma = 5$

This table reports the certainty equivalent return of the in-sample optimal portfolio w^*; the $1/N$ rule; the volatility timing strategy (KO_{VT}) and two versions of the risk-to-reward timing strategy (KO_{RT} and KO_{BT}) proposed by Kirby and Ostdiek (2012) with $\eta = 4$; the sample global minimum variance portfolio $\hat{w}_{g,t}$; and five optimal portfolio rules: the plug-in rule $\hat{w}_{p,t}$, the unbiased rule $\hat{w}_{u,t}$, the BS rule $\hat{w}_{BS,t}$, the MP rule $\hat{w}_{MP,t}$, and the newly developed optimal two-fund rule $\hat{w}_{q,t}$, for $h = 120$ and $\gamma = 5$, using the seven datasets containing excess monthly returns of value-weighted portfolios. The first five columns report the results based on the five datasets obtained from Ken French's website. "NM-V (LT)" and "NM-V (All)" represent the datasets used in Novy-Marx and Velikov (2016), with the former containing the long-side and short-side portfolios of the eight low-turnover anomaly strategies and the latter containing the long-side and the short-side portfolios of all the twenty-three anomaly strategies. One-sided p-values of the performance difference between various portfolio rules and the $1/N$ rule are reported in *italics*.

	Momentum $N = 10$	Size-B/M $N = 25$	Industry $N = 10$	IVOL $N = 10$	OP-Inv $N = 25$	NM-V (LT) $N = 16$	NM-V (All) $N = 46$
w^*	0.0086	0.0131	0.0041	0.0139	0.0158	0.0165	0.0722
$1/N$	0.0003	0.0008	0.0023	−0.0003	0.0015	−0.0005	−0.0015
KO_{VT}	0.0017	0.0024	0.0031	0.0026	0.0029	0.0008	0.0030
	0.0004	*0.0032*	*0.1325*	*0.0028*	*0.0001*	*0.0053*	*0.0000*
KO_{RT}	0.0026	0.0019	0.0019	0.0020	0.0034	0.0018	0.0042
	0.0004	*0.0007*	*0.7464*	*0.0072*	*0.0001*	*0.0011*	*0.0000*
KO_{BT}	0.0028	0.0021	0.0026	0.0010	0.0031	0.0024	0.0033
	0.0000	*0.0000*	*0.1985*	*0.0127*	*0.0000*	*0.0001*	*0.0000*
$\hat{w}_{g,t}$	0.0033	0.0045	0.0025	−0.0011	0.0031	0.0034	0.0020
	0.0024	*0.0100*	*0.4243*	*0.6487*	*0.1763*	*0.0255*	*0.1082*
$\hat{w}_{p,t}$	−0.0032	−0.0371	−0.0086	−0.0058	−0.0367	−0.0161	−0.7351
	0.8142	*1.0000*	*1.0000*	*0.7905*	*1.0000*	*0.9893*	*1.0000*
$\hat{w}_{u,t}$	−0.0002	−0.0159	−0.0067	−0.0014	−0.0183	−0.0087	−0.1808
	0.5645	*0.9995*	*1.0000*	*0.5659*	*0.9992*	*0.9196*	*1.0000*
$\hat{w}_{BS,t}$	0.0058	0.0027	0.0007	0.0055	−0.0001	0.0032	−0.0827
	0.0165	*0.2722*	*0.8976*	*0.1000*	*0.6662*	*0.1636*	*0.9988*
$\hat{w}_{MP,t}$	0.0025	0.0011	0.0019	0.0008	0.0018	−0.0008	0.0022
	0.0407	*0.4214*	*0.6848*	*0.2469*	*0.3969*	*0.6046*	*0.0261*
$\hat{w}_{q,t}$	0.0064	0.0071	0.0018	0.0061	0.0038	0.0053	0.0165
	0.0049	*0.0041*	*0.6767*	*0.0639*	*0.1723*	*0.0273*	*0.0869*

Table IA.2: CER comparison: $h = 240$ and $\gamma = 5$

This table reports the certainty equivalent return of the in-sample optimal portfolio w^*; the $1/N$ rule; the volatility timing strategy (KO_{VT}) and two versions of the risk-to-reward timing strategy (KO_{RT} and KO_{BT}) proposed by Kirby and Ostdiek (2012) with $\eta = 4$; the sample global minimum variance portfolio $\hat{w}_{g,t}$; and five optimal portfolio rules: the plug-in rule $\hat{w}_{p,t}$, the unbiased rule $\hat{w}_{u,t}$, the BS rule $\hat{w}_{BS,t}$, the MP rule $\hat{w}_{MP,t}$, and the newly developed optimal two-fund rule $\hat{w}_{q,t}$, for $h = 240$ and $\gamma = 5$, using the seven datasets containing excess monthly returns of value-weighted portfolios. The first five columns report the results based on the five datasets obtained from Ken French's website. "NM-V (LT)" and "NM-V (All)" represent the datasets used in Novy-Marx and Velikov (2016), with the former containing the long-side and short-side portfolios of the eight low-turnover anomaly strategies and the latter containing the long-side and the short-side portfolios of all the twenty-three anomaly strategies. One-sided p-values of the performance difference between various portfolio rules and the $1/N$ rule are reported in *italics*.

	Momentum $N = 10$	Size-B/M $N = 25$	Industry $N = 10$	IVOL $N = 10$	OP-Inv $N = 25$	NM-V (LT) $N = 16$	NM-V (All) $N = 46$
w^*	0.0098	0.0154	0.0044	0.0146	0.0184	0.0178	0.0518
$1/N$	0.0012	0.0022	0.0029	0.0003	0.0022	−0.0002	−0.0010
KO_{VT}	0.0024	0.0031	0.0032	0.0041	0.0038	0.0018	0.0033
	0.0001	*0.0231*	*0.3307*	*0.0007*	*0.0000*	*0.0000*	*0.0004*
KO_{RT}	0.0034	0.0034	0.0022	0.0035	0.0043	0.0031	0.0046
	0.0001	*0.0002*	*0.8683*	*0.0023*	*0.0001*	*0.0000*	*0.0000*
KO_{BT}	0.0038	0.0033	0.0026	0.0023	0.0034	0.0022	0.0040
	0.0000	*0.0000*	*0.8051*	*0.0024*	*0.0006*	*0.0044*	*0.0002*
$\hat{w}_{g,t}$	0.0042	0.0059	0.0030	0.0025	0.0071	0.0061	0.0070
	0.0017	*0.0015*	*0.4832*	*0.1173*	*0.0016*	*0.0010*	*0.0126*
$\hat{w}_{p,t}$	0.0036	−0.0026	−0.0008	−0.0016	−0.0062	0.0024	−0.3173
	0.2540	*0.8506*	*0.9860*	*0.6043*	*0.9111*	*0.3500*	*1.0000*
$\hat{w}_{u,t}$	0.0044	0.0015	−0.0004	0.0004	−0.0021	0.0043	−0.1765
	0.1705	*0.5685*	*0.9791*	*0.4952*	*0.7808*	*0.2344*	*1.0000*
$\hat{w}_{BS,t}$	0.0075	0.0081	0.0025	0.0053	0.0069	0.0096	−0.1135
	0.0084	*0.0210*	*0.6745*	*0.1795*	*0.0968*	*0.0159*	*0.9999*
$\hat{w}_{MP,t}$	0.0044	0.0033	0.0027	0.0052	0.0057	0.0047	0.0048
	0.0003	*0.1026*	*0.6502*	*0.0031*	*0.0003*	*0.0000*	*0.0215*
$\hat{w}_{q,t}$	0.0077	0.0090	0.0027	0.0056	0.0087	0.0104	−0.0518
	0.0057	*0.0024*	*0.5771*	*0.1611*	*0.0077*	*0.0048*	*0.9890*

Table IA.3: Sharpe ratio comparison: $h = 120$ and $\gamma = 5$

This table reports the Sharpe ratio of the in-sample optimal portfolio w^*; the $1/N$ rule; the volatility timing strategy (KO_{VT}) and two versions of the risk-to-reward timing strategy (KO_{RT} and KO_{BT}) proposed by Kirby and Ostdiek (2012) with $\eta = 4$; the sample global minimum variance portfolio $\hat{w}_{g,t}$; and five optimal portfolio rules: the plug-in rule $\hat{w}_{p,t}$, the unbiased rule $\hat{w}_{u,t}$, the BS rule $\hat{w}_{BS,t}$, the MP rule $\hat{w}_{MP,t}$, and the newly developed optimal two-fund rule $\hat{w}_{q,t}$, for $h = 120$ and $\gamma = 5$, using the seven datasets containing excess monthly returns. The first five columns report the results based on the five datasets obtained from Ken French's website. "NM-V (LT)" and "NM-V (All)" represent the datasets used in Novy-Marx and Velikov (2016), with the former containing the long-side and short-side portfolios of the eight low-turnover anomaly strategies and the latter containing the long-side and the short-side portfolios of all the twenty-three anomaly strategies. One-sided p-values of the performance difference between various portfolio rules and the $1/N$ rule are reported in *italics*.

	Momentum $N = 10$	Size-B/M $N = 25$	Industry $N = 10$	IVOL $N = 10$	OP-Inv $N = 25$	NM-V (LT) $N = 16$	NM-V (All) $N = 46$
w^*	0.2940	0.3624	0.2033	0.3733	0.3985	0.4060	0.8500
$1/N$	0.1308	0.1522	0.1613	0.1252	0.1481	0.1209	0.1034
KO_{VT}	0.1495	0.1667	0.1747	0.1632	0.1744	0.1338	0.1751
	0.0065	*0.0692*	*0.1912*	*0.0194*	*0.0003*	*0.0645*	*0.0000*
KO_{RT}	0.1729	0.1676	0.1475	0.1536	0.1867	0.1665	0.2046
	0.0007	*0.0071*	*0.7971*	*0.0497*	*0.0002*	*0.0005*	*0.0000*
KO_{BT}	0.1777	0.1715	0.1667	0.1373	0.1807	0.1737	0.1859
	0.0001	*0.0000*	*0.2635*	*0.1274*	*0.0000*	*0.0002*	*0.0000*
$\hat{w}_{g,t}$	0.1847	0.2127	0.1595	0.0629	0.1773	0.1843	0.1478
	0.0071	*0.0293*	*0.5256*	*0.9472*	*0.2184*	*0.0603*	*0.2253*
$\hat{w}_{p,t}$	0.2575	0.2171	0.0731	0.3143	0.1641	0.2057	0.5649
	0.0009	*0.0704*	*0.9940*	*0.0003*	*0.3970*	*0.0840*	*0.0000*
$\hat{w}_{u,t}$	0.2608	0.2273	0.0786	0.3142	0.1730	0.2112	0.5699
	0.0006	*0.0416*	*0.9917*	*0.0003*	*0.3392*	*0.0680*	*0.0000*
$\hat{w}_{BS,t}$	0.2697	0.2533	0.1268	0.3058	0.1950	0.2320	0.5776
	0.0001	*0.0078*	*0.8780*	*0.0003*	*0.1976*	*0.0238*	*0.0000*
$\hat{w}_{MP,t}$	0.1744	0.1397	0.1517	0.1240	0.1499	0.0976	0.1548
	0.0384	*0.6889*	*0.6795*	*0.5136*	*0.4729*	*0.8241*	*0.0842*
$\hat{w}_{q,t}$	0.2695	0.2699	0.1444	0.3002	0.2061	0.2392	0.5853
	0.0001	*0.0015*	*0.7266*	*0.0004*	*0.1139*	*0.0118*	*0.0000*

Table IA.4: Sharpe ratio comparison: $h = 240$ and $\gamma = 5$

This table reports the Sharpe ratio of the in-sample optimal portfolio w^*; the $1/N$ rule; the volatility timing strategy (KO_{VT}) and two versions of the risk-to-reward timing strategy (KO_{RT} and KO_{BT}) proposed by Kirby and Ostdiek (2012) with $\eta = 4$; the sample global minimum variance portfolio $\hat{w}_{g,t}$; and five optimal portfolio rules: the plug-in rule $\hat{w}_{p,t}$, the unbiased rule $\hat{w}_{u,t}$, the BS rule $\hat{w}_{BS,t}$, the MP rule $\hat{w}_{MP,t}$, and the newly developed optimal two-fund rule $\hat{w}_{q,t}$, for $h = 240$ and $\gamma = 5$, using the seven datasets containing excess monthly returns. The first five columns report the results based on the five datasets obtained from Ken French's website. "NM-V (LT)" and "NM-V (All)" represent the datasets used in Novy-Marx and Velikov (2016), with the former containing the long-side and short-side portfolios of the eight low-turnover anomaly strategies and the latter containing the long-side and the short-side portfolios of all the twenty-three anomaly strategies. One-sided p-values of the performance difference between various portfolio rules and the $1/N$ rule are reported in *italics*.

	Momentum $N = 10$	Size-B/M $N = 25$	Industry $N = 10$	IVOL $N = 10$	OP-Inv $N = 25$	NM-V (LT) $N = 16$	NM-V (All) $N = 46$
w^*	0.3136	0.3950	0.2115	0.3829	0.4323	0.4252	0.7213
$1/N$	0.1393	0.1659	0.1730	0.1304	0.1606	0.1217	0.1152
KO_{VT}	0.1614	0.1796	0.1791	0.2024	0.1959	0.1536	0.1823
	0.0007	*0.0684*	*0.3547*	*0.0009*	*0.0000*	*0.0001*	*0.0003*
KO_{RT}	0.1889	0.1878	0.1550	0.1867	0.2065	0.1815	0.2153
	0.0001	*0.0005*	*0.8797*	*0.0050*	*0.0001*	*0.0000*	*0.0000*
KO_{BT}	0.1958	0.1883	0.1650	0.1597	0.1853	0.1659	0.2010
	0.0000	*0.0000*	*0.8025*	*0.0109*	*0.0009*	*0.0069*	*0.0004*
$\hat{w}_{g,t}$	0.2043	0.2540	0.1722	0.1610	0.2790	0.2565	0.2809
	0.0026	*0.0010*	*0.5120*	*0.2294*	*0.0014*	*0.0010*	*0.0148*
$\hat{w}_{p,t}$	0.2766	0.2734	0.1232	0.3057	0.2279	0.2853	0.3991
	0.0009	*0.0114*	*0.9307*	*0.0037*	*0.1599*	*0.0160*	*0.0028*
$\hat{w}_{u,t}$	0.2785	0.2799	0.1262	0.3072	0.2356	0.2898	0.4040
	0.0007	*0.0072*	*0.9210*	*0.0033*	*0.1304*	*0.0132*	*0.0024*
$\hat{w}_{BS,t}$	0.2866	0.2966	0.1596	0.3088	0.2725	0.3140	0.4128
	0.0002	*0.0016*	*0.6923*	*0.0026*	*0.0336*	*0.0036*	*0.0017*
$\hat{w}_{MP,t}$	0.2100	0.1854	0.1663	0.2321	0.2449	0.2159	0.2286
	0.0003	*0.1464*	*0.6746*	*0.0044*	*0.0002*	*0.0000*	*0.0300*
$\hat{w}_{q,t}$	0.2868	0.3002	0.1659	0.3084	0.2975	0.3226	0.4206
	0.0001	*0.0007*	*0.6122*	*0.0025*	*0.0065*	*0.0019*	*0.0013*