



# Biomedical Imaging & Analysis

Lecture 6, Part 5. Fall 2014

Basic Image Processing / Filtering (V) – Feature Extraction II

*[Text: Ch: 10, Gonzalez and Woods, Digital Image Processing (3<sup>rd</sup> Edition)]*

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# Image Features

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- Local features
    - local regions with special properties, including points, edges, corners, lines, curves, regions with special properties, etc.
  - Global features
    - global properties of an image, including intensity histogram, frequency domain descriptors, covariance matrix and high order statistics, etc.
    - Obtained from Principal Component Analysis (PCA), Laplace or Fourier Decomposition etc..
  - Depending on applications, various features are useful.
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# Image Features

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High Level Task:

Object Recognition  
Scene classification

**Semantic Gap**

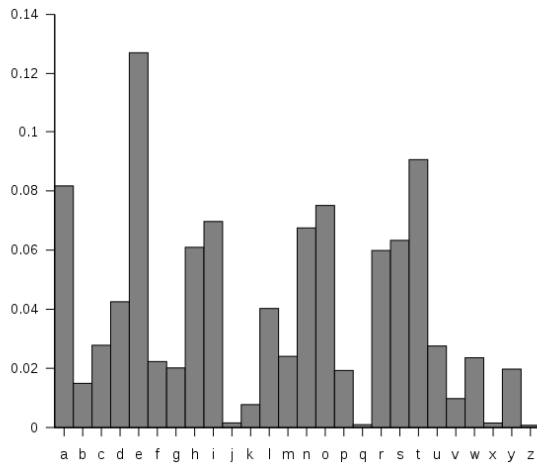
Low Level  
image feature

Pixel intensity, gradient

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# Analogy to text analysis

The regression procedure is appropriate for attributes that organize scenes in a continuous manner (e.g., degree of openness, expansion, ruggedness, and roughness). However, some scene properties refer to a binary classification (e.g., man-made vs. natural, indoor vs. outdoor, objects vs. environments, etc.). The discrimination of two classes can be performed by assigning to the images of each class the attribute values  $s_i = -1$  or  $s_i = 1$  for the two classes respectively. In such a case, the regression parameters (Eq. (12)) are equivalent to the parameters obtained by applying a linear discriminant analysis (see Ripley, 1996; Swets and Weng,



Letter frequency

## High-level goals:

Which author writes this article?

Or what type of content does it represent (eg: science, politics, entertainment)?

Meaning group

Sentence

phrase

word

Letter frequency

# Corner features

- Sources: intersection of image lines, corner patterns in the images, etc
- Stable across sequence of images

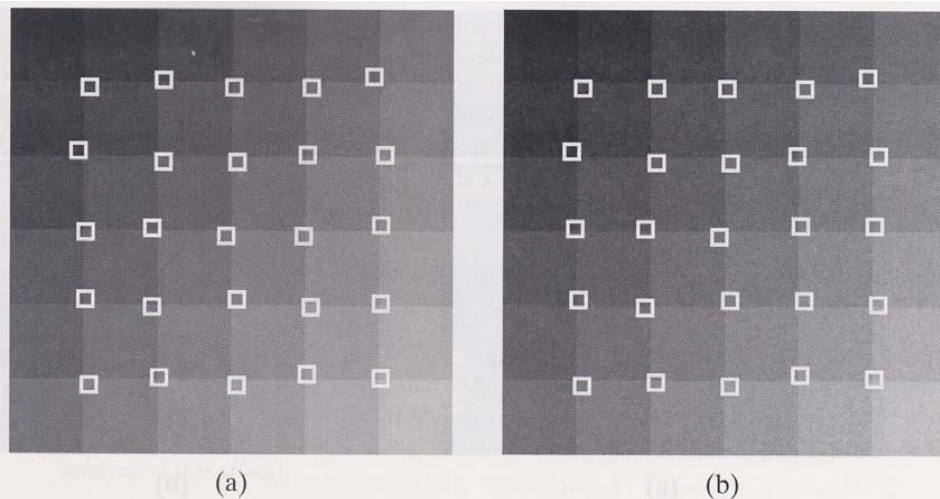
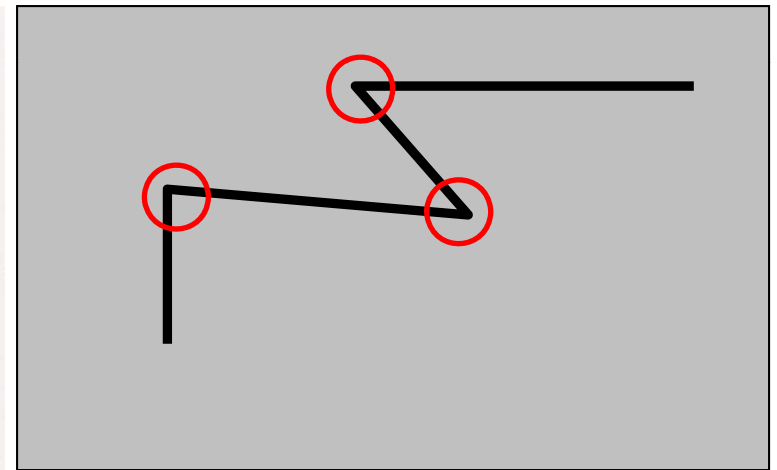


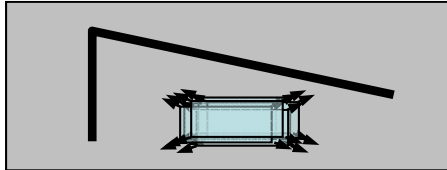
Figure 4.8 Corners found in a 8-bit, synthetic checkerboard image, corrupted by two realizations of synthetic Gaussian noise of standard deviation 2. The corner is the bottom right point of each  $15 \times 15$  neighbourhood (highlighted).



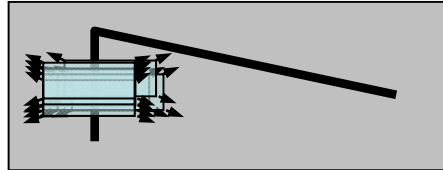
*Eg: Harris corner detector*

C.Harris, M.Stephens. "A Combined Corner and Edge Detector". 1988

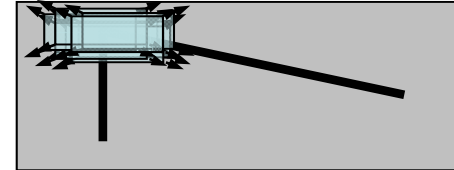
# Harris Detector: Basic Idea



“flat” region:  
no change in all  
directions



“edge”:  
no change along the  
edge direction



“corner”:  
significant change in all  
directions

**Window-averaged change of intensity for the shift  $[u, v]$ :**

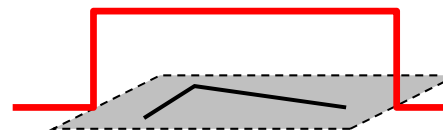
$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

Window  
function

Shifted  
intensity

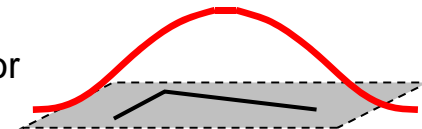
Intensity

Window function  $w(x, y) =$



1 in window, 0 outside

or



Gaussian

# Harris Detector: Mathematics

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Expanding  $E(u,v)$  in a 2<sup>nd</sup> order Taylor series expansion, we have, for small shifts  $[u,v]$ , a *bilinear* approximation:

$$E(u, v) \cong [u, v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

where  $M$  is a  $2 \times 2$  matrix computed from image derivatives:

$$M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

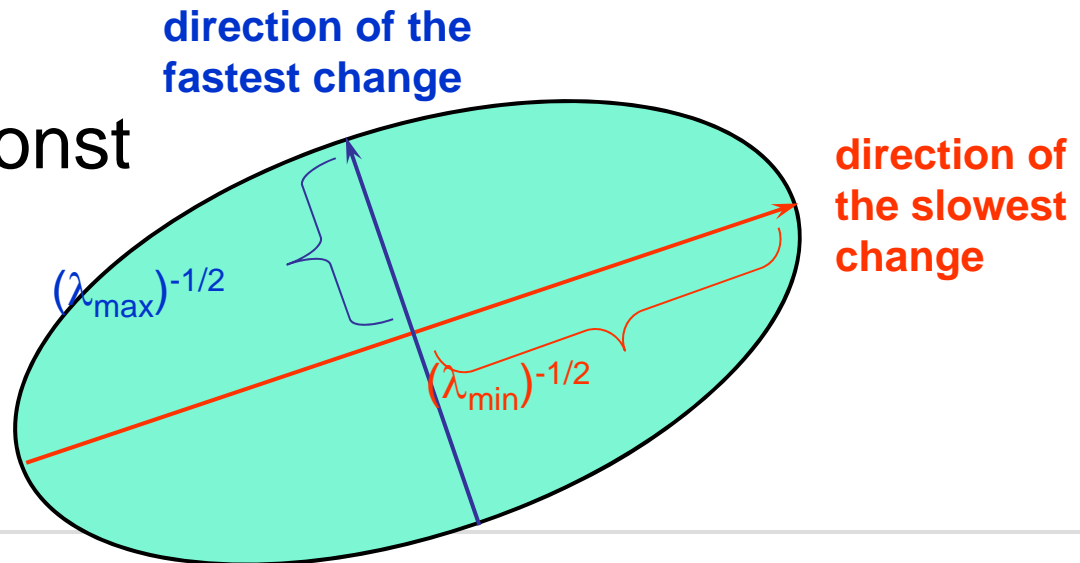
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# Harris Detector: Mathematics

Intensity change in shifting window: eigenvalue analysis

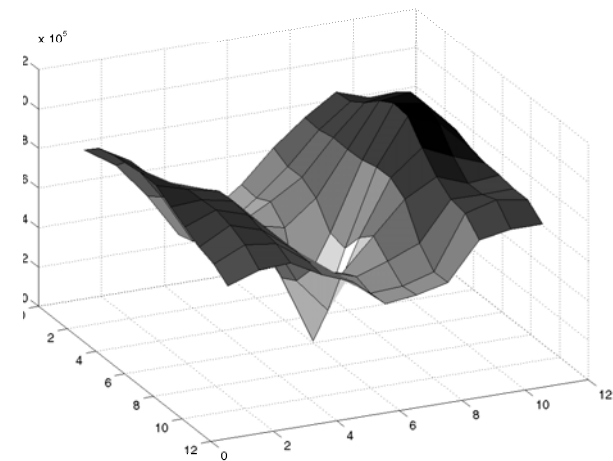
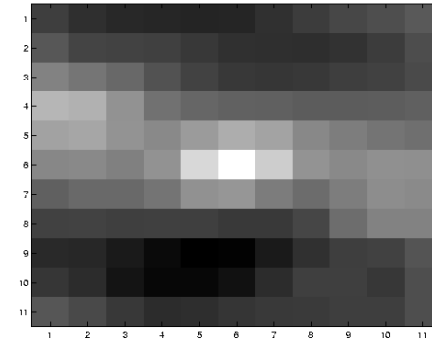
$$E(u, v) \cong [u, v] M \begin{bmatrix} u \\ v \end{bmatrix} \quad \lambda_1, \lambda_2 - \text{eigenvalues of } M$$

Ellipse  $E(u, v) = \text{const}$



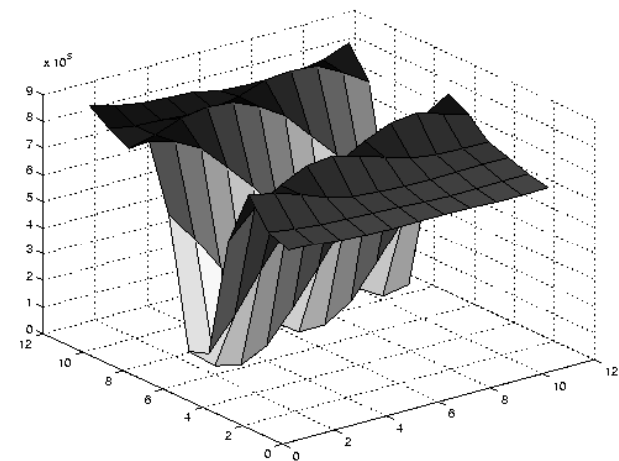
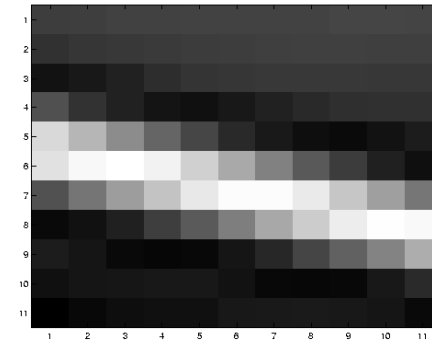


# Selecting Good Features



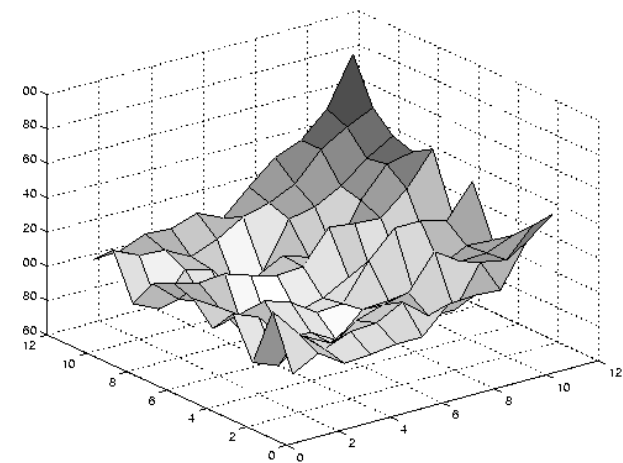
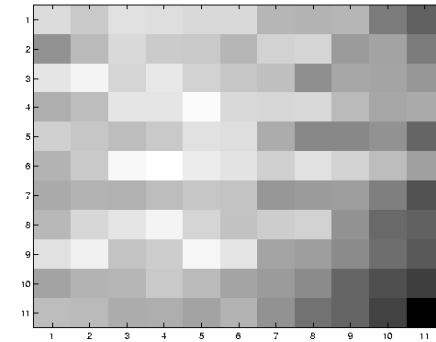
$\lambda_1$  and  $\lambda_2$  are large

# Selecting Good Features



large  $\lambda_1$ , small  $\lambda_2$

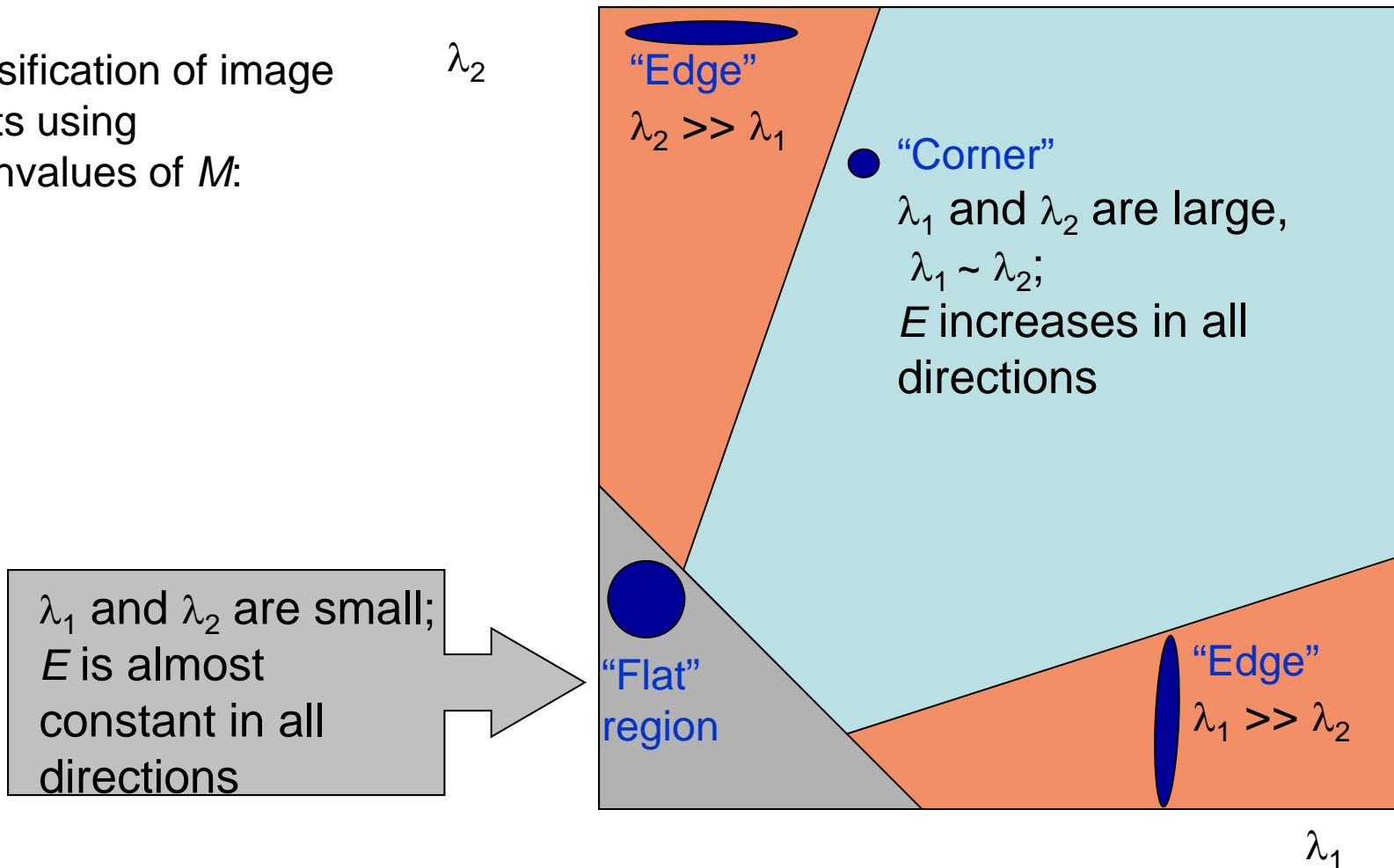
# Selecting Good Features



small  $\lambda_1$ , small  $\lambda_2$

# Harris Detector: Mathematics

Classification of image points using eigenvalues of  $M$ :



# Harris Detector: Mathematics

Measure of corner response:

$$R = \det M - k (\text{trace } M)^2$$

$$\det M = \lambda_1 \lambda_2$$

$$\text{trace } M = \lambda_1 + \lambda_2$$

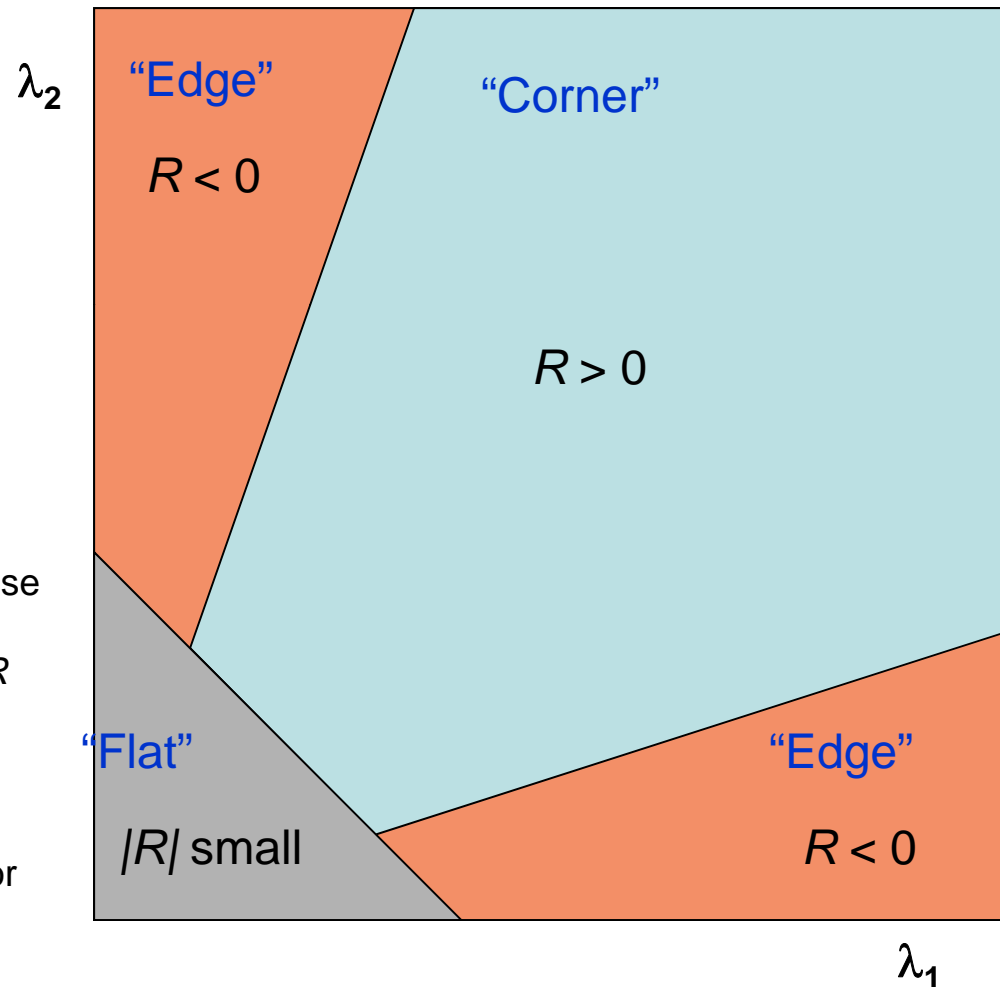
( $k$  – empirical constant,  $k = 0.04$ - $0.06$ )

## The Algorithm:

Find points with large corner response function  $R$  ( $R > \text{threshold}$ )

Take the points of local maxima of  $R$

- $R$  depends only on eigen-values of  $M$
- $R$  is large for a **corner**
- $R$  is negative with large magnitude for an **edge**
- $|R|$  is small for a **flat** region



# Harris Detector: Workflow

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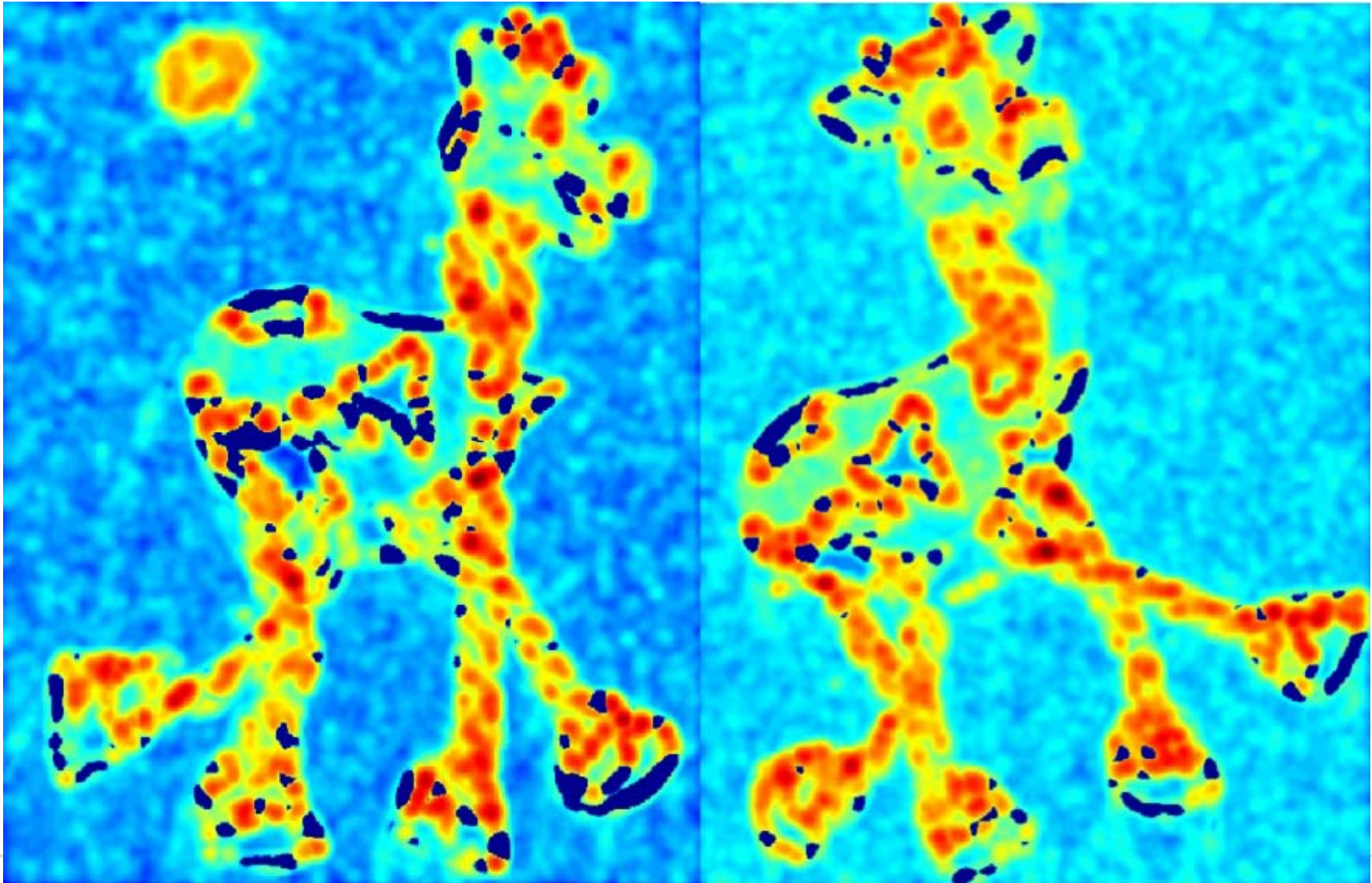




# Harris Detector: Workflow

Compute corner response  $R$

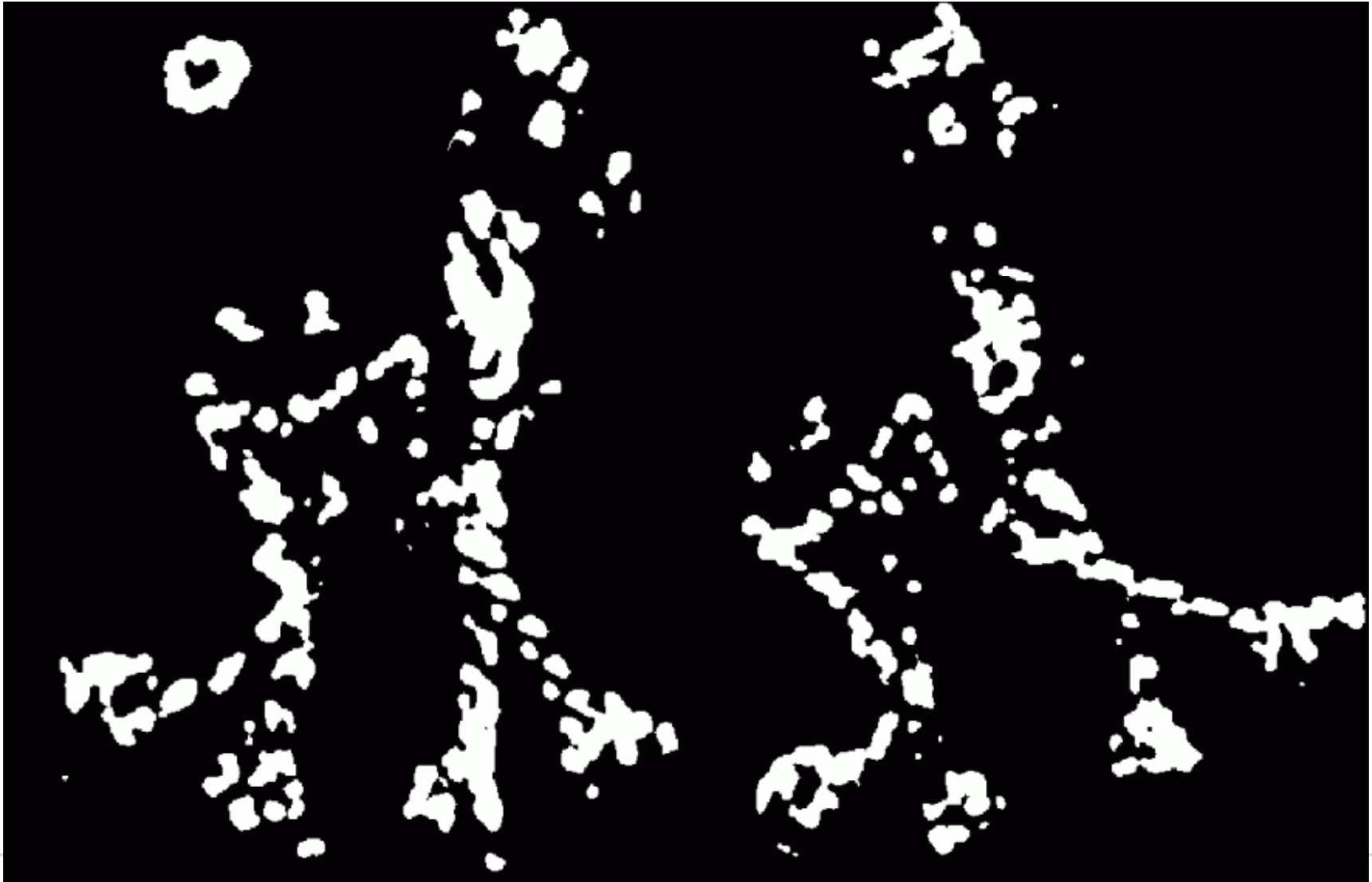
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# Harris Detector: Workflow

Find points with large corner response:  $R > \text{threshold}$

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# Harris Detector: Workflow

Take only the points of local maxima of  $R$

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# Harris Detector: Workflow



# Harris Detector: Summary

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- Average intensity change in direction  $[u, v]$  can be expressed as a bilinear form:

$$E(u, v) \cong [u, v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

- Describe a point in terms of eigenvalues of  $M$ :  
*measure of corner response*

$$R = \lambda_1 \lambda_2 - k (\lambda_1 + \lambda_2)^2$$

- A good (corner) point should have a *large intensity change in all directions*, i.e.  $R$  should be large positive
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# Lucas-Tomasi-Kanade Point (Corner) Detector

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- Basic idea of LTK point detector

$$G = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

if  $\min(\lambda_1, \lambda_2) > \lambda_{threshold}$ , accept the pixel as a point feature

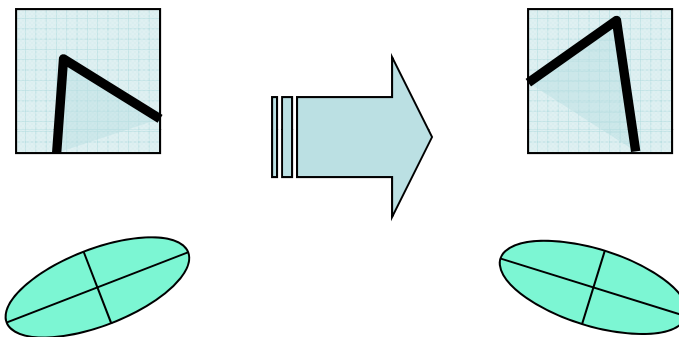
- There is a wide variety of corner detectors.

**Also based on image derivatives and eigen-values...**

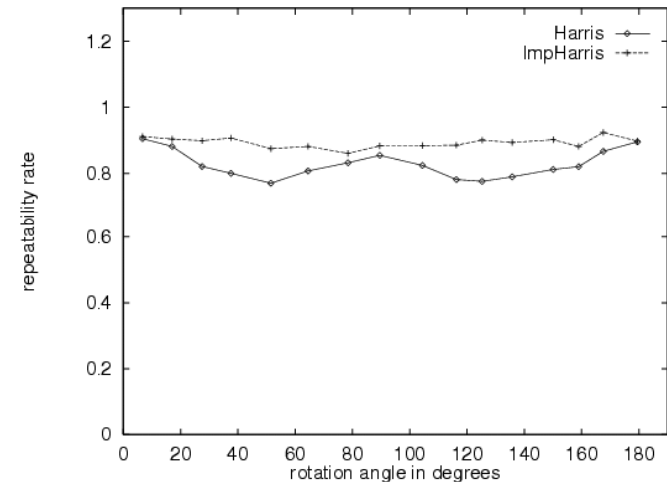
- 1) B. D. Lucas & T. Kanade. An Iterative Image Registration Technique with an Application to Stereo Vision. *International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.
  - 2) C. Tomasi & T. Kanade. Detection and Tracking of Point Features. *Carnegie Mellon University Technical Report CMU-CS-91-132*, 1991.
  - 3) J. Shi & C. Tomasi. Good Features to Track. *IEEE Conf. Computer Vision and Pattern Recognition*, pp. 593–600, 1994.
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# Harris Detector: Some Properties

- Rotation invariance



Ellipse rotates but its shape (i.e. eigenvalues) remains the same



C.Schmid et.al. "Evaluation of Interest Point Detectors". IJCV 2000

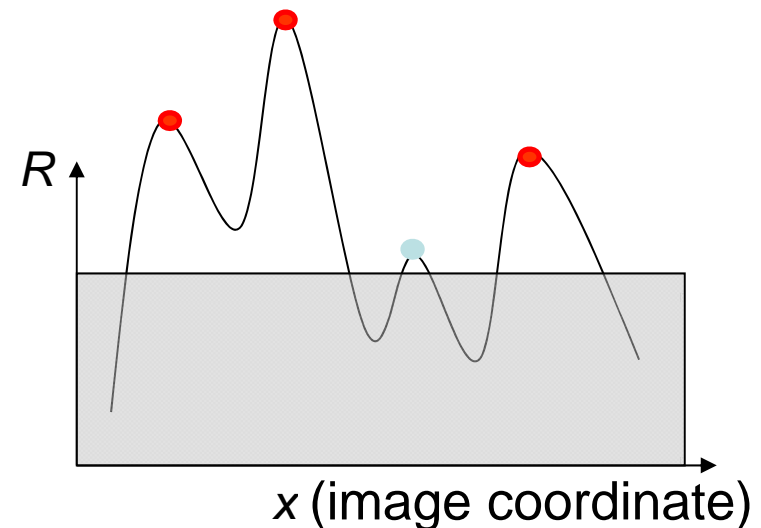
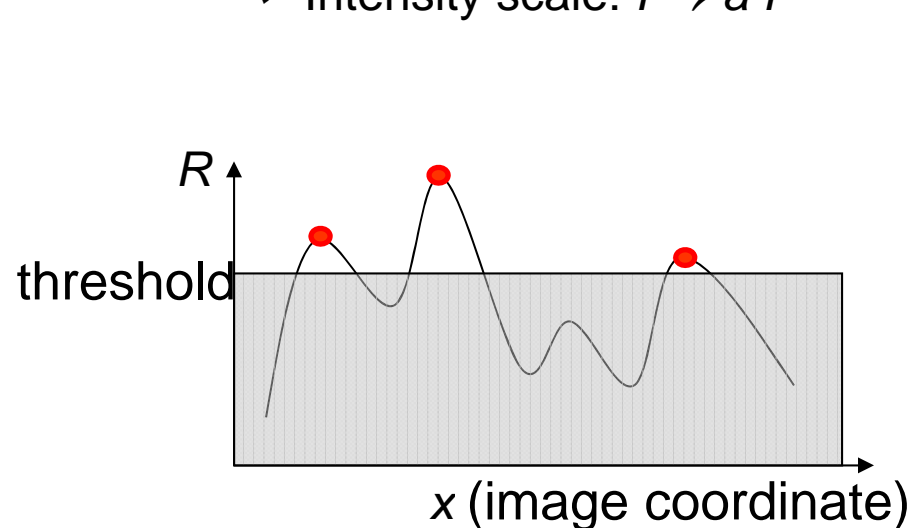
*Corner response  $R$  is invariant to image rotation*

# Harris Detector: Some Properties

- Partial invariance to additive and multiplicative intensity changes

✓ Only derivatives are used  $\Rightarrow$  invariance to intensity shift  $I \rightarrow I + b$



✓ Intensity scale:  $I \rightarrow a I$



# Models of Image Change : Isometries

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- Geometry

- Rotation  
- Similarity (rotation + uniform scale)   
- Affine (scale dependent on direction)     
valid for: orthographic camera, locally planar object

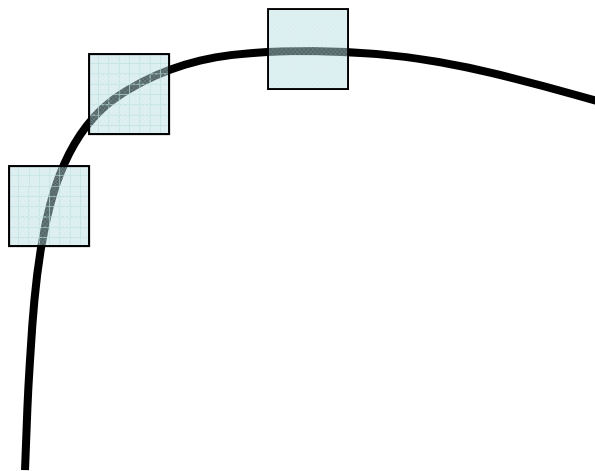
- Photometry

- Affine intensity change ( $I \rightarrow a I + b$ )   

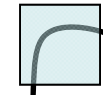
# Harris Detector: Some Properties

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- Not invariant to *image scale*!
  - But, we need to detect *the same* interest points regardless of *image changes*!



All points will be  
classified as **edges**



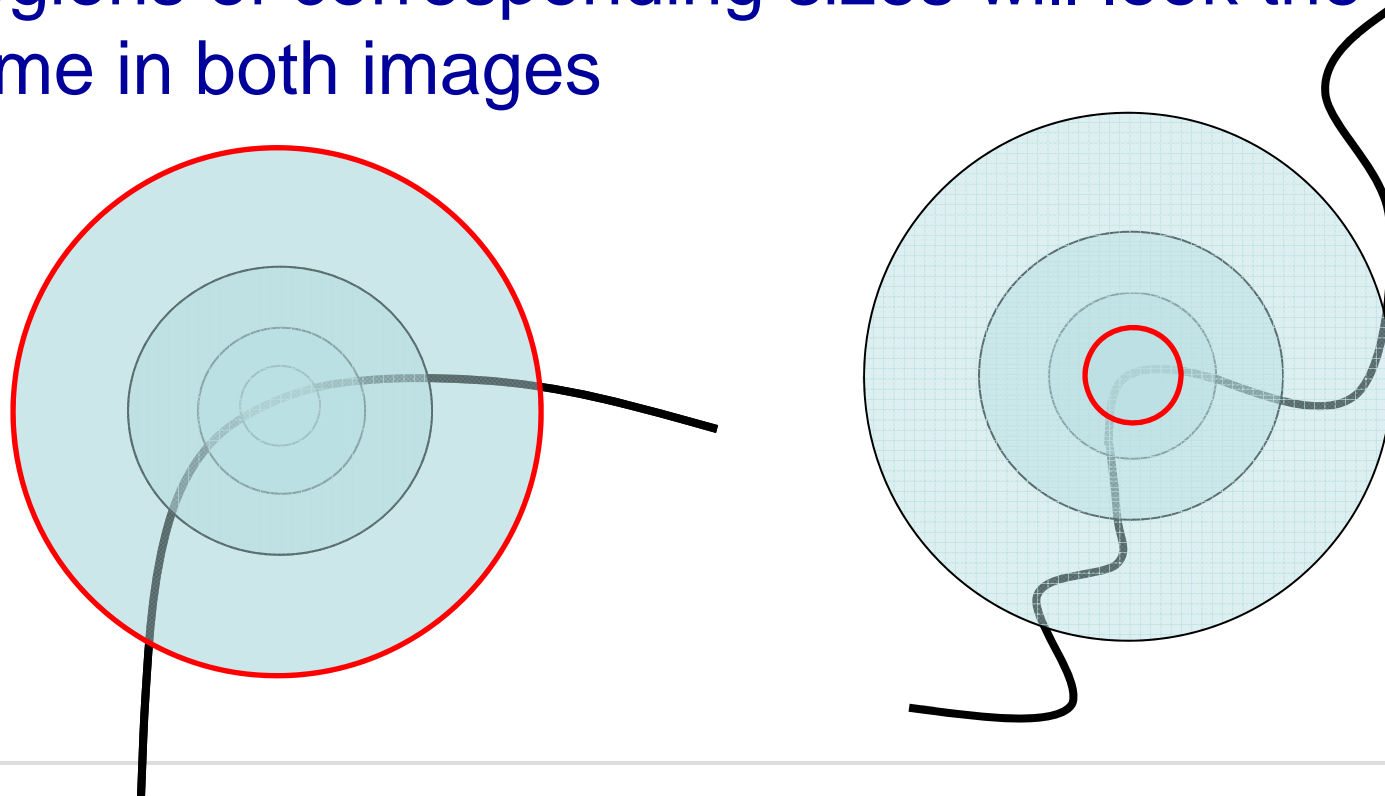
**Corner !**



# Scale Invariant Detection

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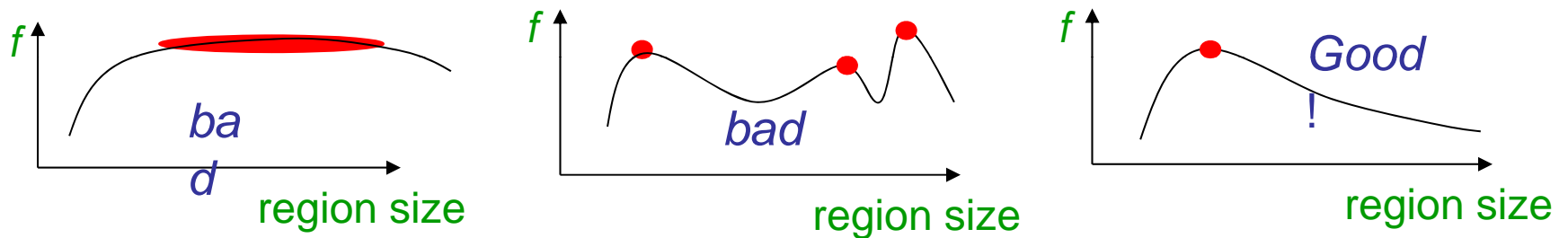
- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



# Scale Invariant Detection. Eg: Local Maxima

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- A “good” function for scale detection:  
has one stable sharp peak



- For usual images: a good function would be a one which responds to contrast (sharp local intensity change)
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# Scale Invariant Detection

- Functions for determining scale  $f = \text{Kernel} * \text{Image}$

Kernels:

$$L = \sigma^2 \left( G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

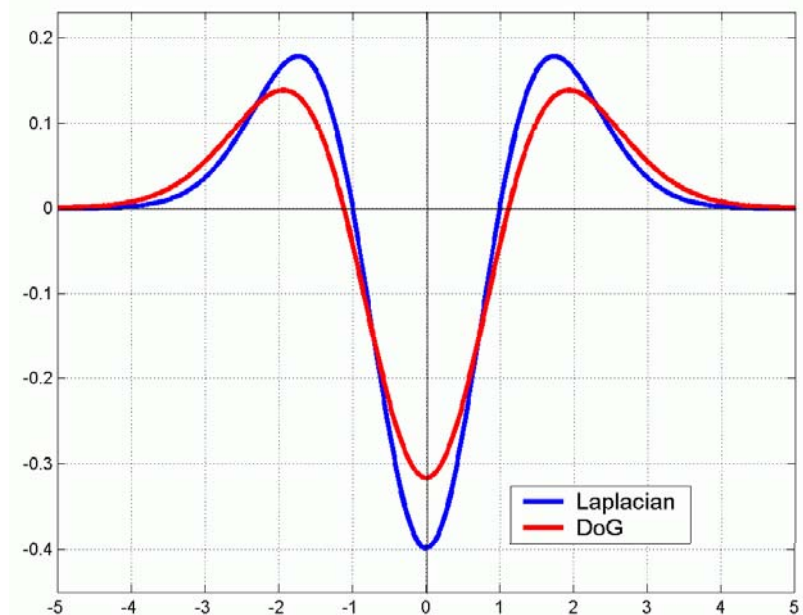
(Laplacian)

$$\text{DoG} = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

where Gaussian

$$G(x, y, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

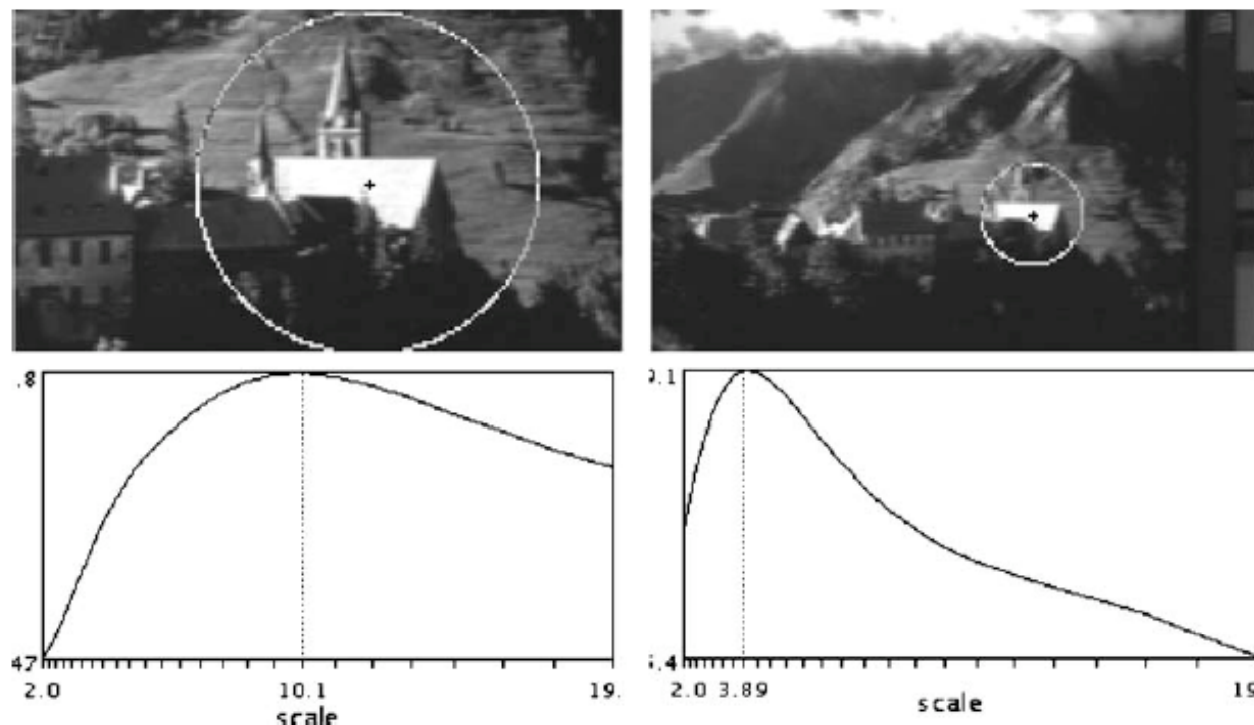


Note: both kernels are invariant to *scale* and *rotation*

# Laplacian of Gaussian for selection of characteristic scale

[http://www.robots.ox.ac.uk/~vgg/research/affine/det\\_eval\\_files/mikolajczyk\\_ijcv2004.pdf](http://www.robots.ox.ac.uk/~vgg/research/affine/det_eval_files/mikolajczyk_ijcv2004.pdf)

Normalize / Rescale detected regions to Fixed Size (Lindeberg et al, 1996) :



*Figure 1.* Example of characteristic scales. The top row shows two images taken with different focal lengths. The bottom row shows the response  $F_{\text{norm}}(x, \sigma_n)$  over scales where  $F_{\text{norm}}$  is the normalized LoG (cf. Eq. (3)). The characteristic scales are 10.1 and 3.89 for the left and right image, respectively. The ratio of scales corresponds to the scale factor (2.5) between the two images. The radius of displayed regions in the top row is equal to 3 times the characteristic scale.