Biomedical Imaging & Analysis

Lecture 2, Part 2. Fall 2014

*Image Formation & Visualization (I): **MRI & k-Space***

*[**Text**:* Joseph P. Hornak, *The Basics of MRI*

http://www.cis.rit.edu/htbooks/mri/inside.htm]

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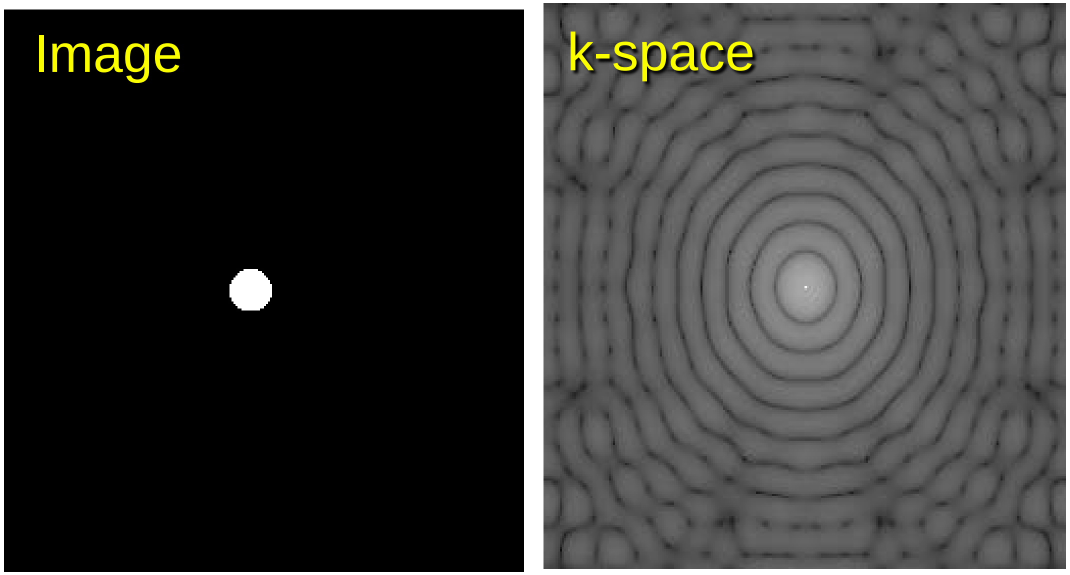
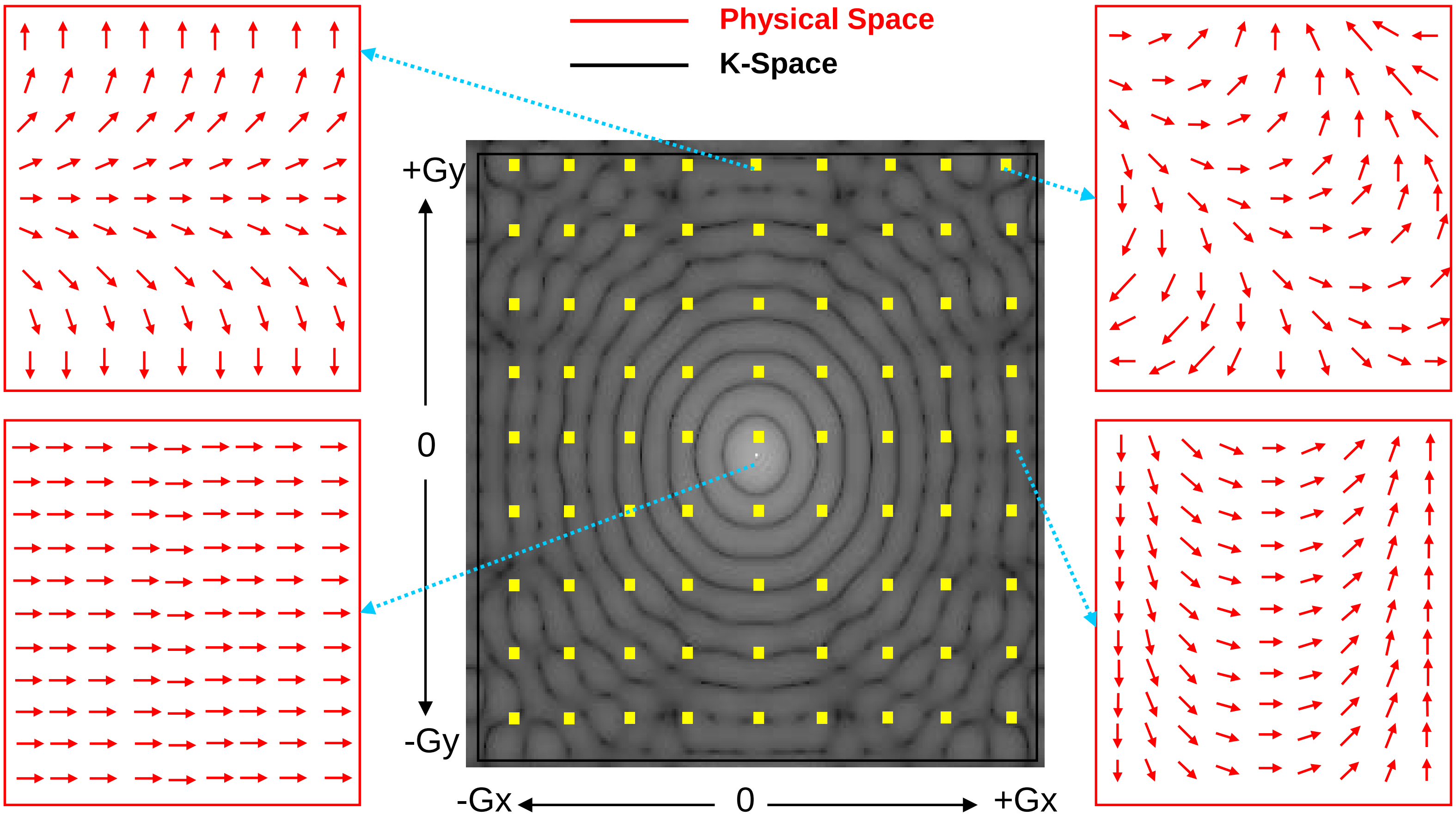
Introduction to k-space
(MR data space)



Introduction to k-space



Contributions of different image locations to the raw k-space data. Each data point in k-space (shown in yellow) consists of the summation of MR signal from all voxels in image space under corresponding gradient fields.

In 2D: Acquired MR Signal

For a given data point in k-space, say (kx, ky), its signal S(kx, ky) is the sum of all the little signal from each voxel I(x,y) in the physical space, under the gradient field at that particular moment

$$S(k_x, k_y) = \int \int I(x, y) e^{-i2\pi(k_x x + k_y y)} dxdy$$

From this equation, it can be seen that the acquired MR signal, which is also in a 2-D space (with kx, ky coordinates), is the Fourier Transform of the imaged object.

$$Kx = \gamma/2\pi \int_0^t Gx(t)\, dt$$

$$Ky = \gamma/2\pi \int_0^t Gy(t)\, dt$$

Fourier Theory

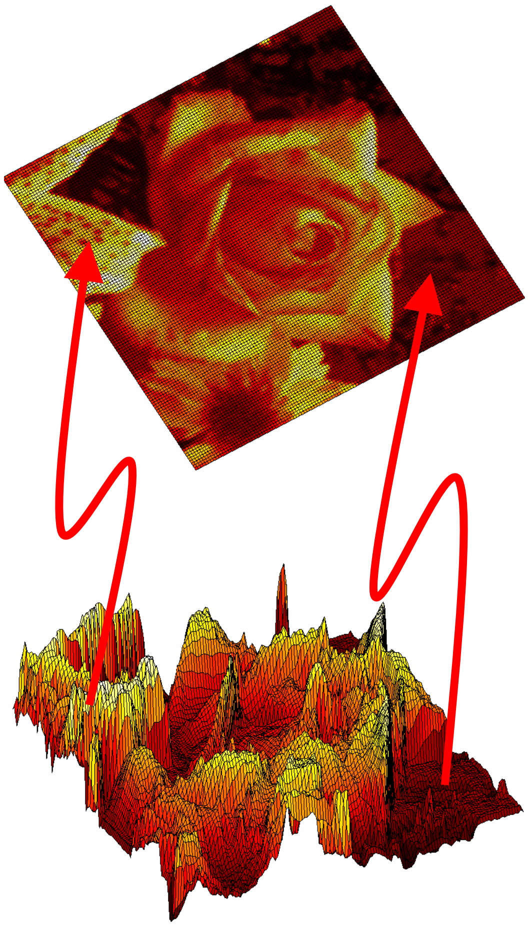
- The physical means of generating and detecting these waves may be quite different for each modality, but there is one common theme that unites them
  - Fourier theory
  - Generally applicable to understanding CT and Ultrasound Imaging as well…

Fourier **Implications**

- Fourier theory explains many aspects of imaging
  - MRI’s natural signal space
  - What determines resolution in CT and SPECT
  - What influences the design and utility of US probes and relationship of probe frequency and resolution
  - Contrast and artifact appearance

Image as a **Signal Waveform**

- The signal intensity view is not one we commonly see.
- However, it is the case that an image is a signal waveform in two dimensions.

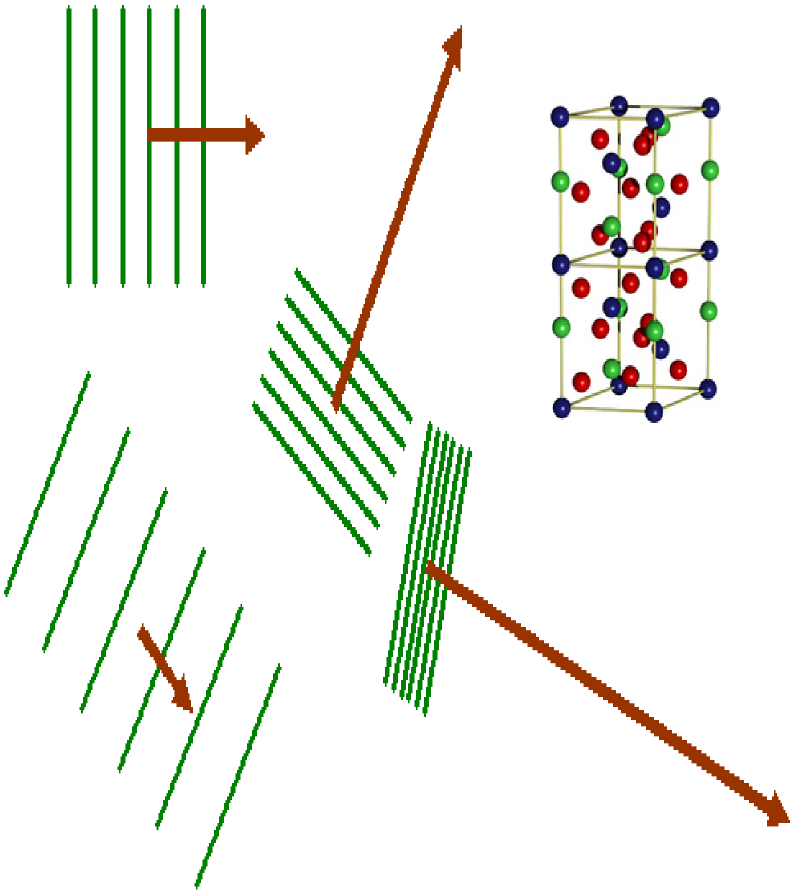


Q & A : Understanding the FT

- How many 7s in 49?
  - Divide 49 by 7
- How many 8s in 102
  - Divide 102 by 8
- How many Xs in Y?
  - Divide Y by X
- How many 23 Hz waveforms in my signal X(t)
  - Divide X(t) by 23Hz
- This describes the basis of Fourier analysis

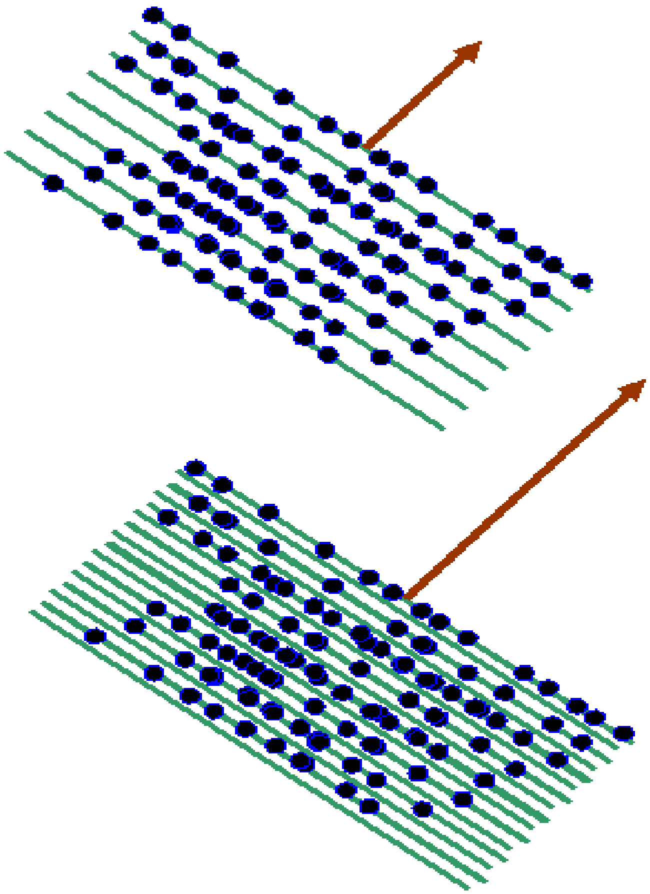
K-Space Terminology

- One of the early uses was in crystallography
  - To find the underlying structure of crystals
- In this approach x-rays (a coherent “light” source) was used to irradiate the crystal and the diffraction pattern observed
- The terminology used to describe this involved using ***k*** as a vector pointing in the direction of travel of the wave



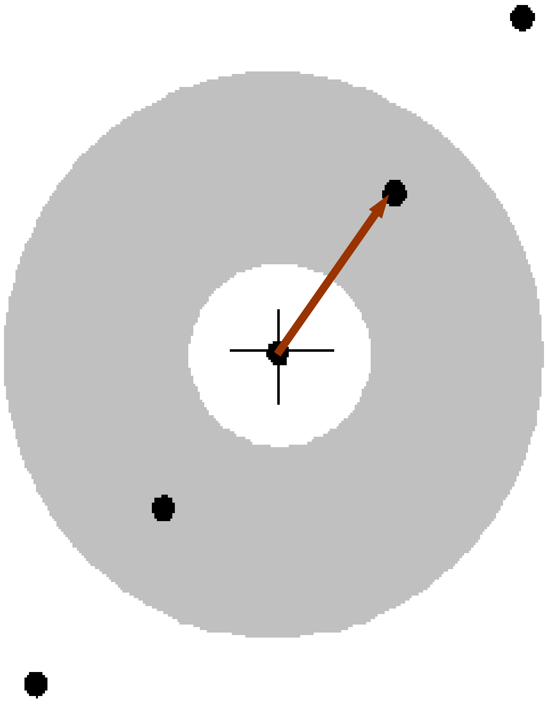
Crystallography

- When the interrogation waveform strongly aligns with the crystal planes, the diffraction K vector is very strong
  - For the fundamental frequency
  - And for harmonics
- When the interrogation wave only weakly aligns with the crystal lattice, the diffraction pattern is weak



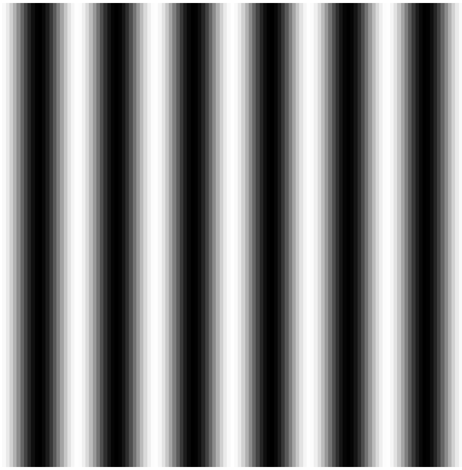
K-Space

- Each of the scattered k vectors can be plotted on a 2D grid, termed k-space
- Crystal…Kristal…K-space
- When each of them is plotted, the amplitude for each point in k-space is found
- This k-space representation is the Fourier transform of the distribution of the scattering atoms
- The k-space data directly relates to the separation of the planes of atoms



Images and Signal Waveforms

- Images are composed of spatial frequency waveforms.
- The Fourier representation describes the k-space response.
- The center of k-space describes our reference.



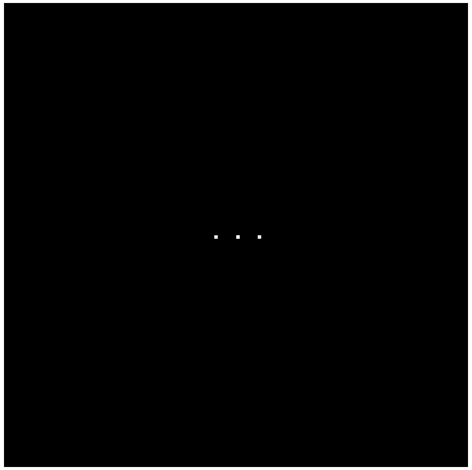
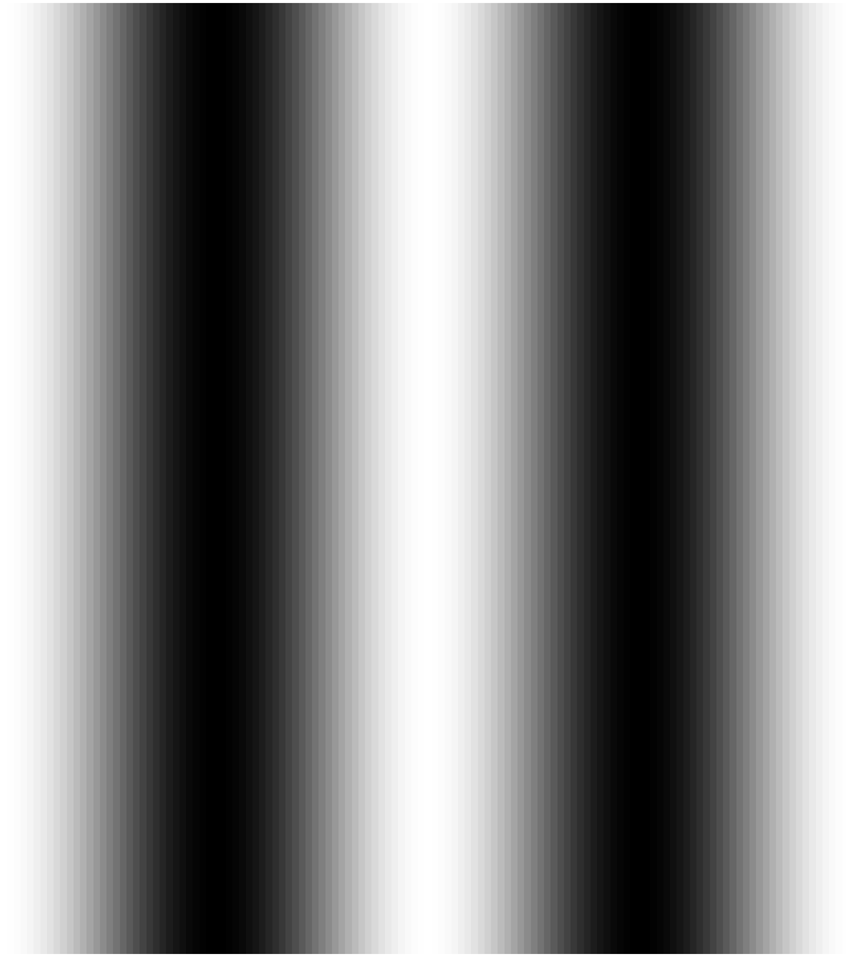


Image-K-space Examples



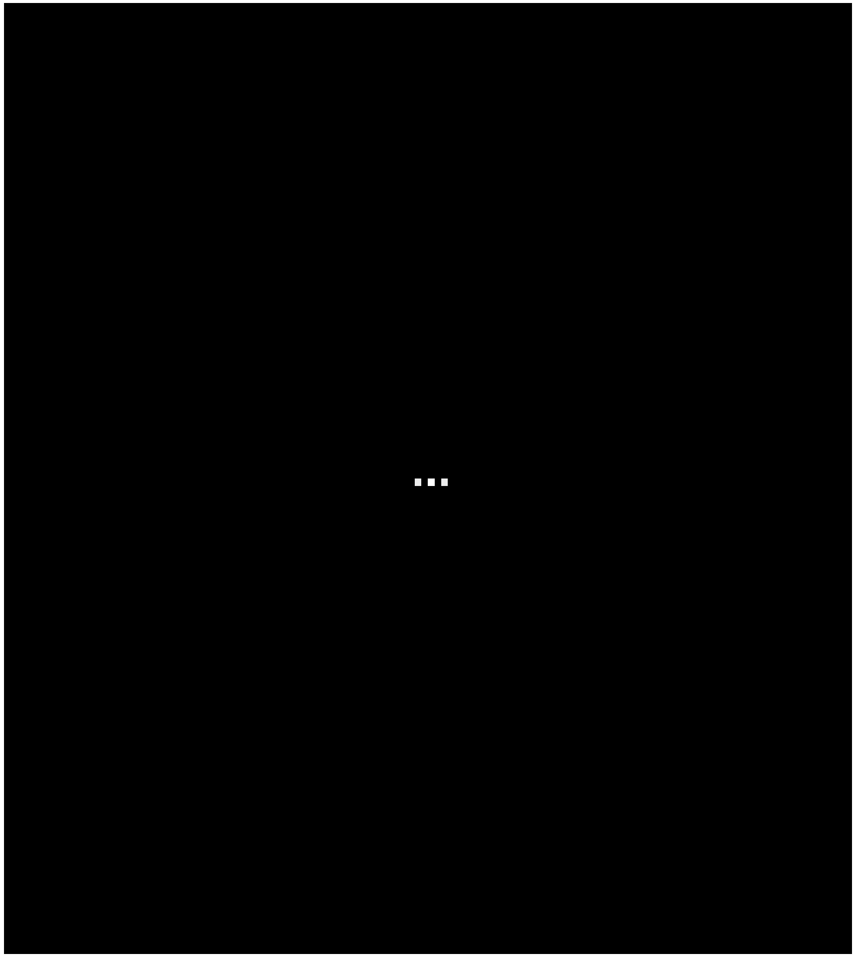
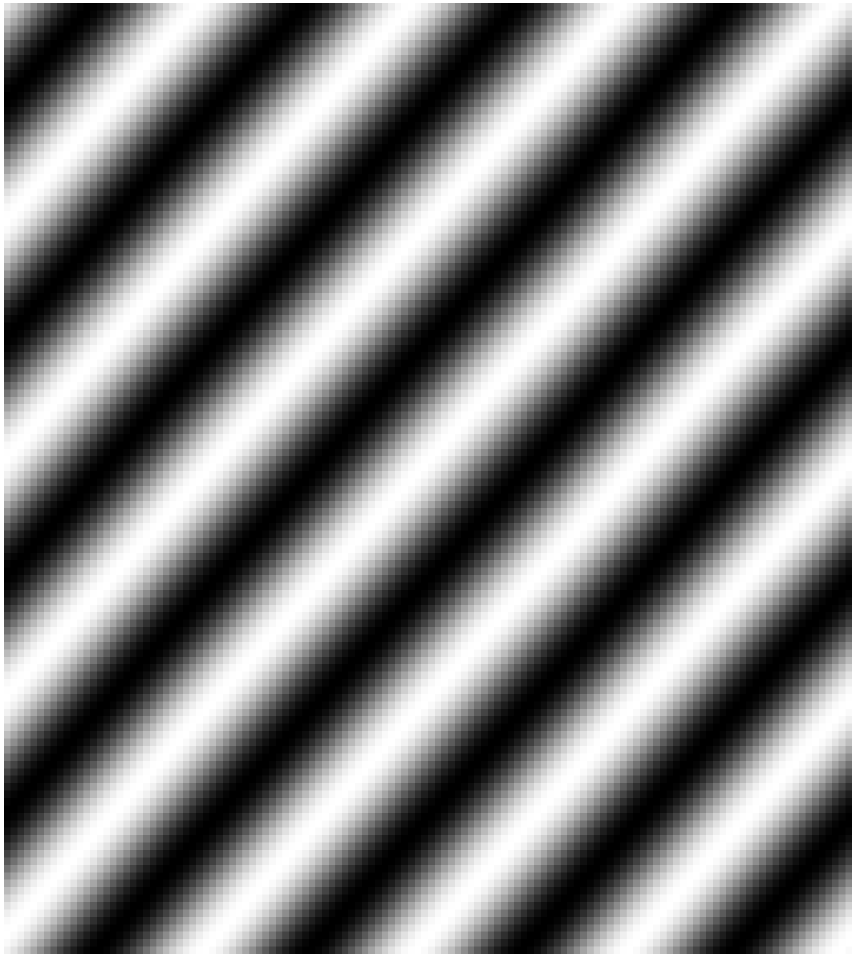


Image-K-space Examples



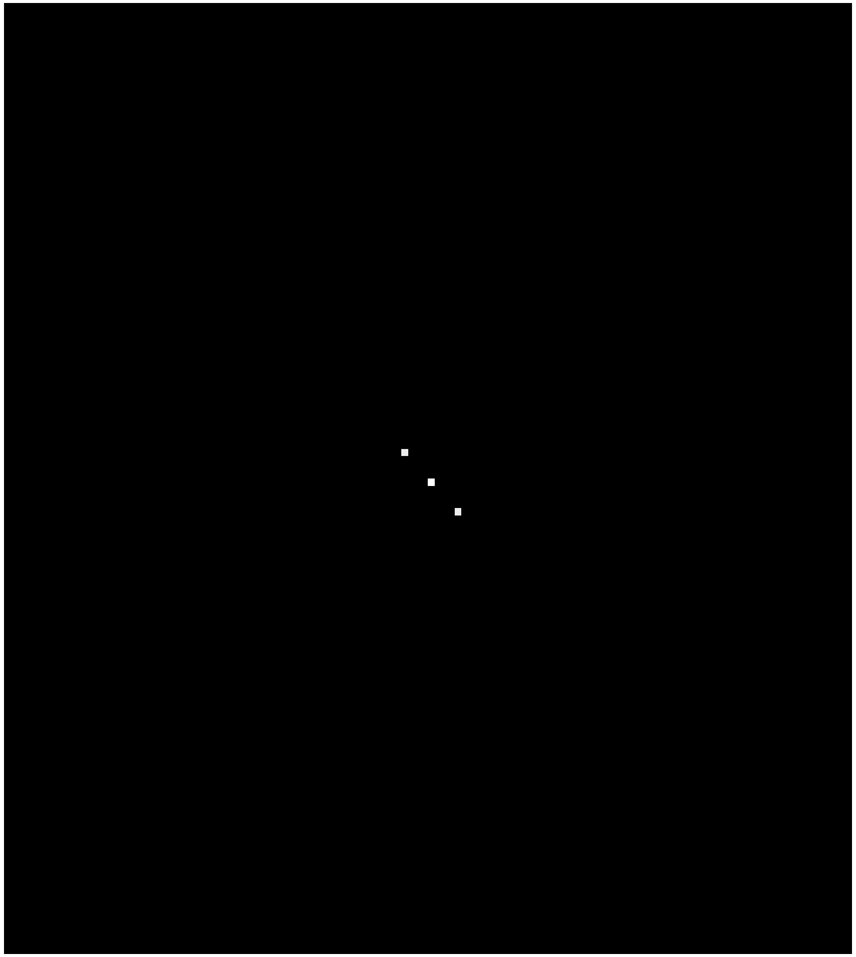
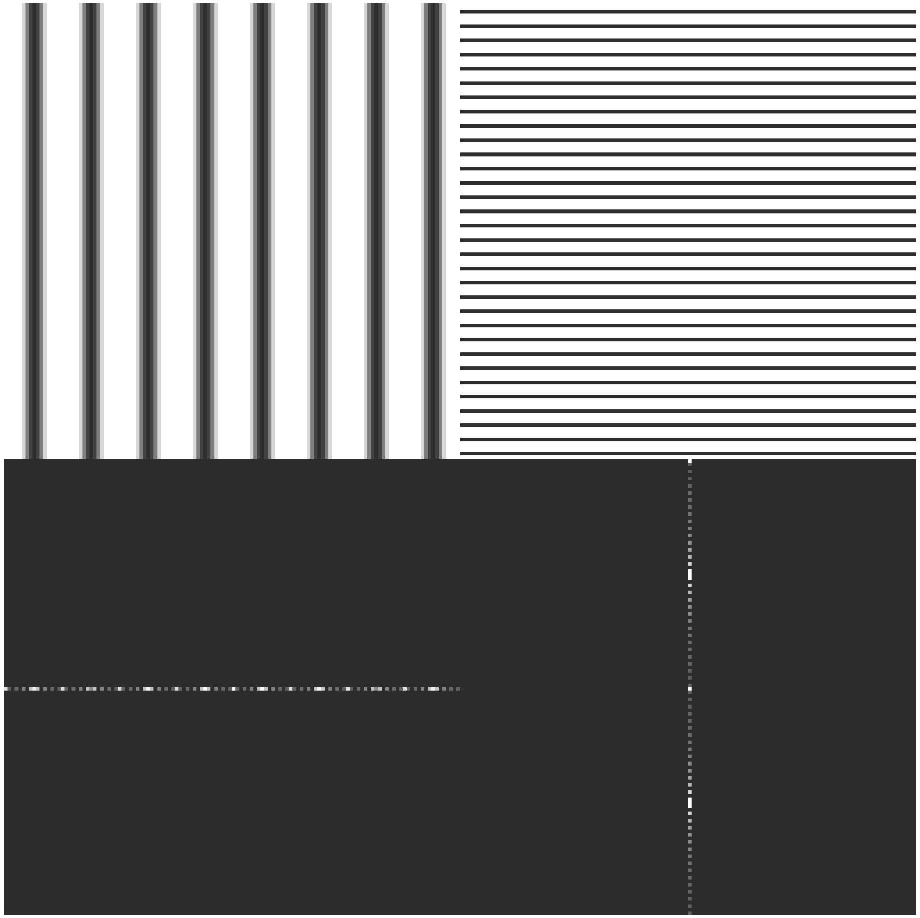


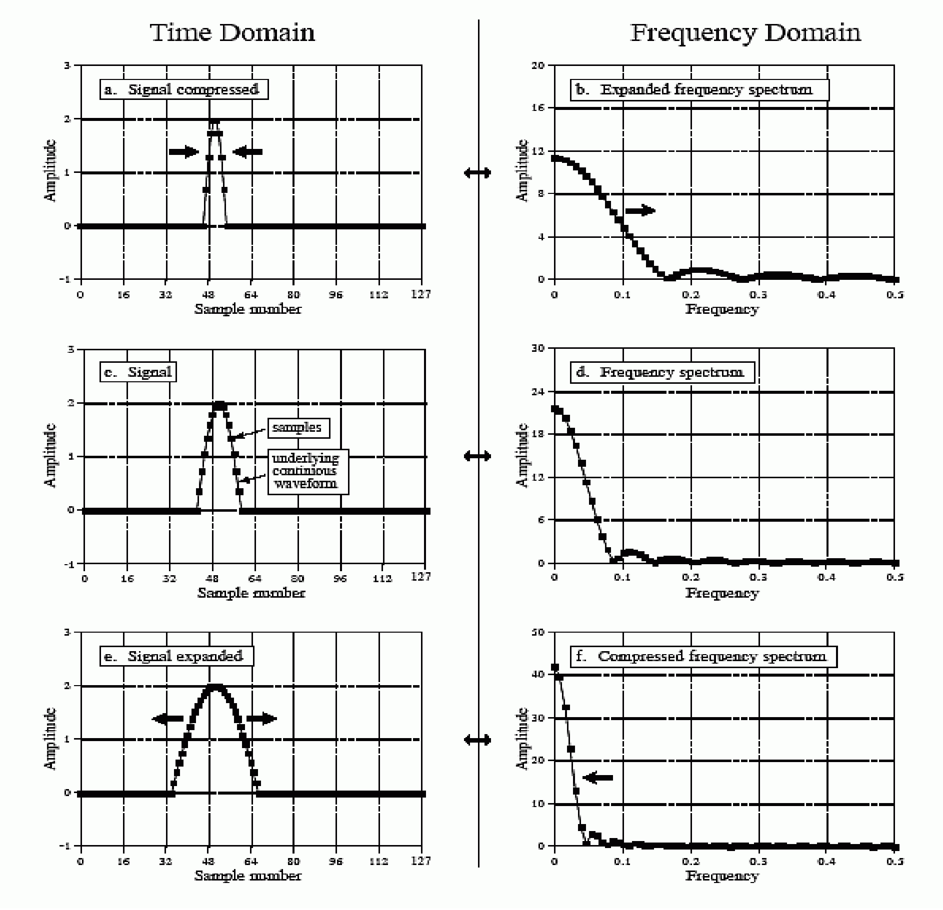
Image-K-space Examples

- In addition to feature orientations
  - Note, wide lines spacing in image corresponds to narrow spacing of k-space features
  - Narrow image spacing has wider k-space spacing

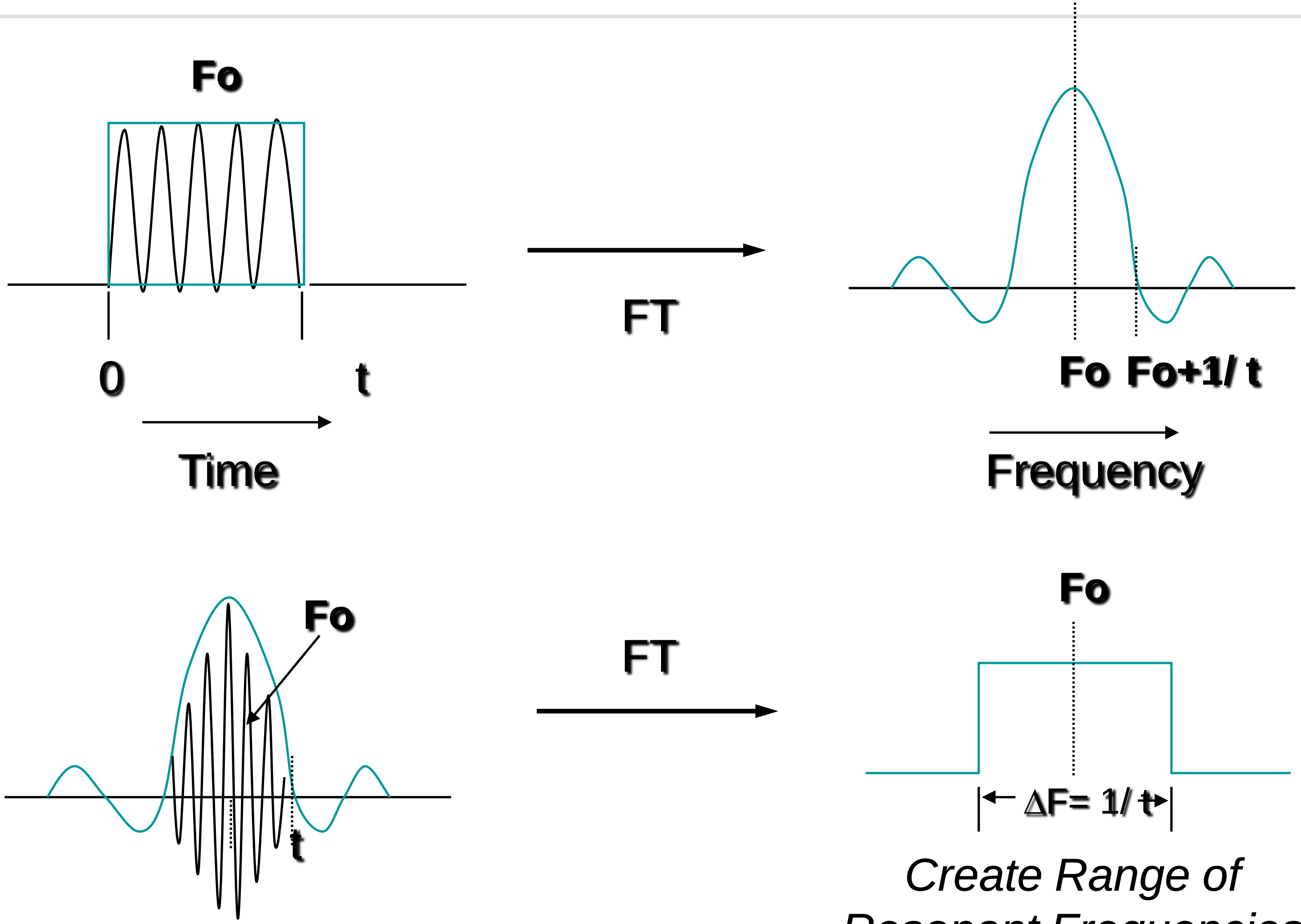


Fourier Properties

- One aspect of the Fourier representation is that sharp features in one domain correspond to wide features in the inverse domain
- And *vice versa*



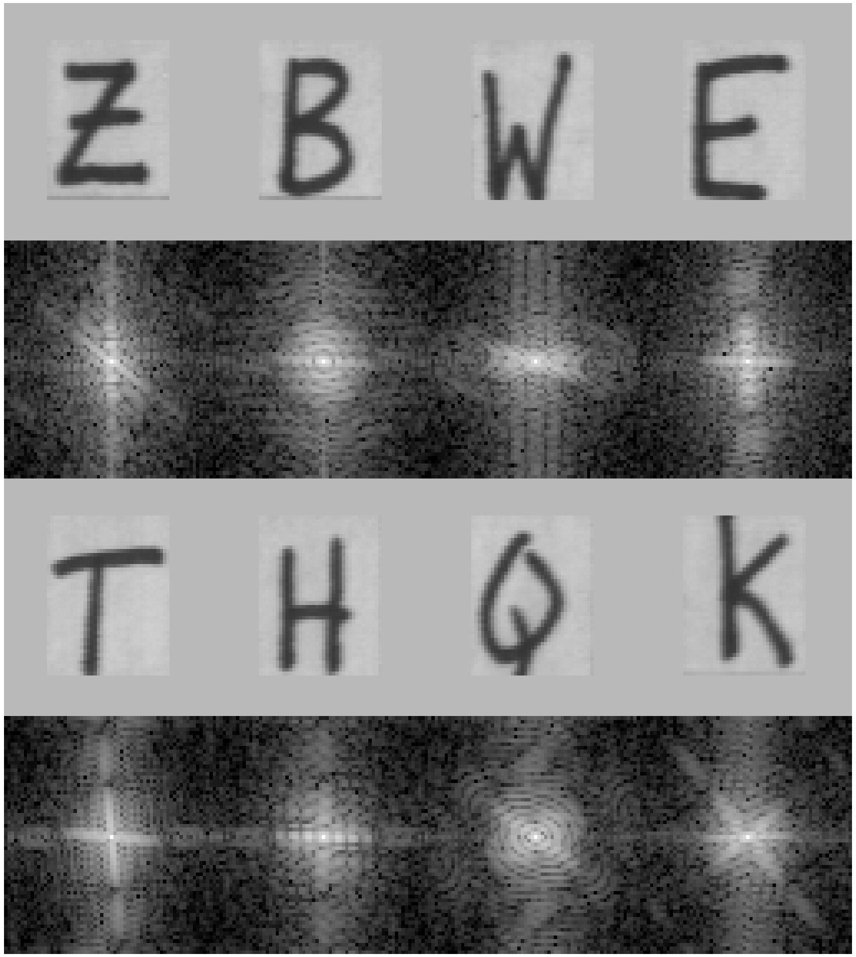
Electromagnetic **Excitation Pulse** (RF Pulse)



Create Range of Resonant Frequencies

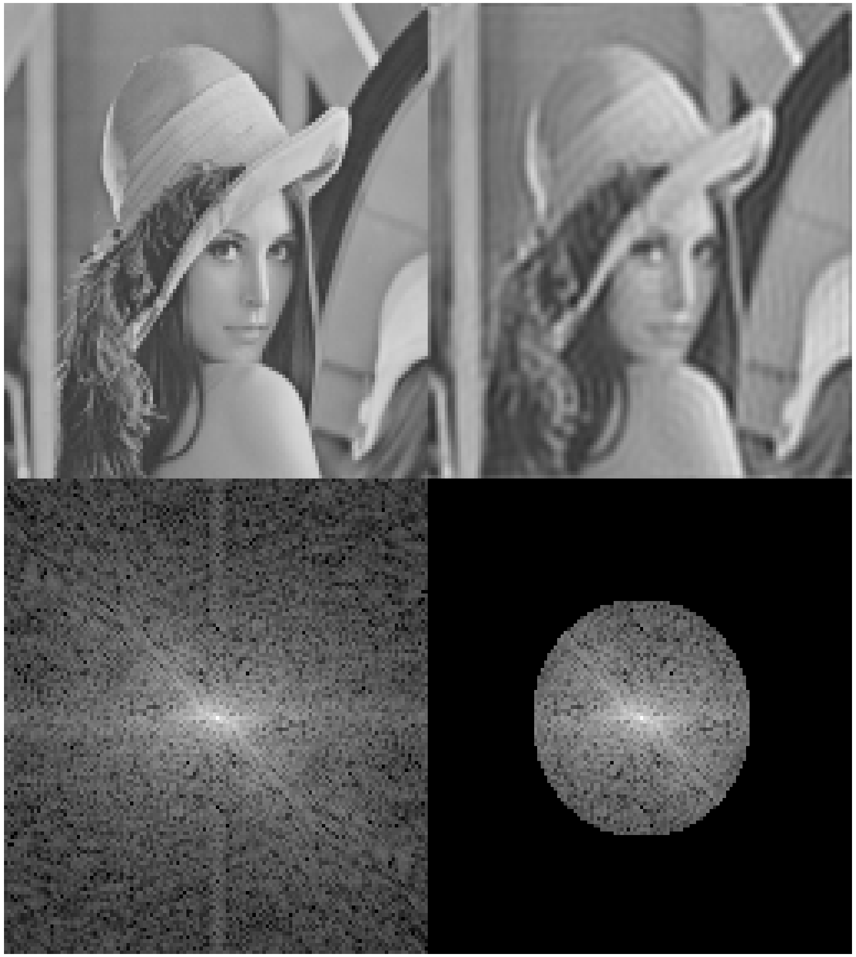
OCR - More Complex Examples

- Note the strong orientation of k-space features for the corresponding “Letter Images”.



K-Space Regions

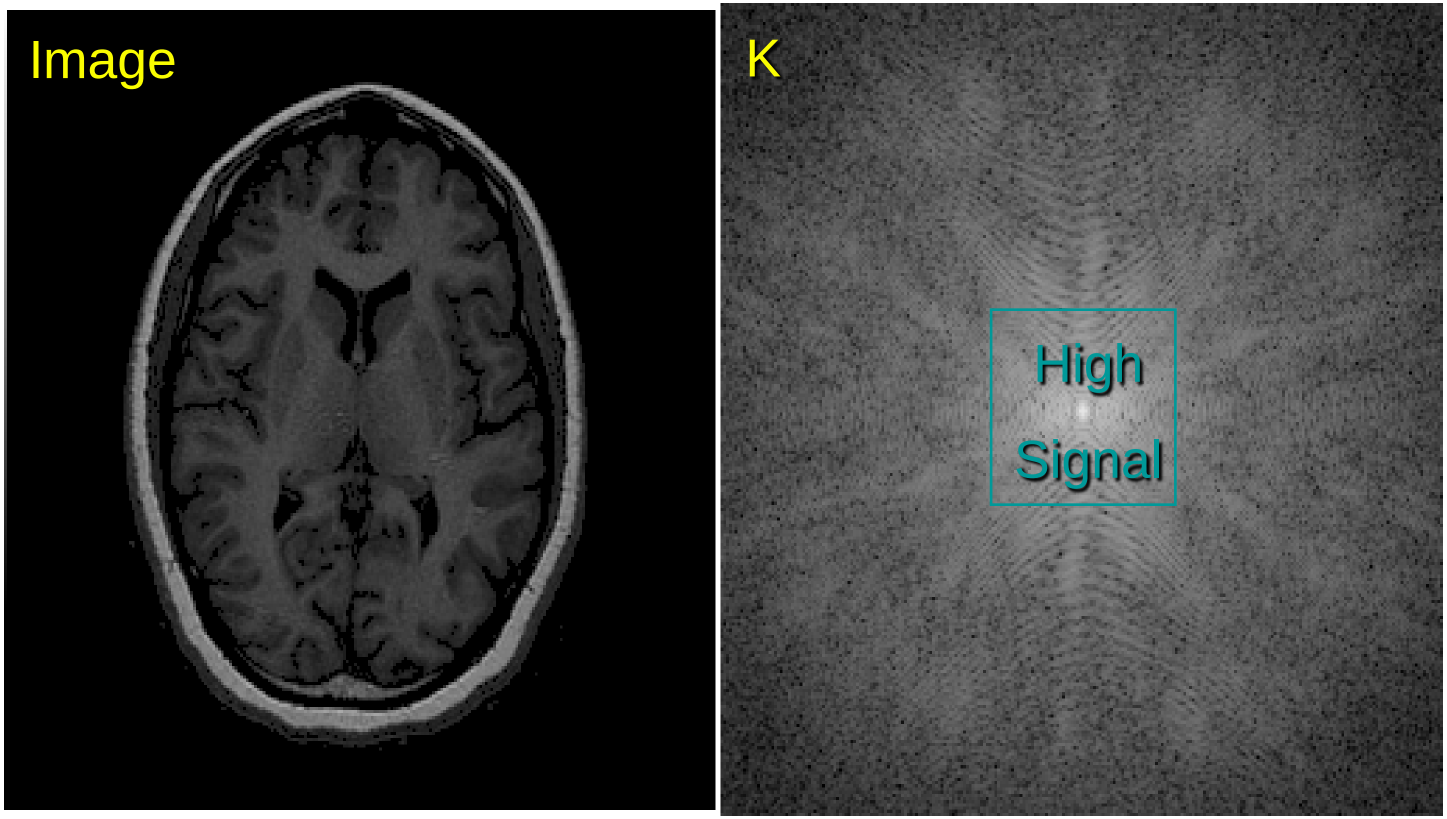
- A wide k-space support domain – *fftshift(fft(Image))* – corresponds with high image detail.
- Note the reduced k-space data corresponds to a low resolution image.
- To move between image and k-space we simply have to perform a Fourier Transform.
- Conversely, higher detail regions are encoded in the outer regions of k-space – useful for *edge-detection*.



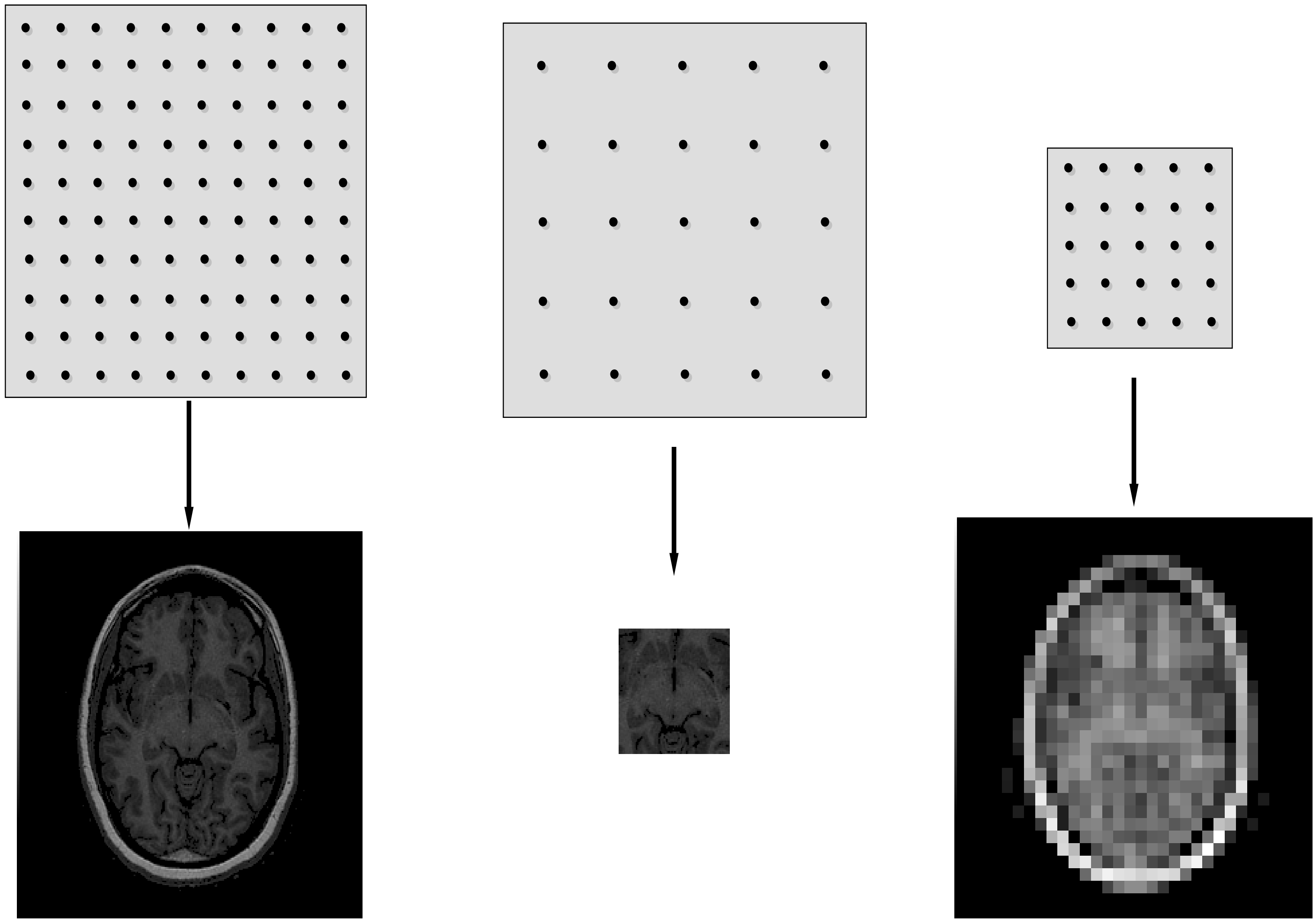
K-Space Regions

- Boost the central region of k-space and we boost the contrast
  - Low-spatial frequency
- By boosting the outer regions of k-space, we boost the sharp detail of the image
  - High spatial frequency
  - Noise is boosted along with the sharp detail

Example – Brain imaging



K-space Sampling and its Effect on Image Space



Calculation of MRI Field of View (FOV) along frequency encoding direction

$\gamma * G_f * FOV_f = BW = 1/\Delta t$

which means $FOV_f = 1/ (\gamma\, G_f\, \Delta t)$

where, BW is the bandwidth for the receiver digitizer.

Calculation of the Field of View (FOV) along phase encoding direction

$\gamma * G_p * FOV_p = N_p / T_p$

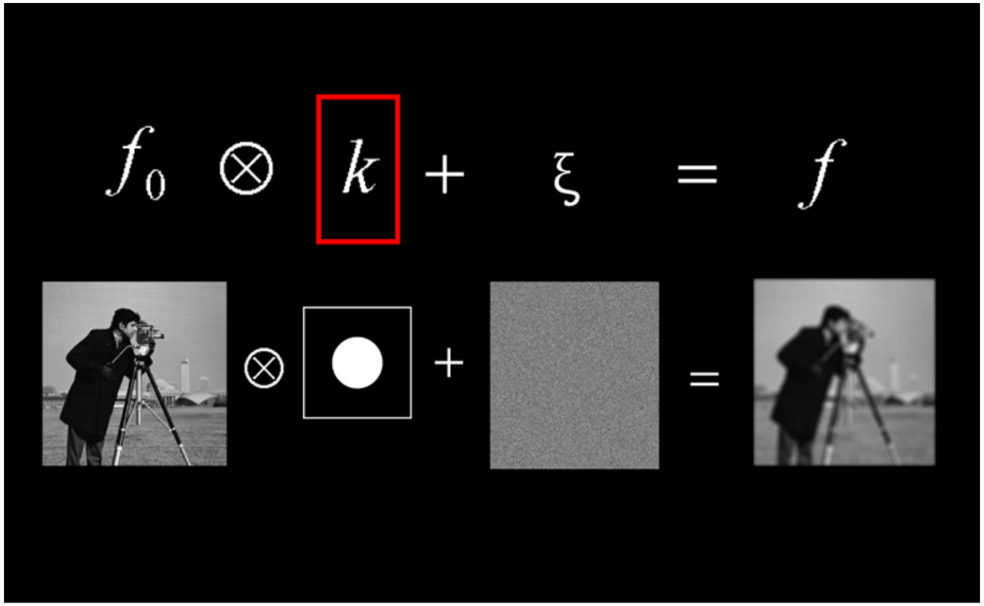
which means $FOV_p = 1/ (\gamma\ G_p\ T_p/N_p)$

$= 1/ (\gamma\ G_p\ \Delta t)$

where T_p is the duration and N_p the number of the phase encoding gradients, Gp is the maximum amplitude of the phase encoding gradient.

Convolution

- The final image generated by an imaging process is the "true image" convoluted with the PSF.



Point Spread Function (PSF)

- The PSF determines the finest resolution that can be detected in the image

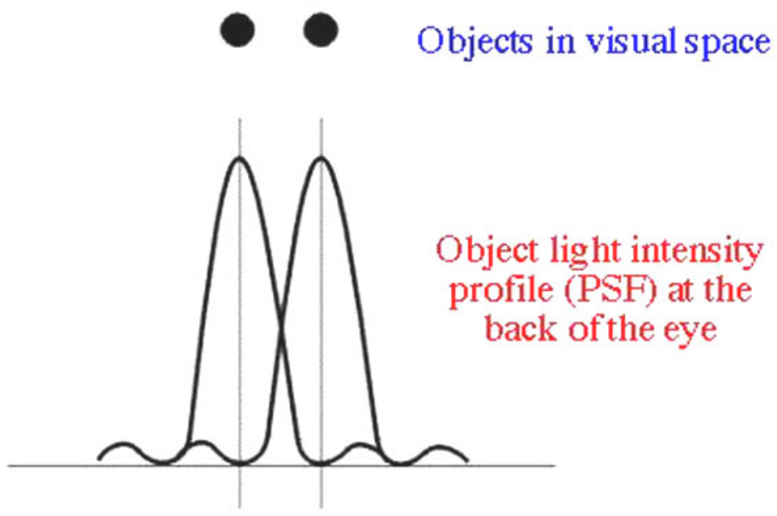
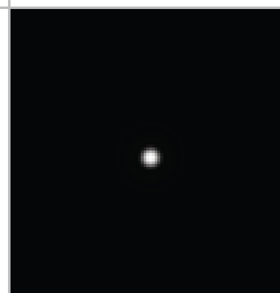
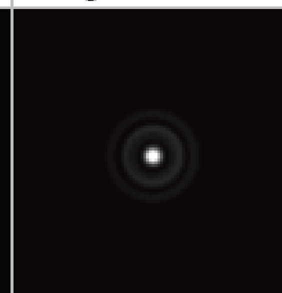
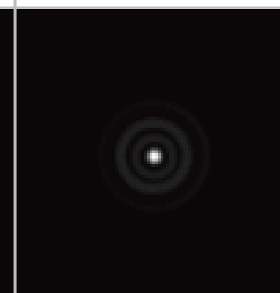
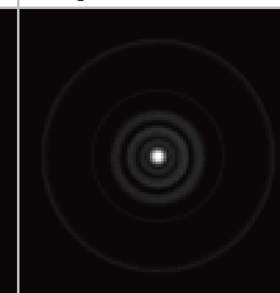
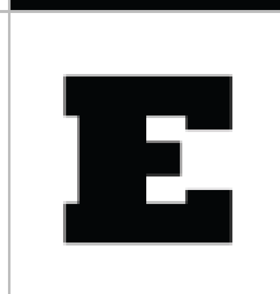
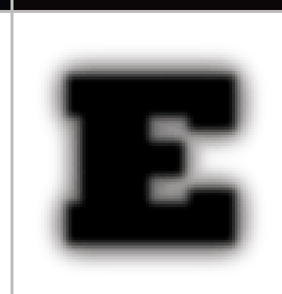
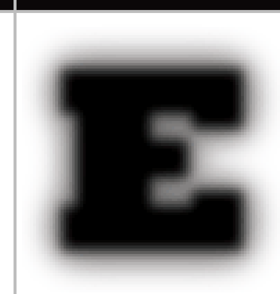
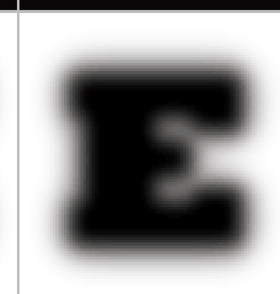


Figure 11. Raleigh's criterion. The criterion states that two points or lines are just resolved if the peak of the point spread function lies on the first trough of the other point spread function.

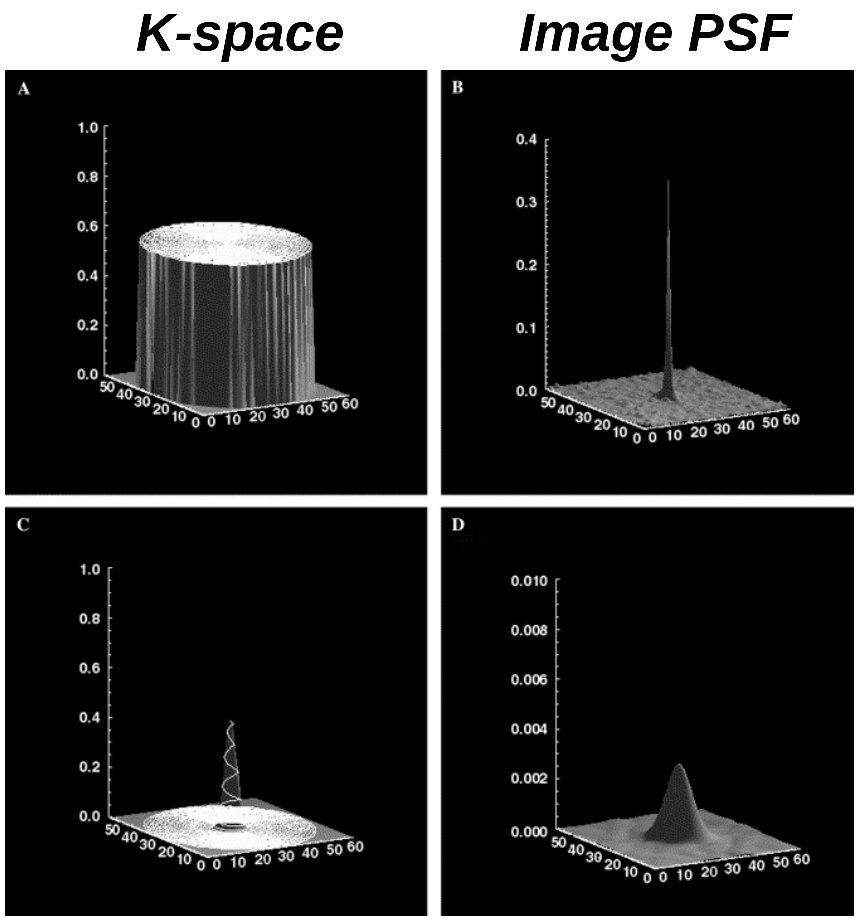
2D PSF

- In an image the PSF is a 2D feature.

Lens	TECNIS® Lens	AcrySof™ IQ IOL	B&L LI61AO IOL	Spherical IOL
Point Spread Function*†				
20/20*				
Average Corneal SA	+.27	+.27	+.27	+.27
Lens SA††	-.27	-.17	0.0	+.15
Total Residual SA	**0.0**	**+.10**	**+0.27**	**+0.42**

Convolution and k-space

- Convolution in image space of the PSF becomes a simple multiplication in the Fourier domain (i.e. k-space).
- Extent and shape of k-space determines PSF.



What's next?

CT & SPECT Imaging

Ultrasound

PSFs & Optical Imaging Systems

Biomedical Imaging & Analysis

Lecture 3, Part 1. Fall 2014

Image Formation & Visualization (II): ***CT, SPECT, Ultrasound***

*[**Text**:]*

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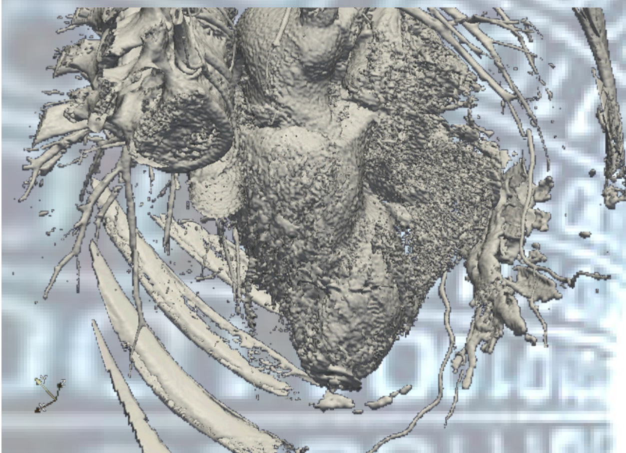

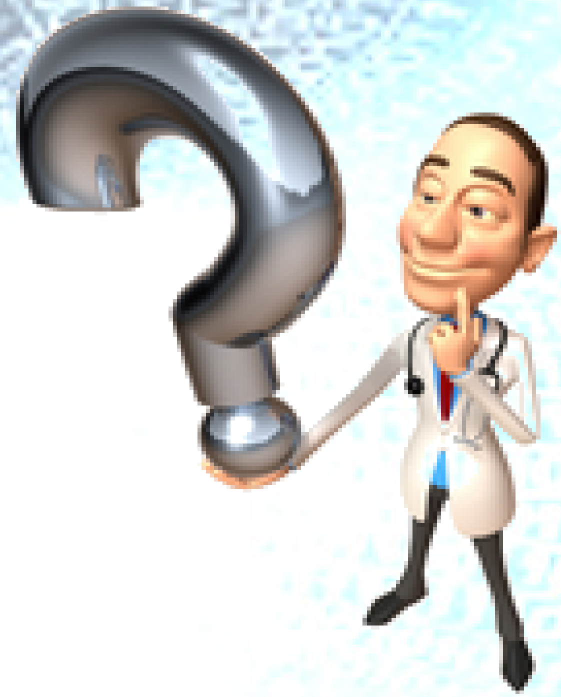

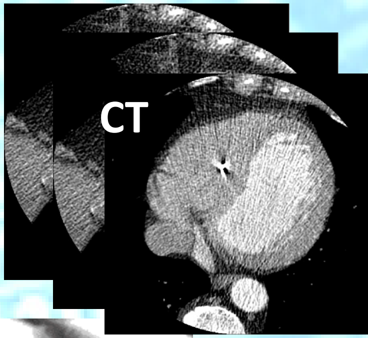

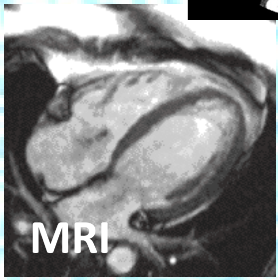

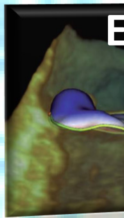

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