Biomedical Imaging & Analysis

Lecture 3, Part 1. Fall 2014

Image Formation & Visualization (II): ***CT, SPECT, Ultrasound***

[Text:]

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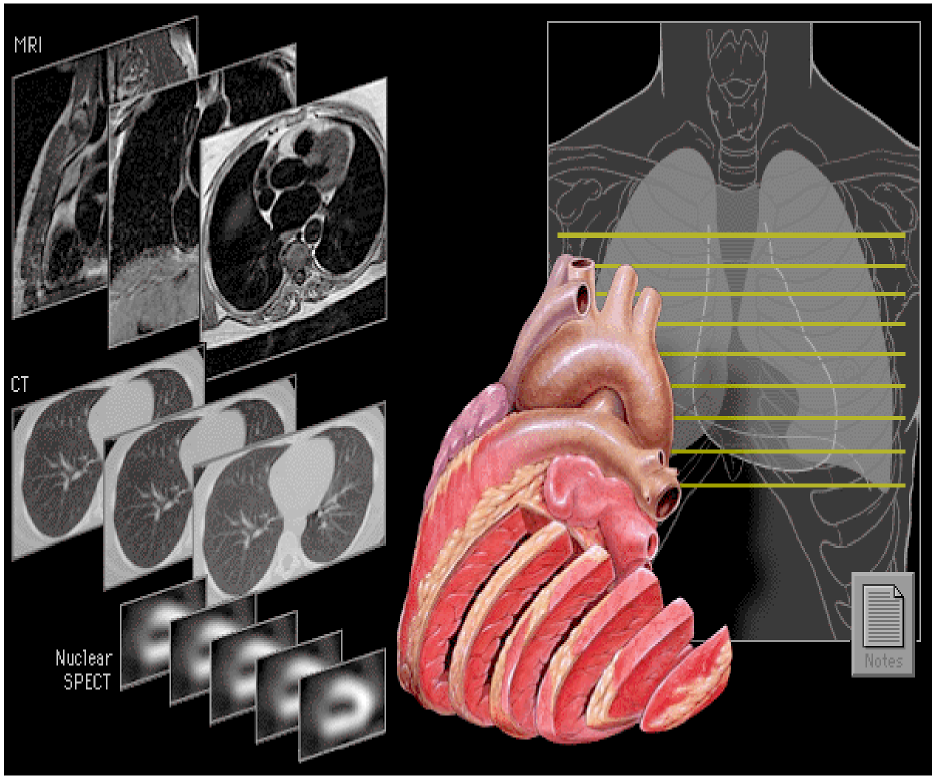
Tomography

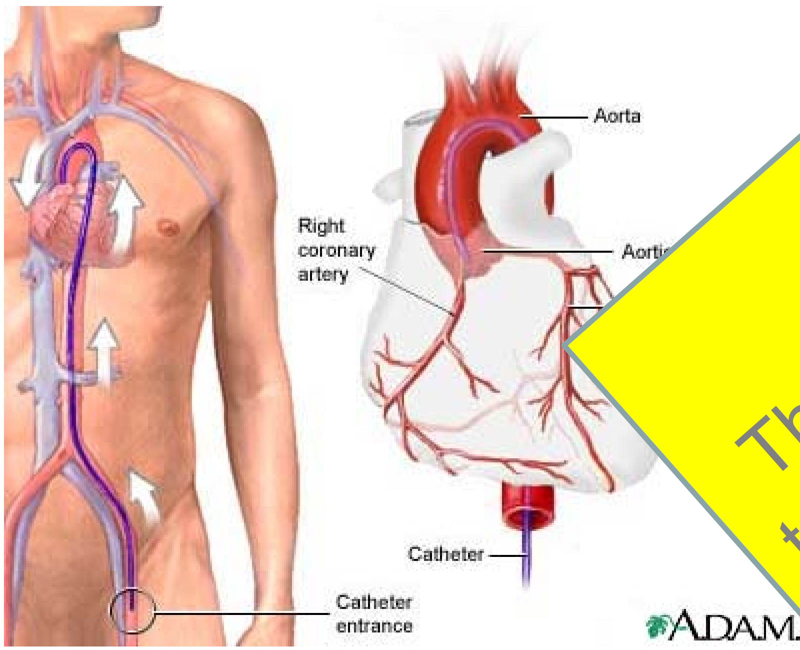
- The word *tomography* is derived from the Greek *tomē* ("cut") or *tomos* ("part" or "section") and *graphein* ("to write").
- "*A technique of x-ray photography by which a single plane is photographed, with the outline of structures in other planes eliminated.*"

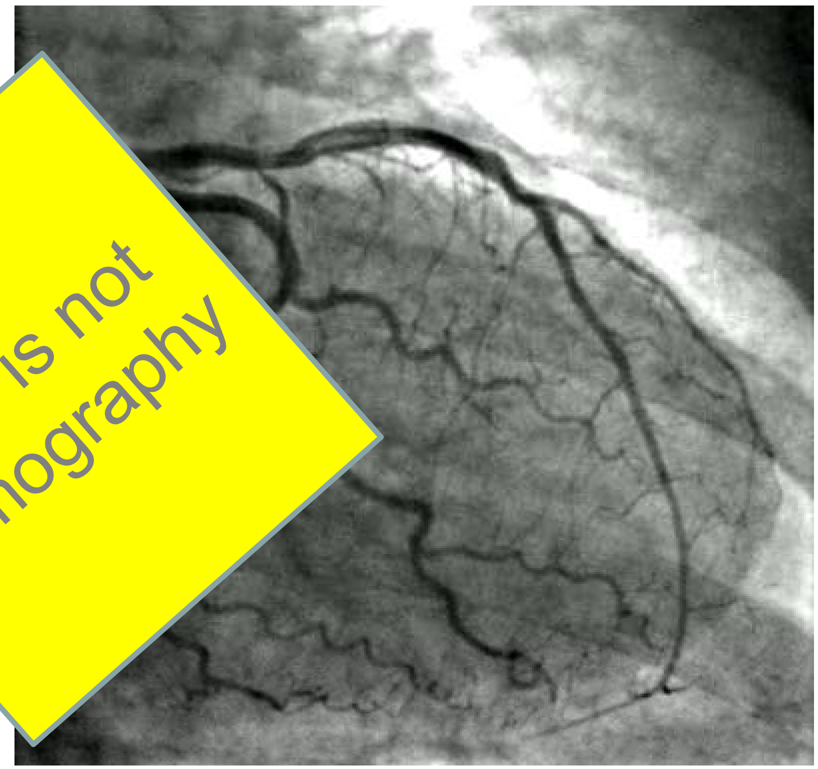


Tomographic Imaging

- Ultimately all tomographic imaging techniques involve the interaction of waves with matter and can be represented in k-space:
  - CT, EM (X-ray)
  - Nuclear, EM (Gamma)
  - MRI, EM (RF)
  - Ultrasound (pressure waves)

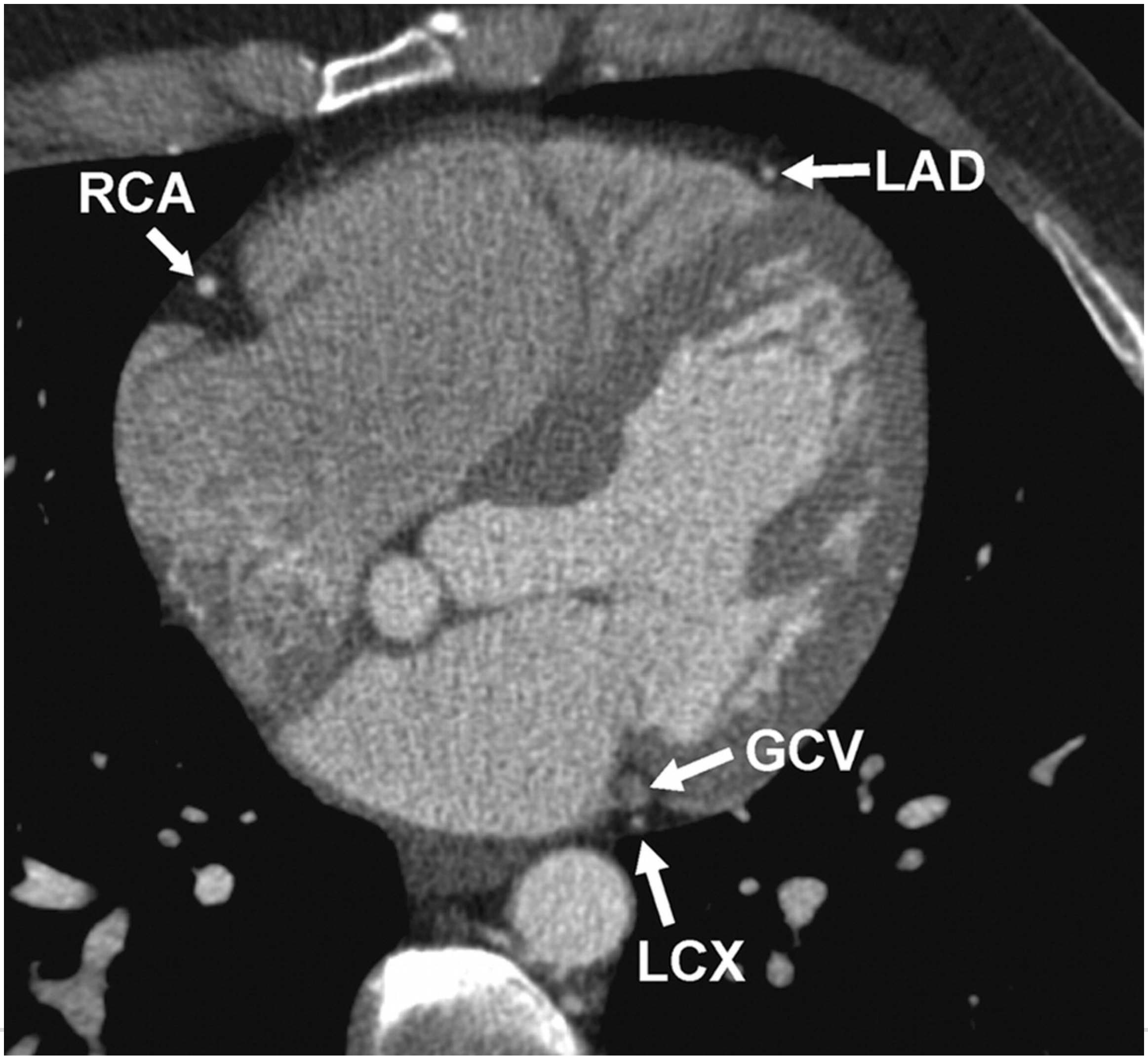
Coronary Artery Angiogram



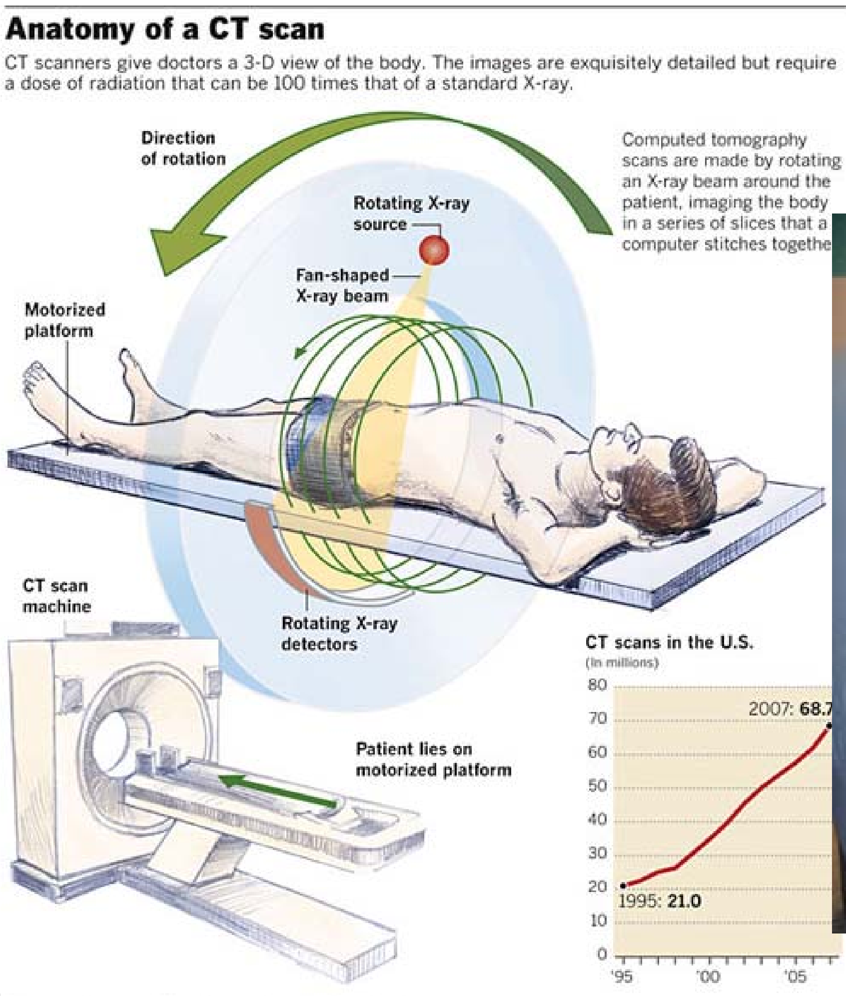


This is not tomography

Computed Tomography (CT)

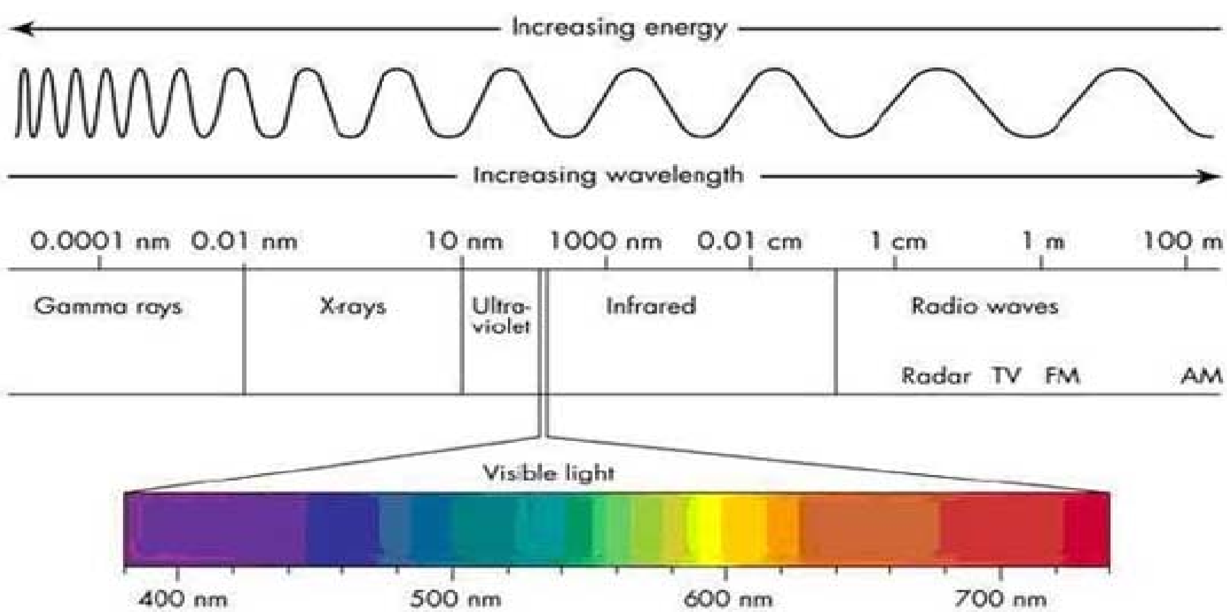


CT Scanner


Anatomy of a CT scan
CT scanners give doctors a 3-D view of the body. The images are exquisitely detailed but require a dose of radiation that can be 100 times that of a standard X-ray.
Direction of rotation
Computed tomography scans are made by rotating an X-ray beam around the patient, imaging the body in a series of slices that a computer stitches togethe
Rotating X-ray source
Fan-shaped X-ray beam
Motorized platform
CT scan machine
Rotating X-ray detectors
Patient lies on motorized platform
CT scans in the U.S.
(In millions)
80
70
60
50
40
30
20
10
0
2007: 68.7
1995: 21.0
'95
'00
'05

Fundamentals of X-ray Physics

- X-ray is an electromagnetic waveform.
- The energy: $$E = h\nu = \frac{hc}{\lambda}$$
- The wavelength and the energy of diagnostic x rays: $\boxed{0.01nm \le \lambda \le 0.1nm}$ $\boxed{12.4keV \le E \le 124keV}$



Interaction of ionizing radiation with matter

1. What is the basis of contrast for x-ray imaging ?
2. By which mechanisms does ionizing radiation interact with matter ?

   Rayleigh scattering

   Compton scattering

   Photoelectric effect

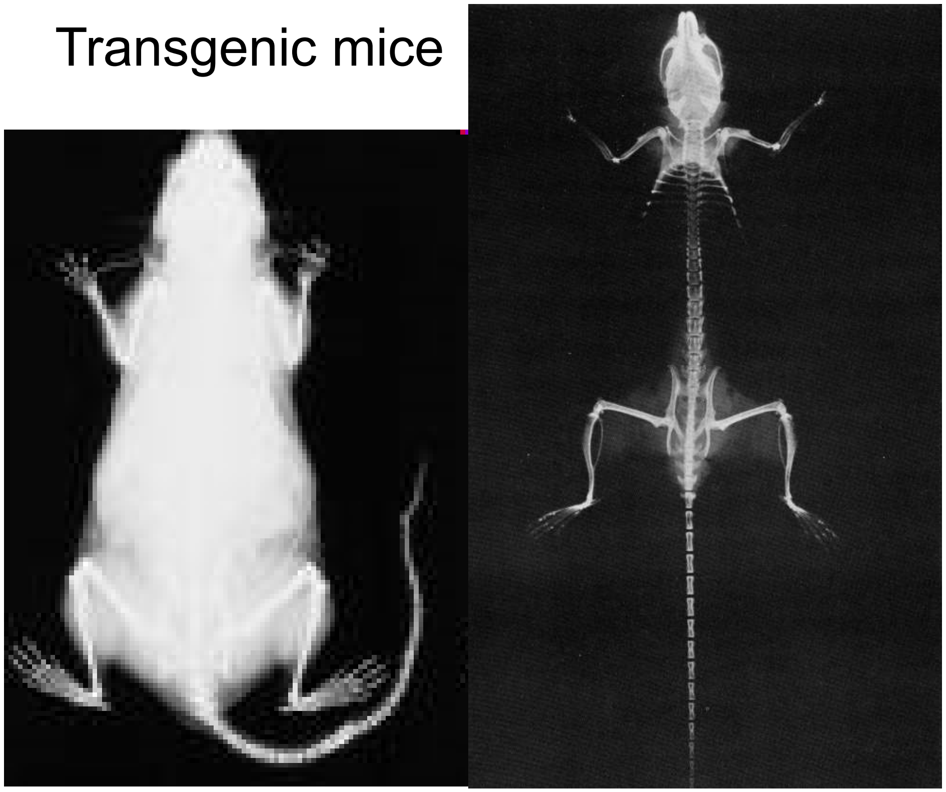
   Pair production
3. How does this interaction depend on the tissue ?

   Energy dependence and effective atomic number Z_{eff}
4. How can we protect ourselves against the biological effects of ionizing radiation ?

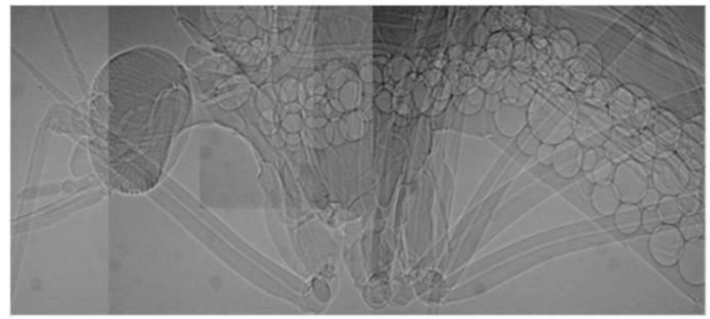
   A radiation protection primer

Imaging using x-rays

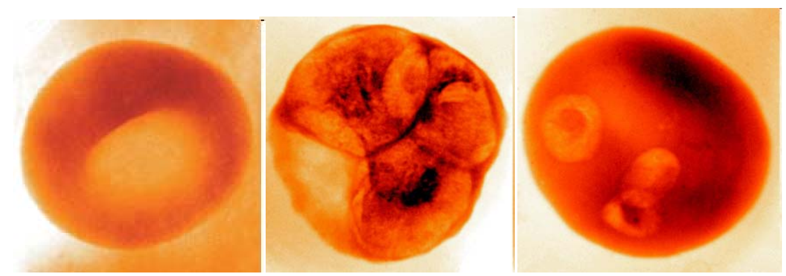
Transgenic mice



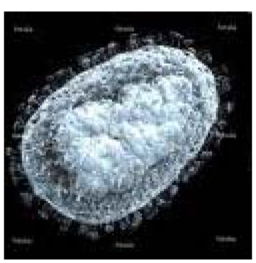
moskito



Evolution of a malaria-infected red blood cell



http://www.cxro.lbl.gov/BL612/bioimaging.html



H1N1

What do we need for bio-imaging ?

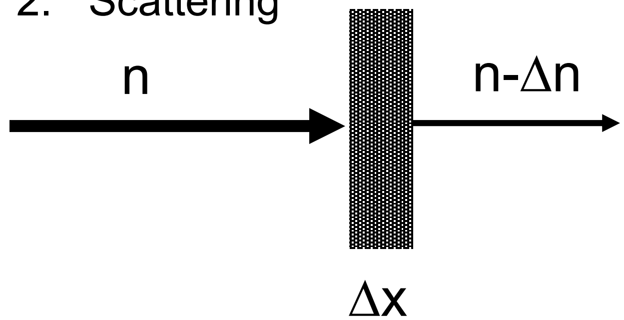
Contrast,

i.e. absorption of x-rays

How can we describe attenuation of x-rays?
Linear attenuation coefficient μ

Fates of the photon (other than transmission):

1. Absorption (transfer of $h\nu$ to lattice)
2. Scattering



Photons are removed according to probability law:

The number of absorbed/scattered photons Δn in a layer with thickness of Δx

$$\Delta n \equiv -\mu n \Delta x$$

μ : linear attenuation coefficient
Unit: [cm^{-1}]

$\mu = f(E_\nu, Z, \rho)$

Consider situation where $\Delta x \to 0$, and n=f(x)

$$dn(x) = -\mu n(x) dx$$

$$\frac{dn(x)}{dx} = -\mu n(x)$$

Solution ?

$$\boxed{n(x) = N_0 e^{-\mu x}}$$

(provided μ is constant in x)

Definition

Half value layer (HVL) $\equiv$ The thickness of a material allowing to pass one half of photons:

$n(x_{HVL}) = N_0/2$

$$N_0 / 2 = N_0 \mathrm{e}^{-\mu(\mathrm{HVL})}$$

$$\mathrm{HVL} = 0.693/\mu$$

Typical HVL values: several cm for tissues, 1-2 cm for aluminum, 0.3 cm for lead

What are typical attenuation coefficients ?

Material	Density (g/cm^3)	Electrons per Mass (e/g) $\times 10^{23}$	Electron Density (e/cm^3) $\times 10^{23}$	μ @ 50 keV (cm^{-1})
Hydrogen	0.000084	5.97	0.0005	0.000028
Water vapor	0.000598	3.34	0.002	0.000128
Air	0.00129	3.006	0.0038	0.000290
Fat	0.91	3.34	3.04	0.193
Ice	0.917	3.34	3.06	0.196
Water	1	3.34	3.34	0.214
Compact bone	1.85	3.192	5.91	0.573

μ/ρ (water) = μ/ρ (ice)

Definition

Mass attenuation coefficient μ/ρ

Unit : [cm^2/g]

(constant for all forms of the same chemical substance, e.g. water)

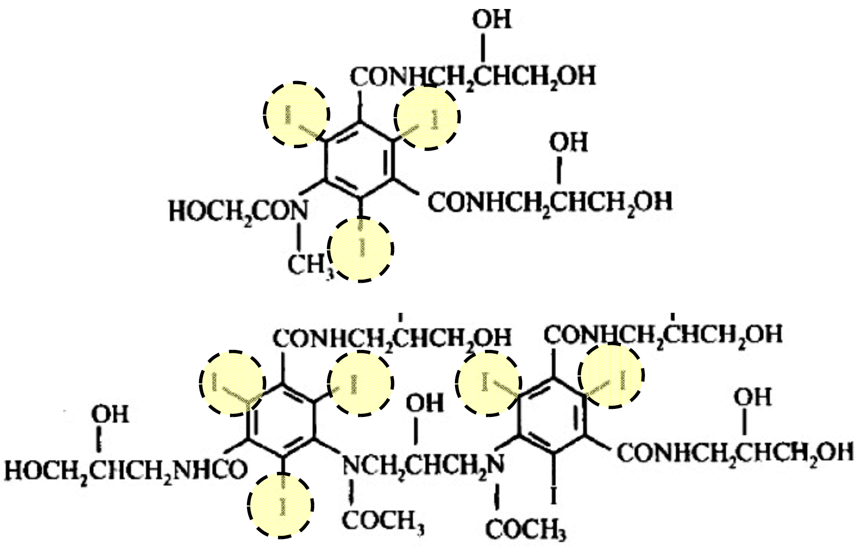
Why do we need more sunscreen in the mountains ?

X-ray contrast agents ?

Exogenously administered substance (by infusion/ingestion)

modifying Z_{eff}
$\Rightarrow$ use high Z compounds

e.g., compounds with multiple iodine atoms, lanthanides etc.



Z_{eff} of (water+10 mmol/kg iodine) = ?

Iodine:

P_I = 10[mmol/kg]×127[mg/mmol] = 0.127%

Z_I = 53

A_I = 127

Calculation of Z_{eff}:

$$\lambda_i = \frac{P_i Z_i / A_i}{\sum_{\substack{\text{all tissue}\\ \text{components } j}} P_j Z_j / A_j}$$

Denominator of λ :

~ denominator λ of pure H_2O

→ λ of H and O are as for water:

λ_H=0.2

λ_O=0.8

$$\lambda_I = \frac{0.127 \cdot 53/127}{55.6} = \frac{0.053}{55.6} = 9.5\times10^{-4}$$

$$Z_{eff}^{3.4} = \underbrace{0.2\cdot1^{3.4} + 0.8\cdot8^{3.4}}_{944} + \underbrace{9.5\cdot10^{-4}\cdot53^{3.4}}_{690}$$

$$Z_{eff} = 8.8$$

$\mu_{PE} \propto Z_{eff}^{3.4}$

$\mu_{PE}(H_2O) \propto 944$

$\mu_{PE}(H_2O+I) \propto 1650$

Quantifying effects of Ionizing Radiation

Three forms of radiation dose

•Absorbed dose D:

•energy deposited by ionizing radiation per unit mass of material:

D = Energy/mass

Units: [Gray[Gy]=1J/kg]

•Equivalent dose *H*:

•= D corrected for effectiveness of radiation to produce biological damage (w_R = radiation weighting factor)

H = Dw_R

Units: [Sievert[Sv] = 1J/kg]

•Effective dose *E* :

•H corrected for sensitivity of tissue T (w_T = tissue weigting factor)

$$E(\mathrm{Sv}) = \sum w_{\mathrm{T}} H_{\mathrm{T}}(\mathrm{Sv})$$

Absorbed dose D depends on

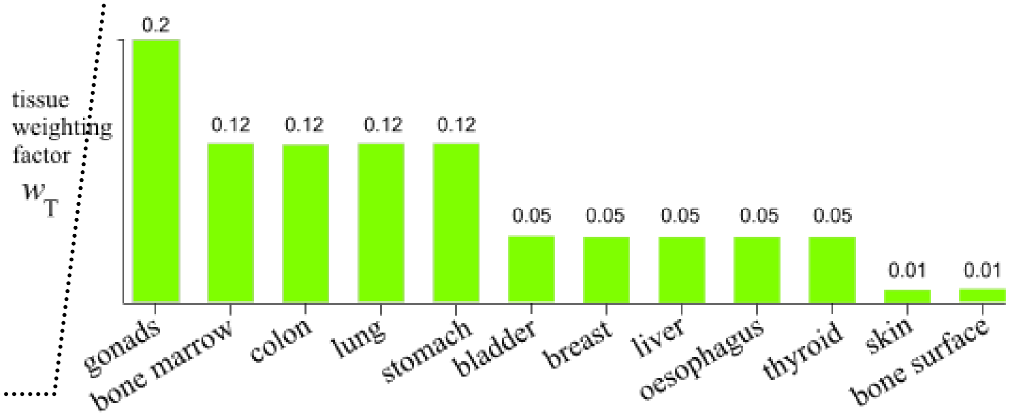
1. Intensity of incident x-ray
2. Duration of exposure

$\mathbf{w_R}$ = 1 (x-rays, β particles)

[20: for α particles]

Bio-imaging:

Equivalent dose D = Absorbed dose H



Typical radiation exposures

Natural, artificial and some examples

Natural	mrem/yr	mSv/yr	%total
Radon	200	2.0	55%
Cosmic	27	0.27	8%
Terrestrial	28	0.28	8%
Internal	39	0.39	11%
Total	**300**	**3**	**82%**

Other			
Occupational	0.9	<0.01	<0.3
Nuclear Fuel Cycle	<1	<0.01	<0.03
Fallout	<1	<0.01	<0.03
Misc.	<1	<0.01	<0.03

Artificial			
Medical X ray	39	0.39	11%
Nuclear Med.	14	0.14	4%
Consumer products	10	0.1	3%
Total	**63**	**0.63**	**18%**

Source of Exposure	
X-rays from TV set (3 cm)	0.500 mrem/hour
Airplane ride (12km)	0.500 mrem/hour
Radionuclides in the body (i.e., K)	39 mrem/year
Building materials (concrete)	3 mrem/year
Drinking Water	5 mrem/year
Pocket watch (radium dial)	6 mrem/year
Eyeglasses (containing thorium)	6 - 11 mrem/year
Transatlantic Airplane roundtrip	5 mrem

http://newnet.lanl.gov/info/dosecalc.asp

http://www.epa.gov/radiation/understand/calculate.html

Chest x-ray, dental x-ray, head & neck	5-20 mrem
Lumbar spinal x-rays	130 mrem
Pelvis x-ray	44 mrem
Hip x-ray	83 mrem
CT (head and body)	1,100 mrem

Useful to know:

100 Röntgen equivalent man (REM) = 1 Sievert

Attenuation : A Closer Look

Beer Lambert Law $$I = I_0 e^{-\mu \cdot ds}$$

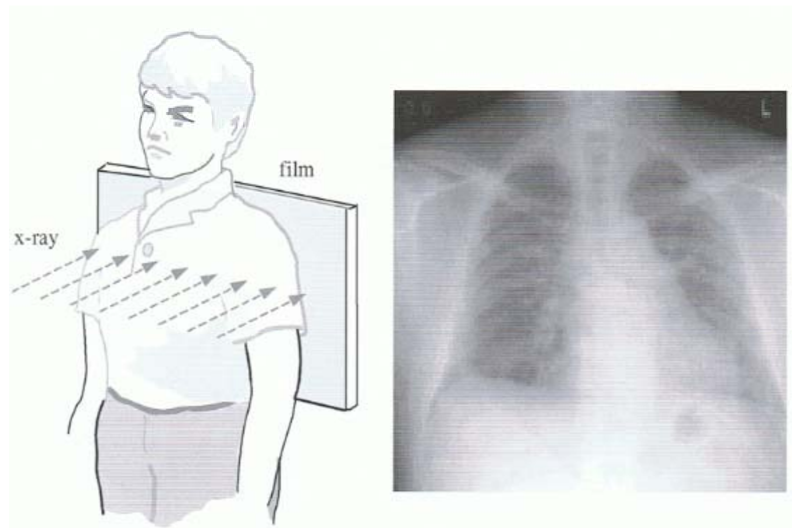
Different material along the ray have different attenuation coefficients:

$$I = I_0 e^{-\mu_1 \cdot ds} e^{-\mu_2 \cdot ds} \ldots e^{-\mu_n \cdot ds}$$
$$= I_0 e^{-\sum_i \mu_i \cdot ds} \approx I_0 e^{-\int \mu(s)\ ds}$$

take the logarithm:

Record the x-ray flux impinging on the x-ray detector after attenuation by a patient

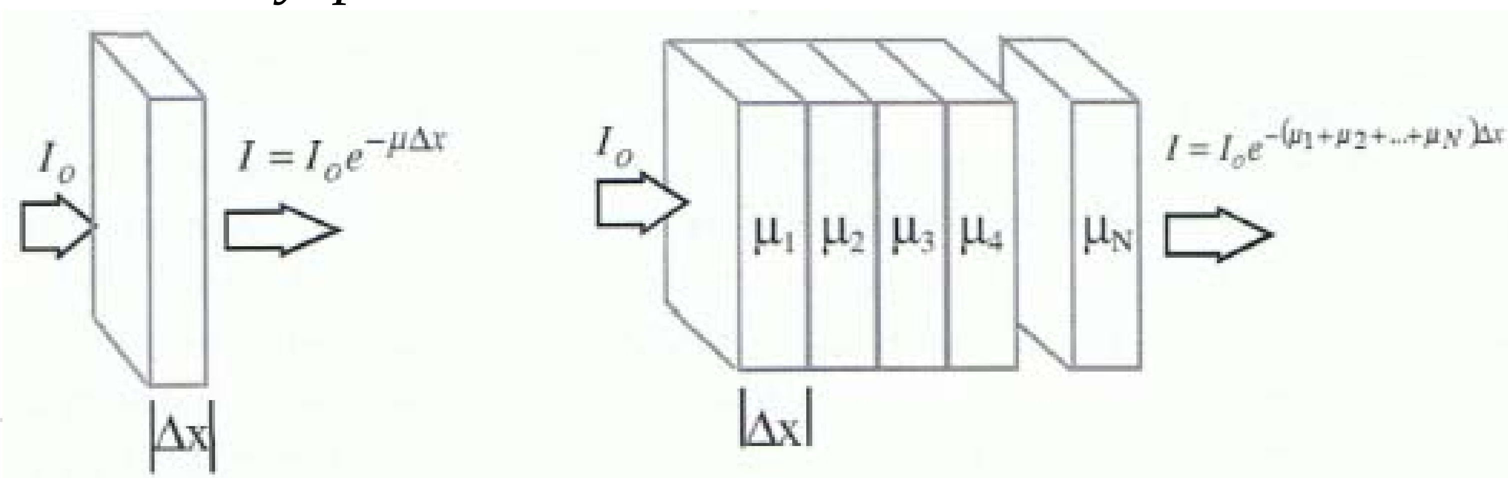
$$I = I_0 e^{-\mu\Delta x} \Rightarrow I = I_0 e^{-\sum_{n=1}^{N}\mu_n \Delta x}$$



$$p = -\ln\left(\frac{I}{I_0}\right) = \sum_{n=1}^{N}\mu_n \Delta x \xrightarrow[\Delta x \to 0]{} \int_L \mu(x)$$

$p - the\ \ projection\ \ measurement$

$L - the\ \ x - ray\ \ path$



The **CT number**

μ - the linear attenuation coefficient of the material

$$[\mu] = m^{-1}$$

$$\boxed{\begin{gathered} CT\ number = \frac{\mu - \mu_{water}}{\mu_{water}} \times 1000 \\ [CT\ number] = HU \end{gathered}}$$

$$\underline{soft\ tissues:}\quad -100HU \le CT\ number \le 60HU$$

$$\underline{cortical\ bones:}\ 250HU \le CT\ number \le 1000HU$$

Defining the Imaging Problem

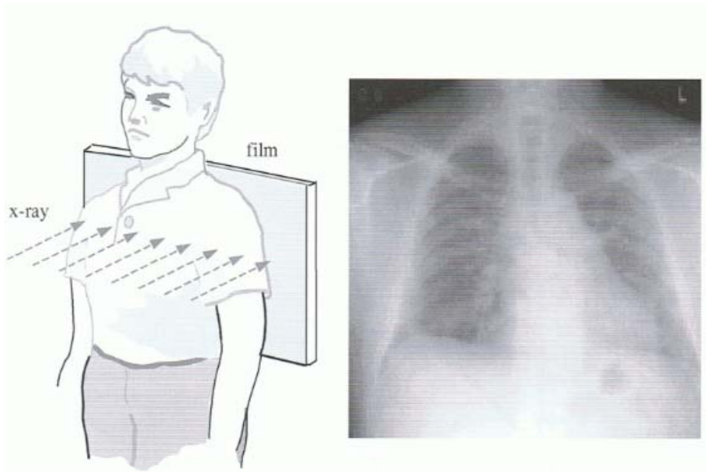
$$\int \mu(s)\, ds \approx \sum \mu_i \cdot ds = \log \frac{I_0}{I}$$

How can we derive the density (or attenuation) distribution μ out of the measured intensities?

Optimization problem..?

Projection Images: The X-ray problem

X-ray images show the sum of the attenuations of the tissue → lots of things we don't want to see overlay the interesting parts



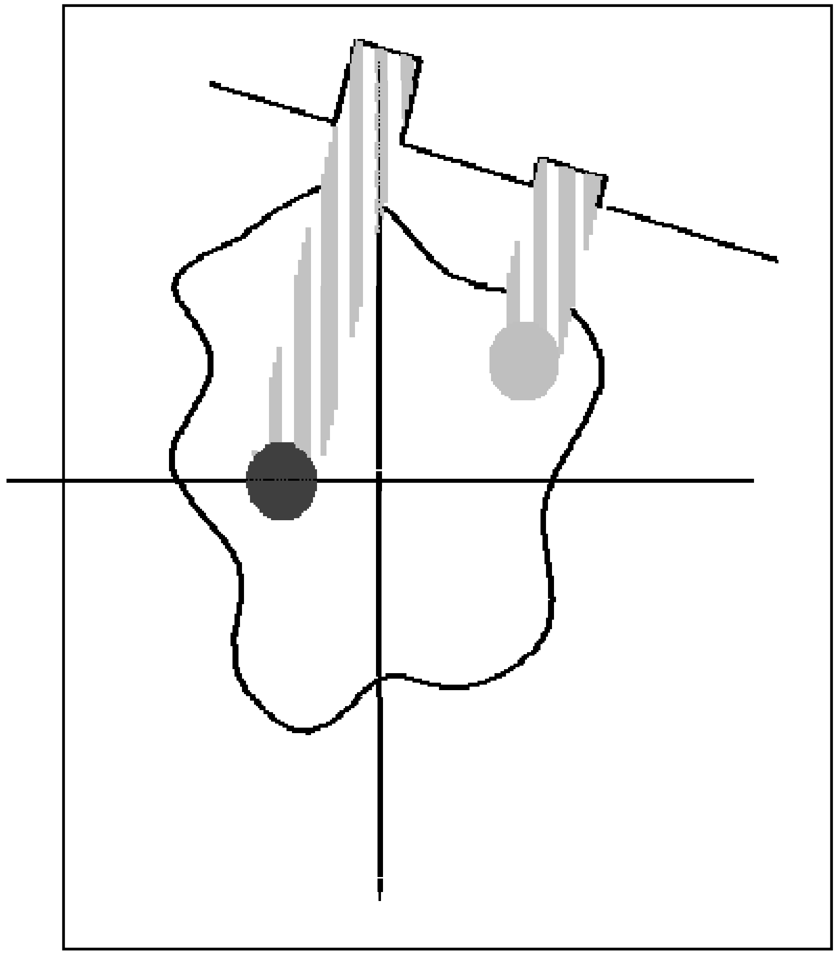
Make an image of just one (section) plane

→ X-ray tomography

Projection **Signal**

- For each slice, a projection signal is represented by a 1D signal…

- The signal represents the sum of the attenuations of the tissue ...



The first prototype CT machine

- Invented by :
Godfrey Hounsfield
in 1971 at the EMI
Labs, UK

- Image Reconstruction
From a system of linear
equations – **Algebraic Reconstruction**...

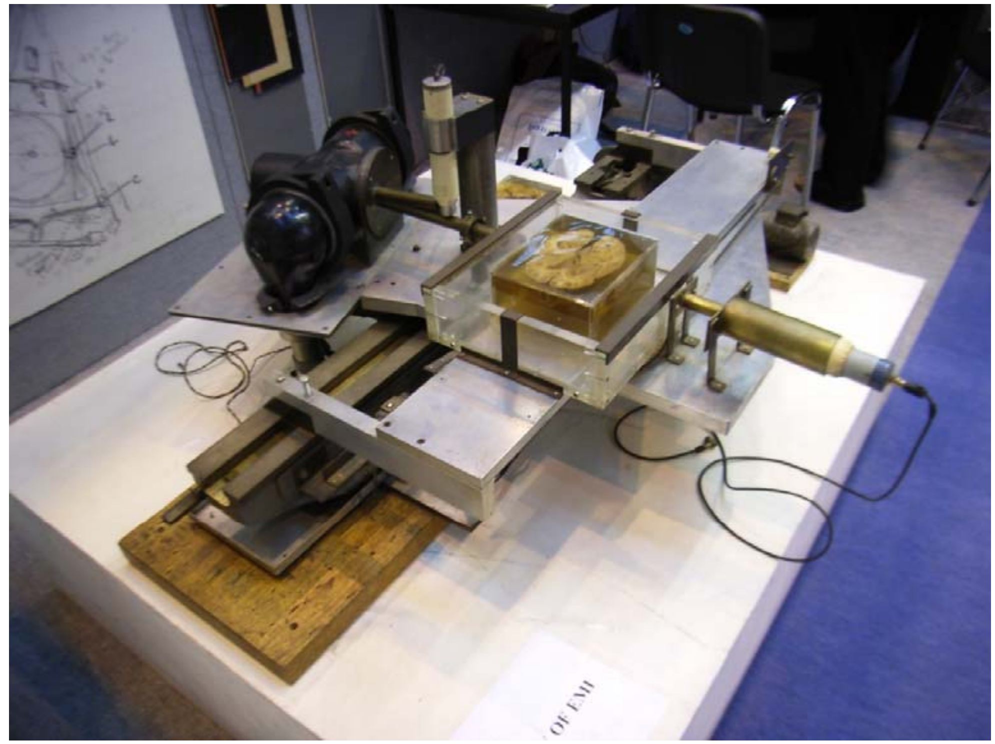
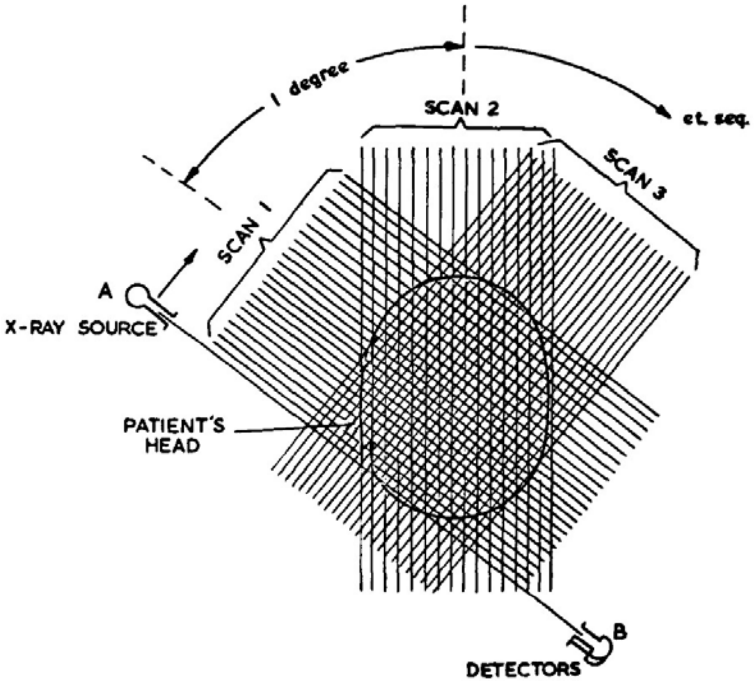


Image from Wikipedia

The EMI Scanner

- First commercial machine, developed by Godfrey Hounsfield at the EMI Labs, UK
- 180x160 = 28,800 readings
- 

The EMI Scanner

High soft tissue contrast !

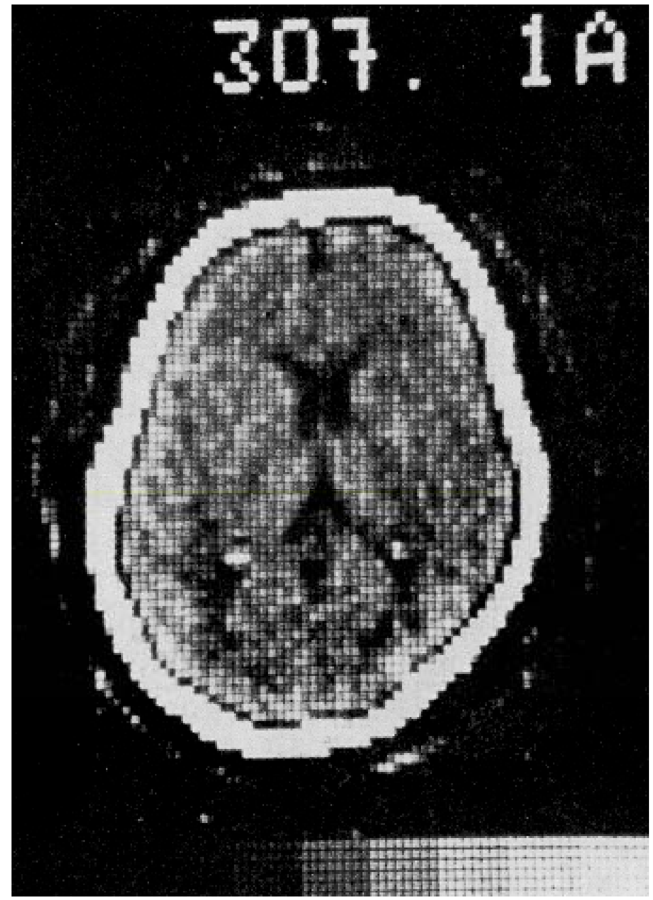
Computerized transverse axial scanning (tomography):
Part I. Description of system

G. N. Hounsfield

Central Research Laboratories of EMI Limited, Hayes, Middlesex

The system is approximately 100 times more sensitive than conventional X-ray systems to such an extent that variations in soft tissues of nearly similar density can be displayed.

Original paper (1973, Br J Rad)



Algebraic Reconstruction

Compute pixel attenuations in a slice from a couple of projections :

Example with 4 pixels

a	b	3
c	d	7
4	6	

a, b, c, d = ?

Attenuations are simply summed up:

$$a + b = 3$$

$$c + d = 7$$

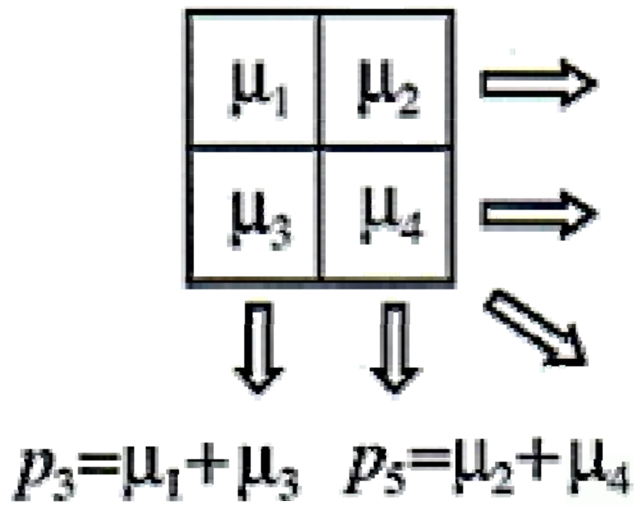
$$a + c = 4$$

$$b + d = 6$$

1	2	3
3	4	7
4	6	

Algebraic Reconstruction

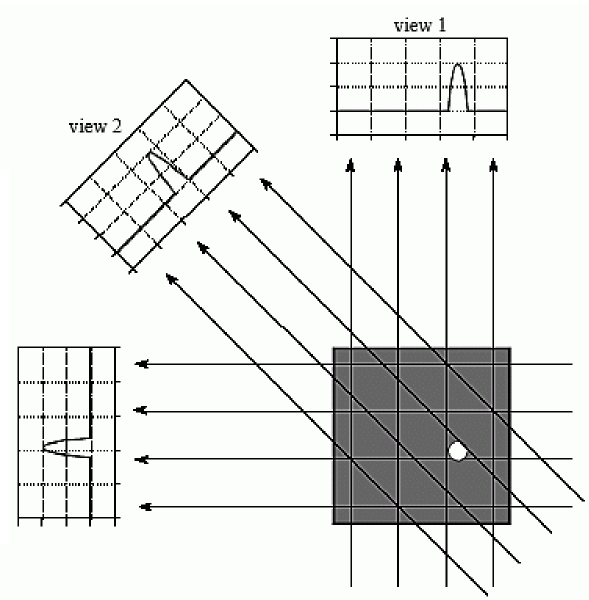
μ_1	μ_2
μ_3	μ_4



$$\left.\begin{aligned} p_1 &= \mu_1 + \mu_2 \\ p_2 &= \mu_3 + \mu_4 \\ p_3 &= \mu_1 + \mu_3 \\ p_4 &= \mu_1 + \mu_4 \end{aligned}\right\}$$

AR Matrix System Formulation & Solving as a least-squares minimization problem:

– **Next Assignment!**



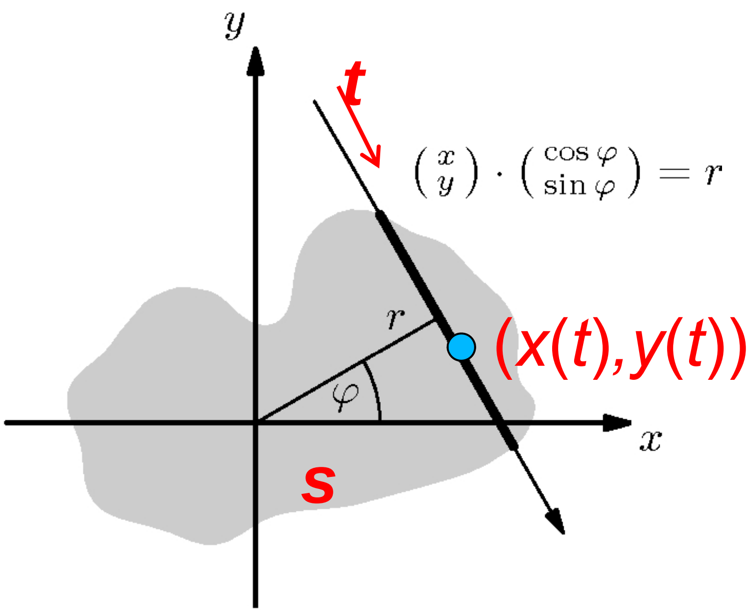
1. Algebraic Reconstruction

- Attenuation of x-rays is a system of equations.
- Right hand side vector is **`(R(1,1),R(1,2),R(1,3),...)'`**
- The solution vector is the density distribution **x**

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & \dots & 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots & 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots & 0 & 0 & 1 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}}_{S} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ x_{n^2} \end{pmatrix} = \begin{pmatrix} \texttt{R(1,1)} \\ \texttt{R(1,2)} \\ \dots \\ \texttt{R(1,n)} \\ \texttt{R(2,1)} \\ \dots \\ \texttt{R(180,n)} \end{pmatrix}$$

- S is called the system matrix
- Every line represents a ray
- To compute S, we have to find the $\mathbf{x_i}$ that are traversed by a line with angle **phi** and radial distance **r**.
- We will use the same idea as with the Radon transform.

Integral along a straight line



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Line parameterization:

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} r \\ t \end{pmatrix} = \begin{pmatrix} r\cos\varphi - t\sin\varphi \\ r\sin\varphi + t\cos\varphi \end{pmatrix}$$

Coordinate systems

Cartesian: (x,y) with s along ray

Radon: (r,φ) with t along ray

Hesse normal form

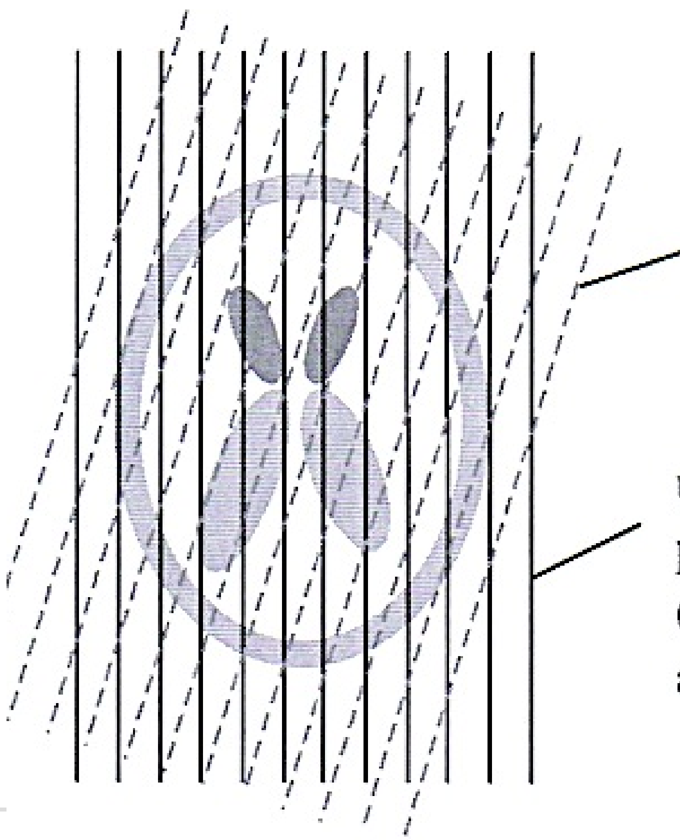
Leads to:

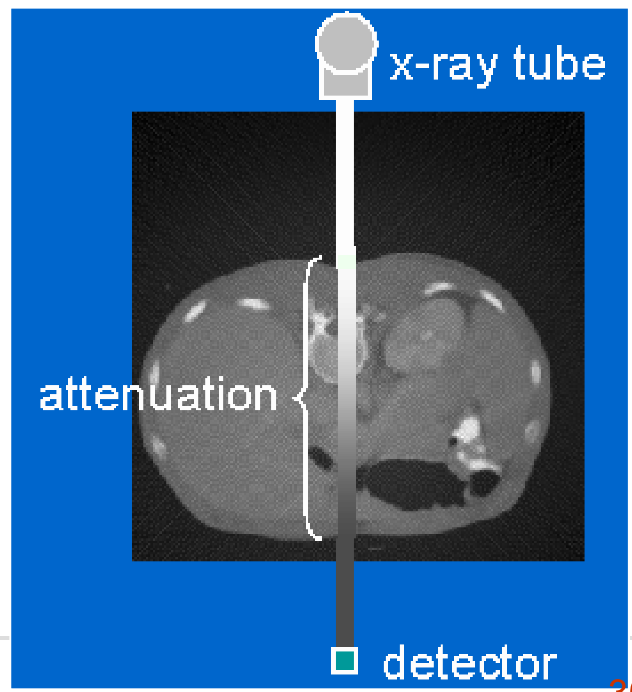
$$\int\limits_{\binom{x}{y}\cdot\binom{\cos\varphi}{\sin\varphi}=r} f(x,y)\,ds = \int_{-\infty}^{\infty} f(r\cos\varphi - t\sin\varphi, r\sin\varphi + t\cos\varphi)\,dt$$

The CT reconstruction problem

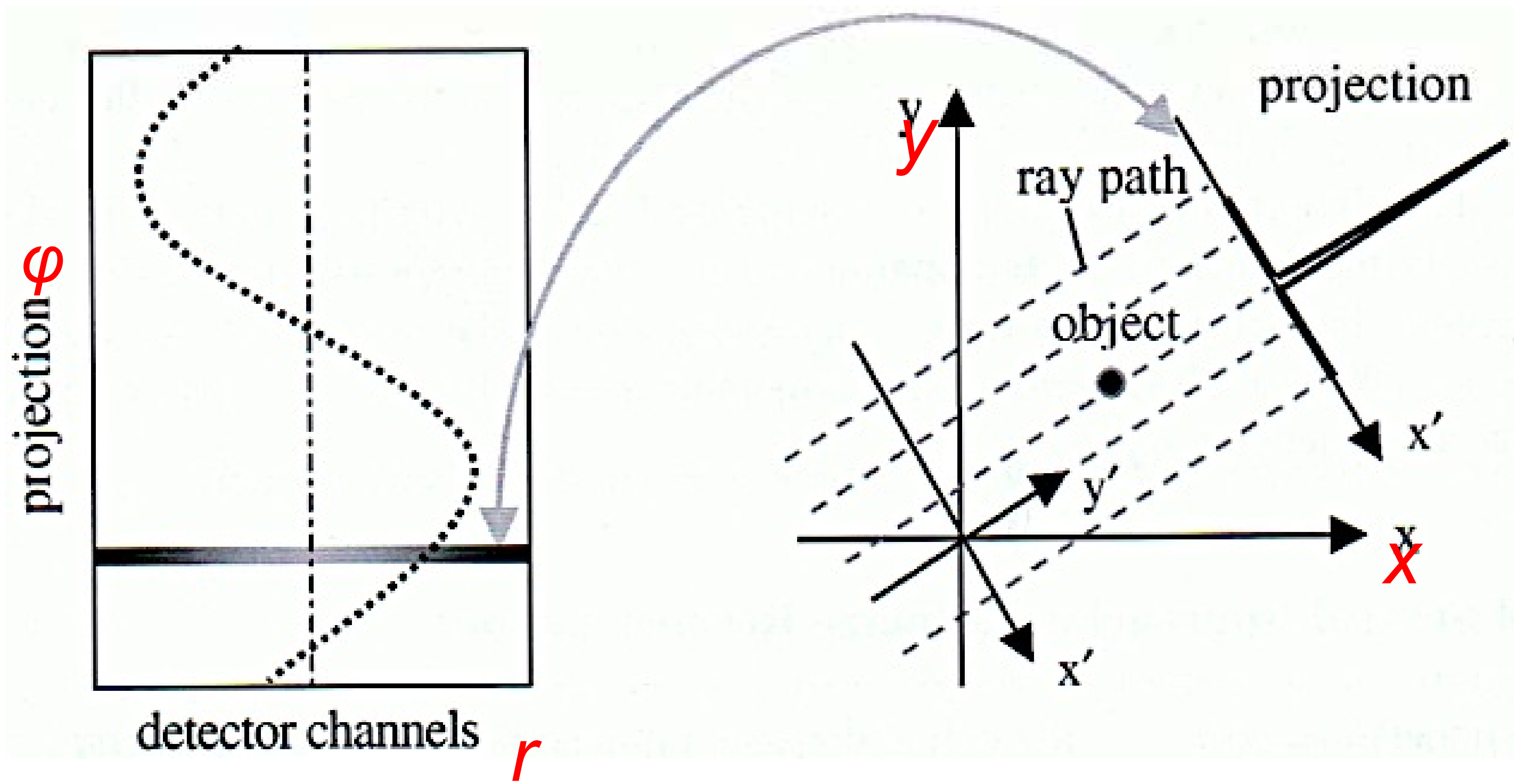
- Given the measured line integrals of an object,
- Estimate or calculate its attenuation distribution

$$\boxed{p = -\ln\left(\frac{I}{I_0}\right) = \int_L \mu(x)\,dx}$$





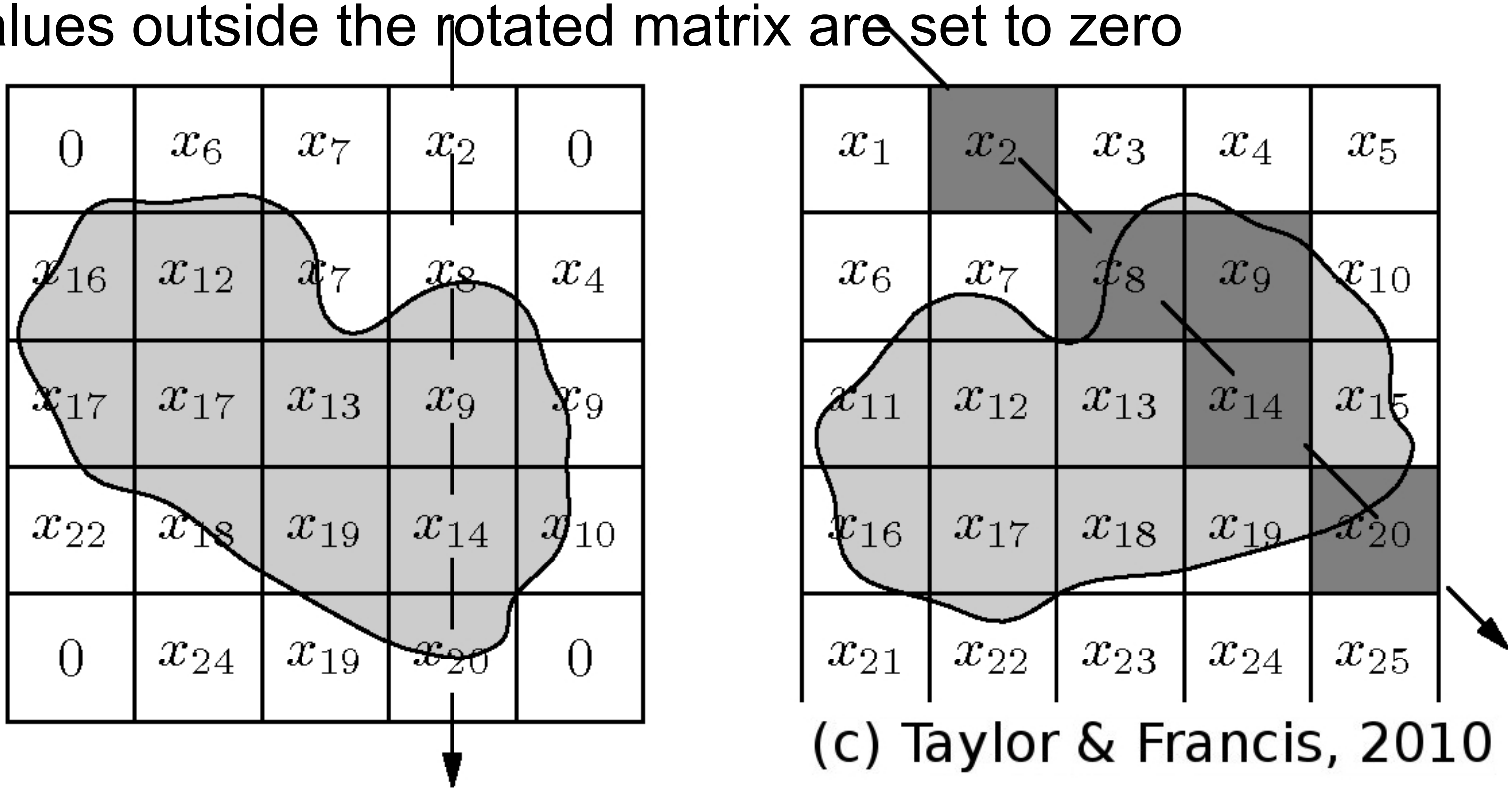
Sinogram



Radon space *Cartesian space*

Sinogram generation – in Matlab

- A line with slope phi crosses the same points as a parallel line through the body rotated by phi clockwise.
- Values outside the rotated matrix are set to zero



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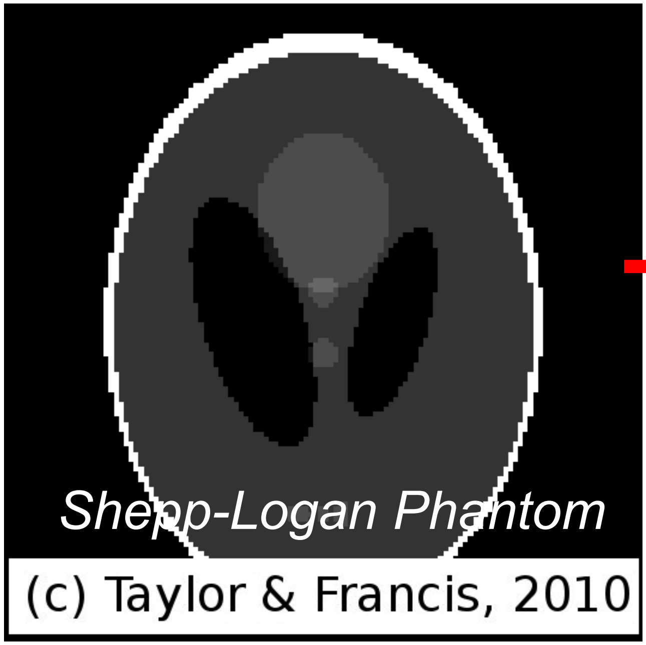
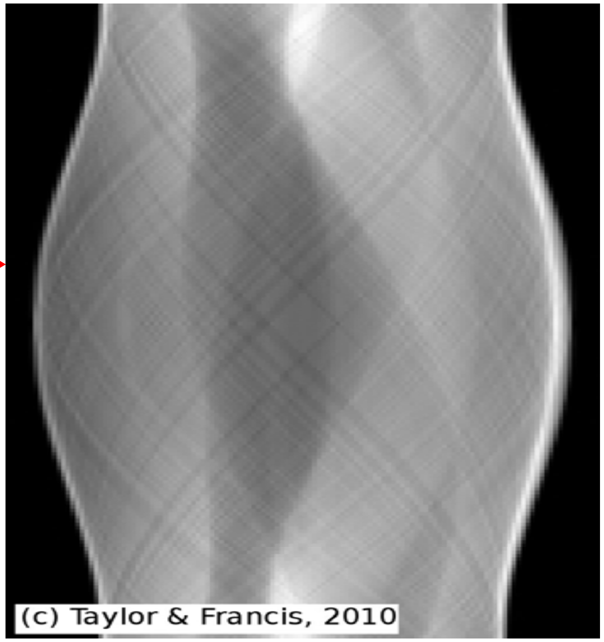
Therefore we first rotate by -phi, then apply the MATLAB function **`sum`**

Sinogram generation – in Matlab

```
function R=radotra(pic,steps)
R=zeros(steps,length(pic));
for k=1:steps
   R(k,:)= sum(imrotate(pic,...
     -(k-1)*180/steps,'bilinear','crop'));
end;
```

Negative angle because of the clockwise rotation

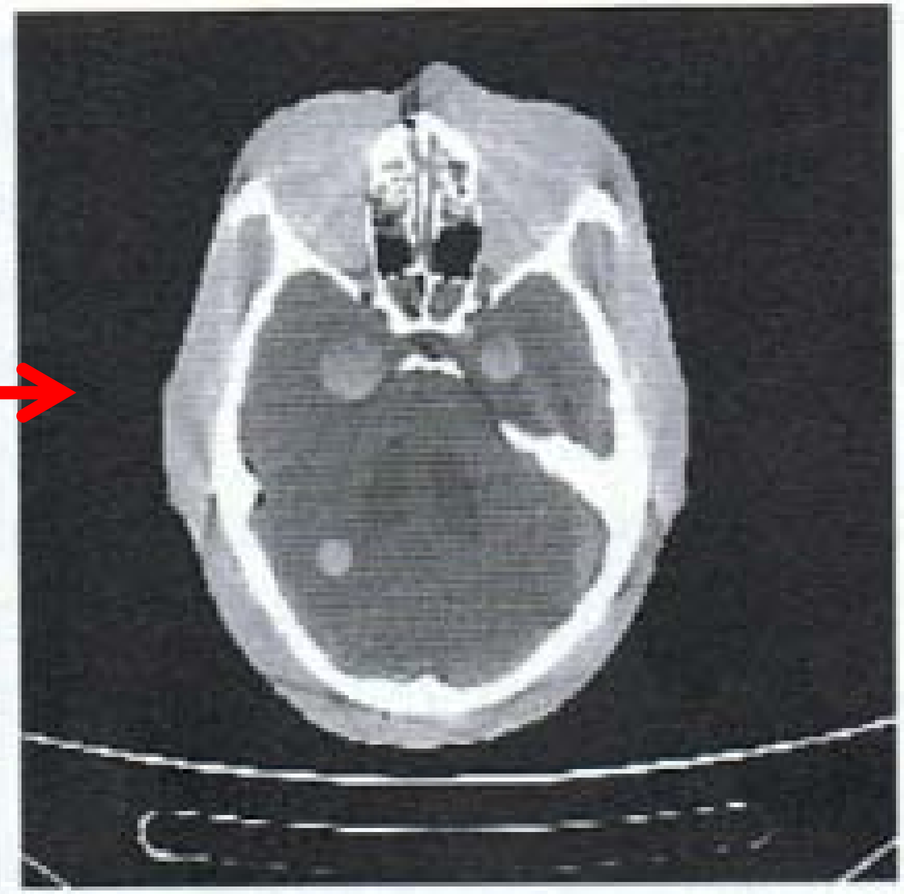




Reconstruction process

Organized Projections lines.

Projections spanning 180^o



Reconstructed Image.

CT image **reconstruction methods**

1. Algebraic reconstruction

2. Direct Fourier method

3. Filtered back-projection

One Formal Projection Image Reconstruction Principle: Vienna, 1917

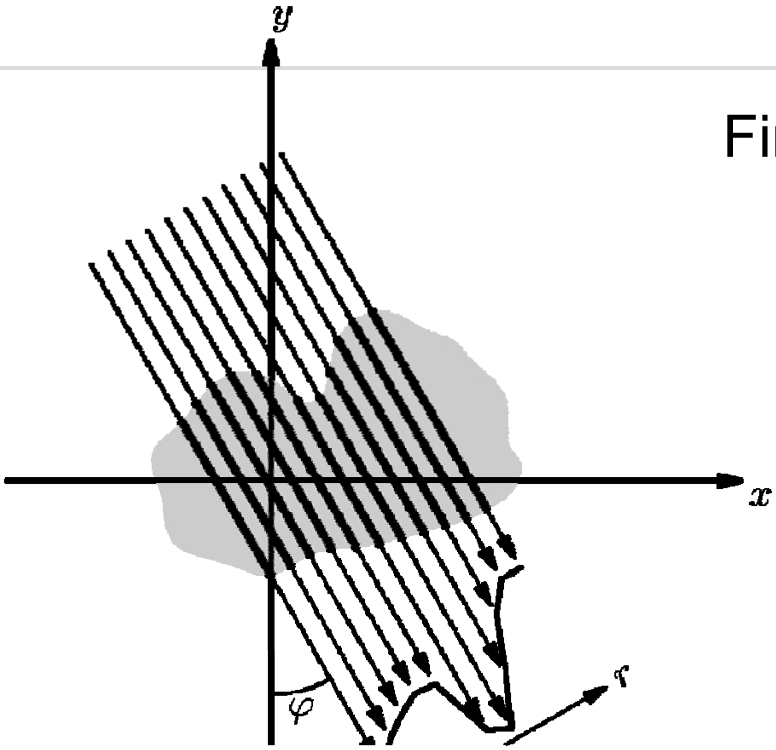
Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten.

Von

Johann Radon.

Integriert man eine geeigneten Regularitätsbedingungen unterworfene Funktion zweier Veränderlichen x, y — eine *Punktfunktion* $f(P)$ in der Ebene — längs einer beliebigen Geraden g, so erhält man in den Integralwerten $F(g)$ eine *Geradenfunktion*. Das in Abschnitt A vorliegender Abhandlung gelöste Problem ist die Umkehrung dieser linearen Funktionaltransformation, d. h. es werden folgende Fragen beantwortet: kann jede, geeigneten Regularitätsbedingungen genügende Geradenfunktion auf diese Weise entstanden gedacht werden? Wenn ja, ist dann f durch F eindeutig bestimmt und wie kann es ermittelt werden?

Definition of the **Radon Transform**



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First generation CT uses parallel beams.

$$f(x,y) \mapsto R_f(\varphi, r)$$

$$R_f(\varphi, r) := \int_{\binom{x}{y}\cdot\binom{\cos\varphi}{\sin\varphi}=r} f(x,y)\, ds$$

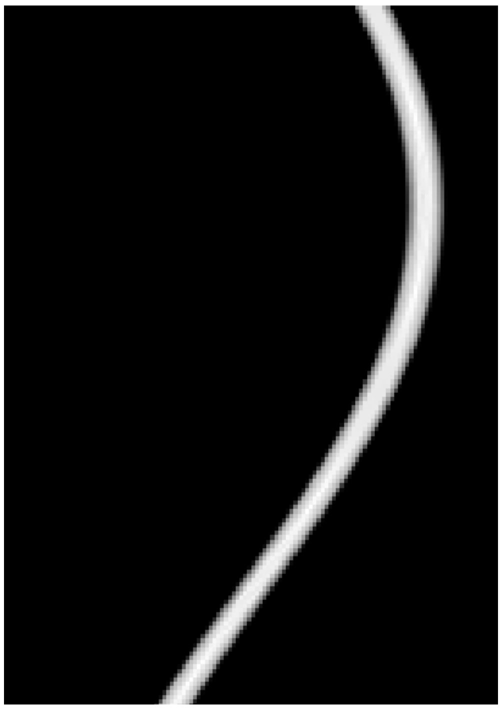
Austrian mathematician Johann Radon (1887-1956) solved the problem of reconstructing the density distribution out of line integrals in 1917.

Radon transform of a point

All lines that go through the point have the same line integral

The others have line integral zero.

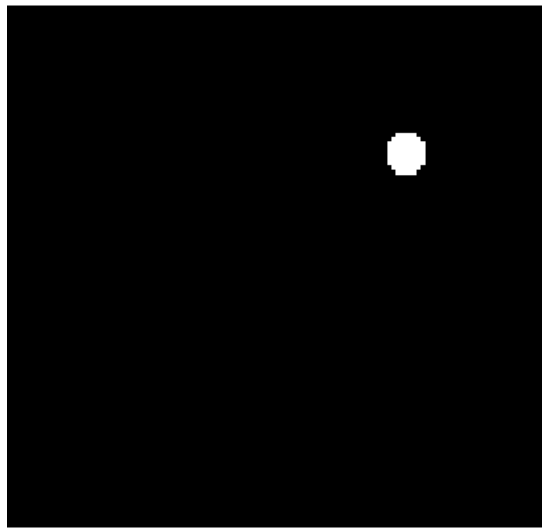
φ



r

Radon space

y



x

Cartesian space

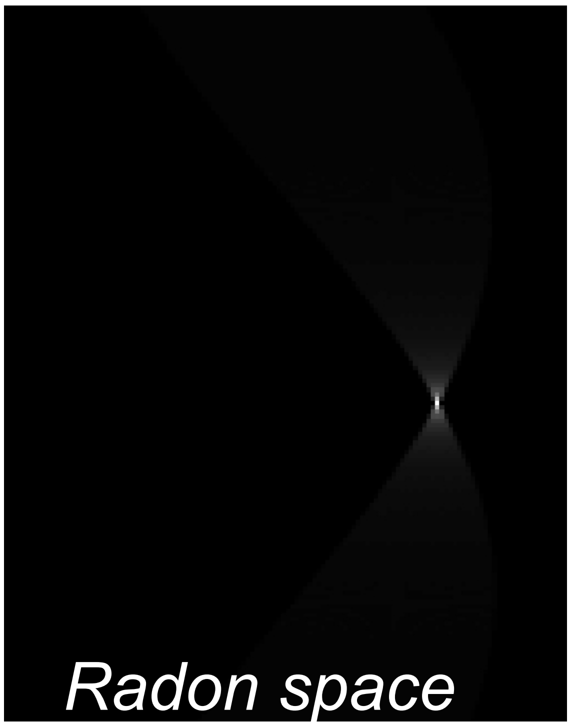
The (r, φ) lines that go through point (x_0,y_0) are:

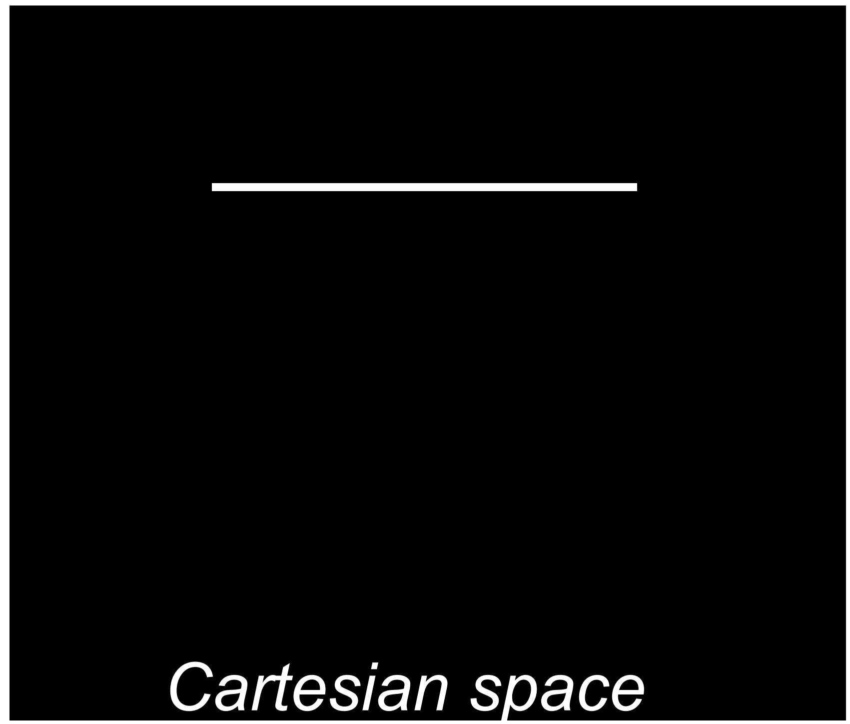
$$r = x_0 \cos\varphi + y_0 \sin\varphi$$

r is a trigonometric function of variable φ

Radon transform of a line

All line integrals will be zero except the one along the line!

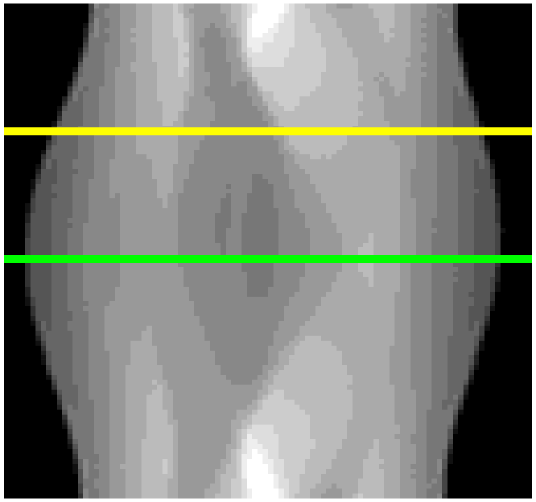


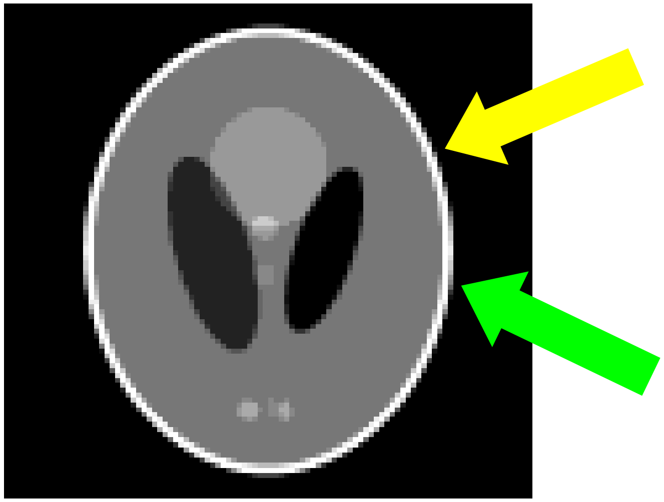


This looks similar to the Hough transform, which is the same as a Radon transform on *binary images*.

CT: Sinogram Generation

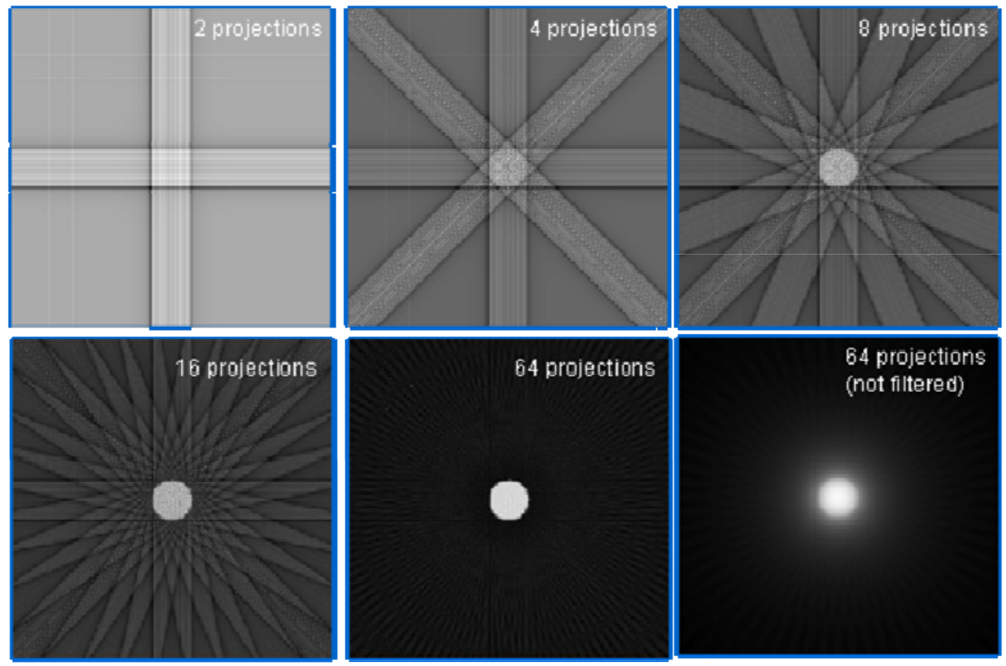
- Each position of the x-ray source provides one projective view of the patient.
- Several hundred (over 180 degrees) projective views are required to generate an image.
- Image = Inverse Radon Transform.





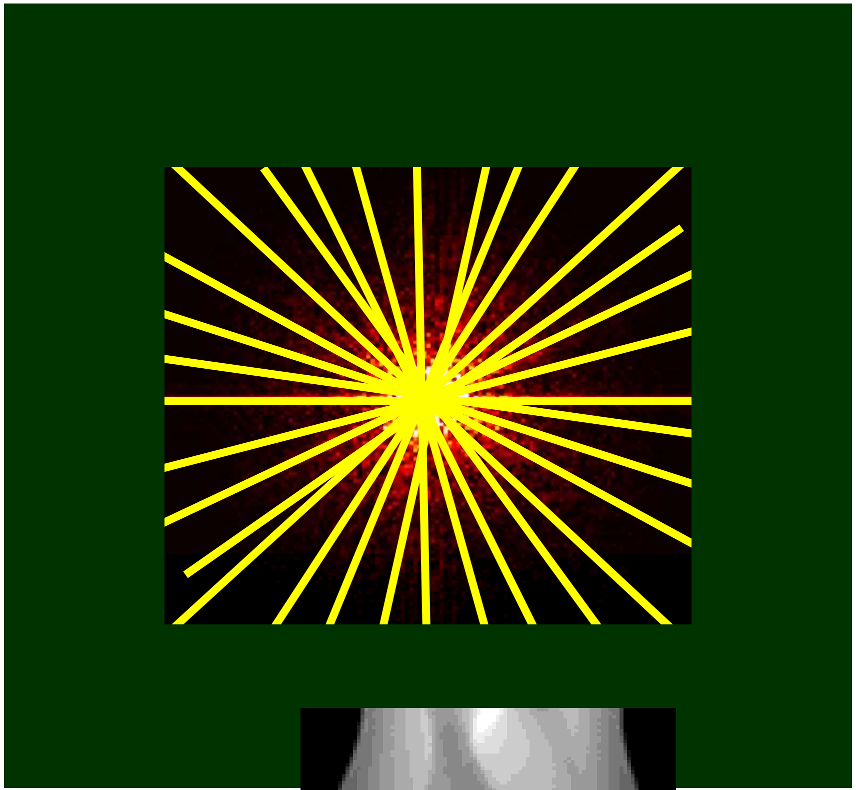
Back-Projection Reconstruction

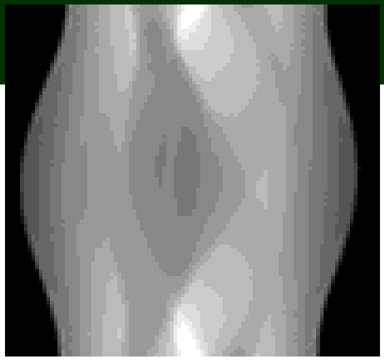
- Back-projecting each projection data into an accumulator matrix builds up the image when sufficient number of projections are used…
- Can be done in k-space.



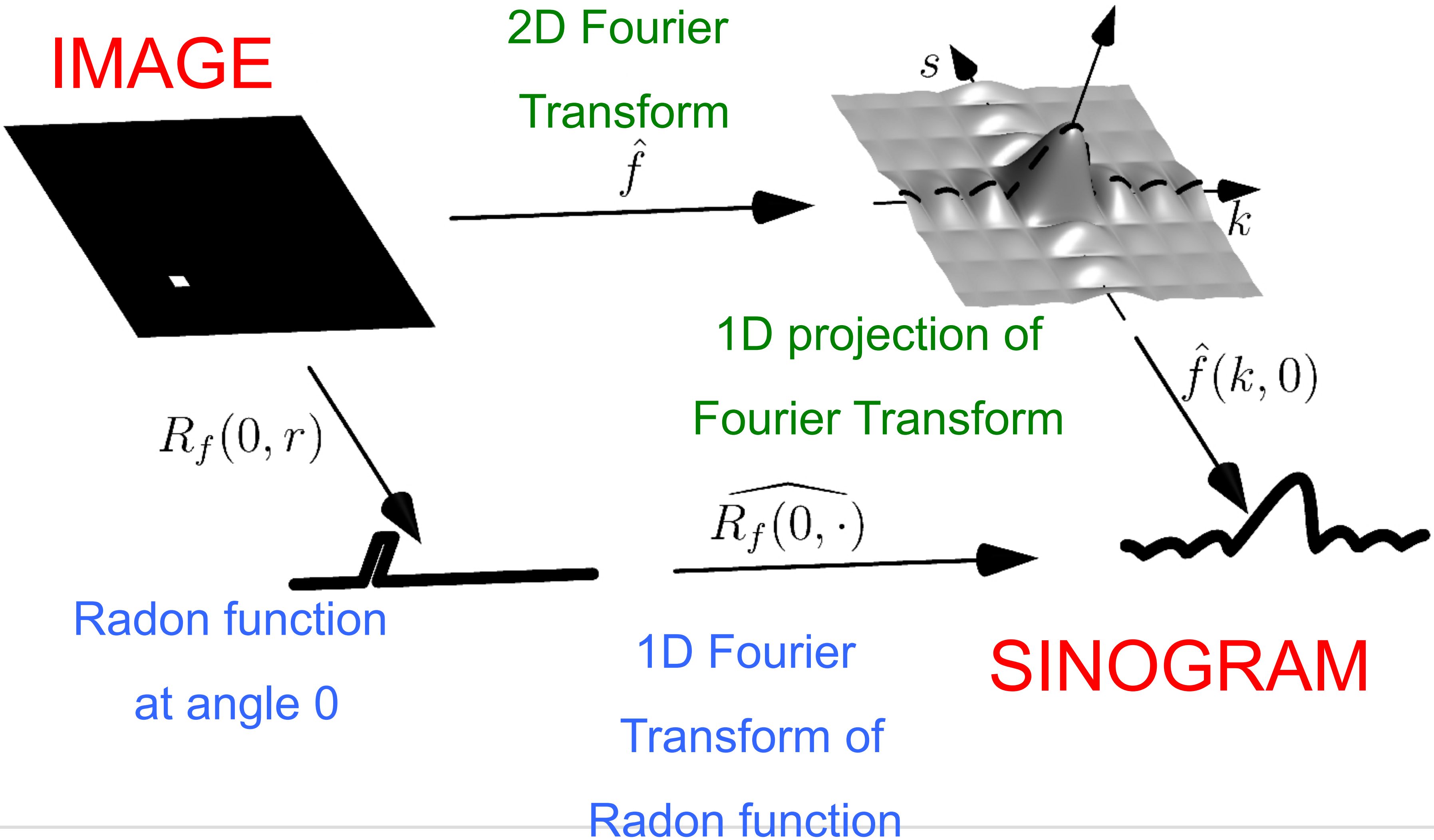
Parseval's theorem

- When viewed in k-space we can appreciate that the series of projections required for CT dramatically over sample the center and dramatically undersampled the outer regions.
- Note, that a uniformly sampled k-space is required to produces images with good resolution features

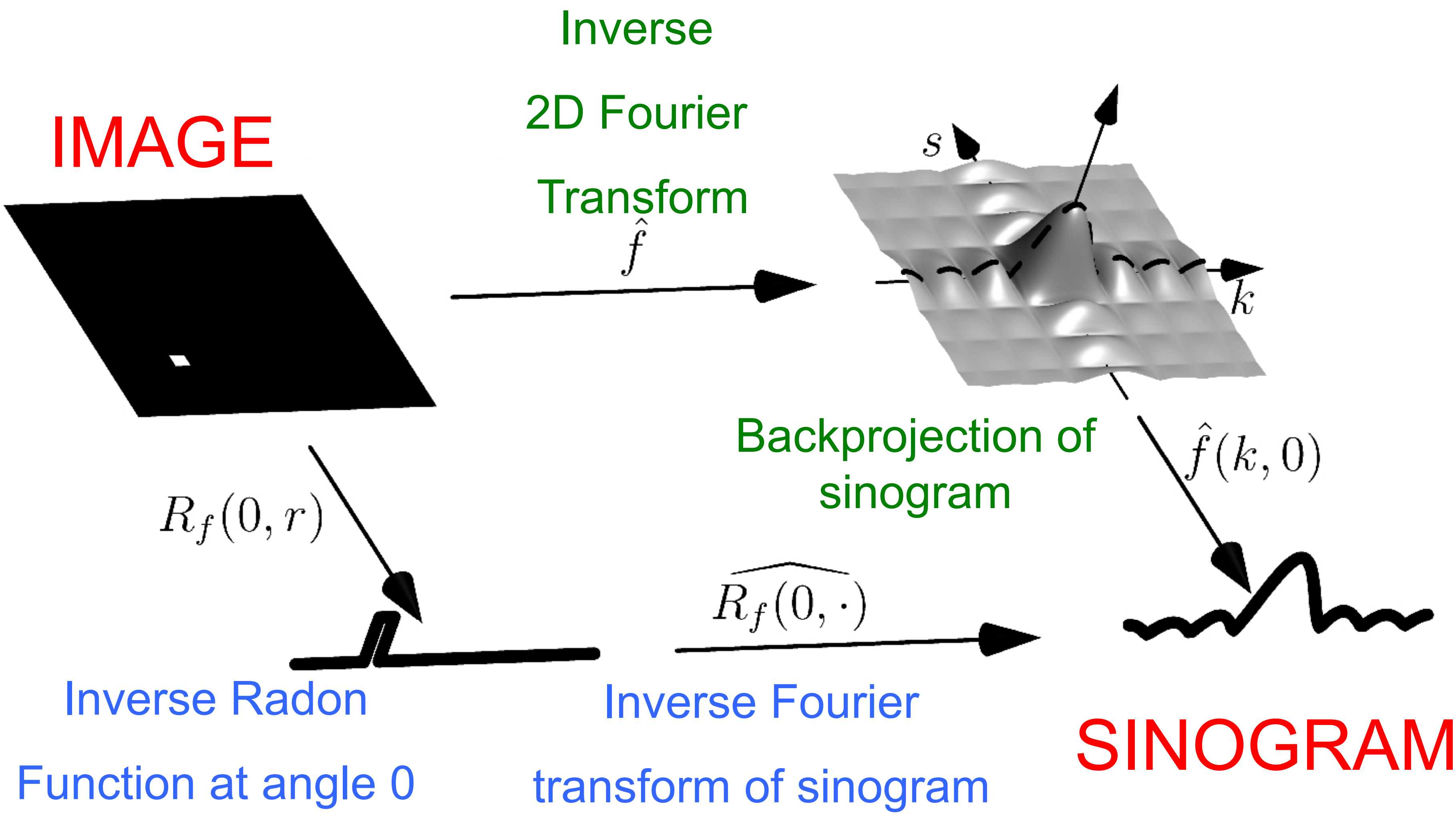




Fourier Slice Theorem (forward)



Fourier Slice Theorem (backwards)



Direct Fourier Projection Reconstruction

Fourier Slice Theorem $$\hat{f}\left(\lambda\begin{pmatrix}\cos\varphi\\ \sin\varphi\end{pmatrix}\right) = \frac{1}{\sqrt{2\pi}}\widehat{R_f(\varphi,\cdot)}(\lambda)$$

- 1D Fourier transforms of the Radon transform for a number of angles

- Inverse 2D Fourier transform of the result

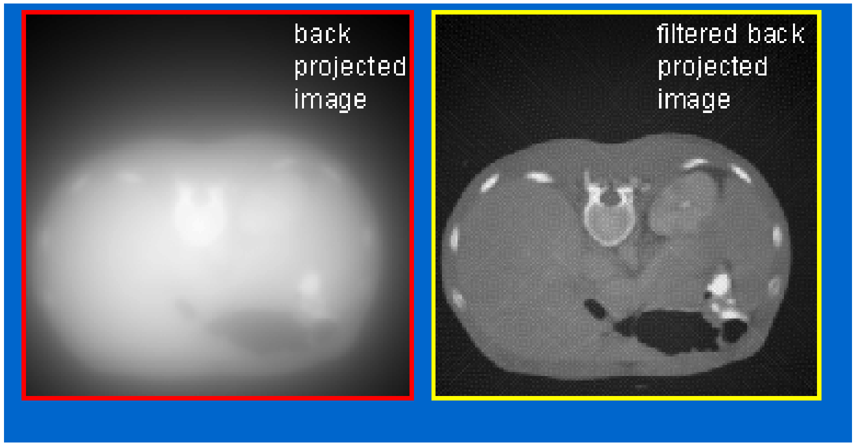
  → reconstructs the Fourier transform.

Limited number of angle steps →

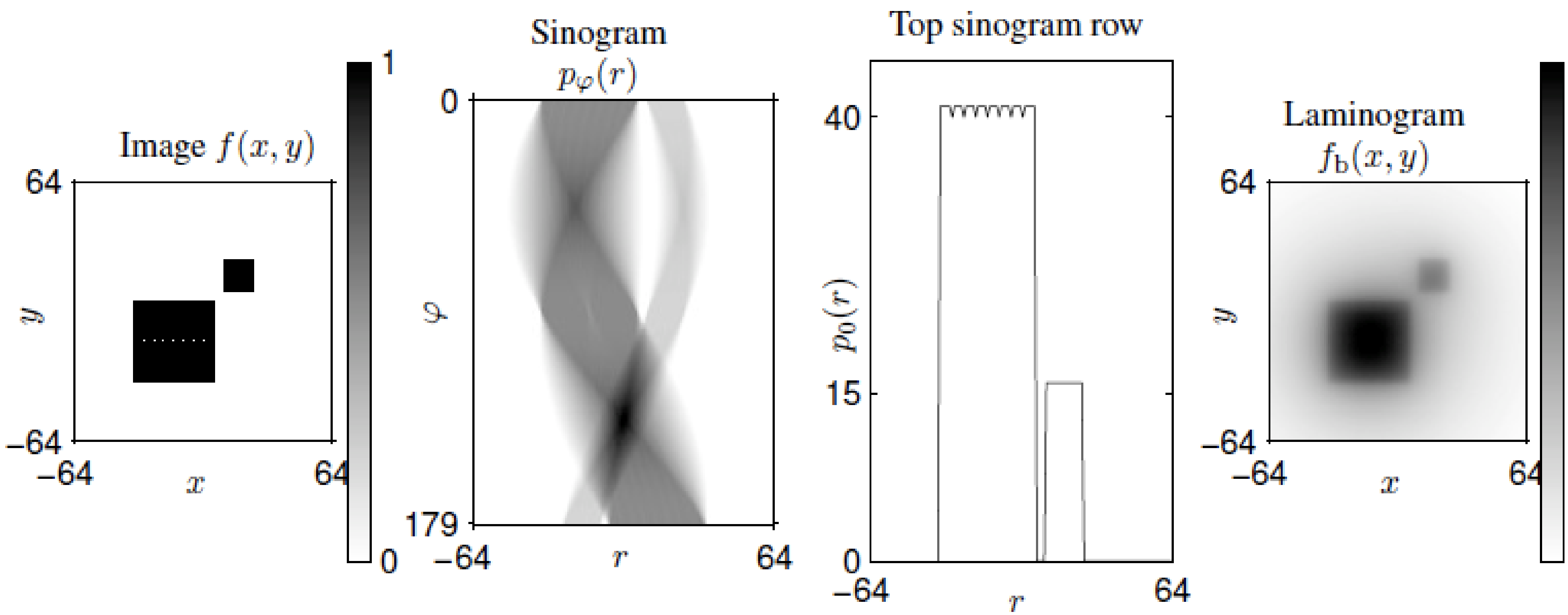
very inaccurate for higher frequencies

Filtered Back Projection (**FBP**)

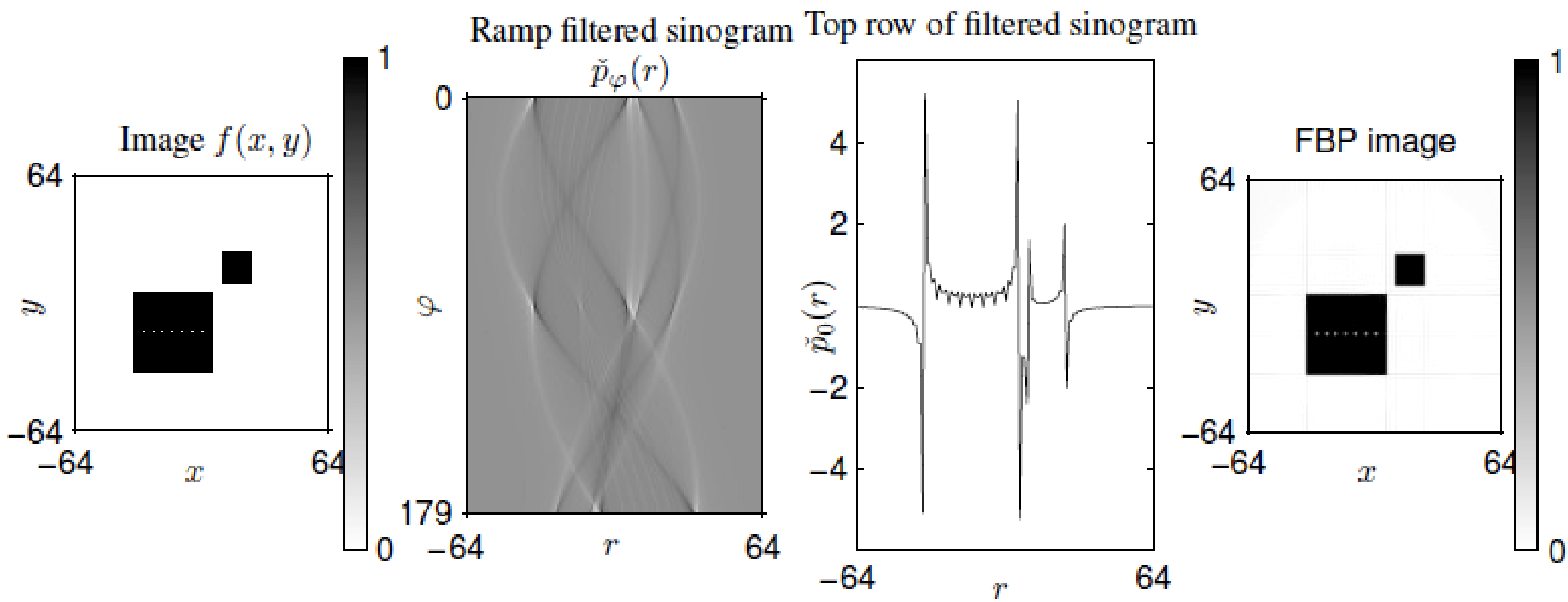
- A common technique of reconstructing CT images is to use Filtered Back Projection
- The Filter refers to the requirement to decrease the data at the center of k-space :
  - To deal with the excess of information at the center of k-space.



Backprojection – no filtering



Backprojection – ramp filtering



Filtering in Fourier Space

- Cleaning a signal from high frequency noise can be done by setting the higher part of its Fourier transform to zero.
- In general we just multiply the Fourier transform of the function with the *frequency response* of a filter.

```
function filt(f,fi)
ff=fftshift(fft(f));
subplot(5,1,1); plot(f);
subplot(5,1,2); plot(abs(ff));
subplot(5,1,3); plot(fi);
subplot(5,1,4); plot(abs(fi.*ff));
subplot(5,1,5); plot(real(ifft(...
     ifftshift(fi.*ff))));
```

Filtered Back Projection, **MATLAB**

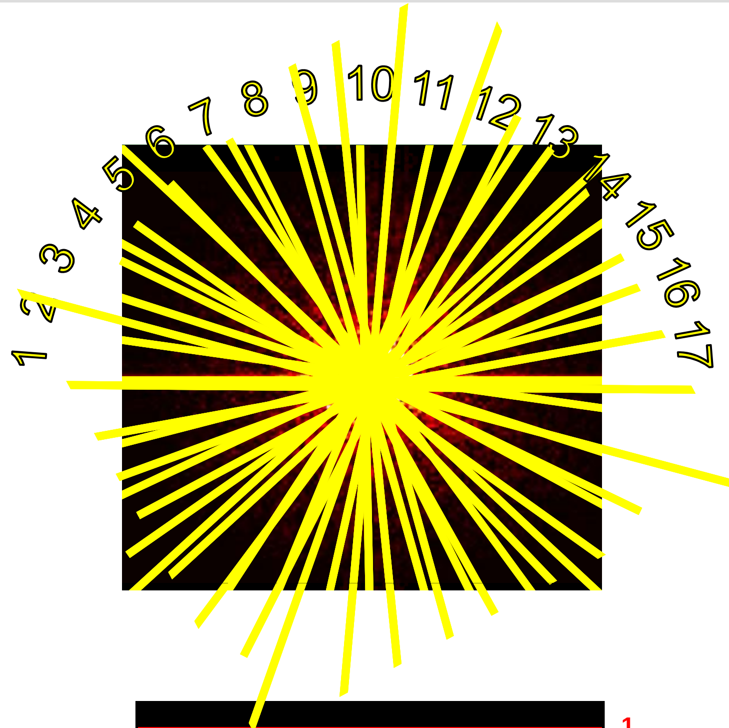
Instead of adding to lines with different angles, we rotate the matrix and add to its columns

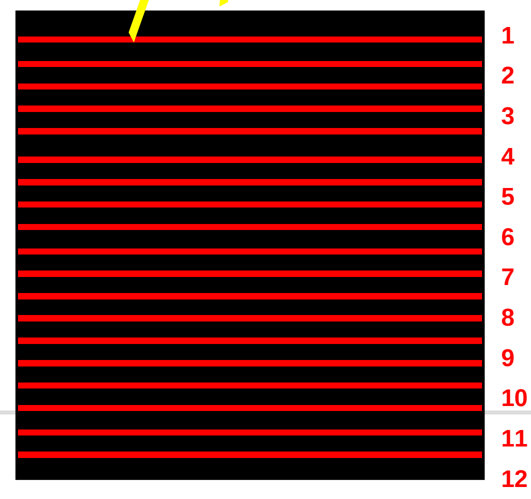
```
function reco=fbp(r,steps,filt)
reco=zeros(128);
for k=1:steps
   q=real(ifft(ifftshift(...
     filt'.*fftshift(fft(r(k,:))))));
   reco = reco + ones(length(q),1)*q;
   reco=imrotate(reco,-180/steps,'bicubic','crop');
end
reco=imrotate(reco,180,'bicubic','crop');
```

Argument **`filt`** is the frequency response of a filter, e.g. a ramp.

Projection Scanning **Efficiency**

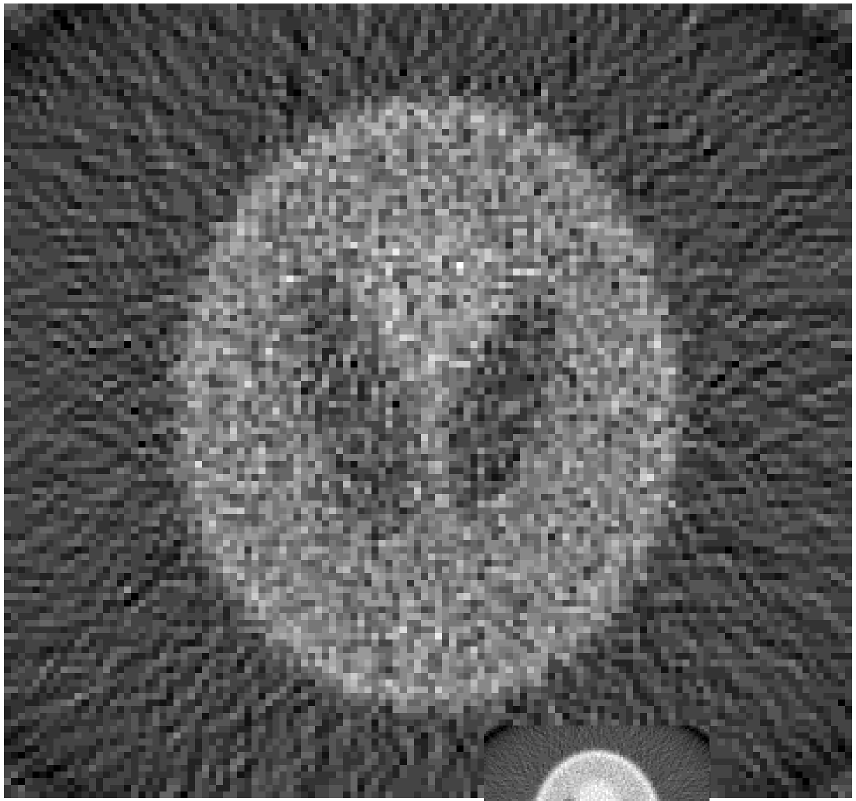
- Linear scanning (MRI) vs Projection scaning (CT):
  - Compared to linear scanning, projection scanning requires about twice the number of lines in k-space to achieve similar resolution.

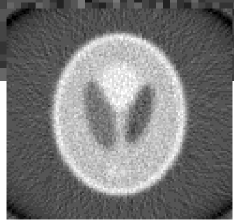




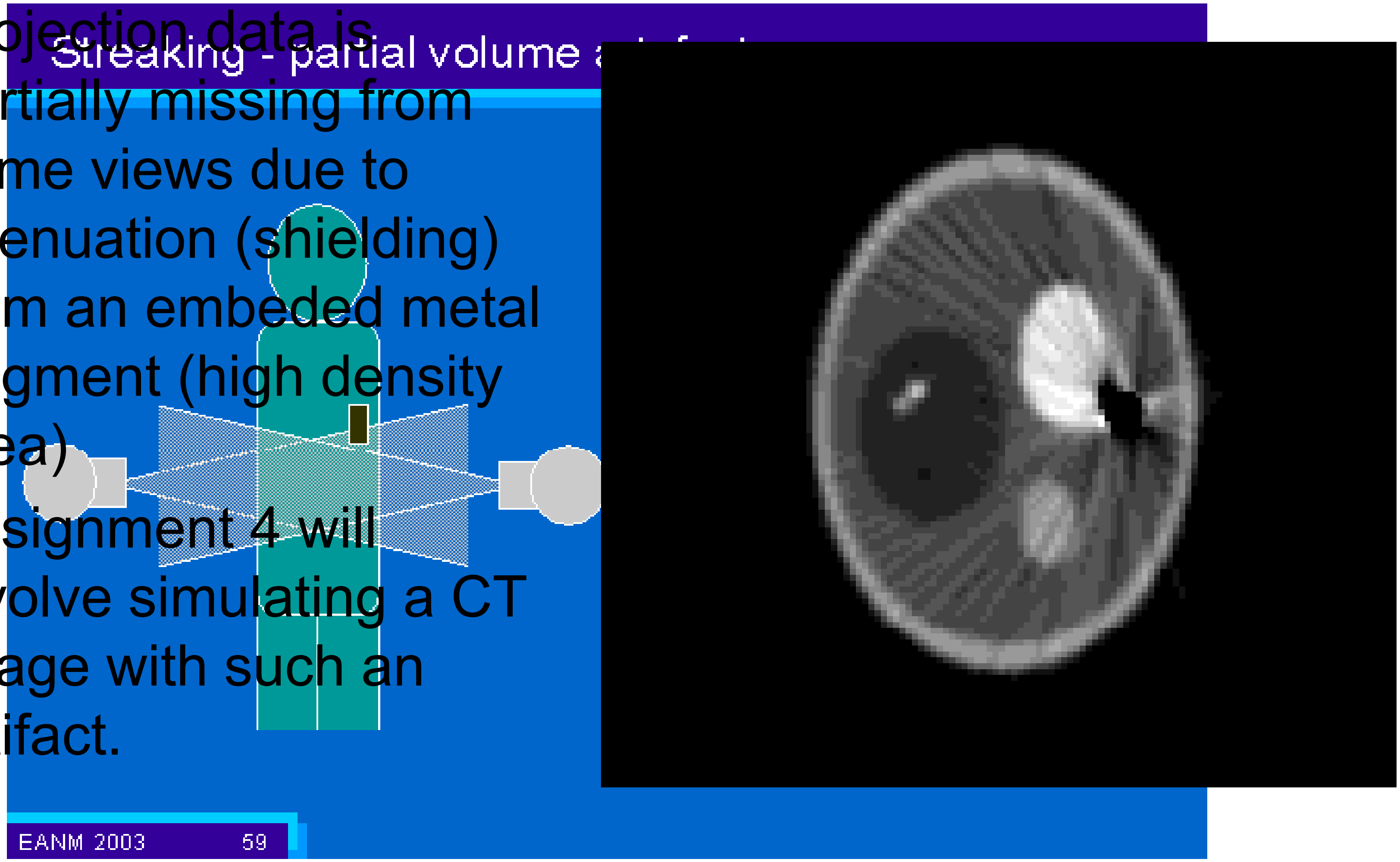
FBP: Counts/SNR/Projections

- FBP images of the Shepp-Logan phantom with finite resolution and limited photon counts.





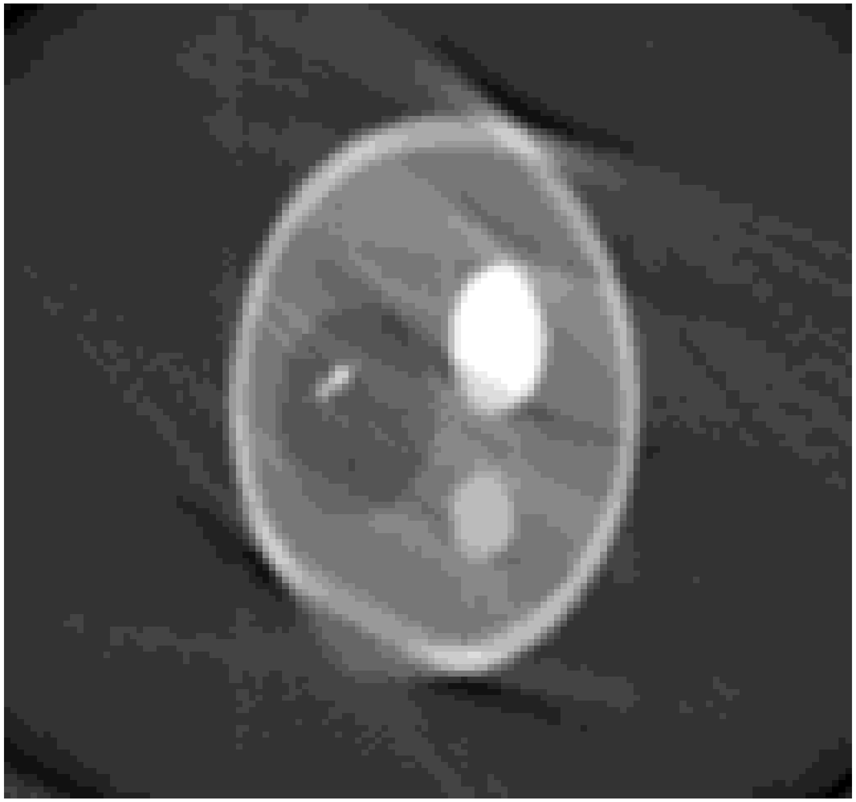
CT: Partial Volume Artifact



- Projection data is partially missing from some views due to attenuation (shielding) from an embeded metal fragment (high density area)
- Assignment 4 will involve simulating a CT image with such an artifact.

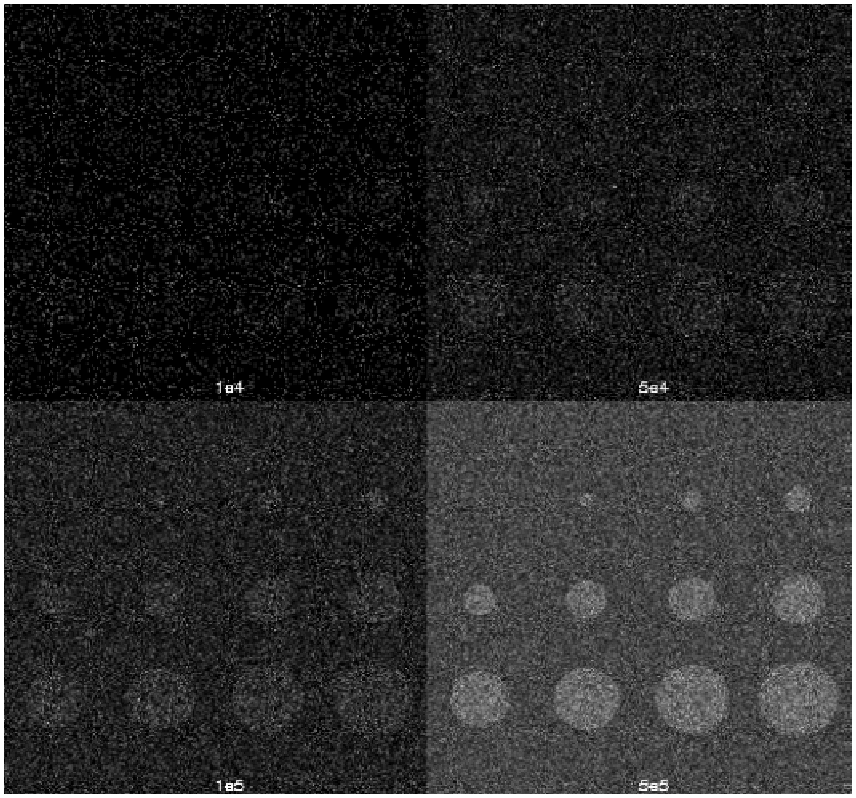
CT: Motion Artifact

- Motion artifact
- Caused by patient motion, such as respiration and heart beat, during data acquisition



CT: Noise

- Noise
- Photon detection is a stochastic process
- Lower noise level can be obtained by increasing the radiation dose, and/or the acquisition time
- Perception of structures depends on the contrast, size, and noise level



CT: Resolution – Detector Size/Density

- Finite resolution
- Finite detector size

  limits the resolution of

  the acquired data