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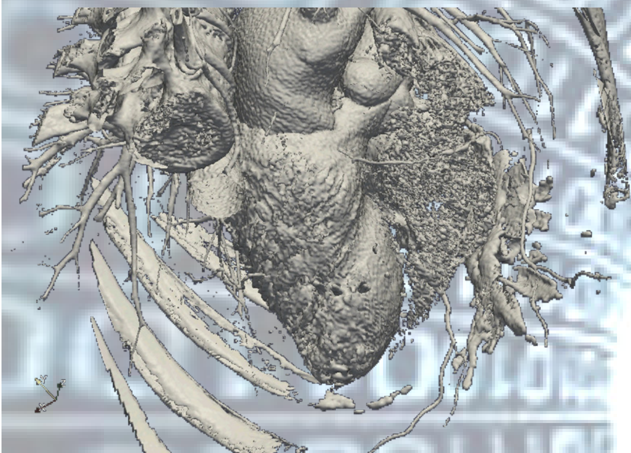
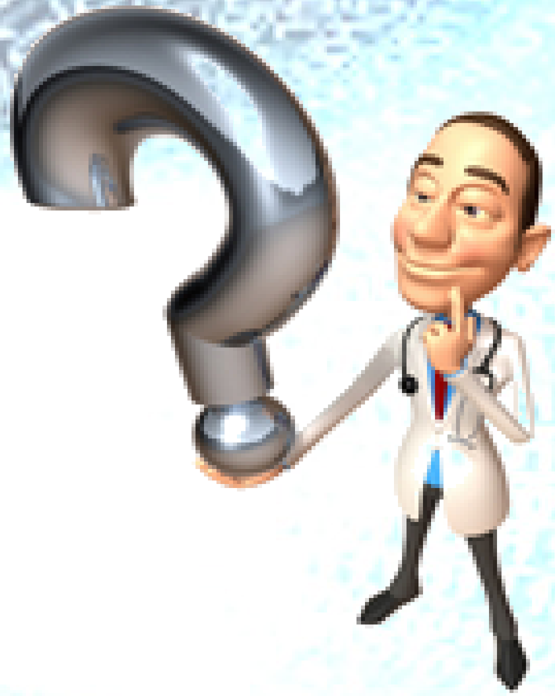
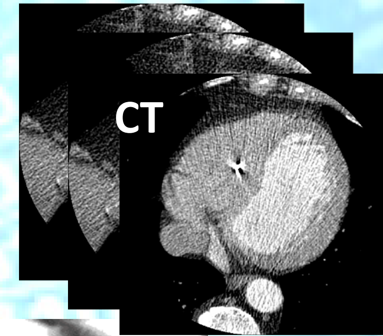
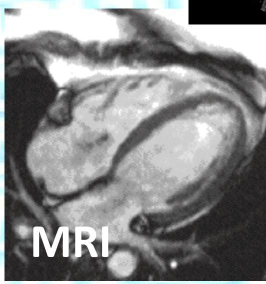
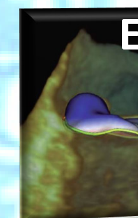
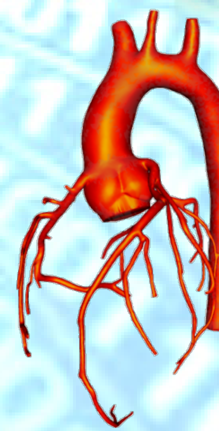












Quiz 4, 20 Oct 2014

Basic Image Filtering,
Gaussians & Edge Detection
(**from Week 5-6**)

QUIZ on Image Filtering (60 pts)

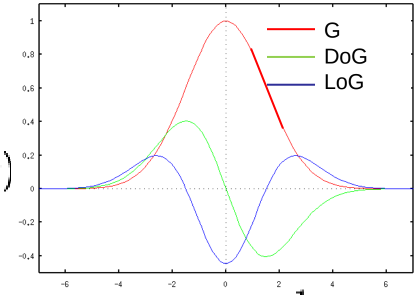
Answer Q1 and Q2.

Q1. Over the last two classes, I have briefly introduced the 'general' form of an optimization problem using the Calculus of "Variations". In your own words define how variational calculus generalizes the optimization problem..? How is it different than minimizing the least-squares curve fitting objective where we optimally fit constant values to some given data..?

HINT: in the following example, E is an energy function which needs to be minimized with respect to C.

$$\min \int E(C(x), C_x(x))dx$$

Q2. We learned about how the difference (i..e first derivative) of the Gaussian (DoG) can serve as a means to accentuate edges. It so happens that the Laplacian (i.e. Second derivative) of the Gaussian (LoG) operator can also help detect edges as 'zero-crossings' in the LoG convoluted image. Given the following knowledge on how gradients apply to create the LoG convolution operator, derive an expression for the LoG operator and fill <u>a 5x5 matrix of values that represents a convolution 'kernel' for the LoG operator</u>.



$$G_\sigma(x,y) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

Since, $\frac{d}{dt}[h(t)*f(t)] = \frac{d}{dt}\int f(\tau)h(t-\tau)d\tau = \int f(\tau)\frac{d}{dt}h(t-\tau)d\tau = f(t)*\frac{d}{dt}h(t)$

Therefore, $\triangle[G_\sigma(x,y)*f(x,y)] = [\triangle G_\sigma(x,y)]*f(x,y) = LoG * f(x,y)$

HINT: Try x and y within the ranges [-2,-1,0,1,2]. Assume a Gaussian with mean = 0 and sigma = 1, and omit $\frac{1}{\sqrt{2\pi\sigma^2}}$,

Calculus of Variations

Generalization of Calculus that seeks to find the path, curve, surface, etc., for which a given Functional has a minimum or maximum.

Goal: find extrema values of integrals of the form

$$\int F(u, u_x)\,dx$$

It has an extremum only if the Euler-Lagrange Differential Equation is satisfied,

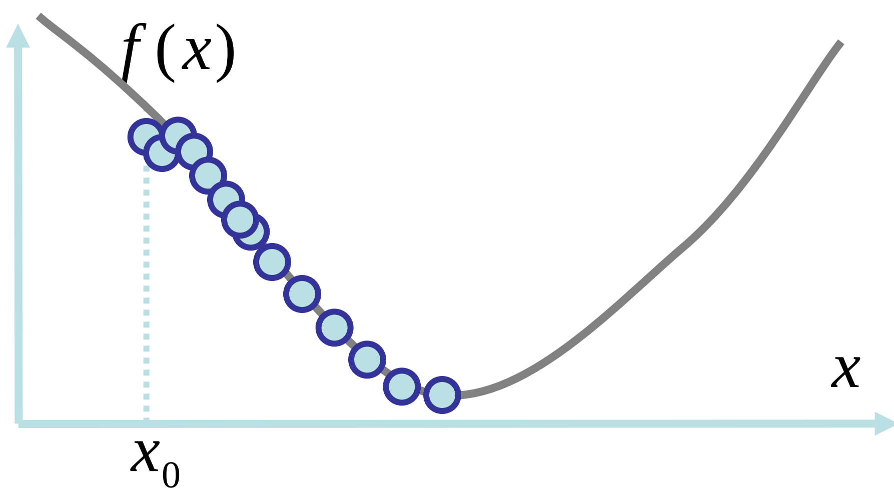
$$\left(\frac{\partial}{\partial u} - \frac{d}{dx}\frac{\partial}{\partial u_x}\right)F(u, u_x) = 0$$

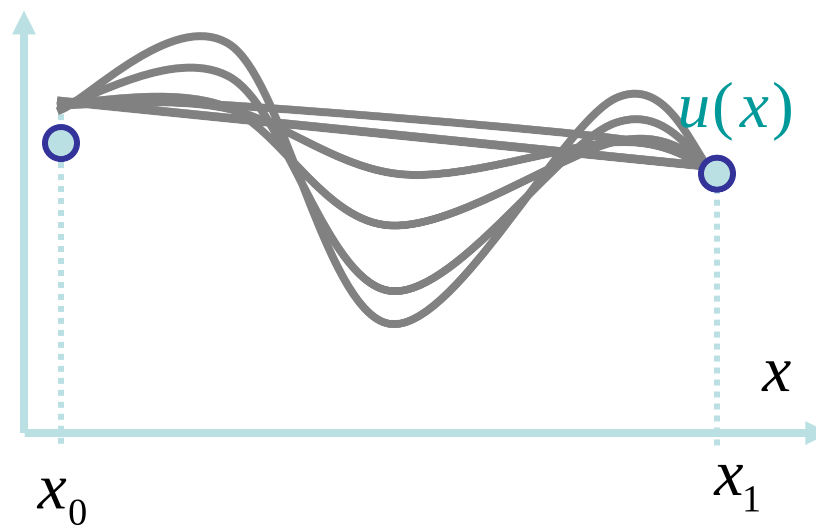
Calculus v/s Calculus of Variations

- Gradient descent process

Calculus $\arg\min_{x} f(x) \Rightarrow x_t = -f_x$

Calculus of variations $\arg\min_{u(x)} \underbrace{\boxed{\int F(u,u_x)dx}}_{E(u)} \Rightarrow u_t = -\overset{\text{Euler-Lagrange}}{\boxed{\frac{\delta E(u)}{\delta u}}}$





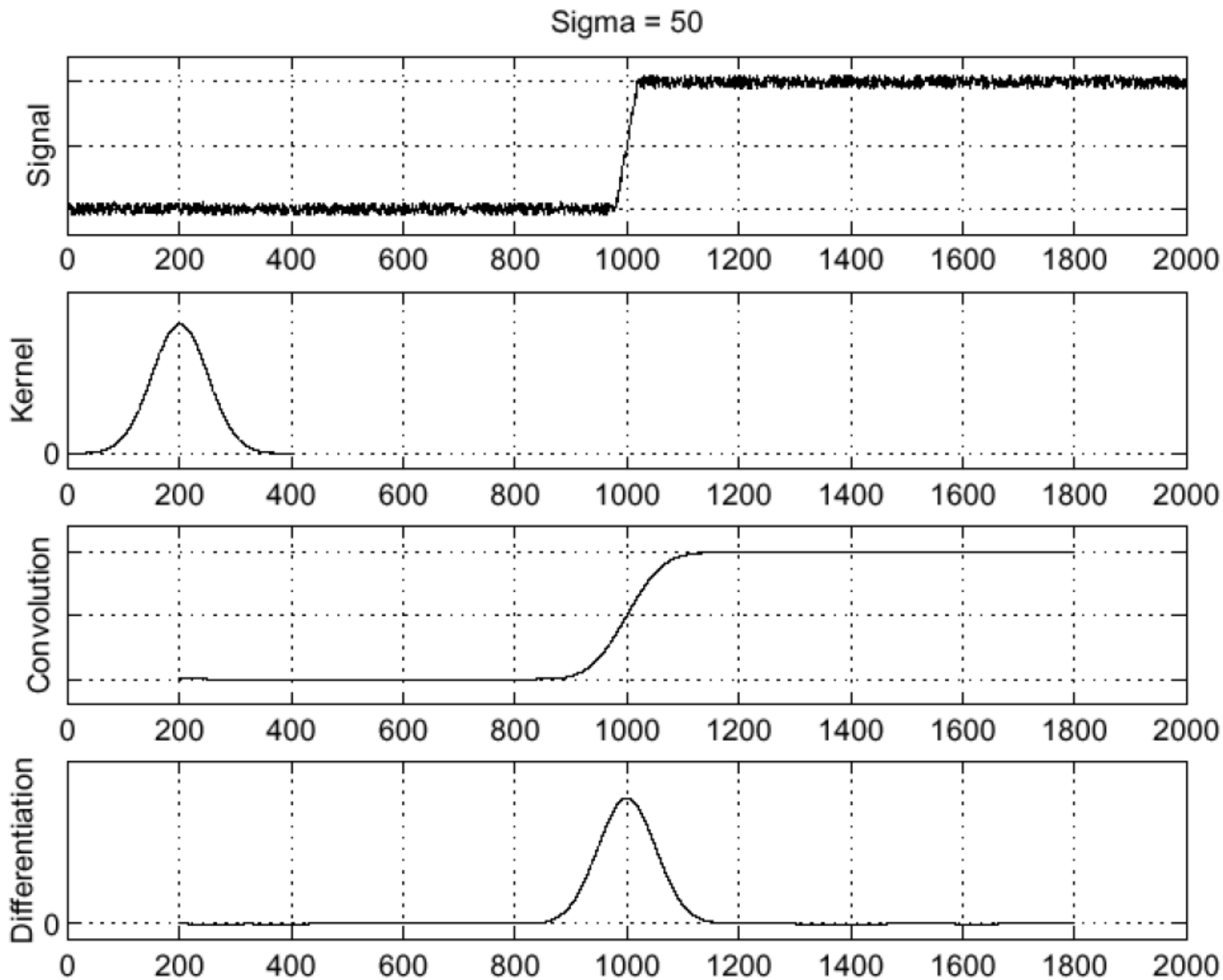
Solution: smooth first

f

h

$h \star f$

$\frac{\partial}{\partial x}(h \star f)$



Where is the edge? Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

Derivative theorem of convolution

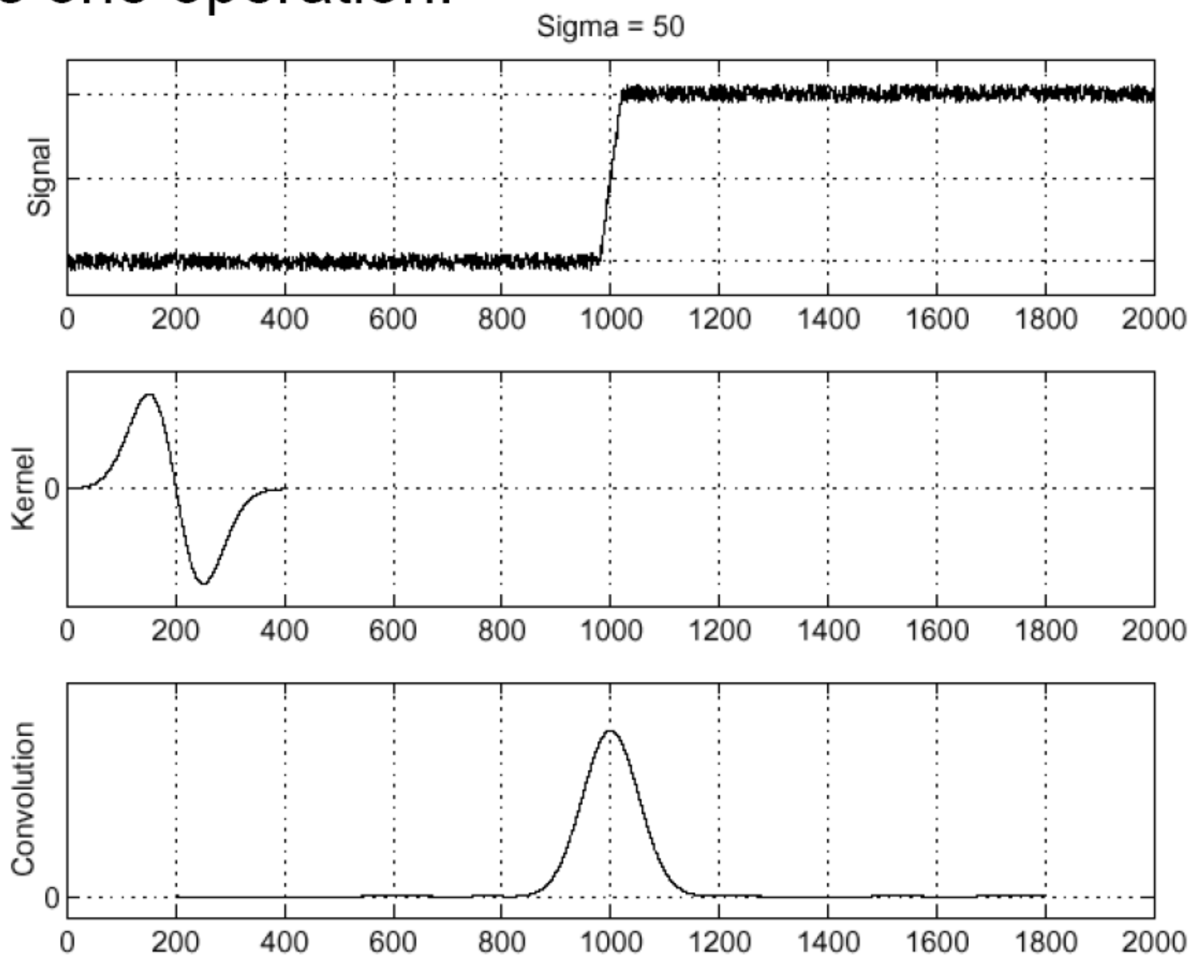
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

This saves us one operation:

f

$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) \star f$



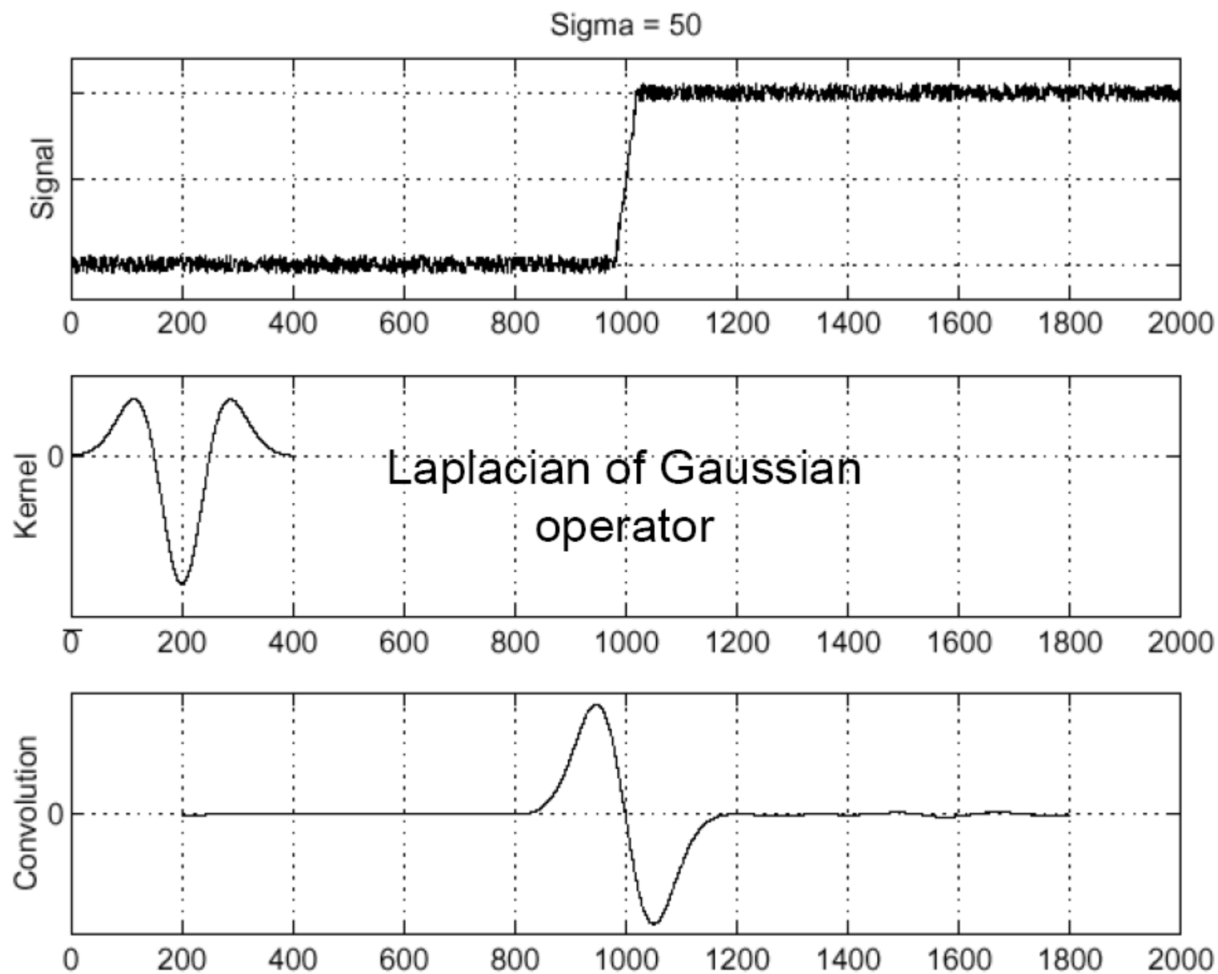
Laplacian of Gaussian

Look for zero-crossings of $\frac{\partial^2}{\partial x^2}(h \star f)$

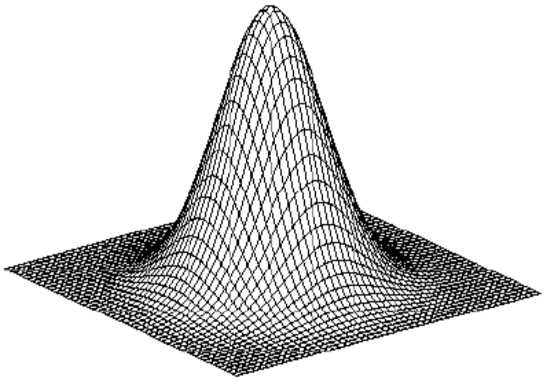
f

$\frac{\partial^2}{\partial x^2}h$

$(\frac{\partial^2}{\partial x^2}h) \star f$

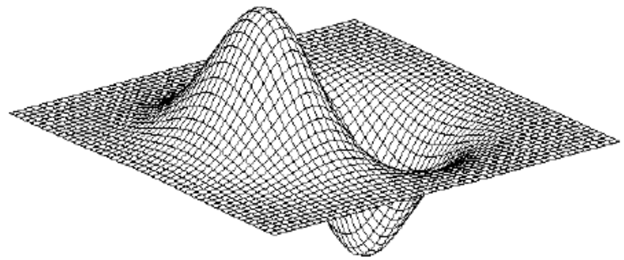


2D edge detection filters



Gaussian

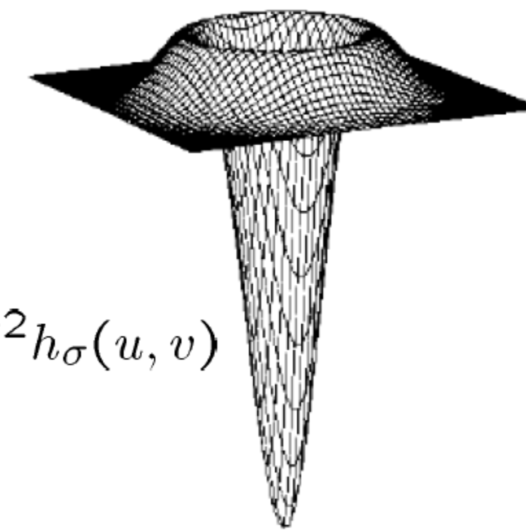
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

∇^2 is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$