Biomedical Imaging & Analysis

Lecture 4, Part 2. Fall 2014

Image Formation & Visualization (III):

Ultrasound image formation

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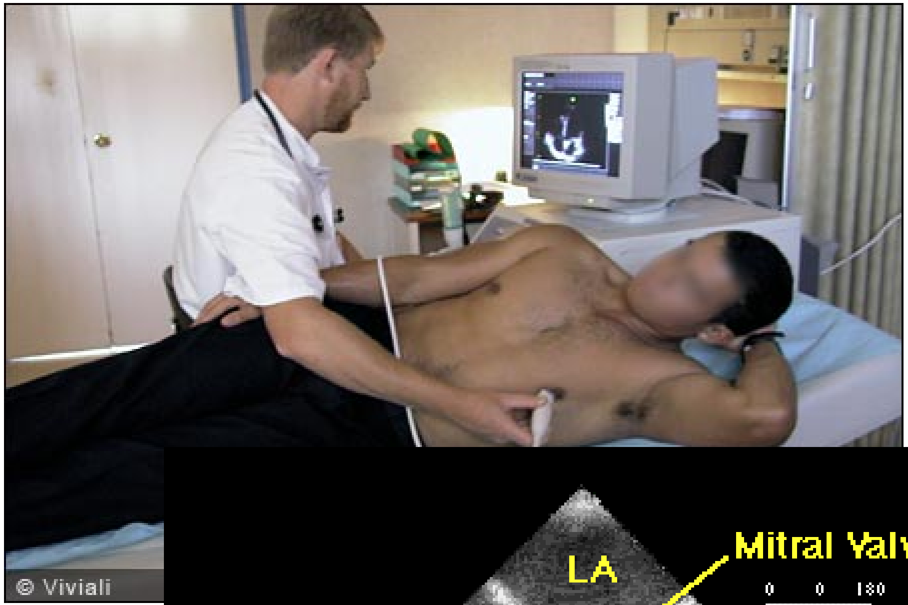
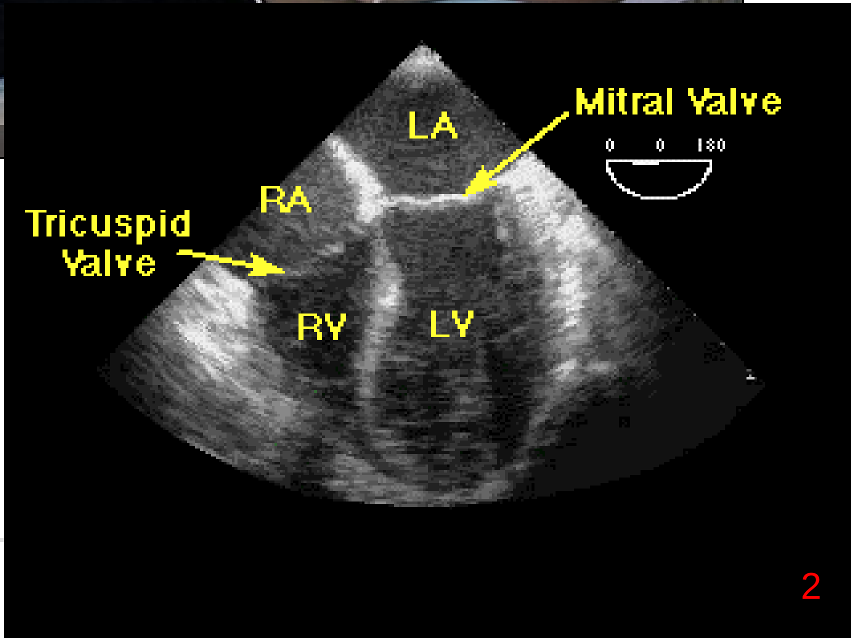
Ultrasound:

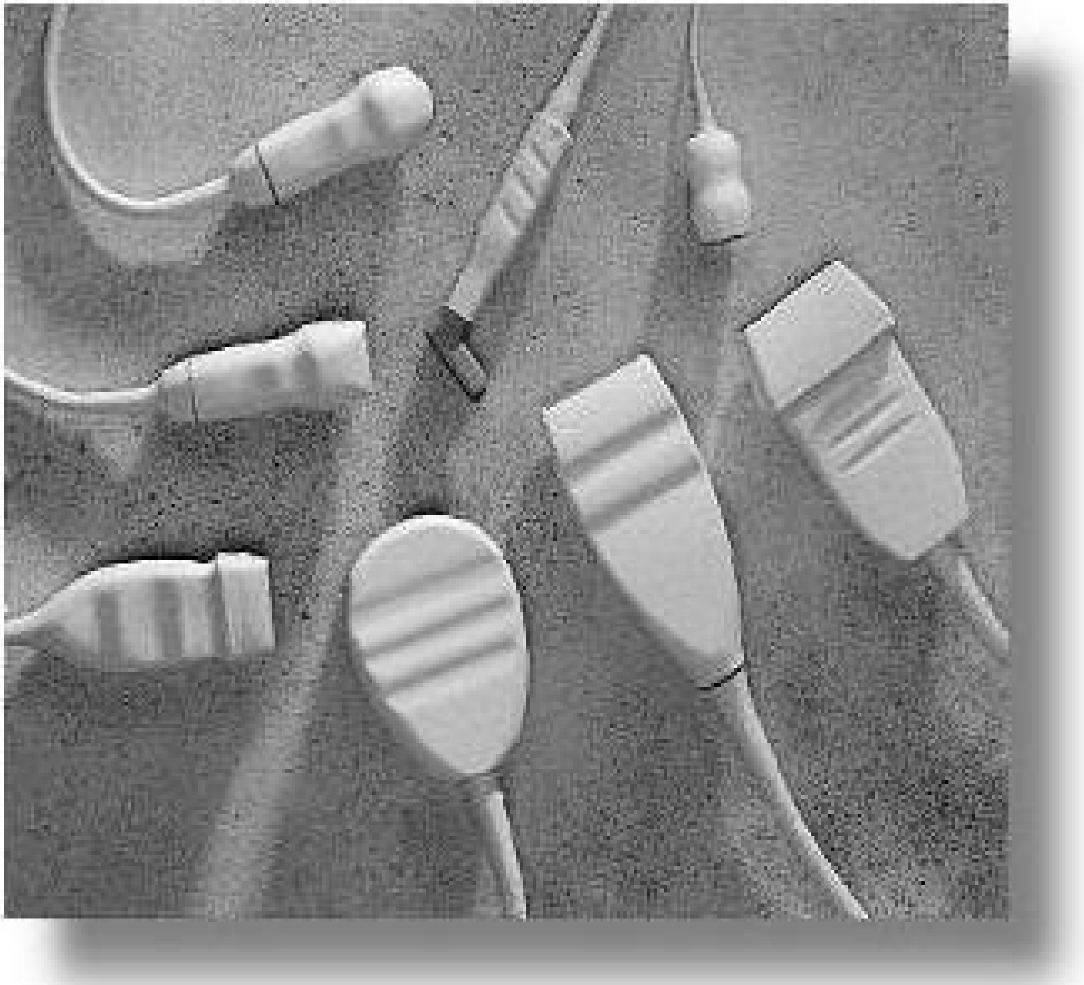
Physics meets Clinic (Part III)





Ultrasound:

Physics meets Clinic (Part III)



Ultrasound Basics

Ultrasound

- sound waves with frequencies above the normal human range of hearing.

Sounds in the range from 20-100kHz

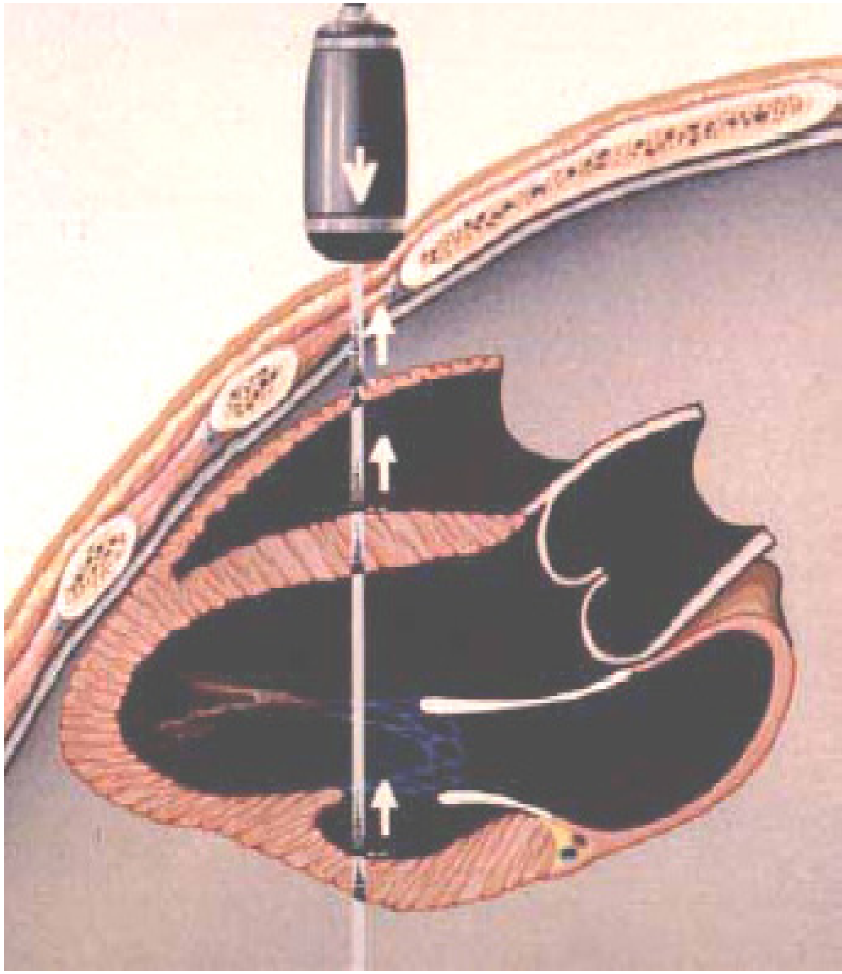
Infrasound

- sounds with frequencies below the normal human range of hearing.

Sounds in the 20-200 Hz range

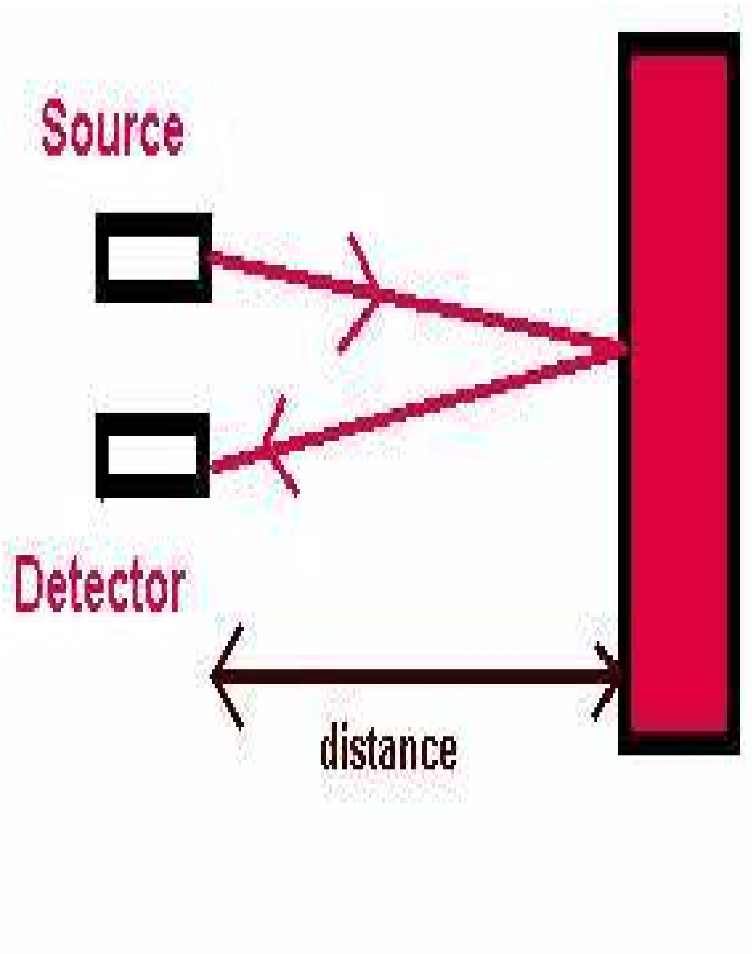
Ultrasound: **Principle**

- Probe sends high-frequency (1-10Mhz) sound waves into body
- Sound waves travel into tissue and get reflected by boundaries
- Reflected waves recorded by probe
- Time of flight gives spatial info of the boundaries
- Frequency of signal depends on a tradeoff: resolution v/s attenuation .



Basic Premise

- The principles of ultrasound are that a pressure wave (ultrasound) is transmitted into the body and the reflected wave is detected
- The time interval between transmission and reception give the distance to the reflector
- Sound is transmitted as a short pulse
- Time of return α distance of reflecting surface from probe
  - Spatial encoding

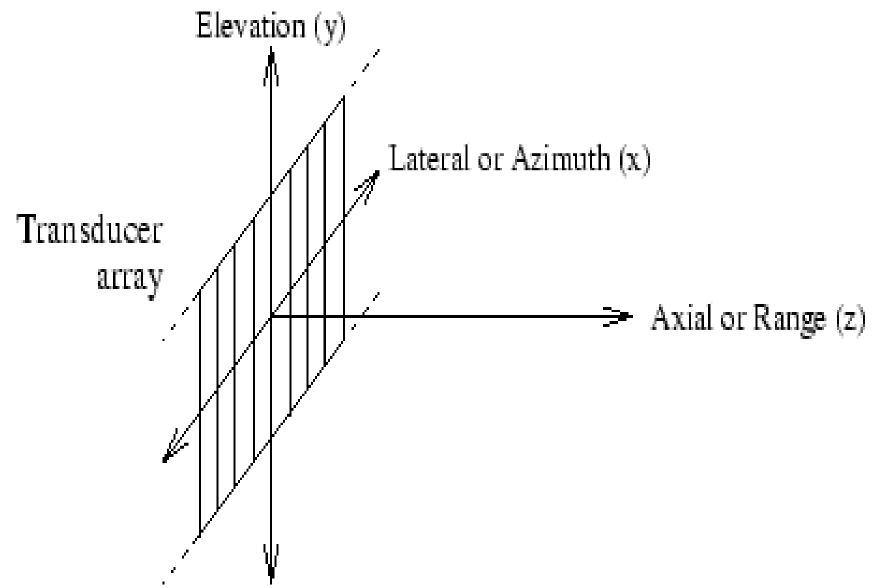


Ultrasound **Transducer**

- **Piezoelectric** – an alternating voltage across the crystal causes it to flex and contract, emitting sound.
- Piezoelectrics also generates alternating voltage in response to a returning sound wave.
- It emits sound waves and receives them.

US Transducer Coordinate System

- The spatial coordinate system used to describe the field and resolution of an ultrasound transducer array.



Speed of Sound

Medium	Velocity m/sec
air (20 C)	343
air (0 C)	331
water (25 C)	1493
sea water	1533
diamond	12000
iron	5130
copper	3560
glass	5640

Acoustic parameters of medium

$$c = \sqrt{\frac{K}{\rho}} \qquad \left[m.s^{-1}\right]$$

Speed of US *c* depends on elasticity and density r of the medium:

K - modulus of compression

in water and soft tissues c = 1500 - 1600 m.s^{-1}, in bone about 3600 m.s^{-1}

Speed of Sound Waves

In gas and liquids:

$$v = \sqrt{\frac{B}{\rho}}$$

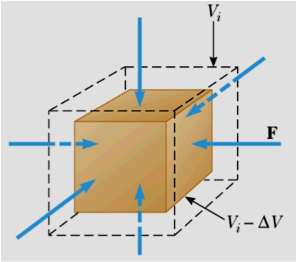
In solids:

$$v = \sqrt{\frac{Y}{\rho}}$$

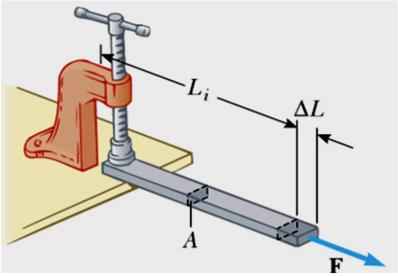
Y… Young's modulus

B… Bulk modulus of medium

ρ…density of material



Bulk modules determines the volume change of an object due to an applied pressure P.



Young's modulus determines the length change of an object due to an applied force F.

$$B = \frac{\text{volume stress}}{\text{volume strain}} = \frac{F/A}{\Delta V/V_i} = \frac{\Delta P}{\Delta V/V_i}$$

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i}$$

1D Wave Equation

- Feynman derives the wave equation that describes the behaviour of sound in matter in one dimension (position x) as:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

- Provided that the speed c is a constant, not dependent on frequency (the dispersion-less case), then the most general solution is: $p = f(ct - x) + g(ct + x)$

1D Wave Equation

- Here, f and g are any two twice-differentiable functions. This may be pictured as the superposition of two waveforms of arbitrary profile, one (f) travelling up the x-axis and the other (g) down the x-axis at the speed c. The particular case of a sinusoidal wave travelling in one direction is obtained by choosing either f or g to be a sinusoid, and the other to be zero, giving:

$$p = p_0 \sin(\omega t \mp kx)$$

- where, *omega* is the angular frequency of the wave and k is its wave number.

3D Scalar Wave Equation

$$\nabla^2 f(x,y,z,t) = \frac{1}{v^2}\frac{\partial^2 f(x,y,z,t)}{\partial t^2}$$

- Solution in Cartesian coordinates

$$\tilde{f}(\mathbf{r},t) = \tilde{A}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

- For a pressure, *p*, and speed of sound, *c*:

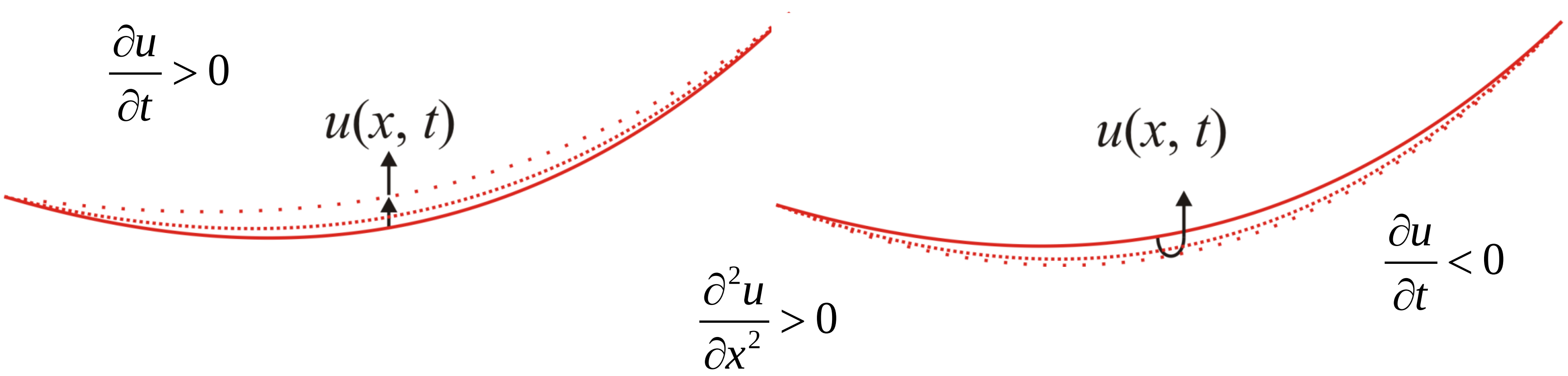
$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

What does this mean – Shape of Wave: Acceleration Proportional to Concavity

We can see this visually:

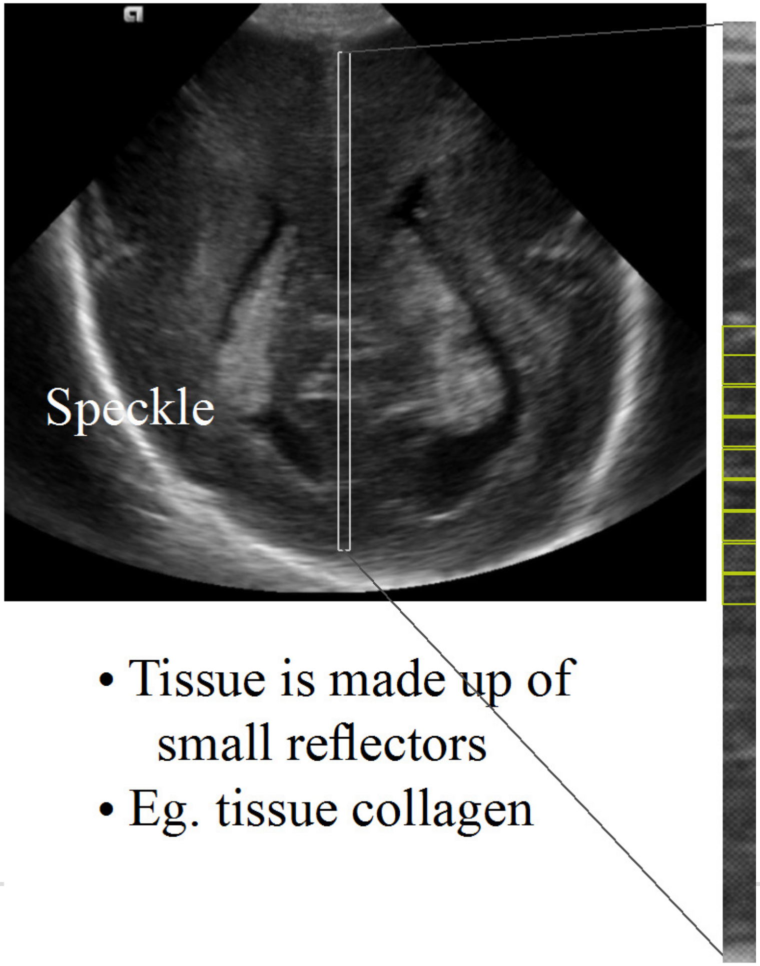
$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

– If the function u is concave up at (x, t), the acceleration of u over time will be positive, and vice versa.



Ultrasound Reflectors

- Tissue itself can be thought of as containing many tiny reflectors. Acoustic signal attenuates with depth.

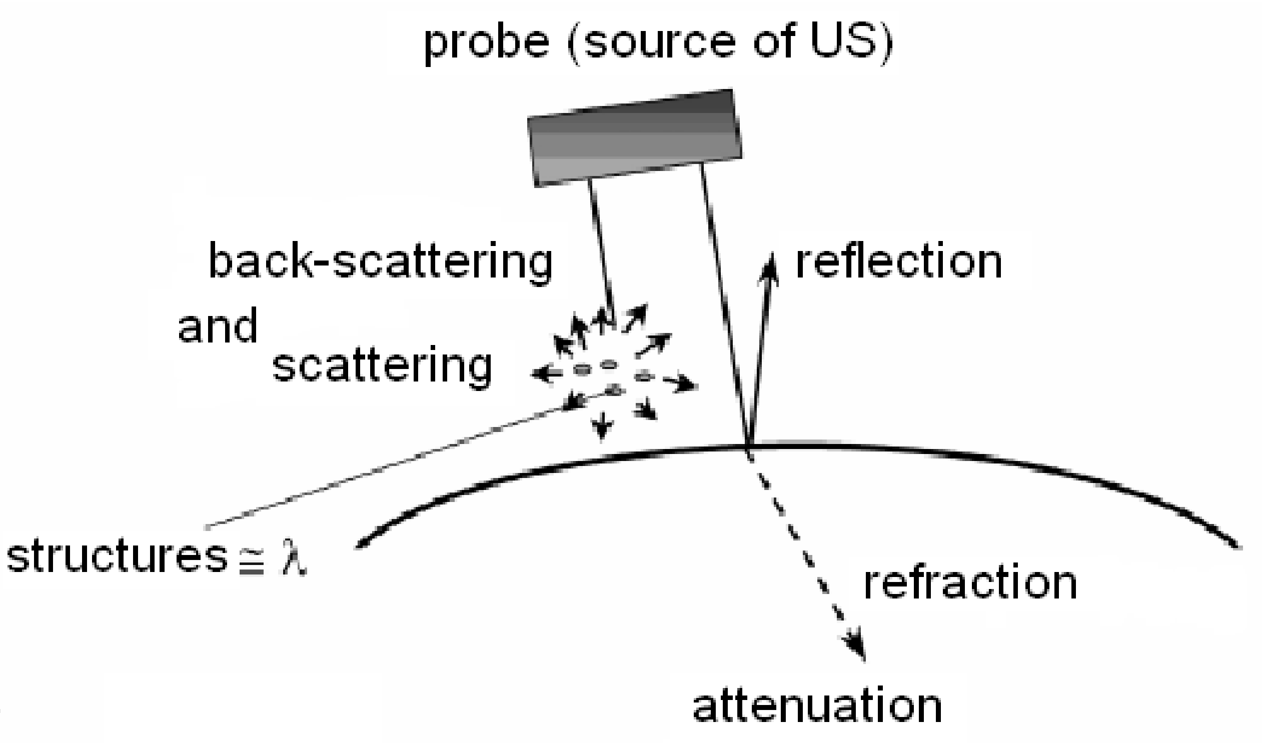


- Tissue is made up of small reflectors
- Eg. tissue collagen

Material!	*Attenuation(dB/cm/MHz)*
Water !!	0.0022
Fat! !	0.5 - 1.8
Soft Tissue!	**1**! ! !
Air! !	12
Bone! !	20

Acoustic parameters of medium:

- Interaction of US with medium – reflection and back-scattering, refraction, attenuation (scattering and absorption)



Reflection, Diffraction, Refraction of US Waves

- **Reflection of sound waves**
  - o Reflection of sound waves off surfaces can lead to the phenomenon of an echo.

- **Diffraction of Sound Waves**
  - Diffraction involves a change in direction of waves as they pass through an opening or around a barrier in their path. The amount of diffraction (the sharpness of the bending) increases with increasing wavelength and decreases with decreasing wavelength.

- **Refraction of Sound Waves**
  - Refraction of waves involves a change in the direction of waves as they pass from one medium to another.
  - Refraction, or bending of the path of the waves, is **accompanied by a change in speed and wavelength** of the waves.

Simulated Ultrasound

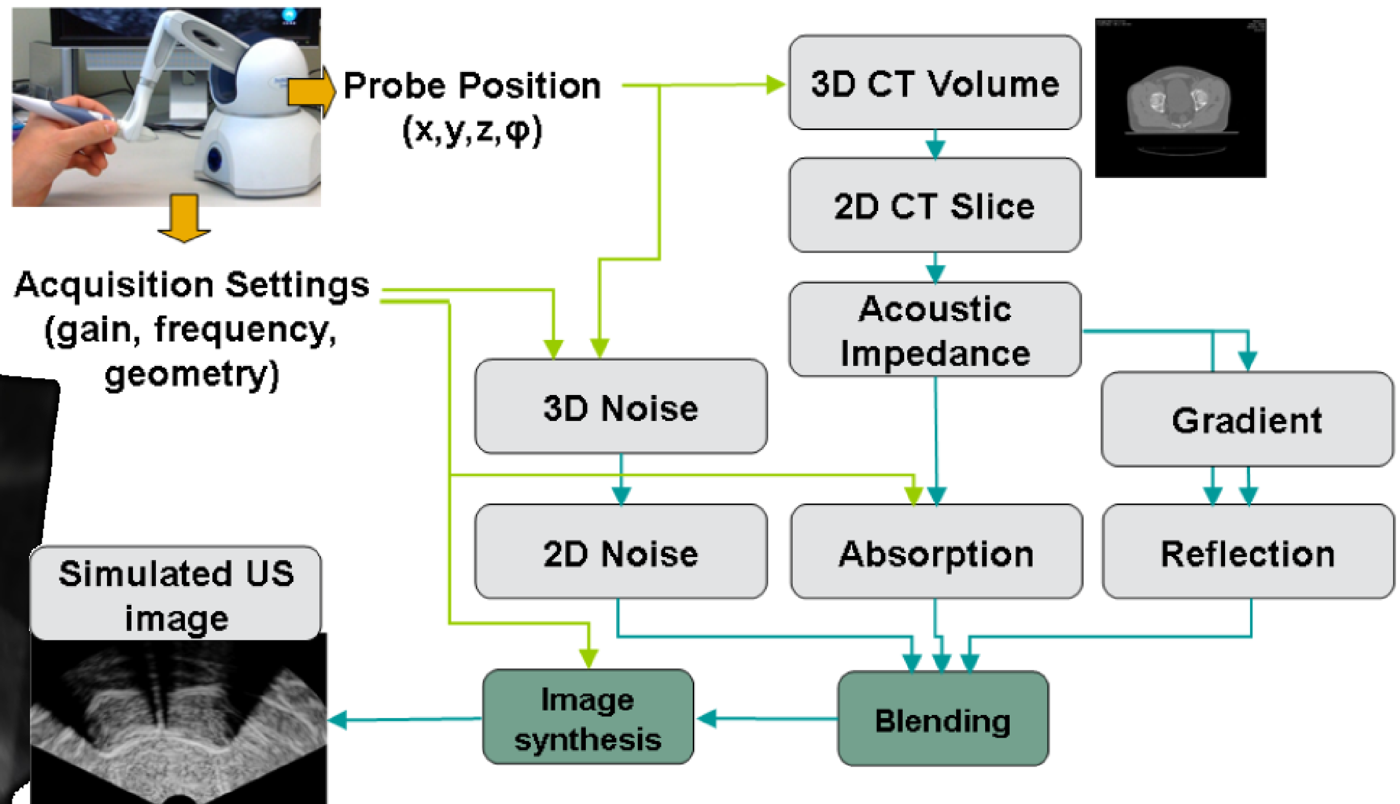


Figure 1. From 3-D CT data, 3-D noise, probe position, and acquisition settings an US image is simulated.

Ultrasound Basics

- Speed of sound is dependent on the medium through which it is traveling
  - For soft tissue, it is 1540 m/s
  - Much faster for bone, much slower in air
  - Have to wait for all of the transmitted energy to return before we can transmit a new line (data collection rate is limited by depth and speed of sound).
- Boundaries between different materials cause much larger reflections:
  - Reflection is larger when **acoustic impedances** are less similar
  - % reflected = $100 * (Z_2 - Z_1)^2 / (Z_2 + Z_1)^2$
  - % transmitted = 100 - %reflected
  - Air has very small Z, soft tissue is 1.62, bone is much larger

Relative sound intensity

Attenuation of US expresses decrease of wave amplitude along its trajectory. It depends on frequency

$$I_x \;=\; I_o\, e^{-2\alpha x} \qquad\qquad \alpha \;=\; \alpha'.f^2$$

I_x – final intensity, I_o – initial intensity, 2x – medium layer thickness (reflected wave travels „to and fro“), α - linear attenuation coefficient (increases with frequency).
Since

$$\alpha = \log_{10}(I_0/I_X)/2x$$

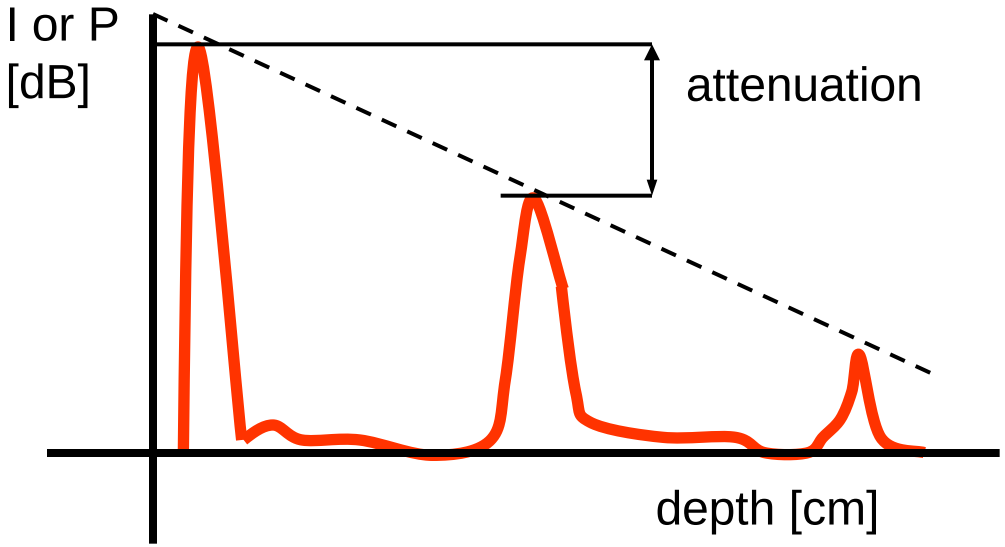
we can express α in units dB/cm. At 1 MHz: muscle 1.2, liver 0.5, brain 0.9, connective tissue 2.5, bone 8.0

Relative sound intensity

- We measure relative sound intensity (energy which is reflected), generally compared to transmitted energy:
  - Relative sound intensity (dB) = $10 * \log(I / I_0)$
- Time Gain Control (TGC) - amplifies signal to offset expected axial attenuation
- Axial Resolution = Spatial Pulse Length / 2 = (n*lambda) / 2
  - If more/less energy has been reflected by a certain depth than expected, that depth will appear darker/brighter than if everything were homogenous.
- Lateral resolution = beam width, smaller with increasing frequency for fixed transducer, but higher frequency has less penetration (higher attenuation)

Attenuation of ultrasound

When expressing intensity of ultrasound in decibels, i.e. as a logarithm of I_x/I_0, we can see the amplitudes of echoes to decrease linearly.



$$\frac{I_x}{I_0} = e^{-2\alpha x} \Rightarrow \ln\frac{I_x}{I_0} = -2\alpha x \Rightarrow \log\frac{I_x}{I_0} = -k' x$$

Acoustic Impedance (Z)

Acoustic impedance: product of US speed *c* and medium density ρ

$$Z = \rho \cdot c \quad (\text{Pa.s/m})$$

$Z.10^{-6}$: muscles 1.7, liver 1.65 brain 1.56, bone 6.1, water 1.48 .

Reflection and Transmission on interfaces in terms of Acoustic Impedance, Z

We suppose perpendicular incidence of US on an interface between two media with different Z - a portion of waves will pass through and a portion will be reflected (the larger the difference in Z, the higher reflection).

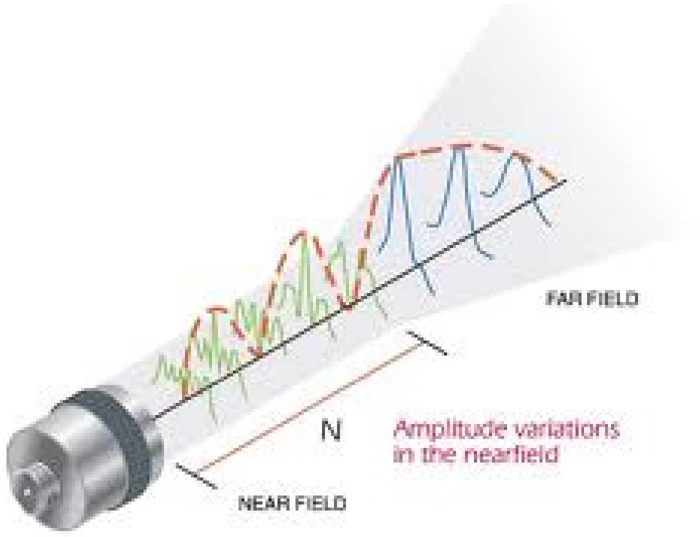
$$R = \frac{P_1}{P} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$D = \frac{P_2}{P} = \frac{2 Z_1}{Z_2 + Z_1}$$

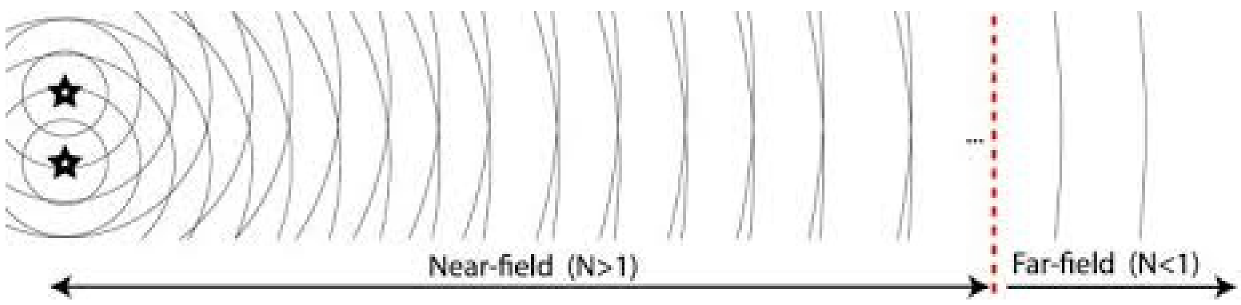
Coefficient of reflection R – ratio of acoustic pressures of reflected and incident waves

Coefficient of transmission D – ratio of acoustic pressures of transmitted and incident waves

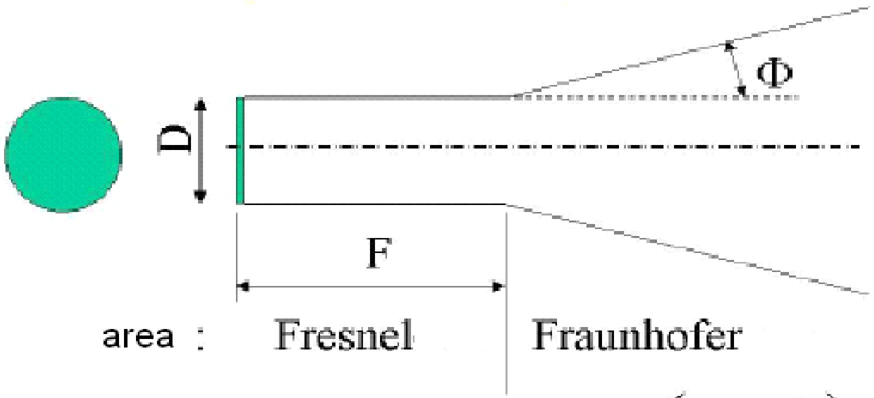
Near field and far field effects



- The adjacent points pressures effect each other. This effect is less in far field.



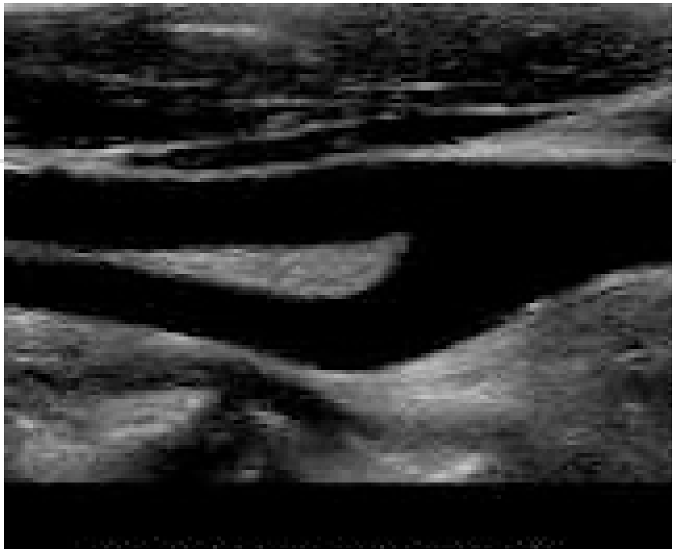
Near field and far field effects



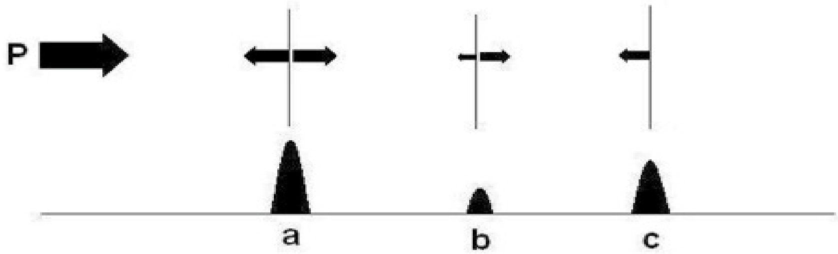
$$F = \frac{D^2}{4\lambda}[m] \qquad \Phi = \arcsin\left(1{,}22\frac{\lambda}{D}\right)[°]$$

- Near field (Fresnel area) – this part of US beam is cylindrical – there are big pressure differences in beam axis
- Far field (Fraunhofer area) – US beam is divergent – pressure distribution is more homogeneous
- Increase of frequency of US or smaller probe diameter cause shortening of near field - divergence of far field increases.

B-Mode US Imaging



- B Mode Imaging
- Produces a 2D grayscale Image.
- Brightness is proportional to amplitude of the reflected sound waves.



•The time at which the signals are received indicates depth.

$$c= 2D/t$$

B-Mode US Imaging

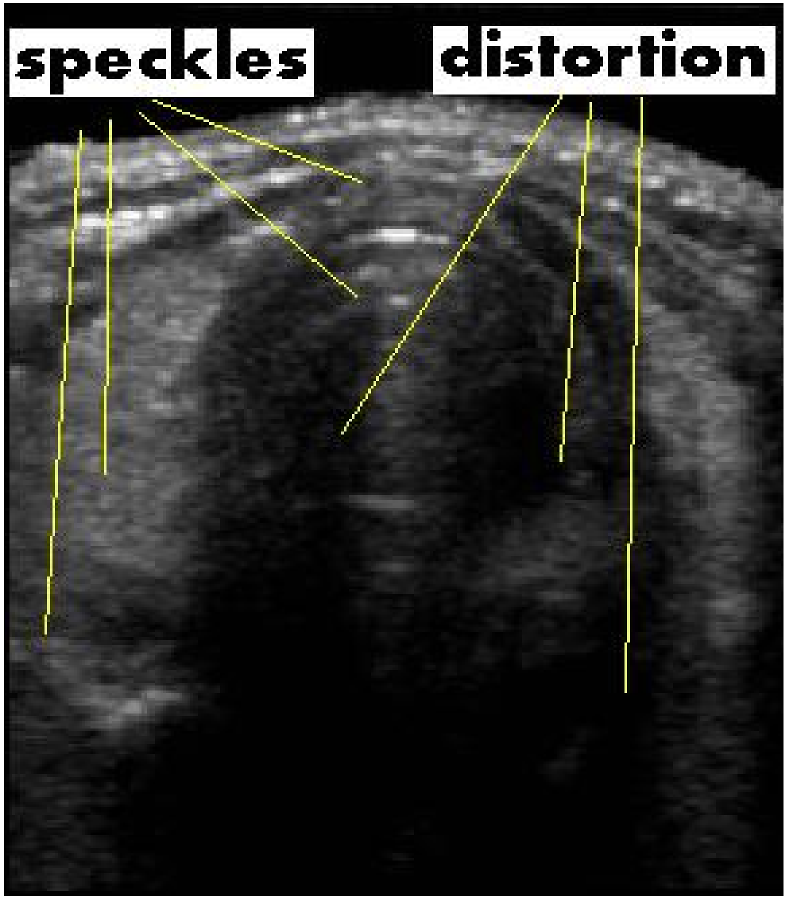
- B-Mode
  - Brightness-mode imaging is the most commonly used method
  - Echo amplitudes are gray-scale encoded, with the highest amplitudes encoded in white
  - Logarithmic scaling is employed to coincide with the eyes ability to discern differing shades of brightness

Ultrasound: What's bright / dark?

- Ultrasound: Dark means low scattering or reflection
  - Fluid (CSF, blood) have fewer scatterers than soft tissue
  - Anything beyond a highly reflecting interface will look dark, since little energy reaches it
  - Anything far from the transducer will appear dark, because most of the energy has been attenuated
- Ultrasound: Bright means high scattering or reflection
  - Sulci, Bone, Choroid plexus all have high scattering
  - Interface between tissue and air or tissue and bone will have very high % reflection

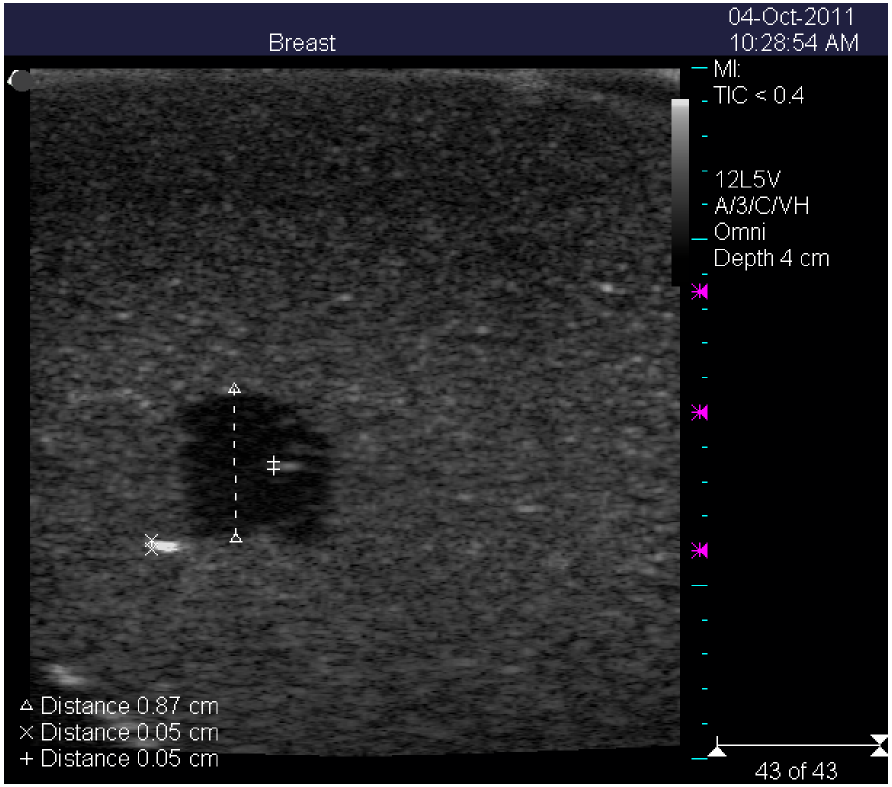
Speckle Removal - Median Filter

- No radiation
- Poor resolution (1mm) non-uniform, distortion, noise
- Low penetration properties
- One 2D slice or several slices (2.5D)
- Relatively cheap, easy to use
- Preoperative and intraoperative use.



- **When are the speckles useful..?**

Examples of US - **tumors**



Cardiac US Image

Figure 35

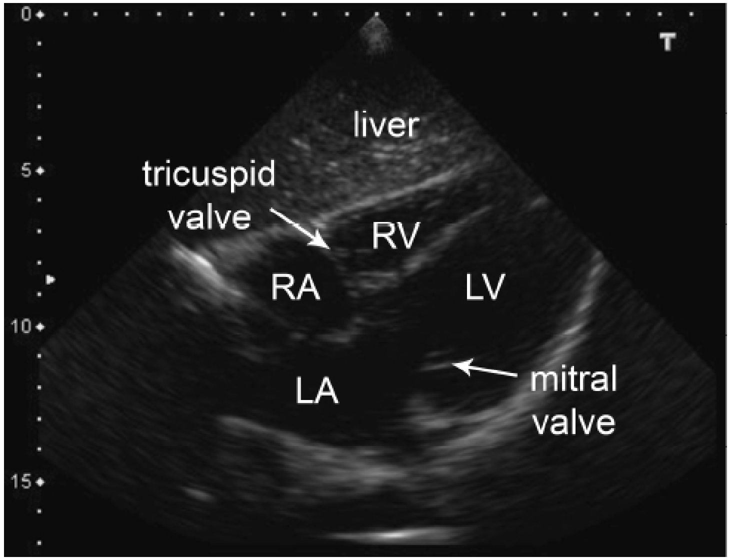
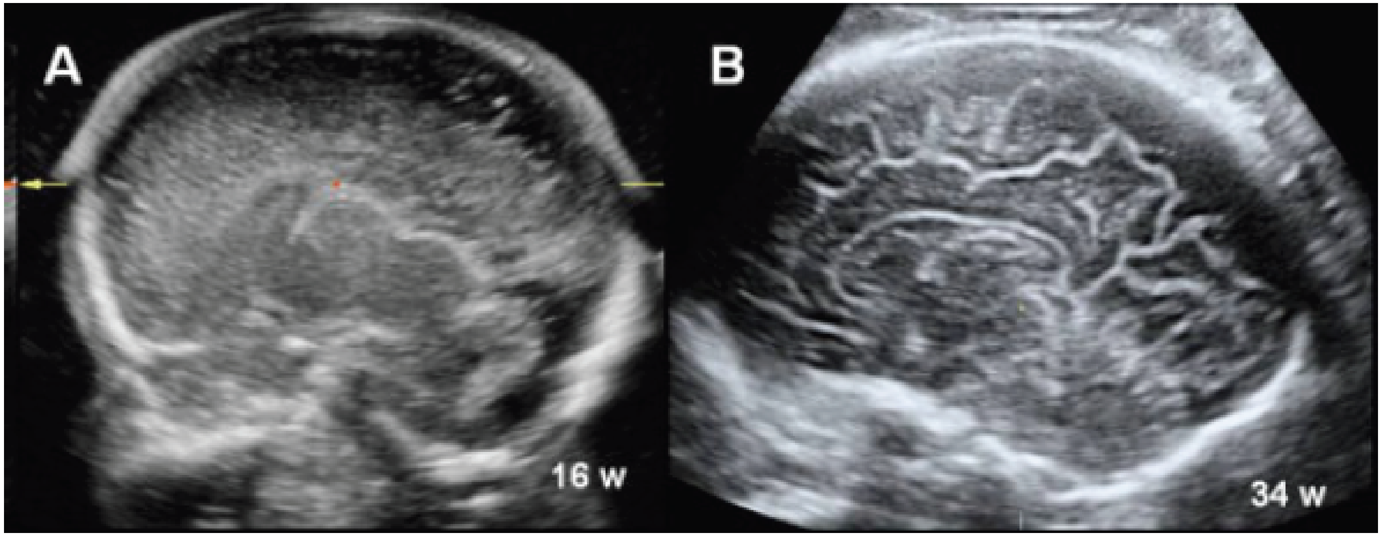


Figure 36

Development of the Cortex

- Initally smooth
- Progression of infoldings (sulci)
  - Sylvian fissure
  - Calcarine fissure

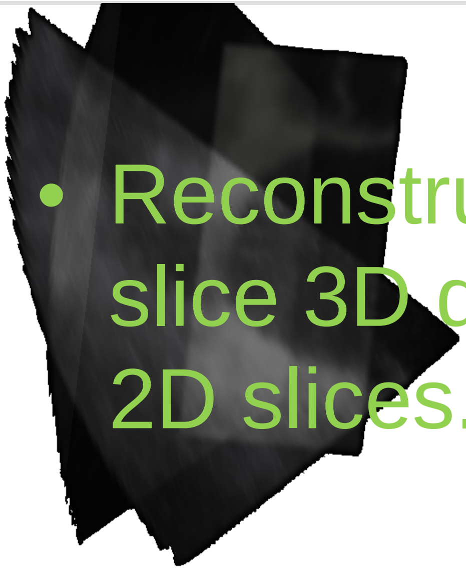


Sagittal View

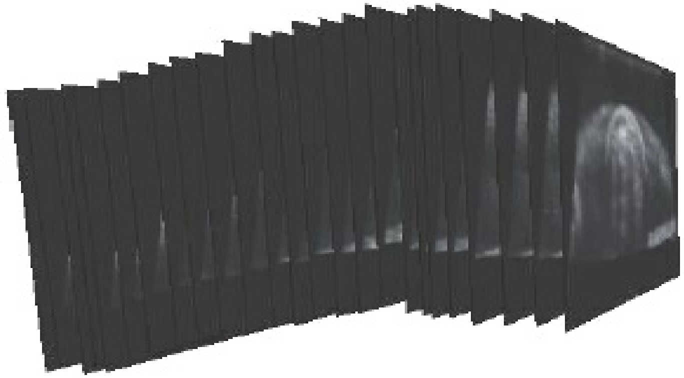
Monteagudo et al., Prenatal Diag 2009


15 Aug 97
11:52:41 am
TIb 0.2 MI 0.6
Fr #97 7.0 cm
C7-4 40R OB/General
HDI
Map 3
150dB/C 3
Persist Med
Fr Rate Med
2D Opt:Res
FETAL NOSE / LIPS
Image courtesy
of ATL

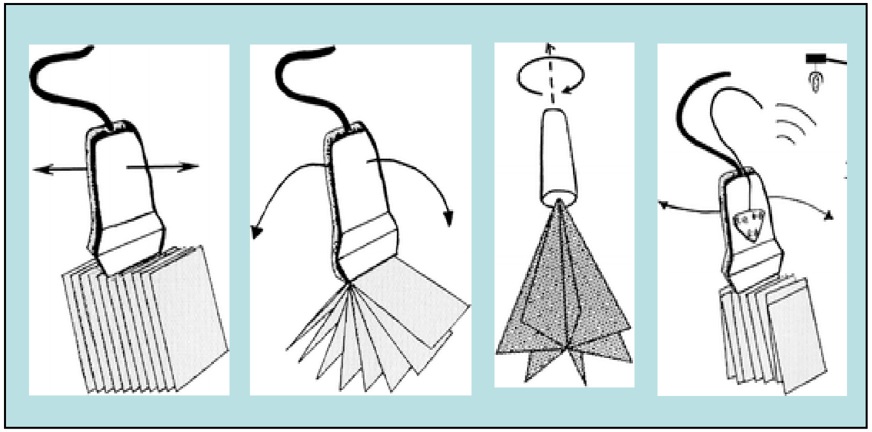
3D ultrasound ..?



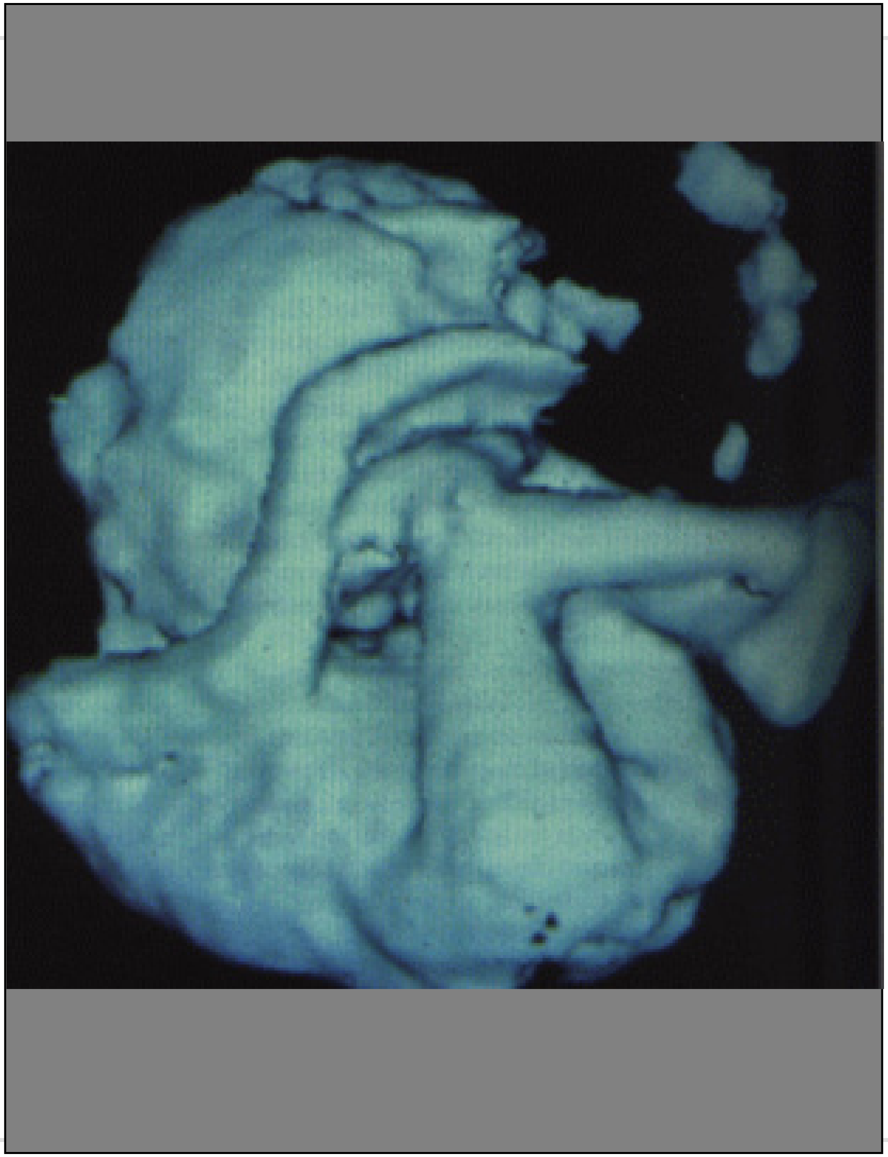
- Reconstruct and re-slice 3D data from 2D slices.

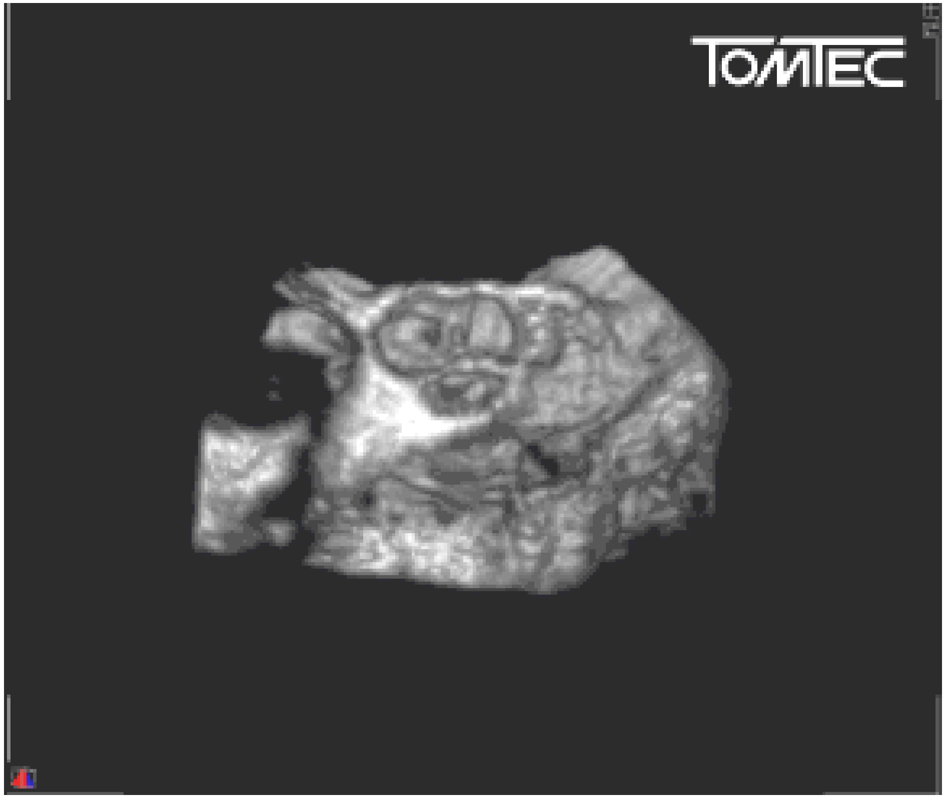


- 3D probe acquisition methods: linear, rotation, fan-like, free-hand.



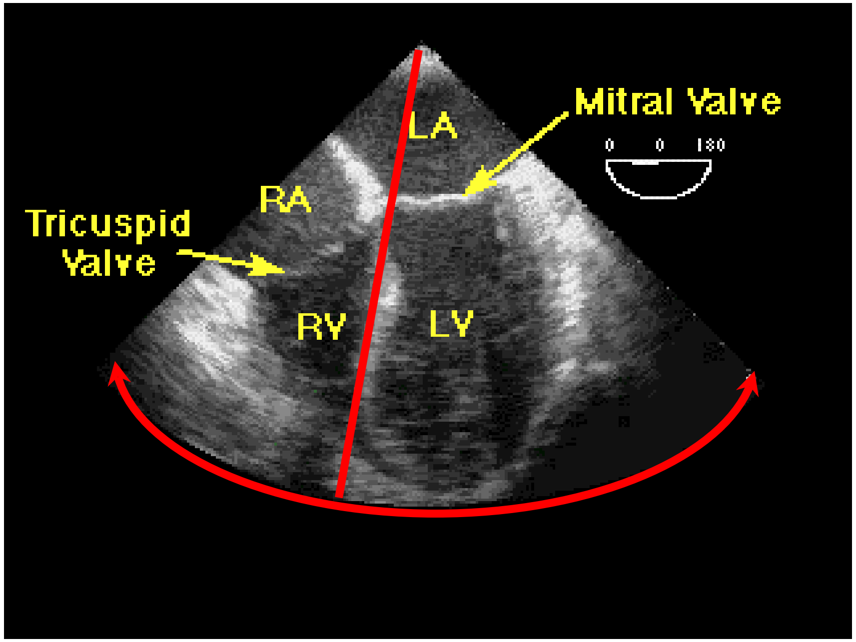
3D ultrasound images





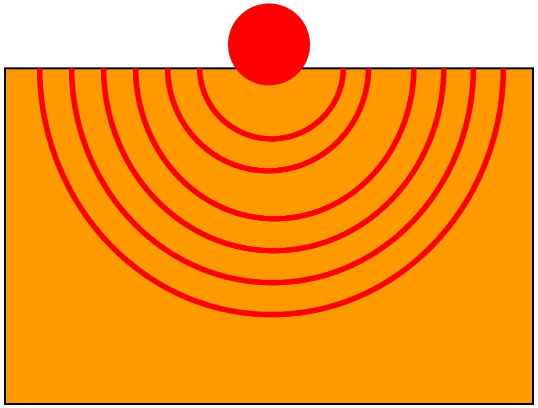
US Beam Forming

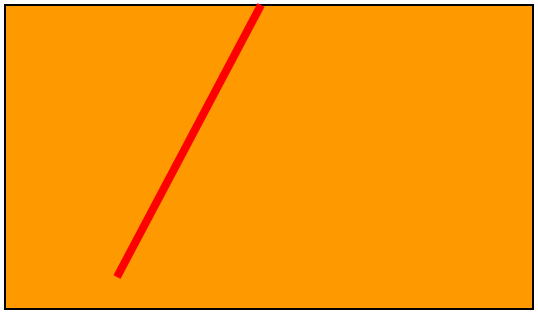
- Restricting the US wave to a beam limits the spatial information to a line perpendicular to the probe
- A 2D image is formed by sweeping the line back and forth to map out a circular segment



Generate a Beam

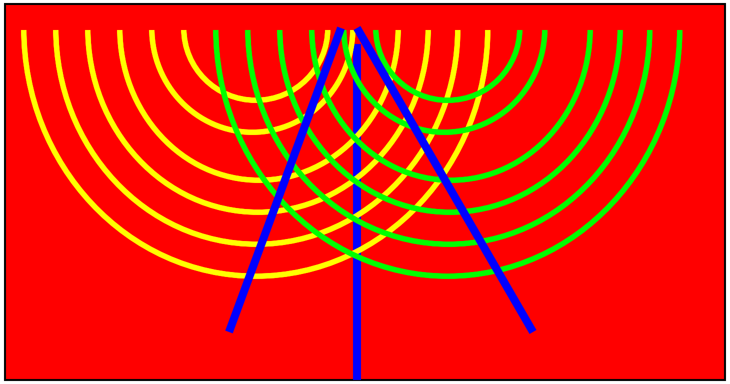
- A piezoelectric transmitter is used to generate the sound waves
- However, sound waves from a single source tend to spread along a circular warfront in tissue
- How do we restrict the sound to make a beam?





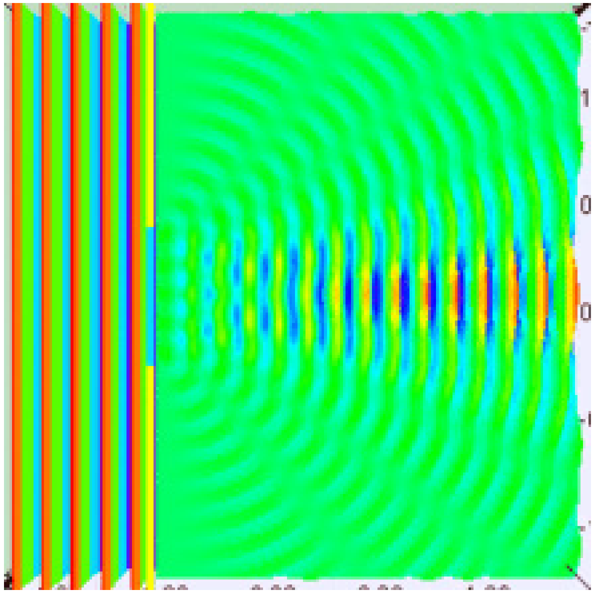
Multi component probe

- Designing the US probe head as an **array** of piezoelectric transducers allows an interference pattern to be established that results in a beam
- However, it also introduces side lobes that can cause significant artifacts



Diffraction & the Sinc Function **(Derivation time!)**

- No-one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them.

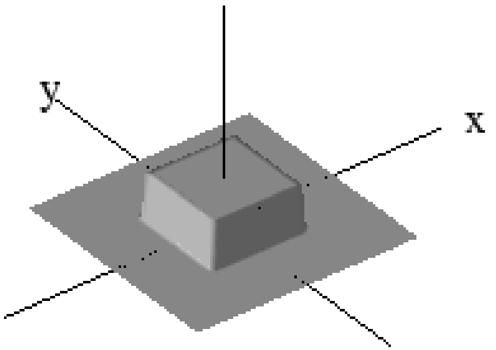


Richard Feynman

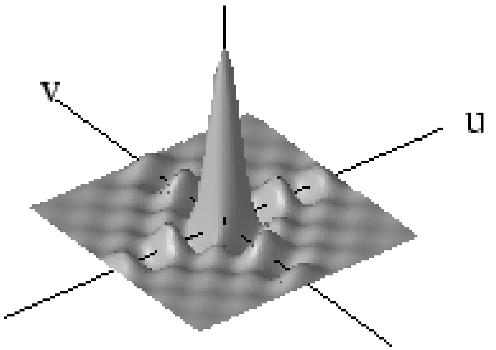
Transmit Design

- An array of multiple signal waveforms can be analyzed by Fourier theory to generate the shape of the transmitted beam:
  - The normalized **sinc function** is the **Fourier transform** of the rectangular function with no scaling.
- The grid spacing of transducer elements can be adjusted based on this Fourier analysis to suppress side lobes

Transducer
$\Pi(x)\Pi(y)$



Transmitted beam
$\mathrm{sinc}(u)\ \mathrm{sinc}(v)$

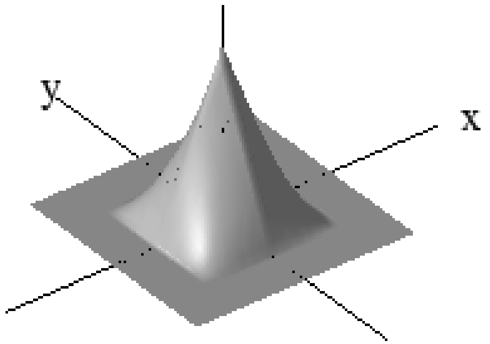


Reception

- When this aperture is used for *both* transmit and receive, as is almost always the case, the sensitivity to the received signal is found by performing a second FT on the signal
  - Further reinforcing the beam response of the US probe.

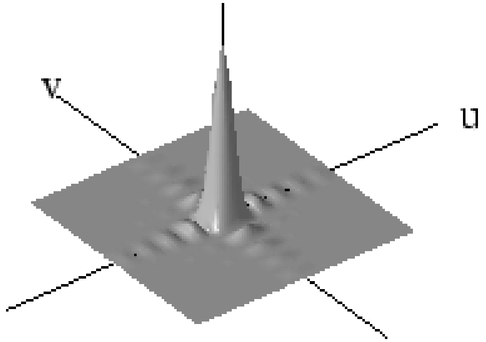
Reflected Signal

$\Lambda(x)\Lambda(y)$



Received beam

$\mathrm{sinc}^2(u)\ \mathrm{sinc}^2(v)$



Summary of US facts

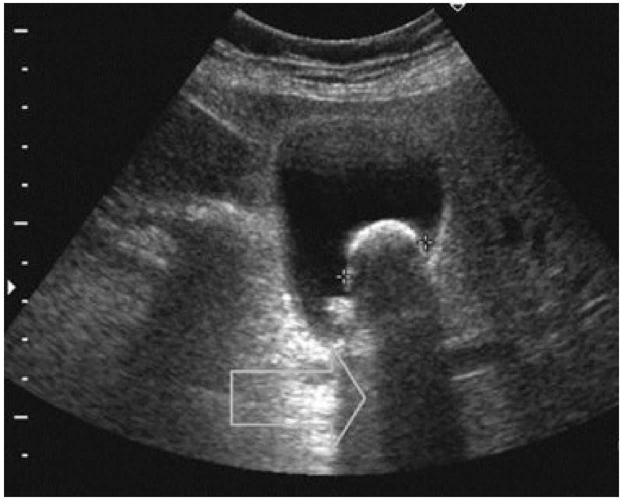
- Sound waves in the body travel at 1,540 m/s
- The best reflections come from structures that are perpendicular to the sound waves
- Reflection coefficient
- Transmission coefficient
- Distance is one measure of signal time
- Amplitude depends on angle and reflection
  - Parallel is the worst
  - Thus US density depends on fiber direction

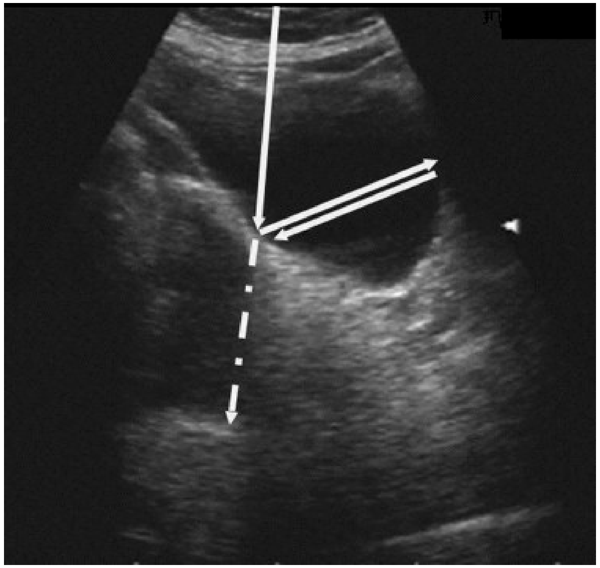
US Quality

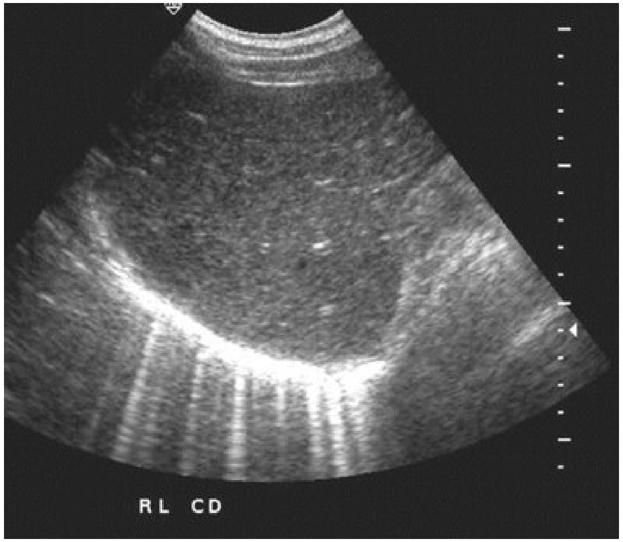
- Signal loss
  - Reflection ~10%
  - Diffracted ~
  - Absorption ~80%
    - Heating, forms limiting factor on amount of energy that can be transmitted, thus defines depth penetration
    - Absorption ~1 dB/cm MHz * (2 for imaging)
    - Thus depth determines frequency that can be used

Ultrasound Artifacts

- Shadows
- Reverberations
- Side lobes



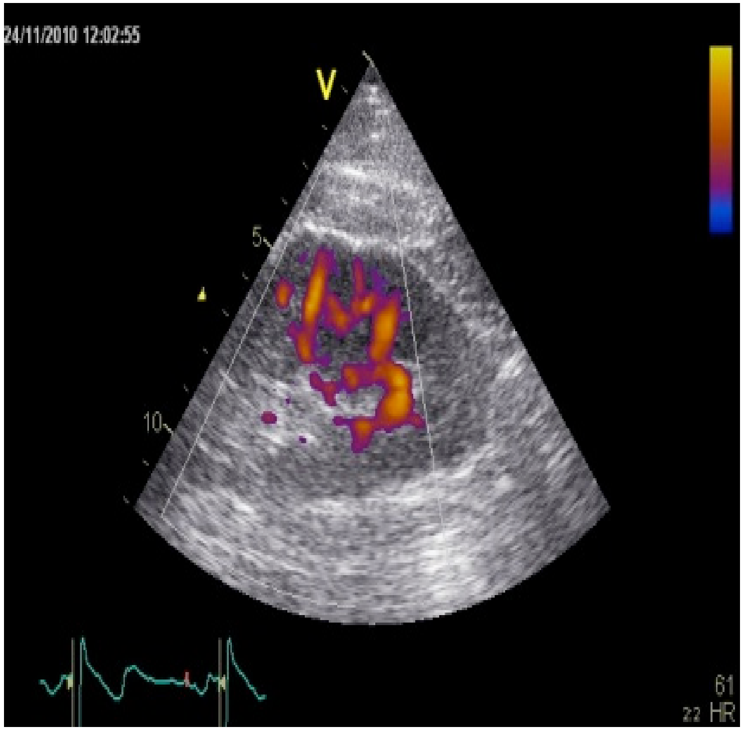




Beyond B-Mode US Imaging

Colour Power Doppler:

- Displays the amplitude of the frequency shift.
- Amplitude is a function of the number of reflectors (RBCs) with that velocity.
- Color is used to determine direction.



Beyond B-Mode US Imaging

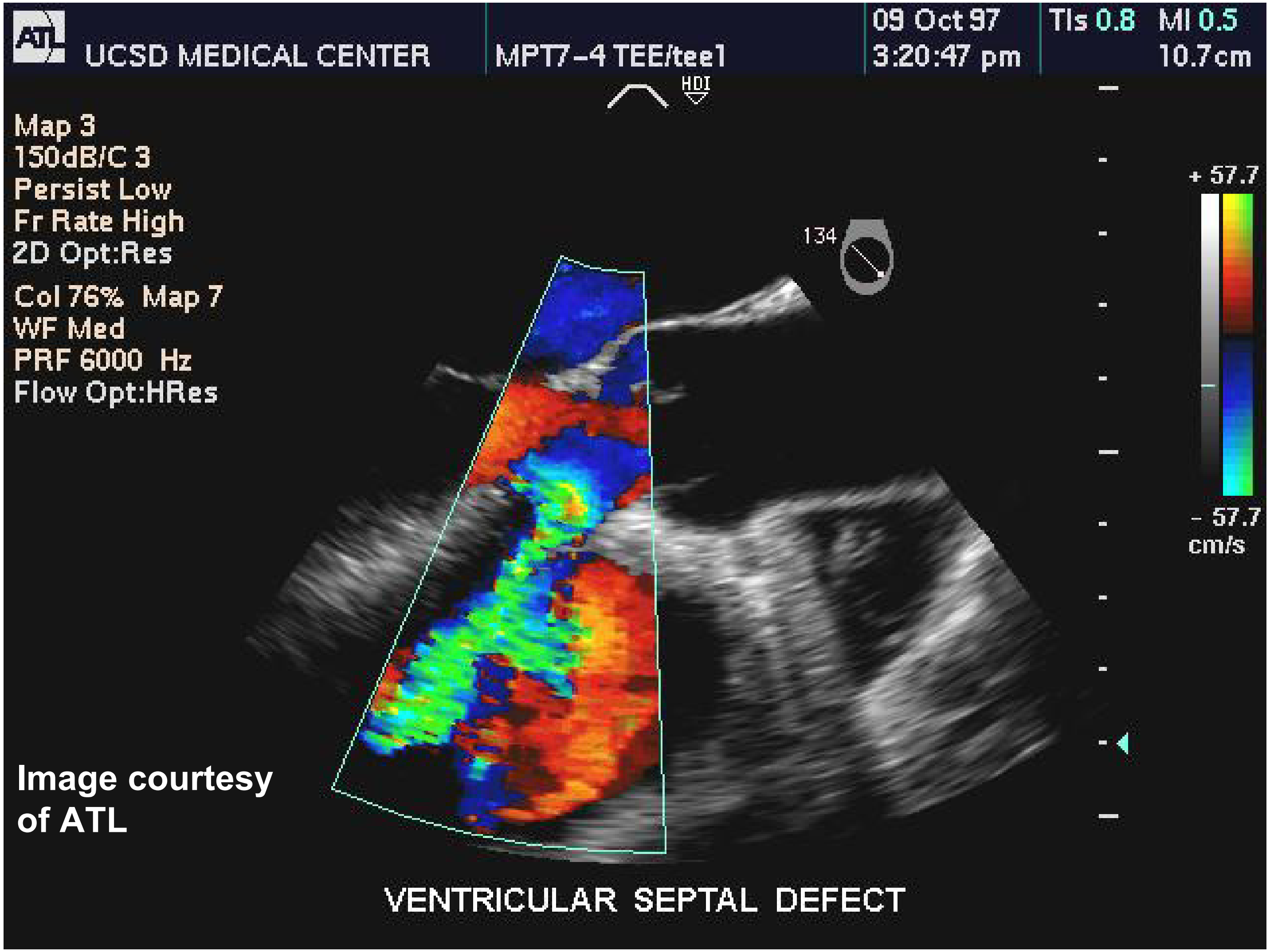
Color Doppler:

- Velocity information is represented by colour and is overlaid onto a 2D B-Mode image.
- Velocity is determined using the Doppler effect:

$$\Delta f = 2f_0 \, (v/c) \cos\alpha$$

where Δf is the Doppler shift frequency, α is the angle between the ultrasound beam and flow vector, $|v|\cos\alpha$ is the component of the velocity of the target towards the transducer, f0 is the transmitted ultrasound frequency and c is the velocity of ultrasound in the tissue.

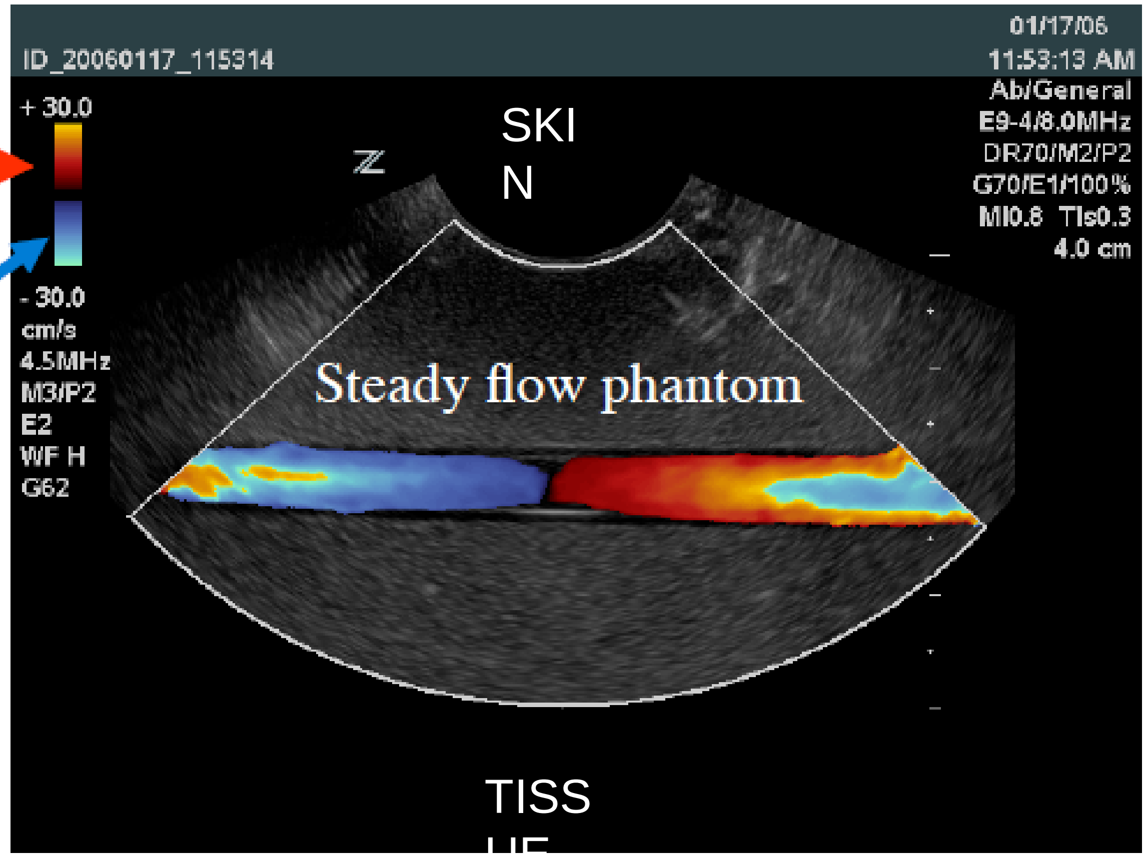
Ultrasound Hemodynamics

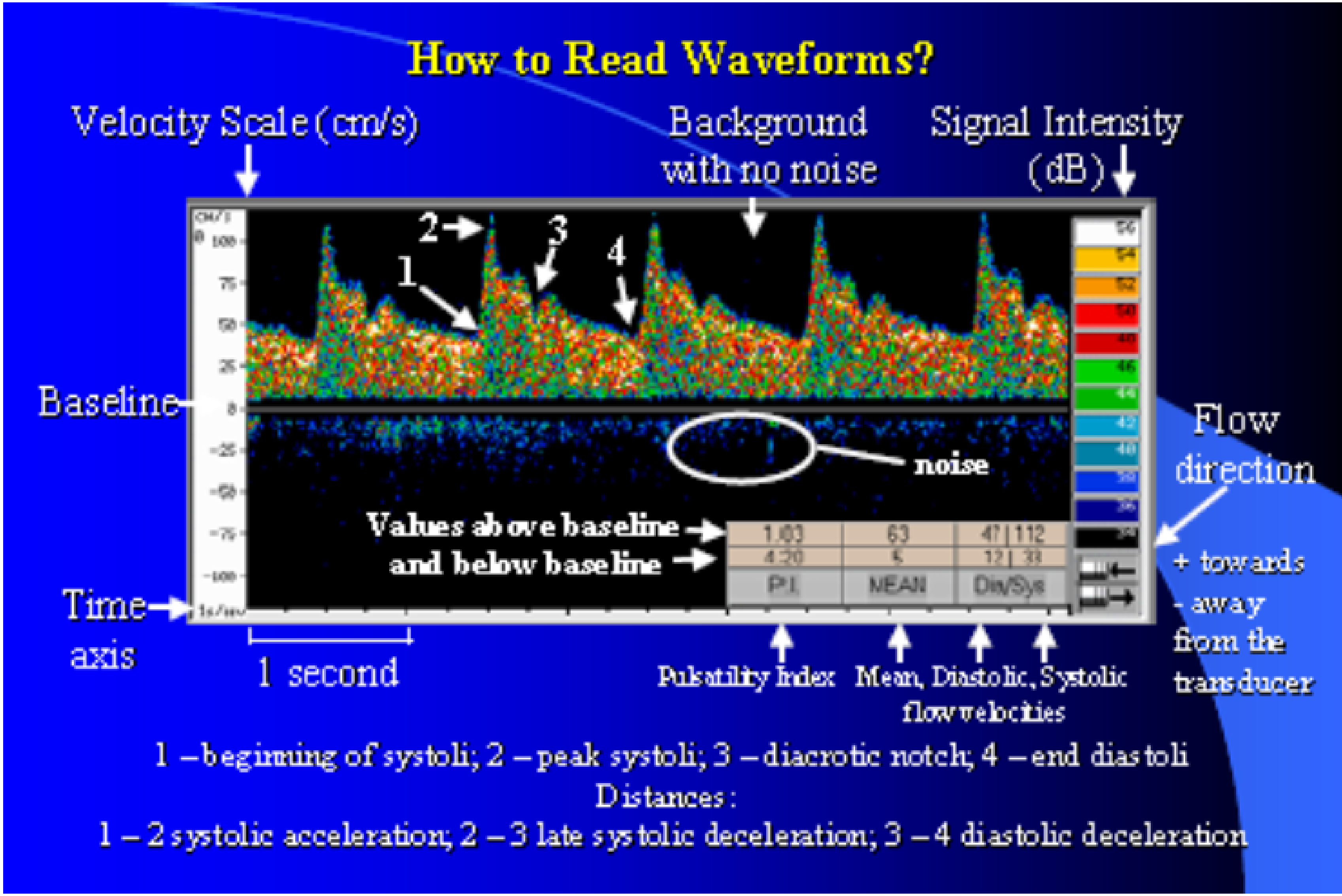


Doppler Angle Effects

Flow towards transducer

Flow away from transducer




How to Read Waveforms?
Velocity Scale (cm/s)
Background with no noise
Signal Intensity (dB)
Baseline
Time axis
1 second
noise
Values above baseline
and below baseline
P.I.
MEAN
Dia/Sys
Flow direction
+ towards
- away from the transducer
Pulsatility Index
Mean, Diastolic, Systolic flow velocities
1 – beginning of systoli; 2 – peak systoli; 3 – diacrotic notch; 4 – end diastoli
Distances:
1 – 2 systolic acceleration; 2 – 3 late systolic deceleration; 3 – 4 diastolic deceleration

Beyond B-Mode US Imaging

- **Pulse Wave (PW) Doppler:**

  - velocity is measured at a specific depth, which can be adjusted. The transducer alternates transmission and reception of ultrasound and waits for echoes from a specific depth.

  - frequency shift in the reflected pulse is measured after a certain time. This will correspond to a certain depth

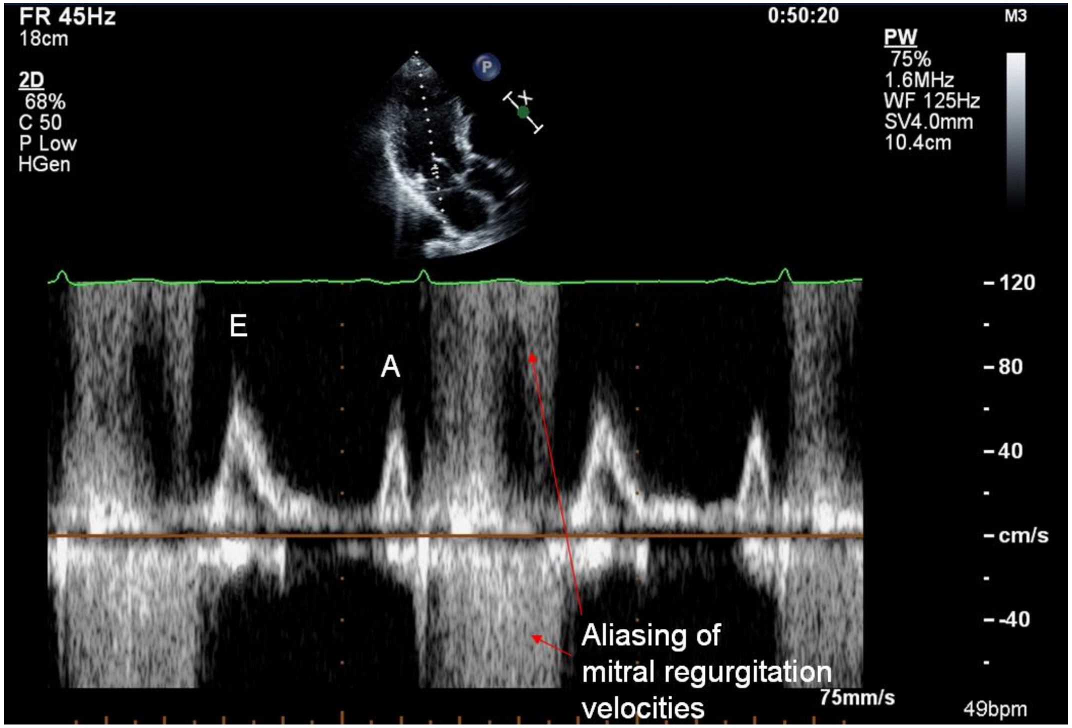
- **Continuous Wave (CW) Doppler:**

  - measures all velocities along the ultrasound beam. It provides no information about depth of the signal.

  - CW allows us to measure blood velocities along an entire line of interrogation. It requires the probe to continuously send out pulses of ultrasound along a line and continuously listen for the multitude of reflected frequency shifts that are coming back

Aliasing of Velocities (PW)

- **Aliasing**
  - When the blood moves too fast, it cannot accurate give the velocity based on Doppler shift and a phenomenon called aliasing occurs, as shown below.

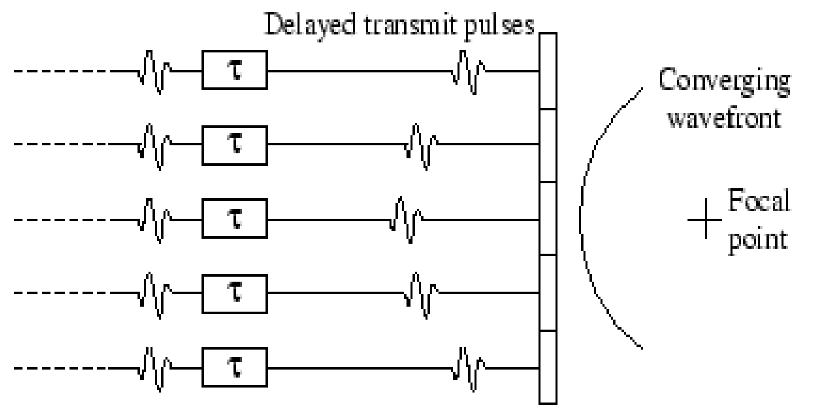


PW with the depth of interest positioned at the mitral valve inflow level.

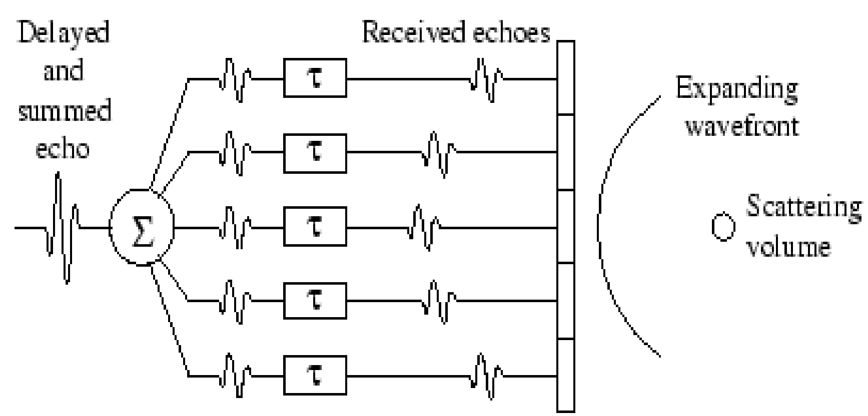
In systole, the mitral regurgitation jet isaliased as the machine cannot easily assign speed or direction to the blood flow. Filled in velocity points therefore occur above and below the baseline.

Beam Focusing (on a Sample Volume)

- However, for these Fourier design principles to work, the US waves have to satisfy a far-field approximation
  - The Sun is far from Earth, and we regard the Sun's radiation as a spherical waterfront

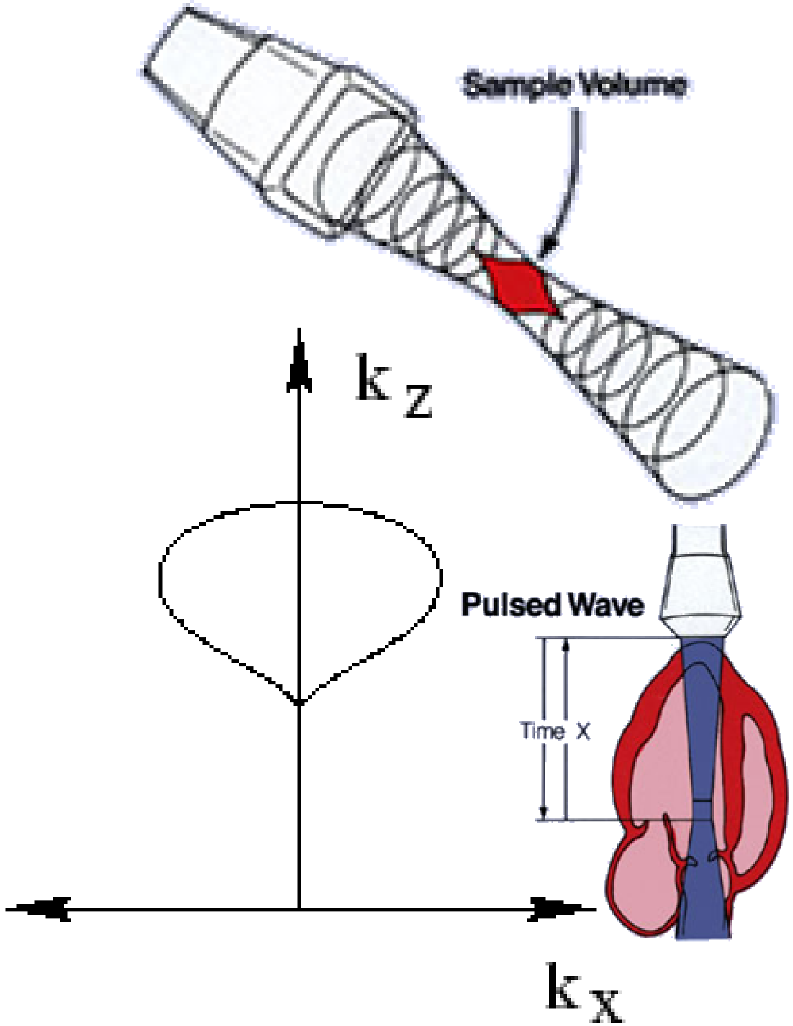


- This is accomplished in an US probe by setting relative delays in each transmission element of the probe to simulate a spherical wave
  - Only true at one focus point



K-Space Response – **Pulsed Wave Doppler**

- The sample volume is really a three-dimensional, teardrop shaped portion of the ultrasound beam.
- Its volume varies with different Doppler machines, different size and frequency transducers and different depths into the tissue. Its width is determined by the width of the ultrasound beam at the selected depth. Its length is determined by the length of each transmitted ultrasound pulse.
- A rectangular aperture (US probe head) has a tear-drop shaped response in *k*-space.
- Within this shape, signal response is not uniform.



2D velocity from ultrasound Transverse Oscillations..?

- *Reading Assignment*:

  Ultrasonic colour Doppler imaging

  David H. Evans, Jørgen Arendt Jensen and Michael Bachmann Nielsen