Notes for MATH 3210: Foundation of Analysis I

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Contents

1 Basic Topology

Metric (distance function) of x_0 and x: $d(x_0, x) = |x - x_0|$

1.1 Metric Space

Metric space X

$$d\colon M \times M \to \mathbb{R} \tag{1}$$

1. $d(x_0, x_1) \geq 0$ where $x_0, x_1 \in X$
2. $d(x_0, x_1) \iff x_0 = x_1$
3. $d(x_0, x_1) = d(x_1, x_0)$ (irrespective of order)
4. $d(x, z) \leq d(x, y) + d(y, z)$ (triangular inequity)

Euclidean metric on $\mathbb{R}^{\nvDash}$: $d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$
Discrete metric: $d(x, y) = \begin{cases} 1 & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$
Open ball of radius ϵ: $B_\epsilon(x_0) = B(x_0, \epsilon) = \{y \in M \mid d(x_0, y) < \epsilon\}$

1.2 Open Sets

1.2.1 Metric Spaces X

$U \subset X$ is open if for any $x \in U$ there exists $\epsilon > 0$ such that $B(x, \epsilon) \subset U$

Open interval is open set; closed interval is not open set.

1.2.2 Topological Spaces

Let U be a family of all sets, X be a set. U is a **topology** on X if

1. $\emptyset$ (Empty set) is always open; X is open.
2. F is a collection of open sets, then $\bigcup_{U \in F} U$ is open. (Union)
3. F is a *finite* collection of open sets, then $\bigcap_{U \in F} U$ is open. (Intersection)

Finite case: $x \in \bigcap_{U \in F} U$, $x \in U$, hence $B(x, \epsilon_U) \subset U$

$$\delta = \min \epsilon_U > 0 \quad \textit{where} \quad U \in F$$
$$B(x, \delta) \subset B(x, \epsilon_U) \subset U$$
$$\therefore B(x, \delta) \subset \bigcap_{U \in F} U$$

Infinite case: There is no minimum. $(-\frac{1}{n}, \frac{1}{n})$ then $n \to 0$, hence not open set.

Topological space: (X, U)

2 Compact Sets

In metric space X, compact sets are closed.

Compact $\Leftrightarrow$ closed and bounded (only for Euclidean metric, $\mathbb{R}^n$)