Contents

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title: "Prediction of inflation rate using SARIMA model"
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Section 1 - Introduction

The goal of this project is to build a statistical model to forecast the interest rate of Canada. By analyzing the historical economic and financial data from the website of international monetary policy(IMF), we would like to discover the underlying mathematical law of inflation. Economic indicators for measuring the inflation rate included the Consumer price index (CPI), the producer price index(PPI) and the GDP deflator. The most commonly used one is CPI, which is defined as the price of one fixed basket of consumer goods and services in one period. CPI include the cost of food, beverages, housing, education, transportation and medical care. The unit of CPI s measured in percentage, which is compared to the based year. For example, if the base year is 2010, then the CPI of 125 means that the price increased by 125 percent compared to the year 2010.

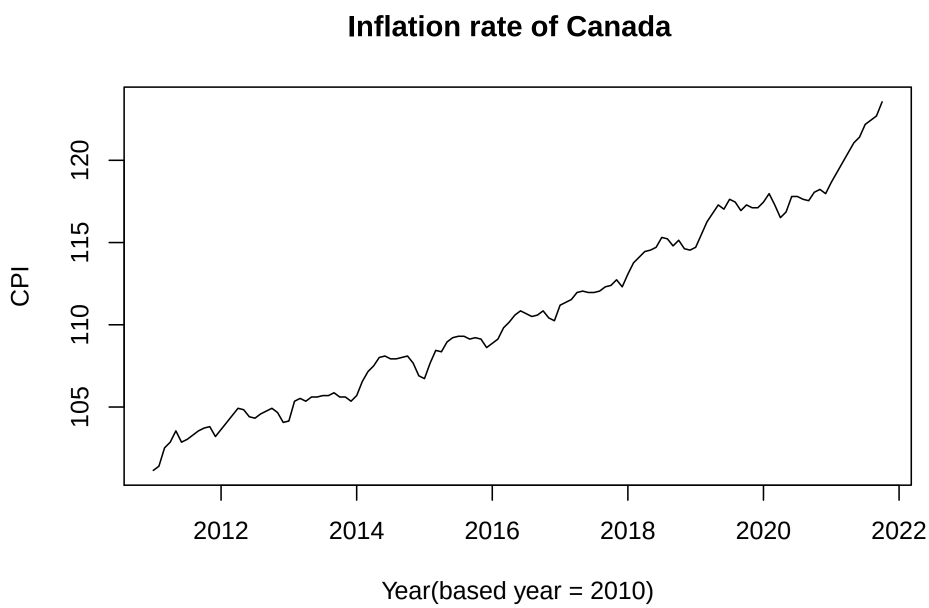
There are many statistical models to predict inflation rates in the existing literature. For example, the Phillips curve shows the inverse trade-off between rates of inflation and rates of unemployment [1]. One other method involves regressions based on implicit expectations derived from asset prices, such as nominal Treasury debt [2, 3]. In this project, we adopt the method that prediction based only on past inflation rate. This leads to time series models such as auto-regressive integrated moving average (ARIMA) models.

We also examine the correlation and causal relation of inflation and its predictors such as government spending. Economists believe that the causes of inflation included increased government spending, demand/supply shocks and unemployment rate. We would like to investigate how strongly these variables are correlated. By examining the correlation of time lag variable, it is possible to find causal relations. We also analyze the US data and examine the cross-correlation with the Canadian data.

Section 2 - Analysis of the time series data

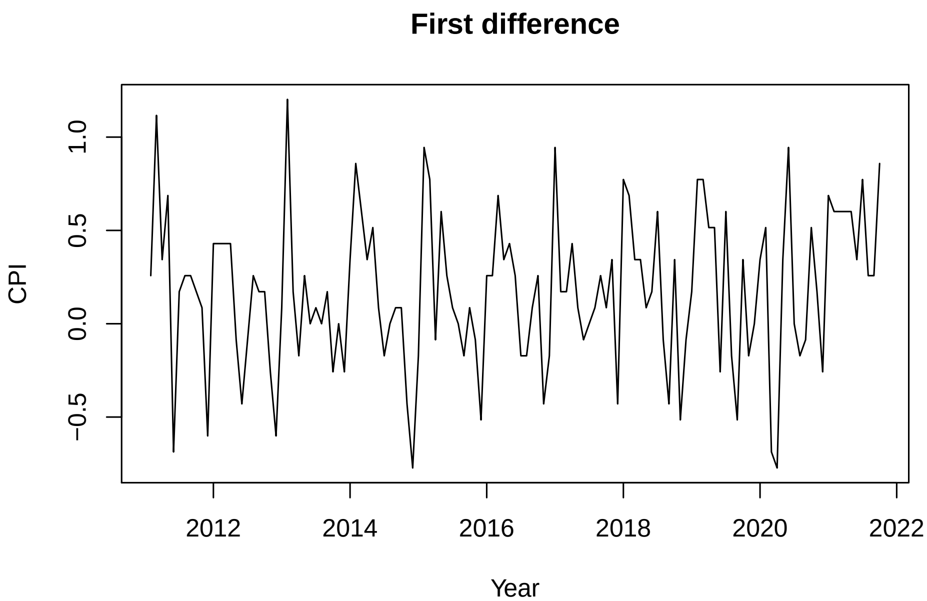
The figure belows shows the CPI for the last ten years, where the unit is in the month.

```
plot(CPI, main = "Inflation rate of Canada  ", xlab = "Year(based year = 2010)", ylab = "CPI
```



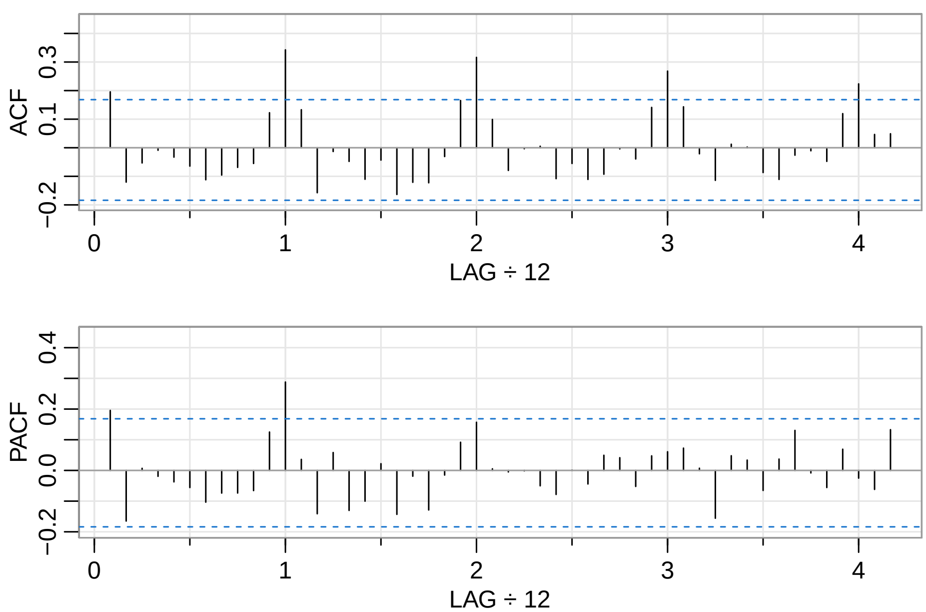
The figure shows an upward trend, and the rate of increase is the fastest over the years 2020 and 2021 because of the pandemic. We apply the difference operator to the data and graph the new data in figure. We can see that the new time series is weakly stationary.

```
stat_CPI = diff(CPI)
plot(stat_CPI, main = "First difference", xlab = "Year", ylab = "CPI")
```



Section 3 - Diagotistic of the model The plots of ACF and PACF are shown in the figure below:

```
acf2(stat_CPI, 50, main = "")
```




```
##       [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10] [,11] [,12]
## ACF    0.2 -0.12 -0.05 -0.01 -0.03 -0.06 -0.11 -0.10 -0.07 -0.06  0.12  0.34
## PACF   0.2 -0.16  0.01 -0.02 -0.04 -0.06 -0.10 -0.07 -0.07 -0.07  0.13  0.29
```

```
##      [,13] [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24]
## ACF   0.13 -0.16 -0.01 -0.05 -0.11 -0.04 -0.16 -0.12 -0.12 -0.03  0.17  0.32
## PACF  0.04 -0.14  0.06 -0.13 -0.10  0.02 -0.14 -0.02 -0.13 -0.02  0.09  0.16
##      [,25] [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36]
## ACF   0.10 -0.08     0  0.01 -0.11 -0.06 -0.11 -0.09  0.00 -0.04  0.14  0.27
## PACF  0.01  0.00     0 -0.05 -0.08  0.00 -0.04  0.05  0.04 -0.05  0.05  0.06
##      [,37] [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48]
## ACF   0.14 -0.02 -0.11  0.01  0.00 -0.09 -0.11 -0.03 -0.01 -0.05  0.12  0.22
## PACF  0.07  0.01 -0.16  0.05  0.03 -0.07  0.04  0.13 -0.01 -0.06  0.07 -0.03
##      [,49] [,50]
## ACF   0.05  0.05
## PACF -0.06  0.13
```

The ACF plot shows the presence of seasonal correlation where the season is equal to 12 months. The PACF shows the seasonal correlation cuts off after 2 seasons. This suggests we fit the model with parameters $P = 2, Q = 0$ and $s = 12$. By observing the figure at the smaller lag, both the ACF and PACF are tailing off. So we try to fit the non-seasonal component with $p = 1$ and $q = 1$. The model we get for the original time series x_t is

$$(1 - \Phi_1 B^{12} - \Phi_2 B^{24})(1 - \phi_1 B)\nabla x_t = \delta + (1 + \theta_1 B)w_t \tag{1}$$

Where $\Phi, \phi, \theta, \delta$ are the coefficients of the model.

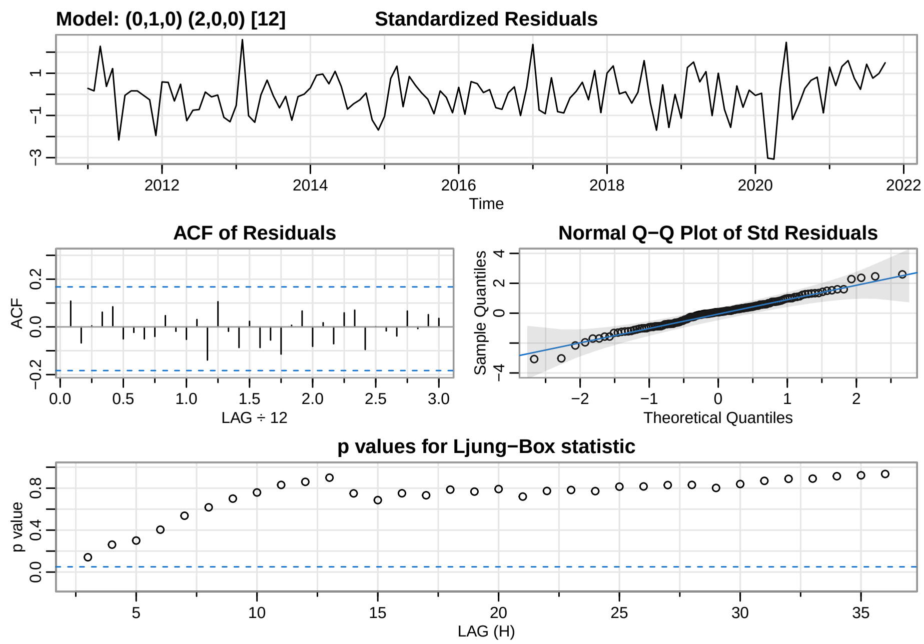
Compare to the other coefficients in the model, the standard error for θ_1, ϕ_1 are quiet large. Fortunately, the P values for these coefficients are large. By eliminating these two we get a simplified model. We fit the model

$$(1 - \Phi_1 B^{12} - \Phi_2 B^{24})\nabla x_t = \delta + w_t \tag{2}$$

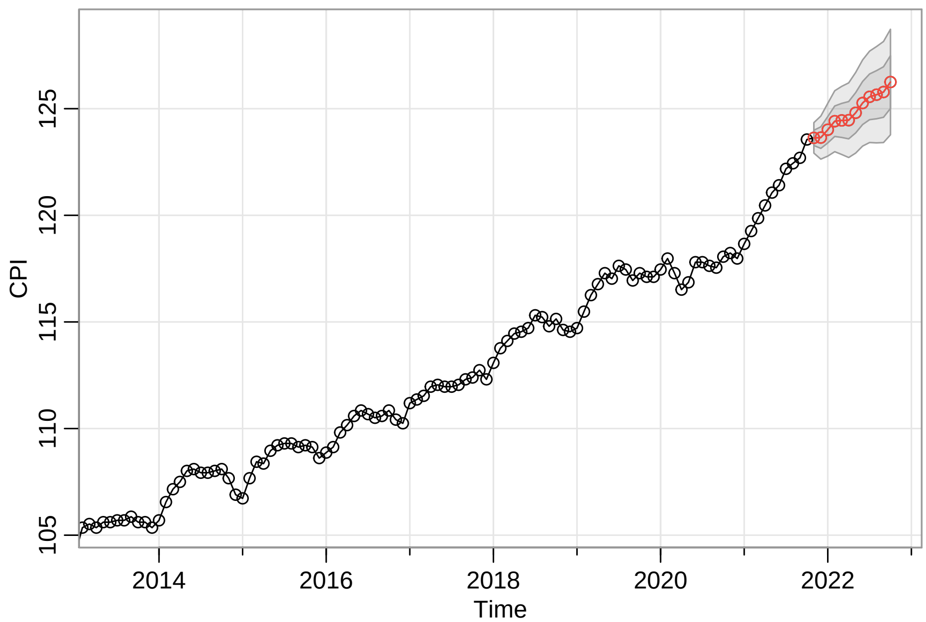
```r
fitted_model = sarima(CPI, 0, 1, 0, 2, 0, 0, 12)
```

```
## initial  value -0.910514 
## iter   2 value -1.002753
## iter   3 value -1.022059
## iter   4 value -1.022995
## iter   5 value -1.023603
## iter   6 value -1.023862
## iter   7 value -1.023898
## iter   8 value -1.023902
## iter   9 value -1.023902
## iter  10 value -1.023902
## iter  10 value -1.023902
## iter  10 value -1.023902
## final  value -1.023902 
## converged
## initial  value -1.010296
```

```
## iter   2 value -1.012201
## iter   3 value -1.013097
## iter   4 value -1.013162
## iter   5 value -1.013207
## iter   6 value -1.013212
## iter   7 value -1.013213
## iter   7 value -1.013213
## iter   7 value -1.013213
## final  value -1.013213
## converged
```




```
sarima.for(CPI, 12, 0, 1, 0, 2, 0, 0 , 12 )
```




```
## $pred
##           Jan      Feb      Mar      Apr      May      Jun      Jul      Aug
## 2021
## 2022 124.0141 124.4131 124.4497 124.4604 124.8077 125.2669 125.5568 125.6563
##           Sep      Oct      Nov      Dec
## 2021                   123.6311 123.6437
## 2022 125.7817 126.2501
##
## $se
##            Jan       Feb       Mar       Apr       May       Jun       Jul
## 2021
## 2022 0.6186300 0.7143324 0.7986479 0.8748750 0.9449730 1.0102186 1.0714986
##            Aug       Sep       Oct       Nov       Dec
## 2021                               0.3571662 0.5051093
## 2022 1.1294587 1.1845863 1.2372600
```

References

[1] Atkeson Andrew, and Lee E.Ohanian *Are Phillip Curves Useful For Forecasting Inflation?* Federal Reserve Bank of Minneapolis Quarterly review 25(1) : 2 - 11 Available at http://www.minneapolisfed.org/research/QR/QR2511.pdf

[2] Mishkin, Frederic S. *What Does the Term Structure Tell us about Future inflation?* Journal of Monetary Economics 25(1) : 77 - 95

[3] Mishkin, Frederic S. *The Information in the Longer-Maturity Term Structure about future inflation.* Quarterly Journal of Economics 105(3) : 815 - 828