When to Push Ads: Optimal Mobile Ad Campaign Strategy under Markov Customer Dynamics

Guokai Li

guokaili@link.cuhk.edu.cn
School of Data Science
The Chinese University of Hong Kong, Shenzhen

Joint work with Pin Gao, Zizhuo Wang

The 15th CSAMSE Conference
July 29, 2023

Outlines

1. Introduction
2. Model Setup
3. Unconstrained Case
4. Budget-Constrained Case

Background: Mobile Ad Campaign

- Mobile ads are sent in various ways to increase customers' engagement.
- The global mobile engagement market (e.g., SMS messages):
  - \$11.75 billion in 2021;
  - Grows at an annual rate 38%;
  - Estimated to approach \$299.00 billion by 2030.

Motivation

- **Deadweight loss**: The customer has already planned to place an order, but the platform sends an ad push.
- **Potential revenue**: The customer is attracted by other platforms, but the platform does not send an ad push to re-engage him.
- When and whom to push ads? (intertemporal & personalized)

Research Questions

- What is the optimal policy for pushing ads?
- How does the type of the customer affect the optimal policy?
- With budget constraints, is there an efficient policy with performance guarantee (e.g., asymptotic optimality)?

Literature Review

RFM Framework

- Hughes [2000]; Fader et al. [2005]; Cui et al. [2006]; Zhang et al. [2015]; Kumar and Srinivasan [2015].

Direct Marketing

- Bertsimas and Mersereau [2007]; Khan et al. [2009]; Wang et al. [2016]; Sun and Zhang [2019]; Liu et al. [2021].

Network Revenue Management

- Gallego and Van Ryzin [1997]; Talluri and Van Ryzin [1998]; Talluri and Van Ryzin [2004]; Gallego et al. [2015].

Restless Bandit

- Whittle [1988]; Brown and Smith [2020]; Brown and Zhang [2022]; Mate et al. [2022].

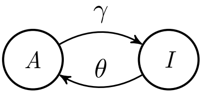
Features of Our Model

1. Partially observed Markovian customer engagement;
2. Ad campaigns as an activation tool.

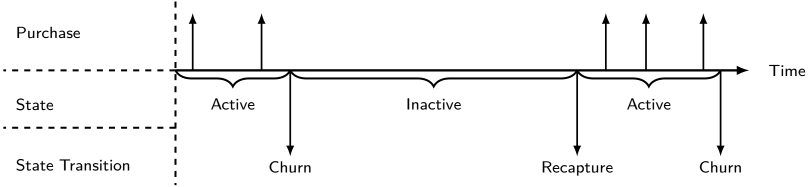
Outlines

Customers' Behavioral Model

- The customer has two states, A (Active) and I (Inactive), and evolves according to a CTMC:



- For example, when talking about the lunch, people whose first thought is Meituan are its active customers.
- An active customer will make purchases according to a Poisson process at rate λ while an inactive customer will not.



- **Main idea**: Activate inactive customers (by ads) to boost profit?

Ad Push Policy

- Given an ad, the customer will instantly become (or stay) active;
- Each purchase brings a revenue r whereas each ad campaign costs c. The corresponding counting processes are $N_p(t)$ and $N_a(t)$;
- The retailer should determine the ad push policy to maximize its long-term expected average profit:

$$\max_{\mu} \left\{ \lim_{T \to \infty} \mathbb{E}\left[\frac{1}{T} \int_0^T r \, dN_p^{\mu}(t) - c \, dN_a^{\mu}(t) \right] \right\}.$$

- With full state information, the retailer will only push ads to inactive customers if profitable;
- However, in practice, the retailer can only observe **partial information** (e.g., purchase history);

Outlines

Ad Push Policy without Budget Constraints

- In this case, we can focus on a representative customer;
- The retailer can only observe the records of purchases and ad campaigns;
- Each time a purchase is observed or an ad campaign is pushed, the customer is certainly active;
- **Silence Time:** The time since the last purchase or ad campaign;
- The optimal policy is a **threshold policy**: The retailer pushes an ad campaign once the silence time approaches the threshold ω.

Objective

- To derive the optimal threshold ω^*, we first rewrite the objective;
- Define the time of a purchase or an ad campaign as an **event time**, and the interval between two adjacent event times as a **cycle**;
- The lengths of cycles are i.i.d. variables κ_i's, and the corresponding profits are also i.i.d. π_i's;
- Then the long-term average profit becomes

$$\Psi(\omega) = \frac{\mathbb{E}[\pi(\omega)]}{\mathbb{E}[\kappa(\omega)]}.$$

- In the following, we study the optimal threshold $\omega^* = \arg\max_{\omega \geq 0} \Psi(\omega)$?

Existence of Optimal Threshold

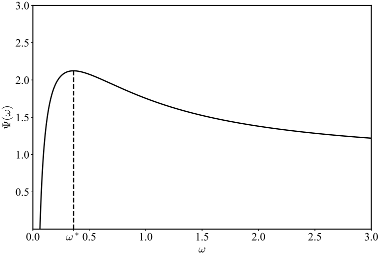
Proposition (Optimal Threshold)

1. *When* $\frac{c}{r} < \frac{\lambda-\gamma-\theta+\sqrt{(\lambda-\gamma-\theta)^2+4\lambda\gamma}}{2(\gamma+\theta)}$*,* $\Psi(\omega)$ *first increases and then decreases in* ω*, and hence there exists a unique maximizer* $\omega^* > 0$*.*
2. *When* $\frac{c}{r} \geq \frac{\lambda-\gamma-\theta+\sqrt{(\lambda-\gamma-\theta)^2+4\lambda\gamma}}{2(\gamma+\theta)}$*,* $\Psi(\omega)$ *increases in* ω*, and there does not exist a maximizer.*

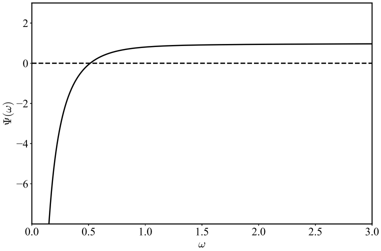
- Moreover, the threshold ω^* can be solved from an equation efficiently. When $\theta = 0$, it has an explicit expression.

Optimal Ad Push Policy

- We illustrate the long-term average profit $\Psi(\omega)$ as the threshold ω varies:



(a) Low Cost ($c = 0.2$)



(b) High Cost ($c = 2$)

- In the following, we first provide some insights for the optimal threshold ω^*, and then analyze its comparative statics.

Insights behind Optimal Threshold

As mentioned, the long-term average profit is $\mathbb{E}[\pi(\omega)]/\mathbb{E}[\kappa(\omega)]$, which implies that each ad campaign should be both effective and timely.

- **Effectiveness**: Push ads later to increase the expected cycle profit $\mathbb{E}[\pi(\omega)]$.

  $$\text{Effectiveness} = \text{Inactive Probability} \times \text{Activation Profit - Ad Cost},$$

  where activation profit measures the profit difference between the two states.

- **Timeliness**: Push ads earlier to reduce the expected cycle length $\mathbb{E}[\kappa(\omega)]$.

- As the threshold ω increases, timeliness $\downarrow$ while effectiveness $\uparrow$. The optimal threshold balance these two countering driving forces.

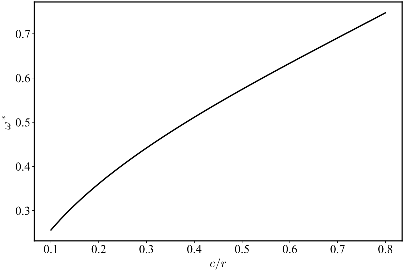
- As parameters change, the effectiveness is significantly affected.

Comparative Statics for Cost c

Effectiveness = Inactive Probability $\times$ Activation Profit - Ad Cost,

Proposition

When $\frac{c}{r} \in (0, \frac{\lambda-\gamma-\theta+\sqrt{(\lambda-\gamma-\theta)^2+4\lambda\gamma}}{2(\gamma+\theta)})$*, the optimal threshold* ω^* *increases in* c *(and decreases in* r*).*



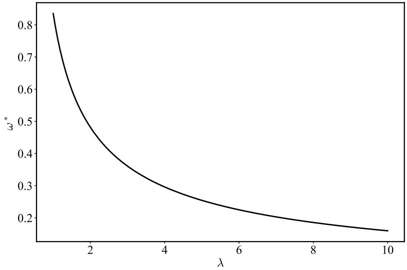
- Ad cost increases;
- Effectiveness is diminished;
- Wait for a longer time to keep balance.

Comparative Statics for Purchase Rate λ

Effectiveness = Inactive Probability $\times$ Activation Profit - Ad Cost,

Proposition

When $\lambda \in \left(\frac{c(c+r)(\gamma+\theta)^2}{r(\gamma c+\gamma r+\theta c)}, \infty\right)$, *the optimal threshold* ω^* *decreases in* λ.



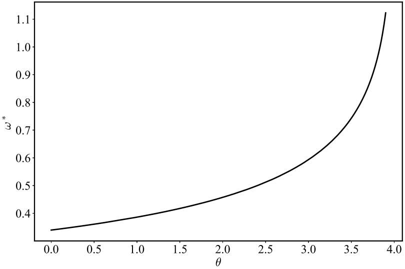
- Both inactive probability and activation profit increase;
- Effectiveness is enhanced;
- Wait for a shorter time.

Comparative Statics for Recapture Rate θ

Effectiveness $=$ Inactive Probability $\times$ Activation Profit - Ad Cost,

Proposition

When $\theta \in \left(0, \frac{\lambda r + \sqrt{(4\gamma + \lambda + 4\gamma\frac{r}{c})\lambda} \cdot r - 2\gamma c - 2\gamma r}{2(c+r)}\right)$, *the optimal threshold* ω^* *increases in* θ.

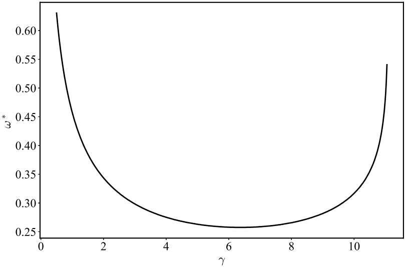


- Both inactive probability and activation profit decrease;
- Effectiveness is diminished;
- Wait for a longer time.

$$\text{Effectiveness} = \text{Inactive Probability} \times \text{Activation Profit - Ad Cost},$$

Proposition

When $\gamma \in \left(0, \frac{\lambda r - \theta c}{c}\right)$ and $\theta = 0$, the optimal threshold ω^* first decreases and then increases in γ.



- Inactive probability increases while activation profit decreases;
- When γ is small, the relative increment of inactive probability is dominating, resulting in enhanced effectiveness.
- When γ is large, the inactive probability is saturated and hence the activation profit dominates.

Outlines

Optimal Policy under Budget Constriants

- In practice, retailer may have a budget for ad campaigns on a cluster of customers (e.g., in Hangzhou).
- Suppose there are M customers (index j) and customer j is characterized by the parameter set $(c_j,\ r_j,\ \lambda_j,\ \gamma_j,\ \theta_j)$.
- Then the problem over time horizon T is

$$\begin{aligned} V_T = \max_{\mu} \quad & \frac{1}{T}\mathbb{E}\Big[\sum_{j=1}^{M} r_j N^{\mu}_{p,j}(T) - c_j N^{\mu}_{c,j}(T)\Big] \\ \text{s.t.} \quad & \sum_{j=1}^{M} c_j N^{\mu}_{c,j}(T) \leq TB \qquad\qquad (a.s.). \end{aligned} \tag{1}$$

  where B measures the relative size of the budget.
- In this case, decisions on different customers are coupled by the budget constraint.

Main Steps to Derive Hueristic Policy

1. In general, the problem is intractable, so we aim to propose a heuristic policy;
2. We first study an infinite-time problem, which possesses good properties;
3. Based on such properties, the infinite-time problem can be efficiently solve;
4. Then we design an easy-to-implement policy, and prove its asymptotical optimality.

Infinite-Time problem

- In order to derive a heuristic policy, we first consider the infinite-time problem.
- In this case, we have the problem as follows:

$$
\begin{aligned}
\bar{V}_\infty(B) = \max_{\omega_1,\omega_2,\ldots,\omega_M} \quad & \sum_{j=1}^{M} \frac{\mathbb{E}[\pi_j(\omega_j)]}{\mathbb{E}[\kappa_j(\omega_j)]} \\
\text{s.t.} \quad & \sum_{j=1}^{M} \frac{\mathbb{E}[\psi_j(\omega_j)]}{\mathbb{E}[\kappa_j(\omega_j)]} \le B \\
& w_j \ge 0, \qquad\qquad \forall\, j \in [M].
\end{aligned} \tag{2}
$$

where $\psi_j(\omega_j) = \min\{\pi_j(\omega_j), 0\}$ is the random ad cost of each cycle of customer j.

Optimal Stationary Policy under Budget Constriants

- We can decompose the problem and consider the single-customer problem with a budget q_j

$$
\begin{aligned}
\phi_j(q_j) = \max_{\omega_j \geq 0} \quad & \frac{\mathbb{E}[\pi_j(\omega_j)]}{\mathbb{E}[\kappa_j(\omega_j)]} \\
\text{s.t.} \quad & \frac{\mathbb{E}[\psi_j(\omega_j)]}{\mathbb{E}[\kappa_j(\omega_j)]} \leq q_j.
\end{aligned} \tag{3}
$$

- Then, the main problem can be equivalently transformed into the following problem

$$
\begin{aligned}
\bar{V}_\infty(B) = \max_{q_1, q_2, \ldots, q_M \geq 0} \quad & \sum_{j=1}^{M} \phi_j(q_j) \\
\text{s.t.} \quad & \sum_{j=1}^{M} q_j \leq B.
\end{aligned} \tag{4}
$$

Optimal Stationary Policy under Budget Constraints

Let ω_j^u and q_j^u denote the maximizer of $\phi_j(+\infty)$ and the corresponding used budget $\frac{\mathbb{E}[\psi_j(\omega_j^u)]}{\mathbb{E}[\kappa_j(\omega_j^u)]}$, respectively.

Proposition (Optimal Ad Push Policy with Budget)

1. *When* $B \geq \sum_{j=1}^{M} q_j^u$, *the optimal solution to* (2) *is* $\omega_j^* = \omega_j^u$ *for any* j.
2. *When* $B < \sum_{j=1}^{M} q_j^u$, *the optimal solution to* (2) *satisfies that* $\omega_j^* > \omega_j^u$ *for any* j. *Moreover, we have* $\sum_{j=1}^{M} q_j^* = B$ *and there exists* $\xi > 0$ *such that the optimal solution satisfies* $\nabla_{q_k}\phi_k(q_k^*) = \xi$ *for all* $q_k^* > 0$, *and* $q_k^* = 0$ *if* $\nabla_{q_k}\phi_k(0) < \xi$.

Therefore, the infinite-time problem can be efficiently solved by searching the gradient $\nabla_q \phi(q^*)$.

BAT Policy for Finite-Time Problem

Algorithm Budget Allocation with Thresholds (BAT)

Compute the optimal budget allocation and the corresponding threshold (q_j^*, ω_j^*) for each customer j in the infinite-time problem (2);
Allocate $Q_j = q_j^* T$ budget to each customer j;
Apply the threshold policy $\varphi^{\omega_j^*}$ to customer j until the allocated budget Q_j is exhausted.

Proposition (Asymptotic Loss)

Let $V_T^{\mathcal{A}}(B)$ be the expected average profit of the BAT policy. We have:

$$V_T(B) - V_T^{\mathcal{A}}(B) = O(1/\sqrt{T}).$$

Extensions

- Strategic customers
  1. Customers may wait for ad campaigns attached with coupons;
  2. A randomized threshold policy can improve the profit.
- Inefficient activation
  1. Customers are activated with probability $p \in (0, 1]$;
  2. Two thresholds: The threshold after a purchase and that after an ad are different.
- Redeeming cost
  1. Redeeming cost only occurs when customers redeem coupons or discounts;
  2. Two thresholds and the validity period of coupons are decision variables.

Summary

We analyze the optimal ad push policy in the presence of customers' Markov dynamics:

1. The optimal policy is a threshold policy, and the optimal threshold can be efficiently solved from an equation.
2. We provide comparative statics for the optimal threshold, and explain the insights.
3. We analyze the problem with budget constraints, and provide an easy-to-implement and asymptotically optimal policy.
4. Moreover, we also enrich the model by incorporating strategic customers, inefficient activation and redeeming cost.

SSRN Paper Link

References I

Arthur Middleton Hughes. *Strategic Database Marketing*. McGraw Hill, New York, 2000.

Peter S Fader, Bruce GS Hardie, and Ka Lok Lee. RFM and CLV: Using iso-value curves for customer base analysis. *Journal of Marketing Research*, 42(4):415–430, 2005.

Geng Cui, Man Leung Wong, and Hon-Kwong Lui. Machine learning for direct marketing response models: Bayesian networks with evolutionary programming. *Management Science*, 52(4): 597–612, 2006.

Yao Zhang, Eric T Bradlow, and Dylan S Small. Predicting customer value using clumpiness: From RFM to RFMC. *Marketing Science*, 34(2):195–208, 2015.

Vineet Kumar and Kannan Srinivasan. Commentary on "Predicting customer value using clumpiness". *Marketing Science*, 34(2):209–213, 2015.

Dimitris Bertsimas and Adam J Mersereau. A learning approach for interactive marketing to a customer segment. *Operations Research*, 55(6):1120–1135, 2007.

Romana Khan, Michael Lewis, and Vishal Singh. Dynamic customer management and the value of one-to-one marketing. *Marketing Science*, 28(6):1063–1079, 2009.

Xinshang Wang, Van-Anh Truong, Shenghuo Zhu, and Qiong Zhang. Dynamic optimization of mobile push advertising campaigns. Technical report, Working paper. Columbia University, New York, NY, 2016.

Yacheng Sun and Dan Zhang. A model of customer reward programs with finite expiration terms. *Management Science*, 65(8):3889–3903, 2019.

Yan Liu, Yacheng Sun, and Dan Zhang. An analysis of "buy x, get one free" reward programs. *Operations Research*, 69(6):1823–1841, 2021.

Guillermo Gallego and Garrett Van Ryzin. A multiproduct dynamic pricing problem and its applications to network yield management. *Operations Research*, 45(1):24–41, 1997.

References II

Kalyan Talluri and Garrett Van Ryzin. An analysis of bid-price controls for network revenue management. *Management Science*, 44(11):1577–1593, 1998.

Kalyan Talluri and Garrett Van Ryzin. Revenue management under a general discrete choice model of consumer behavior. *Management Science*, 50(1):15–33, 2004.

Guillermo Gallego, Richard Ratliff, and Sergey Shebalov. A general attraction model and sales-based linear program for network revenue management under customer choice. *Operations Research*, 63(1):212–232, 2015.

Peter Whittle. Restless bandits: Activity allocation in a changing world. *Journal of Applied Probability*, 25(A):287–298, 1988.

David B Brown and James E Smith. Index policies and performance bounds for dynamic selection problems. *Management Science*, 66(7):3029–3050, 2020.

David B Brown and Jingwei Zhang. Dynamic programs with shared resources and signals: Dynamic fluid policies and asymptotic optimality. *Operations Research*, 70(5):3015–3033, 2022.

Aditya Mate, Lovish Madaan, Aparna Taneja, Neha Madhiwalla, Shresth Verma, Gargi Singh, Aparna Hegde, Pradeep Varakantham, and Milind Tambe. Field study in deploying restless multi-armed bandits: Assisting non-profits in improving maternal and child health. In *Proceedings of the AAAI Conference on Artificial Intelligence*, pages 12017–12025, 2022.