Optimal Emission Regulation under Market Uncertainty

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Environmental Issues Caused by Global Warming

Climate changes have made significant impacts on human lives.

(a) Australian Bushfire

(b) Sinking Island Country Tuvalu

Major countries in the world have started to take actions.

- The main method is to control the emission of greenhouse gas.
- Paris Agreement/Kyoto Protocol.
- Carbon peak and neutrality.

Environmental Regulatory Instruments

Two mainstream types of regulatory instruments:

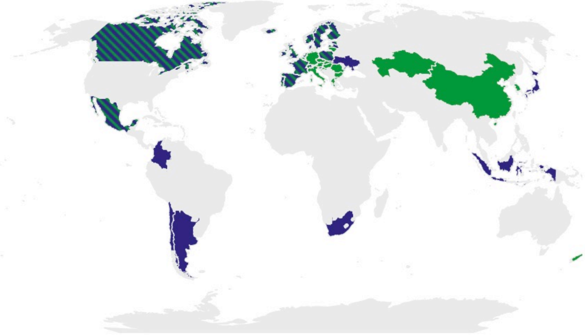
Price Instrument

- The regulator sets a *tax rate* for the greenhouse gas.
- Each firm pays a tax for its emission.

Quantity Instrument

- The regulator imposes a *cap*.
- Each firm's emission amount cannot exceed its holding cap.
- Sometimes firms can trade their caps, which is called the ETS (Emission Trading System) mechanism.

Instruments Implementation Map



- **Carbon Tax**: 27 countries accounting for 5.5% of global greenhouse gas emission.
- **ETS**: 38 countries (including China) accounting for 16.1% of global greenhouse gas emission.

Research Question

Research Question: Which instrument, price or quantity, will lead to a higher social welfare?

1. Differs in industries?
2. Differs in market conditions?

Related Works

In absence of market uncertainty, Montgomery (1972) proves the **equivalence** of the two instruments.

In presence of market **uncertainty**, Weitzman's seminal 1974 paper "Prices vs. Quantities" shows that the equivalence may break down.

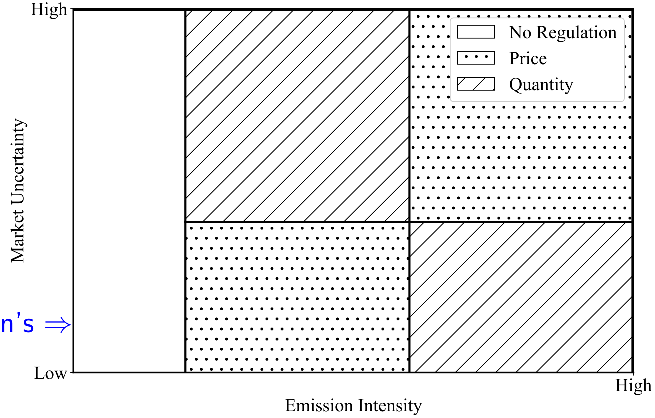
- For high-emitting (low-emitting, reps.) industries, the quantity instrument outperforms (underperforms, resp.) the price instrument.

We revisit Weitzman (1974) by making two different assumptions.

	Weitzman's	Ours
Negative Production	✓	✗
Partially Exercising	✗	✓

We show that the comparison result also depends on the size of market uncertainty.

Overview of Main Results



Weitzman's ⇒

Model Setup

We consider one regulator and one monopolistic firm.

- We consider a linear inverse demand function for the firm: $p(\theta, x) = \theta - x$, where θ is the market size.
- Without considering pollution, firm's profit function at production level x is $R(\theta, x) = \theta x - x^2$.
- Market size θ drawn from $\{\theta^L, \theta^H\} := \{\mu - \sigma, \mu + \sigma\}$ with equal probabilities.
- The pollution damage is defined as $D(x) = d \cdot x^2$, where d measures the **emission intensity**, i.e., emissions per unit of production.

Model Setup

The regulator chooses one of the following two types of instruments:

1. Price Instrument: The regulator sets a tax rate t for per unit of production.
   - The profit of the monopolist will be $(\theta - t)x - x^2$.

2. Quantity Instrument: The regulator sets a cap k for the production level of the monopolist.
   - The production amount of the monopolist is limited by k.

Timing of Events

1. The regulator **ex-ante** commits to a regulatory instrument;
2. The value of market size θ is realized and observed by the firm;
3. The firm chooses its production quantity x based on the **regulation** and the **market size** realization to maximize its profit;
4. The firm's production level, the pollution damage and social welfare are calculated.

The objective of the regulator is to maximize the **expected social welfare**, i.e., to maximize

$$\mathbb{E}_\theta\Big[\underbrace{\theta x - x^2}_{\text{Production Profit}} + \underbrace{\frac{1}{2}x^2}_{\text{Consumer Surplus}} - \underbrace{dx^2}_{\text{Pollution Damage}} \Big].$$

Price Instrument $p(t)$

Given a tax rate t, the firm maximizes its profit

$$\max_{x\geq 0} \big[\underbrace{\theta x - x^2}_{\text{Profit}} - \underbrace{tx}_{\text{Tax}} \big].$$

The firm's optimal production level is given by

$$x^P(\theta, t) = \max\left\{ \frac{\theta - t}{2}, 0 \right\}.$$

The regulator maximizes the expected social welfare

$$\max_{t\geq 0} \quad \mathbb{E}_\theta \big[\theta x^P(\theta, t) - (d + \tfrac{1}{2}) \cdot (x^P(\theta, t))^2 \big].$$

Optimal Price Instrument

Proposition (Optimal Price Instrument)

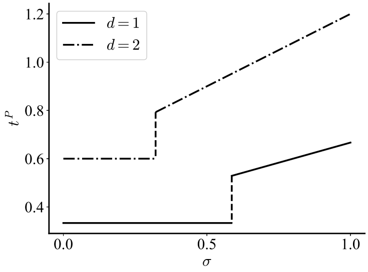
- *When* $d < \frac{1}{2}$, *the optimal tax rate and the corresponding expected social welfare are* $t^P = 0$ *and* $SW^P = \frac{3-2d}{8}(\mu^2 + \sigma^2)$.
- *When* $d \geq \frac{1}{2}$ *and* $\sigma < \frac{\sqrt{2}\mu}{\sqrt{2}-1+2d}$, *the optimal tax rate and the corresponding expected social welfare are* $t^P = \frac{2d-1}{2d+1}\mu$ *and* $SW^P = \frac{4\mu^2+(3+4d-4d^2)\sigma^2}{16d+8}$.
- *When* $d \geq \frac{1}{2}$ *and* $\sigma \geq \frac{\sqrt{2}\mu}{\sqrt{2}-1+2d}$, *the optimal tax rate and the corresponding expected social welfare are* $t^P = \frac{2d-1}{2d+1}(\mu + \sigma)$ *and* $SW^P = \frac{(\mu+\sigma)^2}{8d+4}$.

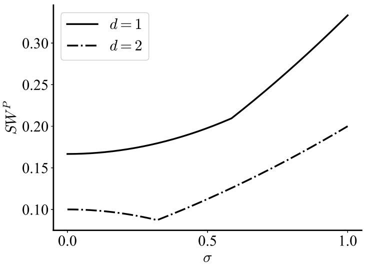
Optimal Price Instrument

$x^P(\theta, t) = \max\left\{\frac{\theta - t}{2}, 0\right\}$

(a) Optimal Tax Rate

(b) Expected Social Welfare





- **Inclusive Phase**: $x^P(\theta^L, t) = \frac{\theta - t}{2} > 0$, concerns both cases;
- **Exclusive Phase**: $x^P(\theta^L, t) = 0$, focuses on the high-demand case.

Quantity Instrument $q(k)$

Given a cap k, the firm maximizes its profit

$$\max_{0 \le x \le k} \left[\theta x - x^2 \right].$$

The firm's optimal production level is given by

$$x^Q(\theta, k) = \min\left\{ \frac{\theta}{2}, k \right\}.$$

The regulator maximizes the expected social welfare

$$\max_{k \ge 0} \quad \mathbb{E}_\theta^{q(k)} \left[\theta x^Q(\theta, k) - (d + \tfrac{1}{2}) \cdot (x^Q(\theta, k))^2 \right].$$

Optimal Quantity Instrument

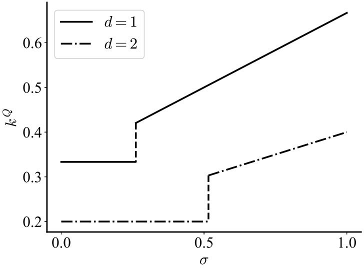
Proposition (Optimal Quantity Instrument)

- *When* $d < \frac{1}{2}$*, the optimal cap and the corresponding expected social welfare are* $k^Q \geq \frac{\mu+\sigma}{2}$ *and* $SW^Q = \frac{3-2d}{8}(\mu^2 + \sigma^2)$.
- *When* $d \geq \frac{1}{2}$ *and* $\sigma < \frac{2d-1}{2d+2\sqrt{2}-1}\mu$*, the optimal cap and the corresponding expected social welfare are* $k^Q = \frac{\mu}{2d+1}$ *and* $SW^Q = \frac{\mu^2}{4d+2}$.
- *When* $d \geq \frac{1}{2}$ *and* $\sigma \geq \frac{2d-1}{2d+2\sqrt{2}-1}\mu$*, the optimal cap and the corresponding expected social welfare are* $k^Q = \frac{\mu+\sigma}{2d+1}$ *and* $SW^Q = \frac{(\mu+\sigma)^2}{8d+4} - \frac{2d-3}{16}(\mu - \sigma)^2$.

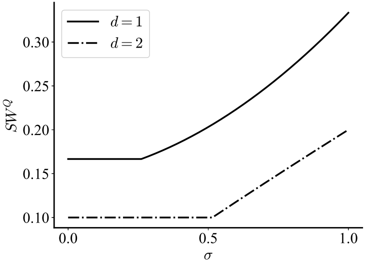
Optimal Quantity Instrument

$$x^Q(\theta, k) = \min\left\{\frac{\theta}{2}, k\right\}$$

(a) Optimal Cap



(b) Expected Social Welfare



- **Inclusive Phase**: $x^Q(\theta^L, k) = k$;
- **Exclusive Phase**: $x^Q(\theta^L, k) = \frac{\theta}{2} < k$.

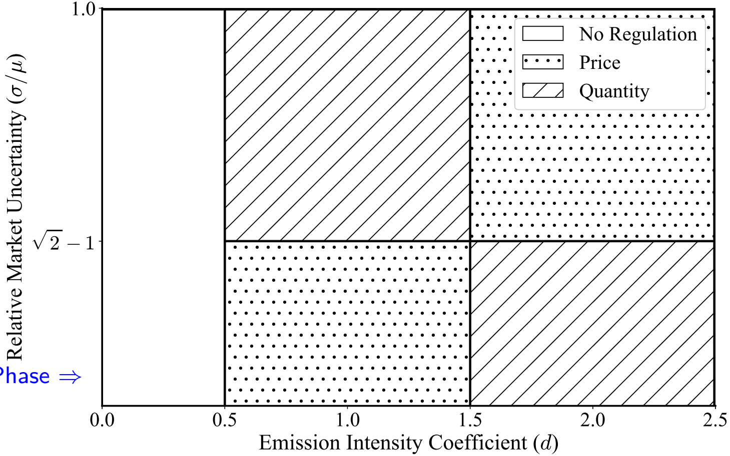
Key Results: Prices versus Quantities

Theorem (Instrument Comparison)

- *When $d \in [0, \frac{1}{2})$, neither instruments should be implemented.*
- *When $d \in [\frac{1}{2}, \frac{3}{2})$, the optimal expected social welfare under the price instrument is higher (lower, resp.) than that under the quantity instrument when $\sigma/\mu < \sqrt{2} - 1$ ($\sigma/\mu > \sqrt{2} - 1$, resp.).*
- *When $d \in [\frac{3}{2}, \infty)$, the optimal expected social welfare under the quantity instrument is higher (lower, resp.) than that under the price instrument when $\sigma/\mu < \sqrt{2} - 1$ ($\sigma/\mu > \sqrt{2} - 1$, resp.).*

Prices versus Quantities

In terms of expected social welfare, the comparison result is illustrated as follows:



Inclusive Phase $\Rightarrow$

Comparison in Inclusive Phase — Small σ

"**Production Flexibility**":

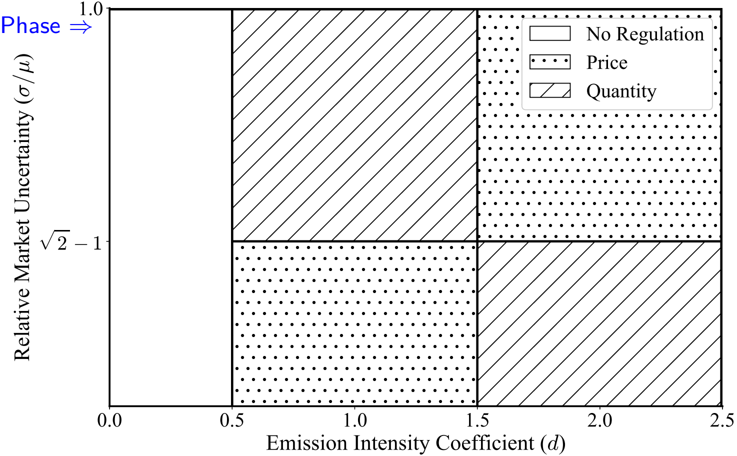
- Price Instrument: $x^P(\theta, t) = \frac{\theta - t}{2}$. ✓
- Quantity Instrument: $x^Q(\theta, k) = k$.

The production flexibility is a **double-edged sword**.

- *Positive Side*: The firm can adaptively choose the production level and generate higher production profit.
- *Negative Side*: The (convex) pollution damage is amplified by the firm's uneven production levels.
- When emission intensity is small, the additional profit outweighs the additional pollution damage, and vice versa.

Prices versus Quantities

Exclusive Phase ⇒



Comparison in Exclusive Phase — Large σ

In this case, both instruments are optimized for the high-demand case θ^H. By Montgomery (1972), the social welfare from θ^H under these instruments is the same.

"**Operations Flexibility**"

- Price Instrument: $x^P(\theta^L, t) = 0$.
- Quantity Instrument: $x^Q(\theta^L, k) = \frac{\theta}{2}$. ✓

However, operations flexibility is also a **double-edged sword**.

- *Positive Side*: The firm can adaptively choose the production level and generate higher production profit.
- *Negative Side*: Additional pollution damage.
- When emission intensity is small, the additional profit outweighs the additional pollution damage, and vice versa.

Managerial Implications

1. Should regulators provide flexibility to the industry (firm)?
   - Double-edged sword;
   - Balance the additional profit and pollution damage;
   - Provide flexibility to **low-emitting** industries;
2. Which regulatory instrument is more flexible for firms under different market conditions?
   - Low Market Uncertainty $\Rightarrow$ Production Flexibility $\Rightarrow$ Price;
   - High Market Uncertainty $\Rightarrow$ Operation Flexibility $\Rightarrow$ Quantity.

- Mismatched regulatory instruments can result in 50% loss of social welfare, especially for **high-emitting** industries.

Extensions

To illustrate the robustness of our results and enrich our insights, we incorporate various practical factors into the base model:

- Hybrid Instrument;
- Abatement Effort;
- Firm Competition;
- Local Pollution Damage;
- Emission Trading System.

Summary

We compare the two mainstreams of emission regulatory instruments in presence of market uncertainty.

1. Both optimal regulatory instruments have two phases as the market uncertainty increases.
2. The comparison of the two instruments depends on both the emission intensity and the market uncertainty.
3. Our results on the hybrid instrument can provide some advice for regulators who have already implemented pure instruments.
4. The emission trading system may not be beneficial to social welfare when the local pollution damage is the main concern.
5. We test the robustness by incorporating firm competition and abatement.