When to Push Ads: Optimal Mobile Ad Campaign Strategy under Markov Customer Dynamics

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Outlines

Background: Mobile Ad Campaign

- Mobile ads are sent in various ways to increase customers' engagement.
- The global mobile engagement market (e.g., SMS messages):
  - $11.75 billion in 2021;
  - Grows at an annual rate 38%;
  - Estimated to approach $299.00 billion by 2030.

Research Questions

- Customers intended to purchase are pushed ads $\Rightarrow$ *deadweight loss*;
- Customers hesitating whether to purchase are not pushed ads, and eventually leave $\Rightarrow$ *revenue loss*;
- When and whom to push ads? (intertemporal & personalized)

- **Research Questions:**
  - What is the optimal policy for pushing ads?
  - How does the type of the customer affect the optimal policy?
  - With budget constraints, is there an efficient policy with good performance?

Literature Review

RFM Framework

- Hughes [2000]; Fader et al. [2005]; Cui et al. [2006]; Zhang et al. [2015]; Kumar and Srinivasan [2015].

Direct Marketing

- Bertsimas and Mersereau [2007]; Khan et al. [2009]; Wang et al. [2016]; Sun and Zhang [2019]; Liu et al. [2021].

Network Revenue Management

- Gallego and Van Ryzin [1997]; Talluri and Van Ryzin [1998]; Talluri and Van Ryzin [2004]; Gallego et al. [2015].

Restless Bandit

- Whittle [1988]; Brown and Smith [2020]; Brown and Zhang [2022]; Mate et al. [2022].

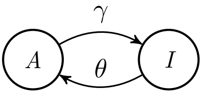
Features of Our Model

1. Partially observed Markovian customer engagement;
2. Ad campaigns as an activation tool.

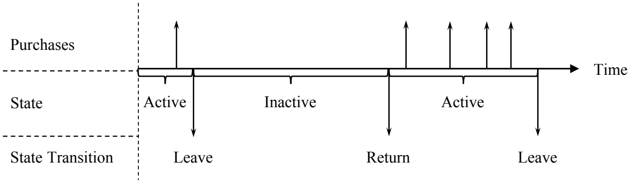
Outlines

Customers' Behavioral Model

- The customer has two states, A (Active) and I (Inactive), and evolves according to a CTMC:



- An active customer will make purchases according to a Poisson process at rate λ while an inactive customer will not.
- λ, γ and θ measure the purchase, the churn and the recapture rates.



- Main idea: Activate inactive customers (by ads) to boost profit?

Ad Push Policy

- Given an ad, the customer will instantly become (or stay) active;
- Each purchase brings a revenue r whereas each ad campaign costs c. The corresponding counting processes are $N_p(t)$ and $N_a(t)$;
- The retailer should determine the ad push policy to maximize its long-term expected average profit:

$$\max_{\mu} \left\{ \lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T} \int_0^T r \, dN_p^{\mu}(t) - c \, dN_a^{\mu}(t) \right] \right\}.$$

- With full state information, the retailer will only push ads to inactive customers if profitable;
- However, in practice, the retailer can only observe **partial information** (e.g., purchase history);

Outlines

Ad Push Policy without Budget Constraints

- In this case, we can focus on a representative customer;
- The retailer can only observe the records of purchases and ad campaigns;
- Each time a purchase is observed or an ad campaign is pushed, the customer is certainly active;
- **Silence Time:** The time since the last purchase or ad campaign;
- The optimal policy is a **threshold policy**: The retailer pushes an ad campaign once the silence time approaches the threshold ω.

Objective

- To derive the optimal threshold ω^*, we first rewrite the objective;
- Define the time of a purchase or an ad campaign as an **event time**, and the interval between two adjacent event times as a **cycle**;
- The lengths of cycles are i.i.d. variables κ_i's, and the corresponding profits are also i.i.d. π_i's;
- Then the long-term average profit becomes

$$\Psi(\omega) = \frac{\mathbb{E}[\pi(\omega)]}{\mathbb{E}[\kappa(\omega)]}.$$

- In the following, we study the optimal threshold $\omega^* = \arg\max_{\omega \geq 0} \Psi(\omega)$?

Existence of Optimal Threshold

Proposition (Optimal Threshold)

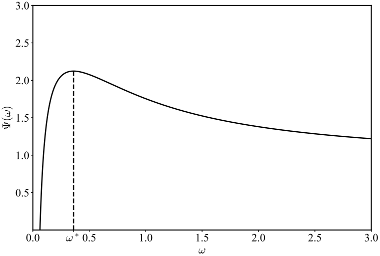
1. *When* $\frac{c}{r} < \frac{\lambda-\gamma-\theta+\sqrt{(\lambda-\gamma-\theta)^2+4\lambda\gamma}}{2(\gamma+\theta)}$*,* $\Psi(\omega)$ *first increases and then decreases in* ω*, and hence there exists a unique maximizer* $\omega^* > 0$*.*

2. *When* $\frac{c}{r} \geq \frac{\lambda-\gamma-\theta+\sqrt{(\lambda-\gamma-\theta)^2+4\lambda\gamma}}{2(\gamma+\theta)}$*,* $\Psi(\omega)$ *increases in* ω*, and there does not exist a maximizer.*

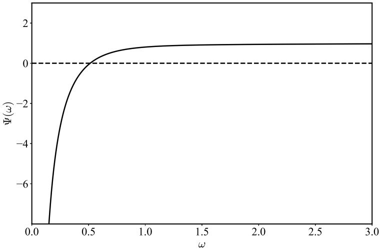
- Moreover, the threshold ω^* can be solved from an equation efficiently. When $\theta = 0$, it has an explicit expression.

Optimal Ad Push Policy

- We illustrate the long-term average profit $\Psi(\omega)$ as the threshold ω varies:



(a) Low Cost ($c = 0.2$)



(b) High Cost ($c = 2$)

- In the following, we first provide some insights for the optimal threshold ω^*, and then analyze its comparative statics.

Insights behind Optimal Threshold

As mentioned, the long-term average profit is $\mathbb{E}[\pi(\omega)]/\mathbb{E}[\kappa(\omega)]$, which implies that each ad campaign should be both effective and timely.

- **Effectiveness**: Push ads later to increase the expected cycle profit $\mathbb{E}[\pi(\omega)]$.

  $$\text{Effectiveness} = \text{Inactive Probability} \times \text{Activation Profit - Ad Cost},$$

  where activation profit measures the profit difference between the two states.

- **Timeliness**: Push ads earlier to reduce the expected cycle length $\mathbb{E}[\kappa(\omega)]$.

- As the threshold ω increases, timeliness $\downarrow$ while effectiveness $\uparrow$. The optimal threshold balance these two countering driving forces.

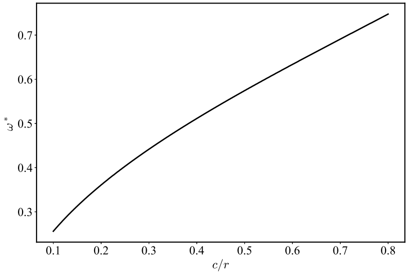
- As parameters change, the effectiveness is significantly affected.

Comparative Statics for Cost c

$$\text{Effectiveness} = \text{Inactive Probability} \times \text{Activation Profit - Ad Cost},$$

Proposition

When $\frac{c}{r} \in (0, \frac{\lambda-\gamma-\theta+\sqrt{(\lambda-\gamma-\theta)^2+4\lambda\gamma}}{2(\gamma+\theta)})$, the optimal threshold ω^* increases in c (and decreases in r).

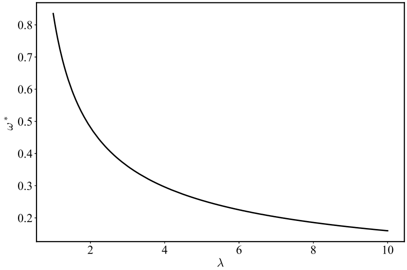


- Ad cost increases;
- Effectiveness is diminished;
- Wait for a longer time to keep balance.

$$\text{Effectiveness} = \text{Inactive Probability} \times \text{Activation Profit - Ad Cost},$$

Proposition

When $\lambda \in (\frac{c(c+r)(\gamma+\theta)^2}{r(\gamma c+\gamma r+\theta c)}, \infty)$*, the optimal threshold* ω^* *decreases in* λ*.*



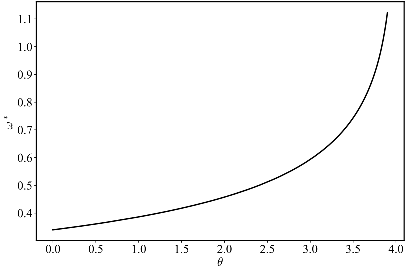
- Both inactive probability and activation profit increase;
- Effectiveness is enhanced;
- Wait for a shorter time.

Comparative Statics for Recapture Rate θ

Effectiveness = Inactive Probability $\times$ Activation Profit - Ad Cost,

Proposition

When $\theta \in \left(0, \frac{\lambda r + \sqrt{(4\gamma + \lambda + 4\gamma\frac{r}{c})\lambda} \cdot r - 2\gamma c - 2\gamma r}{2(c+r)}\right)$*, the optimal threshold* ω^* *increases in* θ*.*



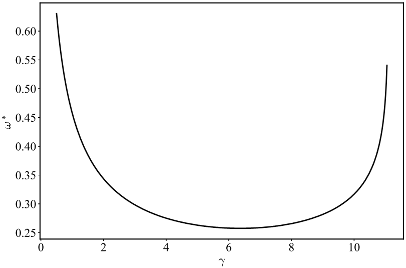
- Both inactive probability and activation profit decrease;
- Effectiveness is diminished;
- Wait for a longer time.

Comparative Statics for Churn Rate γ

Effectiveness $=$ Inactive Probability $\times$ Activation Profit - Ad Cost,

Proposition

When $\gamma \in \left(0, \frac{\lambda r - \theta c}{c}\right)$ and $\theta = 0$, the optimal threshold ω^* first decreases and then increases in γ.



- Inactive probability increases while activation profit decreases;
- When γ is small, the relative increment of inactive probability is dominating, resulting in enhanced effectiveness.
- When γ is large, the inactive probability is saturated and hence the activation profit dominates.

Outlines

Optimal Policy under Budget Constriants

- In practice, retailer may have a budget for ad campaigns on a cluster of customers (e.g., in Hangzhou).
- Suppose there are M customers (index j) and customer j is characterized by the parameter set $(c_j,\ r_j,\ \lambda_j,\ \gamma_j,\ \theta_j)$.
- Then the problem over time horizon T is

$$\begin{aligned} V_T = \max_{\mu} \quad & \frac{1}{T}\mathbb{E}\Big[\sum_{j=1}^{M} r_j N^{\mu}_{p,j}(T) - c_j N^{\mu}_{c,j}(T)\Big] \\ \text{s.t.} \quad & \sum_{j=1}^{M} c_j N^{\mu}_{c,j}(T) \le TB \qquad\qquad (a.s.). \end{aligned} \tag{1}$$

where B measures the relative size of the budget.

- In this case, decisions on different customers are coupled by the budget constraint.

Infinite-Time problem

- In order to derive a heuristic policy, we first consider the infinite-time problem.
- In this case, we have the problem as follows:

$$
\begin{aligned}
\bar{V}_\infty(B) = \max_{\omega_1,\omega_2,\ldots,\omega_M} \quad & \sum_{j=1}^{M} \frac{\mathbb{E}[\pi_j(\omega_j)]}{\mathbb{E}[\kappa_j(\omega_j)]} \\
\text{s.t.} \quad & \sum_{j=1}^{M} \frac{\mathbb{E}[\psi_j(\omega_j)]}{\mathbb{E}[\kappa_j(\omega_j)]} \le B \\
& w_j \ge 0, \qquad\qquad \forall\, j \in [M].
\end{aligned}
\tag{2}
$$

where $\psi_j(\omega_j) = \min\{\pi_j(\omega_j), 0\}$ is the random ad cost of each cycle of customer j.

Optimal Stationary Policy under Budget Constriants

- We can decompose the problem and consider the single-customer problem with a budget q_j

$$\begin{aligned} \phi_j(q_j) = \max_{\omega_j \geq 0} \quad & \frac{\mathbb{E}[\pi_j(\omega_j)]}{\mathbb{E}[\kappa_j(\omega_j)]} \\ \text{s.t.} \quad & \frac{\mathbb{E}[\psi_j(\omega_j)]}{\mathbb{E}[\kappa_j(\omega_j)]} \leq q_j. \end{aligned} \tag{3}$$

- Then, the main problem can be equivalently transformed into the following problem

$$\begin{aligned} \bar{V}_\infty(B) = \max_{q_1, q_2, \ldots, q_M \geq 0} \quad & \sum_{j=1}^{M} \phi_j(q_j) \\ \text{s.t.} \quad & \sum_{j=1}^{M} q_j \leq B. \end{aligned} \tag{4}$$

Optimal Stationary Policy under Budget Constraints

Let ω_j^u and q_j^u denote the maximizer of $\phi_j(+\infty)$ and the corresponding used budget $\frac{\mathbb{E}[\psi_j(\omega_j^u)]}{\mathbb{E}[\kappa_j(\omega_j^u)]}$, respectively.

Proposition (Optimal Ad Push Policy with Budget)

1. *When* $B \geq \sum_{j=1}^{M} q_j^u$, *the optimal solution to* (2) *is* $\omega_j^* = \omega_j^u$ *for any* j.
2. *When* $B < \sum_{j=1}^{M} q_j^u$, *the optimal solution to* (2) *satisfies that* $\omega_j^* > \omega_j^u$ *for any* j. *Moreover, we have* $\sum_{j=1}^{M} q_j^* = B$ *and there exists* $\xi > 0$ *such that the optimal solution satisfies* $\nabla_{q_k}\phi_k(q_k^*) = \xi$ *for all* $q_k^* > 0$, *and* $q_k^* = 0$ *if* $\nabla_{q_k}\phi_k(0) < \xi$.

Therefore, the infinite-time problem can be efficiently solved by searching the gradient $\nabla_q \phi(q^*)$.

BAT Policy for Finite-Time Problem

Algorithm Budget Allocation with Thresholds (BAT)

Compute the optimal budget allocation and the corresponding threshold (q_j^*, ω_j^*) for each customer j in the infinite-time problem (2);
Allocate $Q_j = q_j^* T$ budget to each customer j;
Apply the threshold policy $\varphi^{\omega_j^*}$ to customer j until the allocated budget Q_j is exhausted.

Proposition (Asymptotic Loss)

Let $V_T^{\mathcal{A}}(B)$ be the expected average profit of the BAT policy. We have:

$$V_T(B) - V_T^{\mathcal{A}}(B) = O(1/\sqrt{T}).$$

Extensions

- Strategic customers
  1. Customers may wait for ad campaigns attached with coupons;
  2. A randomized threshold policy can improve the profit.
- Inefficient activation
  1. Customers are activated with probability $p \in (0, 1]$;
  2. Two thresholds: The threshold after a purchase and that after an ad are different.
- Redeeming cost
  1. Redeeming cost only occurs when customers redeem coupons or discounts;
  2. Two thresholds and the validity period of coupons are decision variables.

Summary

We analyze the optimal ad push policy in the presence of customers' Markov dynamics:

1. The optimal policy is a threshold policy, and the optimal threshold can be efficiently solved from an equation.
2. We provide comparative statics for the optimal threshold, and explain the insights.
3. We analyze the problem with budget constraints, and provide an easy-to-implement and asymptotically optimal policy.
4. Moreover, we also enrich the model by incorporating strategic customers, inefficient activation and redeeming cost.

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