

Higher-Order Functions

Announcements

Designing Functions

Describing Functions

A function's **domain** is the set of all inputs it might possibly take as arguments.

A function's **range** is the set of output values it might possibly return.

A pure function's **behavior** is the relationship it creates between input and output.

```
def square(x):  
    """Return X * X."""
```

x is a number

square returns a non-negative real number

square returns the square of x

A Guide to Designing Function

Give each function exactly one job, but make it apply to many related situations

```
>>> round(1.23)      >>> round(1.23, 1)      >>> round(1.23, 0)      >>> round(1.23, 5)
1                    1.2                    1                    1.23
```

Don't repeat yourself (DRY). Implement a process just once, but execute it many times.



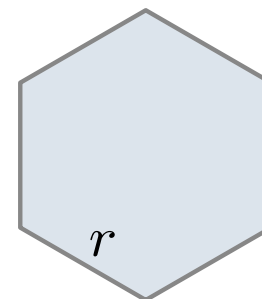
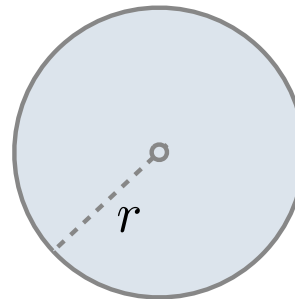
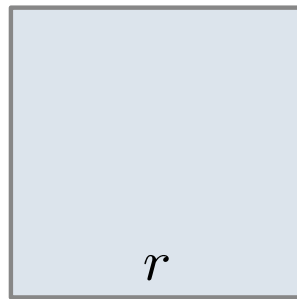
(Demo)

Generalization

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



Area:

$$1 \cdot r^2$$

$$\pi \cdot r^2$$

$$\frac{3\sqrt{3}}{2} \cdot r^2$$

Finding common structure allows for shared implementation

(Demo)

Higher-Order Functions

Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^5 k = 1 + 2 + 3 + 4 + 5 = 15$$

$$\sum_{k=1}^5 k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$\sum_{k=1}^5 \frac{8}{(4k-3) \cdot (4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} = 3.04$$

(Demo)

Summation Example

```
def cube(k):  
    return pow(k, 3)
```

Function of a single argument
(*not* called "term")

```
def summation(n, term):  
    """Sum the first n terms of a sequence.
```

A formal parameter that will
be bound to a function

```
>>> summation(5, cube)
```

```
225
```

```
"""
```

```
    total, k = 0, 1
```

```
    while k <= n:
```

```
        total, k = total + term(k), k + 1
```

```
    return total
```

The cube function is passed
as an argument value

0 + 1 + 8 + 27 + 64 + 125

The function bound to term
gets called here

Functions as Return Values

(Demo)

Locally Defined Functions

Functions defined within other function bodies are bound to names in a local frame

A function that
returns a function

```
def make_adder(n):  
    """Return a function that takes one argument k and returns k + n.
```

```
>>> add_three = make_adder(3)
```

```
>>> add_three(4)
```

```
7
```

```
"""
```

```
def adder(k):  
    return k + n
```

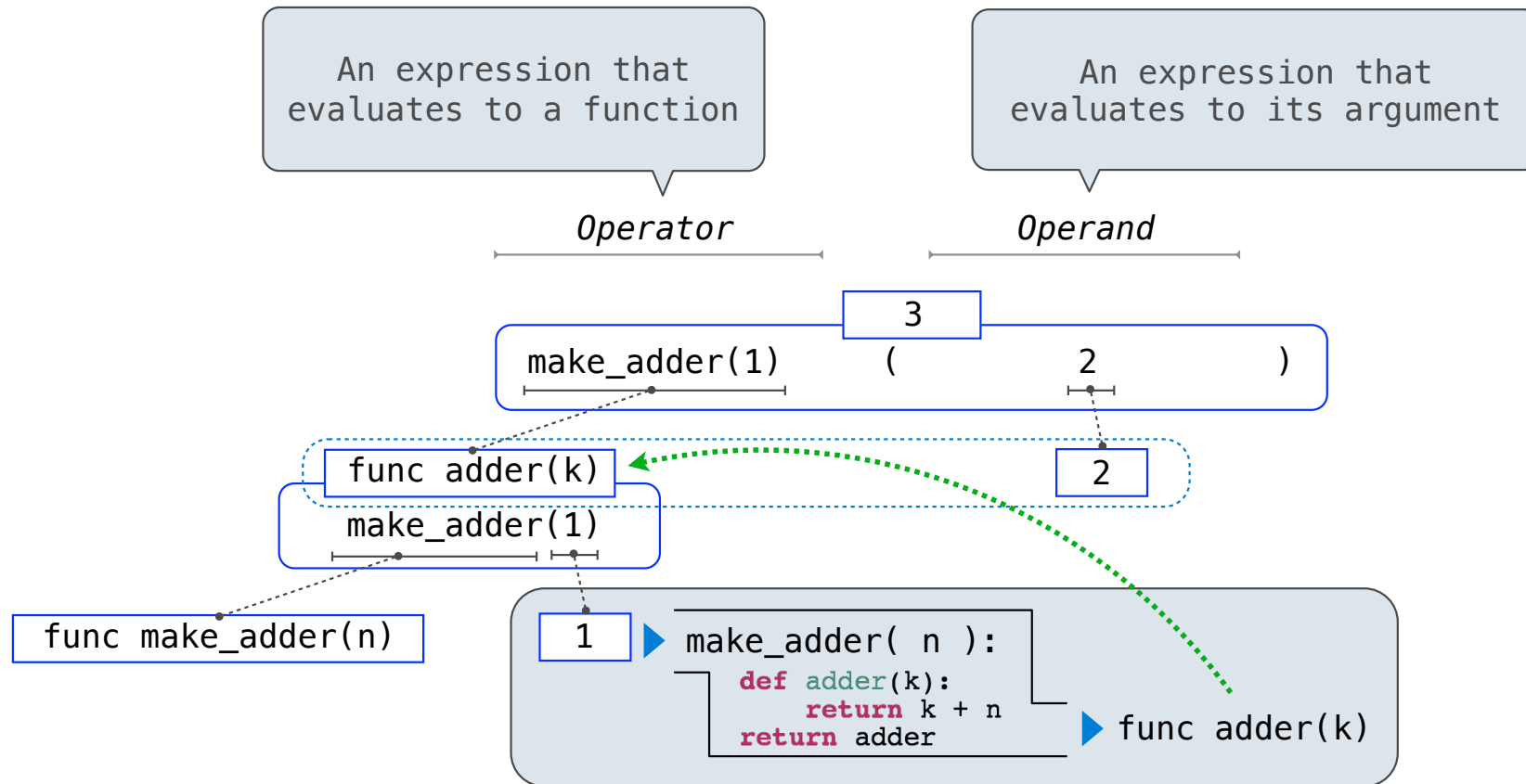
```
return adder
```

The name add_three is bound
to a function

A def statement within
another def statement

Can refer to names in the
enclosing function

Call Expressions as Operator Expressions



Lambda Expressions

(Demo)

Lambda Expressions

```
>>> x = 10
```

An expression: this one evaluates to a number

```
>>> square = x * x
```

Also an expression: evaluates to a function

```
>>> square = lambda x: x * x
```

Important: No "return" keyword!

A function

with formal parameter *x*

that returns the value of `"x * x"`

```
>>> square(4)  
16
```

Must be a single expression

Lambda expressions are not common in Python, but important in general
Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements



`square = lambda x: x * x`

VS



`def square(x):
 return x * x`

- Both create a function with the same domain, range, and behavior.
- Both bind that function to the name `square`.
- Only the `def` statement gives the function an intrinsic name, which shows up in environment diagrams but doesn't affect execution (unless the function is printed).

