Chapter 1

System Model

1.1 System Model

The system that will be considered throughout this work aims to represent the downlink of a canonical cellular network, comprising several identical cells, layed out over a regular hexagonal grid.

When studying a cellular network, the cells located at the edge of the network will not experience the same conditions as the cells in the center of the network. A typical way to deal with this situation is to consider a scenario that wraps around Figure 1.1 so that cells on one side of the scenario affect cells on the opposite side. Another option is to consider a scenario with more cells than necessary, and then analyze the behavior of the cells located within the center of the network Figure 1.2, so that the exterior cells account for the interference, equaling the conditions of all the cells in the network.

Each ~~the~~ cell in the system under study will be served with a single Base Station (BS) that is equipped with t antennas. Each of the users considered in the system has r antennas.

A system with M BS and N users can then be modelled as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1.1}$$

$\mathbf{y}$ represents the signal received at all the users and is defined as

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} \in \mathbb{C}^{Nr \times 1} \tag{1.2}$$

where $\mathbf{y}_i \in \mathbb{C}^{r \times 1}$ is the signal received at the i-th user.

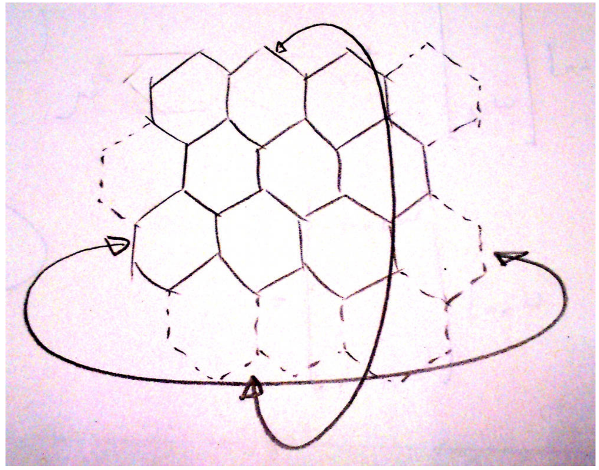

Figure 1.1: Wrap around scenario. Cells on one side of the scenario influence the cells on the other side as if they were next to each other.

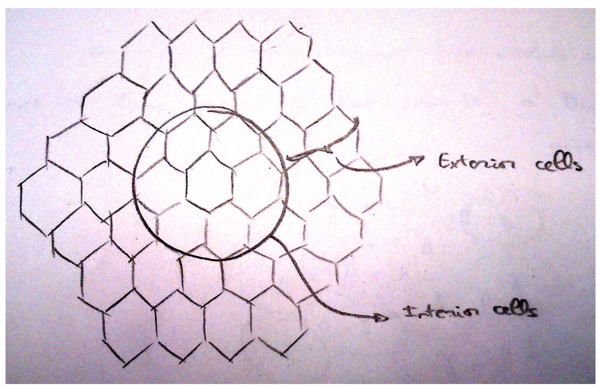

Figure 1.2: Oversized scenario. The behavior of the network is analyzed in the central cells of the network, and the exterior cells compensate the network edge effects.

$\mathbf{H}$ is the channel matrix from all the BS to all the users, with the following structure

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \cdots & \mathbf{H}_{1M} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{N1} & \cdots & \mathbf{H}_{NM} \end{bmatrix} \in \mathbb{C}^{Nr \times Mt} \tag{1.3}$$

where $\mathbf{H}_{ij} \in \mathbb{C}^{r \times t}$ represents the channel matrix from the j-th BS to the i-th user. It will include the path loss due to propagation, small scale ~~fast~~ fading, shadowing, and any other characteristic of the radio channel that needs to be taken into consideration.

$\mathbf{x}$ is the signal transmitted from all the BS, and it is composed of

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_M \end{bmatrix} \in \mathbb{C}^{Mt \times 1} \tag{1.4}$$

where $\mathbf{x}_j \in \mathbb{C}^{t \times 1}$ is the signal transmitted by the j-th BS. Additionally, the power transmitted by the j-th BS can be calculated from the transmitted signal as

$$P_{j,\text{tx}} = \text{Tr}\left(\mathbf{x}_j \mathbf{x}_j^H\right) = \mathbf{x}_j^H \mathbf{x}_j \tag{1.5}$$

and each BS will have an independent power constraint

$$P_{j,\text{tx}} \leq P_{j,\max} \tag{1.6}$$

Finally $\mathbf{n}$ represents the Additive White Gaussian Noise (AWGN) at all the receivers

$$\mathbf{n} = \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_N \end{bmatrix} \in \mathbb{C}^{Nr \times 1} \tag{1.7}$$

with $\mathbf{n}_i \in \mathbb{C}^{r \times 1}$ accounts for the gaussian noise at the i-th receiver. Throughout this work $\mathbf{n}_i$ is considered to be formed by *iid* entries, drawn from a zero mean, σ_i^2 variance gaussian distribution, $\mathbf{n}_i \sim \mathcal{N}\left(\mathbf{0}, \sigma_i^2 \mathbf{I}\right)$. The noise variance will be assumed the same for all the receivers.

In a general scenario, there may be cooperation among the BS in the system so that the information intended for a particular user will be transmitted by several or all the BS. Or, equivalently, each BS transmits a combination of the information of several users

$$\mathbf{x}_j = \mathbf{W}_{j1}^{(\text{tx})} \mathbf{s}_1 + \cdots + \mathbf{W}_{jN}^{(\text{tx})} \mathbf{s}_N \tag{1.8}$$

where $\mathbf{s}_i \in \mathbb{C}^{\ell \times 1}$ is the vector of information symbols to be transmitted to user i, with ℓ being the number of simultaneous symbols or streams to be transmitted to that user. $\mathbf{W}_{ji}^{(\text{tx})} \in \mathbb{C}^{t \times \ell}$ is the precoding matrix used at the j-th transmitter for the data of the i-th user.

It will be assumed that the information symbols are independent and drawn from a Gaussian distribution of zero mean and unit energy, *i.e.* $\mathbf{s}_i \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$, so that the transmitted power in (1.5) will be determined by the precoding matrix used.

$$P_{j,\text{tx}} = \sum_{i=1}^{N} \text{Tr}\left(\mathbf{W}_{ji}^{(\text{tx})} \mathbf{W}_{ji}^{(\text{tx}),H}\right) \tag{1.9}$$

The choice of the precoding matrices and of the receiving filter will determine the transmission strategy used. This ~~work~~ will focus mainly on Block Diagonalization (BD) [1] which is described in Section 1.2.

On the receiver side, no cooperation among the users will be considered, so each user may perform, independently, additional processing of the received signal by applying a linear filter or equalizer

$$\hat{\mathbf{s}}_i = \mathbf{W}_i^{(\mathrm{rx})}\mathbf{y}_i \tag{1.10}$$

where $\mathbf{W}_i^{(\mathrm{rx})} \in \mathbb{C}^{r\times\ell}$ is the equalizer used at the i-th receiver.

Combining (1.1)–(1.8) it is possible to rewrite (1.1) as

$$\mathbf{y} = \mathbf{H}\mathbf{W}^{(\mathrm{tx})}\mathbf{s} + \mathbf{n} \tag{1.11}$$

where

$$\mathbf{s} = \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_N \end{bmatrix} \in \mathbb{C}^{N\ell\times 1} \tag{1.12}$$

and the global precoding matrix is

$$\mathbf{W}^{(\mathrm{tx})} = \begin{bmatrix} \mathbf{W}_{11}^{(\mathrm{tx})} & \cdots & \mathbf{W}_{1N}^{(\mathrm{tx})} \\ \vdots & \ddots & \vdots \\ \mathbf{W}_{M1}^{(\mathrm{tx})} & \cdots & \mathbf{W}_{MN}^{(\mathrm{tx})} \end{bmatrix} \in \mathbb{C}^{Mt\times N\ell} \tag{1.13}$$

and it can be partitioned as

$$\mathbf{W}^{(\mathrm{tx})} = \left[\mathbf{W}_1^{(\mathrm{tx})}, \ldots, \mathbf{W}_N^{(\mathrm{tx})}\right] \tag{1.14}$$

with

$$\mathbf{W}_i^{(\mathrm{tx})} = \begin{bmatrix} \mathbf{W}_{11}^{(\mathrm{tx})} \\ \vdots \\ \mathbf{W}_{M1}^{(\mathrm{tx})} \end{bmatrix} \in \mathbb{C}^{Mt\times\ell} \tag{1.15}$$

The channel matrix can then be partitioned as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_N \end{bmatrix} \tag{1.16}$$

where $\mathbf{H}_i \in \mathbb{C}^{r \times Mt}$ is the channel matrix from all the BS to the i-th user.

As it has already been mentioned, receiver cooperation is not going to be considered, so looking at a particular user, *e.g.* the i-th user, the signal that is received will be

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{W}^{(\mathrm{tx})} \mathbf{s} + \mathbf{n}_i \tag{1.17}$$

which can be rewritten as

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{W}_i^{(\mathrm{tx})} \mathbf{s}_i + \underbrace{\sum_{\substack{j=1 \\ j \neq i}}^{N} \mathbf{H}_i \mathbf{W}_j^{(\mathrm{tx})} \mathbf{s}_j}_{\text{Interference}} + \mathbf{n}_i \tag{1.18}$$

so that it can be readily seen how other users' data appear as an interference term that degrades the signal ~~received~~.

Using (1.18), the ergodic (mean) rate for the i-th user is given by [2], [3]

$$R_i = \mathbb{E}\left\{ \log_2 \left| \mathbf{I} + \mathbf{H}_i \mathbf{W}_i^{(\mathrm{tx})} \mathbf{W}_i^{(\mathrm{tx}),H} \mathbf{H}_i^H \mathbf{R}_{i,IN}^{-1} \right| \right\} \tag{1.19}$$

where $\mathbf{R}_{i,IN} \in \mathbb{C}^{r \times r}$ is the covariance matrix of the noise plus the interference term in (1.18)

$$\mathbf{R}_{i,IN} = \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} \mathbf{H}_i \mathbf{W}_j^{(\mathrm{tx})} + \mathbf{n}_i \right) \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} \mathbf{H}_i \mathbf{W}_j^{(\mathrm{tx})} + \mathbf{n}_i \right)^H \tag{1.20}$$

In the rest of the work, there are a set of assumptions that will be made, mainly to guarantee the feasibility of some of the results obtained:

- The number of users will be the same as the number of BS, *i.e.* $N = M$.
- The total number of antennas transmitting will be greater or equal than the total antennas at the receiver side, this is $Mt \geq Nr$.
- The number of streams transmitted to each user must be $\ell \leq r$.
- There is no correlation, neither at the transmitters nor at the receivers, so that $\mathbf{H}$ is full rank or, equivalently, $\operatorname{rank}(\mathbf{H}) = \min(Nr, Mt)$. As $Mt \geq Nr$, then $\operatorname{rank}(\mathbf{H}) = Nr$.

1.2 Block Diagonalization

One possibility to cancel the interuser interference is to diagonalize the channel matrix. Perfect diagonalization is only possible if $Mt \geq Nr$ [4], and it is achieved using the following precoding matrix

$$\mathbf{W}^{(\mathrm{tx})} = \mathbf{H}^{\dagger} \tag{1.21}$$

This solution is optimum only when every user has only one antenna. In the case under study, with multiantenna receivers, complete diagonalization of the channel matrix is suboptimal since each user is able to coordinate the processing of its received signal.

In [1] is stated that the optimum solution under the constraint that all interuser interference be zero is obtained with $\mathbf{H}\mathbf{W}^{(\mathrm{tx})}$ being block diagonal. In [1], BD is proposed as an algorithm to obtain a precoding matrix that is able to block diagonalize the channel matrix. This algorithm is described next.

In order to meet the condition of zero interuser interference, it is necessary to cancel the interference term in (1.18), and this is equivalent to meet the following

$$\mathbf{H}_i \mathbf{W}_j^{(\mathrm{tx})} = \mathbf{0} \quad \forall j \neq i \tag{1.22}$$

Let $\widetilde{\mathbf{H}}_i$ be the channel matrix $\mathbf{H}$ without $\mathbf{H}_i$, *i.e.*

$$\widetilde{\mathbf{H}}_i = \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_{i-1} \\ \mathbf{H}_{i+1} \\ \vdots \\ \mathbf{H}_N \end{bmatrix} \in \mathbb{C}^{(N-1)r \times Mt} \tag{1.23}$$

Then the condition (1.22) can be obtained making $\mathbf{W}_i^{(\mathrm{tx})}$ lie in the null space or kernel of $\widetilde{\mathbf{H}}_i$. This is possible only if the dimension of the null space is greater than zero, *i.e.* $\operatorname{rank}\left(\ker\left(\widetilde{\mathbf{H}}_i\right)\right) > 0$.

Now, with the dimensions of $\widetilde{\mathbf{H}}$, the rank of its null space is

$$\operatorname{rank}\left(\ker\left(\widetilde{\mathbf{H}}_i\right)\right) = Mt - \operatorname{rank}\left(\widetilde{\mathbf{H}}_i\right) \tag{1.24}$$

But it is assumed that $\mathbf{H}$ is full rank, ergo $\operatorname{rank}\left(\widetilde{\mathbf{H}}_i\right) = (N-1)r = \widetilde{L}_i$, and then

$$\operatorname{rank}\left(\ker\left(\widetilde{\mathbf{H}}_i\right)\right) = Mt - \widetilde{L}_i > 0 \tag{1.25}$$

so it is guaranteed that a precoding matrix $\mathbf{W}_i^{(\mathrm{tx})}$ that lies in the null space of $\widetilde{\mathbf{H}}_i$ exists.

The simplest way to obtain such $\mathbf{W}_i^{(\mathrm{tx})}$ involves using the Singular Value Decomposition (SVD) of the matrix $\widetilde{\mathbf{H}}_i$.

Let $\widetilde{\mathbf{H}}_i$ be decomposed as

$$\widetilde{\mathbf{H}}_i = \widetilde{\mathbf{U}}_i \widetilde{\mathbf{\Lambda}}_i \left[\widetilde{\mathbf{V}}_i^{(1)}, \widetilde{\mathbf{V}}_i^{(0)}\right]^H \tag{1.26}$$

where $\widetilde{\mathbf{V}}_i^{(0)} \in \mathbb{C}^{Mt\times(Mt-\widetilde{L}_i)}$ contains the last $Mt-\widetilde{L}_i$ right singular vectors of $\widetilde{\mathbf{H}}_i$, corresponding to the singular values equal to zero. $\widetilde{\mathbf{V}}_i^{(0)}$ forms an orthonormal basis of the null space of $\widetilde{\mathbf{H}}_i$, and thus its columns can be used to cancel the interuser interference

$$\widetilde{\mathbf{H}}_i \widetilde{\mathbf{V}}_i^{(0)} = \mathbf{0} \tag{1.27}$$

Using these matrices as precoding the result is

$$\widehat{\mathbf{H}} = \mathbf{H}\left[\widetilde{\mathbf{V}}_1^{(0)}, \ldots, \widetilde{\mathbf{V}}_N^{(0)}\right] = \begin{bmatrix} \mathbf{H}_1\widetilde{\mathbf{V}}_1^{(0)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{H}_N\widetilde{\mathbf{V}}_N^{(0)} \end{bmatrix} \tag{1.28}$$

that, as it can be seen, has a block diagonal structure, which gives the name to the algorithm proposed in [1].

The next problem that BD solves is the maximization of the sum rate of the system, given the block diagonal structure in (1.28). The precoding matrix $\mathbf{W}_i^{(\mathrm{tx})}$ will be considered to be

$$\mathbf{W}_i^{(\mathrm{tx})} = \widetilde{\mathbf{V}}_i^{(0)} \mathbf{W}_i' \mathbf{P}_i^{1/2} \tag{1.29}$$

where $\mathbf{W}_i' \in \mathbb{C}^{(Mt-\widetilde{L}_i)\times\ell}$ will take care of the rate maximization, and $\mathbf{P}_i = \operatorname{diag}\{p_{i1}, \ldots, p_{il}\} \in \mathbb{R}^{\ell\times\ell}$ will contain the power allocated to each of the parallel streams that need to be sent to the i-th user.

Introducing (1.29) into (1.19) the ergodic capacity simplifies to

$$R_i^{\text{no interf}} = \mathbb{E}\left\{\log_2\left|\mathbf{I} + \frac{1}{\sigma_i^2}\widehat{\mathbf{H}}_i\mathbf{W}_i'\mathbf{P}_i\mathbf{W}_i'^{,H}\widehat{\mathbf{H}}_i^H\right|\right\} \tag{1.30}$$

where $\widehat{\mathbf{H}}_i = \mathbf{H}_i \widetilde{\mathbf{V}}_i^{(0)} \in \mathbb{C}^{r \times (Mt - \widetilde{L}_i)}$.

In order to maximize the rate, consider the SVD

$$\widehat{\mathbf{H}}_i = \widehat{\mathbf{U}}_i \begin{bmatrix} \widehat{\mathbf{\Lambda}}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \left[\widehat{\mathbf{V}}_i^{(1)}, \widehat{\mathbf{V}}_i^{(0)} \right]^H \tag{1.31}$$

where $\widehat{\mathbf{\Lambda}}_i = \operatorname{diag}\left\{ \hat{\lambda}_{i1}^{1/2}, \ldots, \hat{\lambda}_{ir}^{1/2} \right\} \in \mathbb{C}^{r \times r}$ contains the non zero singular values of $\widehat{\mathbf{H}}_i$, which has rank $\left(\widehat{\mathbf{H}}_i\right) = r$. And $\widehat{\mathbf{V}}_i^{(1)} \in \mathbb{C}^{(Mt - \widetilde{L}_i) \times r}$ contains the first r right singular vectors of $\widehat{\mathbf{H}}_i$, and it will be used as $\mathbf{W}_i'$, yielding the following precoding matrix

$$\mathbf{W}_i^{(\mathrm{tx})} = \widetilde{\mathbf{V}}_i^{(0)} \widehat{\mathbf{V}}_i^{(1)} \mathbf{P}_i^{1/2} \tag{1.32}$$

The BD also provides the receiver filter to be used at each user which will be

$$\mathbf{W}_i^{(\mathrm{rx})} = \widehat{\mathbf{U}}_i^H \tag{1.33}$$

Using all of the above, the rate that the i-th user can obtain is given by the expression

$$R_i^{\mathrm{BD}} = \mathbb{E}\left\{ \sum_{k=1}^{\ell} \log_2 \left(1 + \frac{\hat{\lambda}_{ik}^{1/2} p_{ij}}{\sigma_i^2} \right) \right\} \tag{1.34}$$

where the only parameters left to be computed are the power allocated to each of the ℓ streams of each user, and different options to do it will be discussed in Section 1.3.

1.3 Power Allocation

Bibliography

[1] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Transactions on Signal Processing*, vol. 52, pp. 461–471, February 2004.

[2] T. M. Cover and J. A. Thomas, *Elements of information theory.* John Wiley & Sons, 2012.

[3] B. Holter, "On the capacity of the MIMO channel: A tutorial introduction," in *Proc. IEEE Norwegian Symposium on Signal Processing*, pp. 167–172, 2001.

[4] G. Caire and S. Shamai (Shitz), "On the achievable throughput of a multiantenna gaussian broadcast channel," *IEEE Transactions on Information Theory*, vol. 49, pp. 1691–1706, July 2003.