Minimum-Energy Robotic Exploration: A Formulation and an Approach

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***Abstract*—This technical correspondence is concerned with the problem of autonomous robot exploration in a 3-D world. It provides a better understanding of exploration problems with regard to their mathematical formulation, and present a means to evaluate the performance of future algorithms designed to solve it. In addition, an approach capable of being implemented on articulated robotic systems is proposed. The presented algorithm possesses completeness properties, and typically chooses navigation paths that increase the robot's knowledge of its environment while expending minimal energy. It is compared to other exploration approaches capable of being implemented on high-degree-of-freedom robots. The comparison, performed in simulation, shows the significance of reducing energy during the exploration process.**

***Index Terms*—Energy consumption, path planning, robot sensing systems.**

I. Introduction

Exploration in robotics is the problem of moving a robot in an unknown environment to acquire information. The problem of exploration encompass a wide range of applications mentioned by the following motivational examples. The robotics community has been primarily interested in exploration for the purpose of providing robots with the capability of autonomously building maps, i.e., representations of the environment based on spatial information gathered by sensors over time [1]. Robots can then use maps to navigate and operate in the environment. Robots can become extremely valuable tools in areas where it is dangerous for humans to operate, e.g., disaster areas affected by natural or technological hazards, or areas containing land mines. Acar *et al.* [2] developed an algorithm that guides a robot in a minefield to detect and possibly clear mines. Other applications in dangerous environments include the inspection of underwater structures. Englot and Hover [3] devise a planning algorithm used for the sensory coverage of complex underwater surfaces such as the hull of a ship.

Energy conservation is an important aspect of exploration and, more generally, robot motion. Because exploring robots may operate on a limited power source, reducing the amount of energy expended during the exploration is valuable. Energy-efficient movements can augment the lifetime of a robot operating on a small energy source. The idea of minimizing energy during the exploration is proposed by Mei *et al.* [4]. The amount of energy expended by a robot can be divided into mechanical and electrical energy [5]. When compared to other components, the actuators have a high power consumption [6]. Therefore, this technical correspondence is focused on the minimization of the amount of energy dissipated through the robot motors.

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The following is a statement of contribution.

1) The exploration problem is formulated as a multioptimization problem in Section II. The formulation is applied on articulated robots operating under geometric and energy constraints. The formulation creates a natural means to evaluate competing approaches to the problem.
2) A probabilistically complete approach to the exploration problem is proposed in Section III. The approach uses a cost approximation method that quantifies the amount of energy expended to move an articulated robot along a path in the configuration space. The cost approximation method is combined with the existing motion planning algorithm RRT* to find low-energy and collision-free paths.
3) Using the problem formulation, a methodology for systematically evaluating the efficiency of exploration algorithms is presented in Section VI. This methodology is applied to evaluate exploration algorithms in 3-D simulated environments. The results of the comparative evaluation show the significance of minimizing energy.

II. Related Work

A. Exploration in Topological Maps

Early approaches in robotic exploration were conducted by Kuipers and Byun [7] and Dudek *et al.* [8]. They propose a hierarchical map, characterized by a topological layer represented as a graph, where vertices are distinctive places in the environment. Traditionally, mapping methods are divided into topological maps that model the connectivity of distinctive places in the environment, and metric maps that model its geometric properties [9]. Exploration methods that use metric maps (e.g., occupancy grid maps) are examined below.

B. Exploration With Mobile Robots

Yamauchi [10] introduces an exploration method that uses an occupancy grid and is based on the concept of frontiers, regions of the environment that lie between the explored space and the unexplored space; the robot explores by traveling to these regions. Other methods [11]–[13] include a navigation cost to the expected information gain at a position in the map; the robot moves to the position that maximizes the difference between information gain and cost. A comparison between different exploration strategies for mobile robots is conducted by Holz *et al.* [14]. The criterion used in their comparative evaluation is the total length of the robot's trajectory.

C. Exploration With Manipulator Robots

The methods cited above are designed toward mobile robots and are thus not necessarily practical for an articulated robot. Other exploration methods concentrate on manipulator robots that have a sensor attached to their end effector. This type of robot is used to create a 3-D model of an object of interest in the environment [15, 16] by using the next best view (NBV) paradigm. The goal is to determine the next (best) sensor configuration to scan the object with a minimum number of scans. A NBV approach is also used by Wang and Gupta [17] to explore the robot's configuration space with a manipulator. Their work introduces a measure, called configuration space entropy, that expresses the robot's ignorance of its configuration space. This measure is used to find the next sensor configuration that maximizes the robot's knowledge of its configuration space. Renton *et al.* [18] explore by scanning selected targets in the workspace, where a target is a 3-D point in the unknown space. This exploration strategy is less constrained than other NBV approaches because the manipulator's sensor is not required to reach an exact configuration (since a target can be viewed by an infinite number of configurations). The NBV methods produce, at each iteration, one or more desired operational space configurations for a sensor. These methods rely upon a planner to find a collision-free path to a goal configuration. Other methods (discussed below) for exploration with a manipulator only use an external planner in exceptional cases.

A function that maps a configuration to a rating is introduced by Kruse *et al.* [19]. Their exploration approach searches along the gradient of the rating function to find a goal configuration. In most cases, the goal configuration can be reached through a linear path. The rating function uses a weight that specifies the importance given to acquire new information at the expense of navigation cost. The rapidly-exploring random tree (RRT) algorithm [20] (which has been employed heavily in motion planning for exploration of high-dimensional spaces) is used as an integral part of the exploration approach [21]. RRTs bias search into the largest Voronoi regions [22], which makes them efficient for solving motion planning problems. The RRT-based method expands a RRT in a predefined subset of the configuration space. The subset contains all the configurations included in an n-ball of a predefined radius. This constrained exploration is necessary to execute a "world-based depth-first traversal" of the robot's physical world and to prevent continuous backward and forward motion between distant configurations.

D. Energy-Optimal Motion Planning

In contrast to the work cited above, our technical correspondence focuses on minimizing actuator energy dissipation during the exploration. The energy-optimal planning problem has seen an interest in industrial applications to reduce manufacturing costs. The optimal control problem of finding a minimum-energy point-to-point trajectory is solved on a three-degree-of-freedom industrial robot using numerical methods [23]. Galicki [24] applies the negative formulation of Pontryagin's maximum principle to guide a planar robot in a 2-D environment with obstacles. Gregory *et al.* [25] solve two planning problems on a planar robot manipulator with two degrees of freedom: the path tracking and the intersection-free trajectory planning problems. In their solution, the optimal control problem is reformulated into a calculus of variations problem.

Energy-optimal robot exploration has been studied by Mei *et al.* [4] who use the concept of frontiers [10] in order to direct a mobile robot via a locomotion strategy toward minimizing energy. They compare their method to a simple greedy strategy. A vein of research by Willow Garage [26], [27] has motivated our technical correspondence. Their use of multiple sensors—3-D laser range sensors in particular—to construct models of indoor environments for motion planning raises the question of what algorithms should be used to explore an environment efficiently.

III. Problem Definition

The robotic exploration problem has been studied extensively. It has been phrased by Thrun *et al.* [13] as a decision-theoretic problem. However, it is typically presented without a complete formulation, i.e., a mathematical description of the robotic system, the exploration objectives and the physical constraints.

The energy-optimal exploration problem (EEP) is formulated here as a bi-objective optimization. It is informally defined as follows. Given a robot initial state (i.e., its initial configuration and velocity vectors), the goal is to maximize its knowledge of an unknown environment while minimizing the amount of energy expended during that process. We require that the exploration itself does not cause the robot to contact obstacles; we assume that the environment is static (in order to avoid contact) and that the robot's configuration is known throughout the exploration process, i.e., there is no proprioceptive uncertainty. The transformation to a formal description follows.

A. Notation

Let $\mathcal{W}$ be a bounded region in a 3-D space that represents the robot's physical world. The portion of $\mathcal{W}$ unknown to the robot at time t is denoted $\mathcal{W}_t^{\mathrm{u}}$. The known portion, at time t, is subdivided into the free space and the occupied (also called obstacle) space, respectively denoted $\mathcal{W}_t^{\mathrm{free}}$ and $\mathcal{W}_t^{\mathrm{obs}}$. We have

$$\mathcal{W} = \mathcal{W}_t^{\mathrm{u}} \cup \mathcal{W}_t^{\mathrm{free}} \cup \mathcal{W}_t^{\mathrm{obs}} \tag{1}$$

where $\mathcal{W}_t^{\mathrm{u}}$, $\mathcal{W}_t^{\mathrm{free}}$, and $\mathcal{W}_t^{\mathrm{obs}}$ are pairwise disjoint. We denote their respective volumes by $|\mathcal{W}_t^{\mathrm{u}}|$, $|\mathcal{W}_t^{\mathrm{free}}|$, and $|\mathcal{W}_t^{\mathrm{obs}}|$.

Let $\mathcal{C}$ be the n-dimensional configuration space of the robot. $\mathcal{C}_t^{\mathrm{free}}$ denotes the free configuration space at time t. Let $\mathbf{q} \in \mathcal{C}$ be a configuration. The physical space occupied by the robot at $\mathbf{q}$ is denoted by $\mathcal{R}(\mathbf{q})$. We have $\mathbf{q} \in \mathcal{C}_t^{\mathrm{free}}$ if and only if $\mathcal{R}(\mathbf{q})$ does not intersect with $\mathcal{W}_t^{\mathrm{u}}$ and $\mathcal{W}_t^{\mathrm{obs}}$.

B. Exploration Algorithm and Termination

The exploration is carried out by an algorithm that is applied to the robot's knowledge of the world at time t (i.e., $\mathcal{W}_t^{\mathrm{u}}$, $\mathcal{W}_t^{\mathrm{free}}$, and $\mathcal{W}_t^{\mathrm{obs}}$) to obtain a feasible control $\mathbf{u}(t)$. The algorithm stops when a termination criterion is reached; let T denote the time length of the exploration. $|\mathcal{W}_T^{\mathrm{u}}|$ represents the volume of the region unexplored at T. Ideally, the robot keeps exploring until the totality of the world is known, i.e., $|\mathcal{W}_T^{\mathrm{u}}| = 0$. However, this termination criterion is impractical: regions in the unknown space might be unobservable or unreachable due to the robot's geometry. We propose an alternative criterion for terminating the exploration in Section V.

The control vector $\mathbf{u}(t)$ represents the torque vector at time t denoted by $\tau(t)$. Using the rough approximation of integral-squared torque to energy dissipated in the motors as suggested by Chevallereau *et al.* [28] and Gregory *et al.* [25], the total energy expended by the actuators is defined as

$$J_T := \int_0^T \tau(t)^{\mathsf{T}} \tau(t)\, \mathrm{d}t \tag{2}$$

where J_T is equal to zero if no torque is applied to the robot and is positive otherwise.

C. Problem Formulation

We can quantify the informal statement given at the beginning of this section by proposing the problem of finding $\tau(t)$, $0 \le t \le T$, to

$$\text{minimize} : [|\mathcal{W}_T^{\mathrm{u}}|, J_T]^{\mathsf{T}} \tag{3}$$

$$\text{subject to} : (\mathbf{q}(0), \dot{\mathbf{q}}(0)) = (\mathbf{q}_0, \dot{\mathbf{q}}_0) \tag{4}$$

$$\mathbf{q}(t) \in \mathcal{C}_t^{\mathrm{free}} \ \forall t \tag{5}$$

$$\ddot{\mathbf{q}} = \mathbf{M}(\mathbf{q})^{-1} \{\tau - \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{g}(\mathbf{q})\} \tag{6}$$

where $(\mathbf{q}_0, \dot{\mathbf{q}}_0)$ is the robot's initial state, $\mathbf{M}(\mathbf{q})$ is the robot's generalized inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ denotes the Coriolis and centrifugal forces, and $\mathbf{g}(\mathbf{q})$ is the vector of gravity forces. Equation (6) is the well-known equation for multirigid-body dynamics.

A solution to (3) is a set of functions (known as the Pareto optimal set) instead of a single optimal solution. We transform EEP into a single-objective optimization problem by applying the constrained objective function method [29]. More importance is awarded to the first criterion $|\mathcal{W}_T^{\mathrm{u}}|$; the second criterion J_T is used to form an additional constraint. The resulting problem consists of finding $\tau(t)$, $0 \leq t \leq T$, to

$$\text{minimize} : |\mathcal{W}_T^{\mathrm{u}}| \tag{7}$$

$$\text{subject to} : \text{Equations (4, 5, 6)}$$

$$J_T \leq J^{\max} \tag{8}$$

where the parameter $J^{\max}$ is an upper bound for the amount of energy expended during the exploration.

When using the constrained objective function method we choose to place the second criterion J_T as a constraint. As mentioned above, $|\mathcal{W}_T^{\mathrm{u}}|$ is treated as a more important criterion, however, there are other arguments in favor of this decision. Given that the workspace $\mathcal{W}$ is bounded, then for a large $J^{\max}$, the function $|\mathcal{W}_T^{\mathrm{u}}|$ will surely converge to a single value; which makes the comparison between different exploration methods easier. This can be seen in the results section. Furthermore, the value of $J^{\max}$ can have a practical significance, e.g., it can represent the robot's total amount of power supply.

IV. Existing Approaches

This section describes exploration algorithms that can be used on robots that possess multiple degrees of freedom and can operate in a 3-D workspace. The following existing approaches (evaluated in Section VI) optimize different criteria to carry out an efficient exploration, however, they do not explicitly minimize energy.

A. Rating Functions

The rating functions (RF) approach, described by Kruse *et al.* [19], introduces a function that maps a configuration $\mathbf{q} \in \mathcal{C}^{\mathrm{free}}$ to a rating. The rating combines the predicted amount of new information obtained at $\mathbf{q}$ and the Euclidean distance between the current configuration $\mathbf{q}_0$ and $\mathbf{q}$. At each iteration of the algorithm, the gradient of the rating function is calculated, and safe configurations along the gradient are examined. The algorithm chooses the one with the highest rating. When a local maximum is attained, the search proceeds to a random configuration that has a positive rating.

If $\mathbf{q}$ cannot be reached through a linear path, a path planning algorithm must be called, e.g., the RRT planner.

B. Maximal Expected Entropy Reduction

The maximal expected entropy reduction (MER) method is described by Wang and Gupta [17] and is a continuation of the work by Yu and Gupta [30]. This technical correspondence introduces a measure, called configuration space entropy, that expresses the robot's ignorance of its configuration space. MER defines a function of a sensor's configuration $\mathbf{s}$, called the expected entropy reduction function, and denoted $\widetilde{ER}$. The goal is to move the robot to the configuration that maximizes $\widetilde{ER}(\mathbf{s})$ at each iteration. $\widetilde{ER}(\mathbf{s})$ is approximated by performing a summation of marginal entropies over a large enough number of samples in $\mathcal{C}$.

MER performs a sensor-based incremental construction of a probabilistic road map [31] whose nodes are configurations in $\mathcal{C}^{\mathrm{free}}$. MER uses this roadmap to plan a path to the next desired sensor's configuration.

C. Sensor-Based Exploration Tree

The sensor-based exploration tree (SET) was developed by Freda *et al.* [21]. At each iteration, SET computes the robot's local free boundary (LFB) at the current configuration $\mathbf{q}_0$. The LFB contains all points of the frontier that are both in $\mathcal{C}^{\mathrm{free}}$ and can be viewed from the sensor at an admissible configuration. A configuration $\mathbf{q}$ is admissible if the sensor position at $\mathbf{q}$, denoted $s(\mathbf{q})$, is within a maximum distance ρ of $s(\mathbf{q}_0)$, and the line segment between $s(\mathbf{q})$ and $s(\mathbf{q}_0)$ does not intersect a known obstacle.

Let $\mathcal{D}$ denote the subset of $\mathcal{C}$ containing all the configurations included in the n-ball of radius δ and center $\mathbf{q}_0$. If the LFB is not empty, SET locally expands a RRT (rooted at $\mathbf{q}_0$) in $\mathcal{D}$. Then, the algorithm extracts from the RRT all admissible configurations. The robot moves to the admissible configuration that maximizes the predicted amount of new information obtained by the sensor. If the LFB is empty, the robot backtracks to a previous configuration.

If the RRT fails to find a configuration that increases the predicted information, a second tree is expanded in $\mathcal{D}$ without collision detection. Only the admissible configurations are added to the lazy tree. The robot moves to a safe configuration in the lazy tree that increases the predicted information after a path planner is called. In the case where both the RRT and the lazy tree fail, the robot backtracks to a previous configuration.

V. Proposed Algorithm

The algorithms described in the previous section do not attempt to explicitly find exploration paths that reduce energy. To address EEP we propose an alternative exploration algorithm described here.

A. Effectiveness of Exploration Paths

Let $\mathbf{q}_0$ be the robot current configuration and $\sigma = (\mathbf{q}_0, ..., \mathbf{q}_k)$ be a path in $\mathcal{C}^{\mathrm{free}}$. We seek to develop a utility function U that predicts the effectiveness of the move along a path σ. This measure combines the predicted volume of the region explored by the robot and the predicted amount of energy required to reach $\mathbf{q}_k$.

1) Predicted Volume of the Region Explored: Let $\mathbf{s}_k$ denote the sensor's operational space configuration [i.e., a point in $SE(3)$] at $\mathbf{q}_k$. Given $\mathbf{q}_k$, $\mathbf{s}_k$ can be computed using the robot's forward kinematics. The observable portion of $\mathcal{W}$ from $\mathbf{s}_k$ is called the field of view (FOV). We denote by v_k the predicted increase in volume attained by moving to $\mathbf{q}_k$, we have $v_k \geq 0$. The quantity v_k is determined by finding the portion of the FOV that intersects $\mathcal{W}^{\mathrm{u}}$ and is not obstructed by $\mathcal{R}(\mathbf{q}_k)$ and $\mathcal{W}^{\mathrm{obs}}$. This is illustrated in Fig. 1. Note that v_k does not incorporate exploration due to sensor views at intermediate configurations between $\mathbf{q}_0$ and $\mathbf{q}_k$.

2) Predicted Energy: The predicted amount of energy required to move along the path σ is denoted by c_k and is given by

$$c_k = \begin{cases} \epsilon & \text{if } k = 0 \\ c_{k-1} + \hat{J}_k & \text{if } k > 0 \end{cases} \tag{9}$$

where ϵ is a positive infinitesimal quantity, and $\hat{J}_k$ represents the approximated amount of energy needed to travel from $\mathbf{q}_{k-1}$ to $\mathbf{q}_k$ and is calculated below. We have $c_k > 0$.

The amount of energy $\hat{J}_k$ is computed as follows. Let Π:$[0, t_f] \rightarrow \mathbb{R}^n$ be a point-to-point trajectory, with zero velocities at end points, interpolating $\mathbf{q}_{k-1}$ and $\mathbf{q}_k$ at 0 and t_f, respectively. The trajectory Π is discretized into m points ($m > 1$) evenly spaced in time from 0 to t_f, and t_f is given by $t_f = (m-1)\Delta t$, where m and Δt are exploration parameters. We calculate the cost of moving along Π as

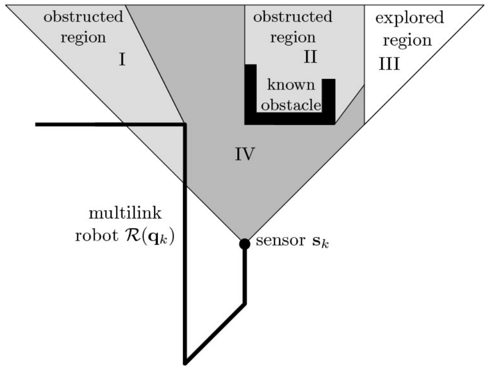

Fig. 1. FOV of a sensor placed on the end effector of a multilink robot, in a 2-D workspace. The FOV is subdivided into four regions. Region I is unknown and obstructed by the physical space occupied by the robot $\mathcal{R}(\mathbf{q}_k)$. Region II is unknown and obstructed by the known occupied space $\mathcal{W}^{\text{obs}}$. Region III intersects with the known free space $\mathcal{W}^{\text{free}}$. Region IV is the region of interest, it is in the FOV, and is neither explored nor obstructed. v_k is calculated by finding the area of Region IV.

an approximation of the integral-squared torque given by

$$\hat{J}_k = \sum_{i=0}^{m-2} \tau_k(t_i)^{\mathsf{T}} \tau_k(t_i)\, \Delta t \tag{10}$$

where the torque vector $\tau_k(t_i)$ is determined by solving the inverse dynamics problem at $t_i = i\Delta t$.

3) Utility Function: The predicted effectiveness of the move along the path σ is given as a function of v_k and c_k as follows:

$$U(\sigma) = f(v_k, c_k). \tag{11}$$

The function U is a utility function that combines information gain and cost to articulate the exploration objectives. Thrun *et al.* [13] and Kruse *et al.* [19] express U as a weighted sum, whereas Gonzalez-Banos and Latombe [12] choose an exponential weighted utility function. In this technical correspondence, we choose the weighted product method. This type of function is used in multi-objective optimization when dealing with quantities having different orders of magnitude [32], such as v_k and c_k. Thus, we have

$$U(\sigma) = {v_k}^{\alpha} {c_k}^{\alpha - 1} \tag{12}$$

where α is a weight with $0 < \alpha < 1$. A large α means that maximizing information gain is favored over minimizing cost. The function $U(\sigma)$ is equal to zero if $v_k = 0$ and is positive otherwise.

B. Path Planning

The probabilistic roadmap [33] and the RRT [34] are popular motion planning algorithms. They have been used in the context of exploration (Section IV). Such algorithms are shown to lack in asymptotic optimality with respect to a given cost function [e.g., (9)] [35]. The nonoptimality of RRT is avoided by using RRT*, shown to be asymptotically optimal.

We denote by τ-RRT* the RRT* algorithm that uses the cost in (9), and by d-RRT* the RRT* algorithm that uses a Euclidean metric. We compare the respective running time of the two planning algorithms. Fig. 2 shows that the running time of τ-RRT* slightly increases when compared to the running time of d-RRT*.

C. Algorithm

At each iteration Algorithm 1 expands a tree $\mathbf{T}$, from $\mathbf{q}_0$, using the τ-RRT* algorithm. The total number of configurations sampled by

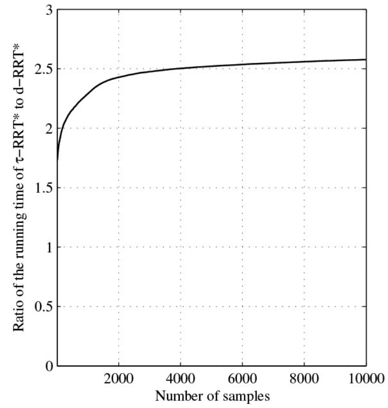

Fig. 2. Ratio of the running time of τ-RRT* to d-RRT* is shown for an increasing number of samples (i.e., number of iterations). The results are obtained from 100 trials.

Algorithm 1 Exploration Algorithm

```
1:  exploration-completed ← false
2:  while exploration-completed = false do
3:      q0 ← current configuration of the robot
4:      T ← τ-RRT*(q0, K)
5:      σ* ← path in T that maximizes (12)
6:      if U(σ*) > 0 then
7:          move the robot along σ*
8:      else
9:          exploration-completed ← true
10:     end if
11: end while
```

τ-RRT* is K (an exploration parameter). The robot chooses the path σ^* in $\mathbf{T}$ that maximizes the utility function in (12). We assume that, while moving along the path σ^*, the robot senses the environment and updates a map (the robot's current representation of the physical world). The processes of sensing the environment and updating the map are described in the next section.

The algorithm terminates if $U(\sigma^*)$ is equal to zero, i.e., $v_k = 0$ for all the configurations added to $\mathbf{T}$. Such condition signifies that the algorithm cannot find an informative configuration, i.e., a configuration that reduces the size of $\mathcal{W}^{\text{u}}$.

D. Running Time

Building the τ-RRT* is performed in $O(K \log K)$-time [35], where K is the number of samples. Finding σ^* is performed in $O(K)$-time with a preorder traversal of $\mathbf{T}$.

E. Completeness Analysis

We begin by defining the notion of completeness in the context of exploration. Let $\mathcal{W}'(\mathbf{q}_0)$ represent the explorable region of $\mathcal{W}$ given the initial configuration $\mathbf{q}_0$. A complete algorithm explores $\mathcal{W}'(\mathbf{q}_0)$ entirely, and we have

$$\mathcal{W}_0^{\text{u}} \setminus \mathcal{W}_T^{\text{u}} = \mathcal{W}'(\mathbf{q}_0). \tag{13}$$

We now show that Algorithm 1 possesses probabilistic completeness.

Theorem 1: The probability that Algorithm 1 explores $\mathcal{W}'(\mathbf{q}_0)$ converges to 1 as the number of samples K approaches infinity.

Proof: Suppose that Algorithm 1 terminates with

$$\mathcal{W}_0^{\text{u}} \setminus \mathcal{W}_T^{\text{u}} \subset \mathcal{W}'(\mathbf{q}_0) \tag{14}$$

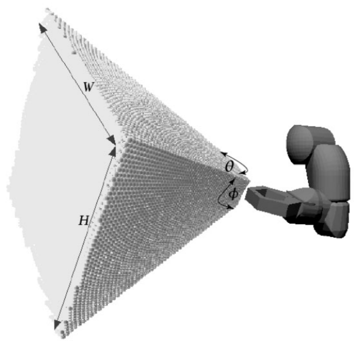

Fig. 3. Virtual 3-D laser range scanner uses a resolution of 50×50 (i.e., $H = W = 50$) and a FOV defined by $\theta = \phi = 0.8$ rad.

then there exists a point $P \in \mathcal{W}'(\mathbf{q}_0)$ such that $P \notin \mathcal{W}_0^{\mathrm{u}} \setminus \mathcal{W}_T^{\mathrm{u}}$ (i.e., P is not explored).

Since P is in the explorable region, there exists at least one configuration $\mathbf{q}$ such that $\mathbf{q}$ is reachable from $\mathbf{q}_0$ and P is in the robot's FOV at $\mathbf{q}$. We say that $\mathbf{q}$ is informative. At each iteration, Algorithm 1 creates a RRT tree that expands in the configuration space. For a number of samples K approaching infinity, the probability that a RRT reaches $\mathbf{q}$ converges to 1 [36].

Algorithm 1 terminates if no informative configuration is found. Therefore, for a number of samples K approaching infinity the probabilities that $\mathbf{q}$ is not found and that P is not explored converge to 0. ■

VI. Evaluation

The evaluation is made entirely in simulation, which allows us to obtain results from multiple trials repeated on different environments. Simulation permits ready design of heterogeneous environments for experimentation and accurate reproduction of initial conditions.

A. Simulated Environment

The robot is dynamically simulated using Featherstone's method for articulated bodies [37]. The simulated robot is an anthropomorphic manipulator with a fixed base: the arm possesses six degrees of freedom and the gripper two degrees of freedom. A simulated range sensor is positioned on the wrist of the arm. The "sensor" provides a 2-D array of depth readings using the z-buffer algorithm. The manipulator and the sensor's FOV are depicted in Fig. 3.

The depth readings provided by the sensor update an octree representation of the environment. The octree encodes both the unknown space $\mathcal{W}^{\mathrm{u}}$ and the occupied space $\mathcal{W}^{\mathrm{obs}}$. At the start of the exploration, all of the leaves of the octree are initialized to the unknown state. The robot at configuration $\mathbf{q}$ is said to be in collision, i.e., $\mathbf{q} \notin \mathcal{C}^{\mathrm{free}}$, if its geometric model intersects with the octree.

Three environments, with varied obstacle sizes and shapes, are employed. The first environment (E1), shown in Fig. 4(a), is spacious and contains large obstacles. Fig. 4(b) shows a cluttered space (E2), for which it is harder to plan a path between two configurations. The total volume of free space is comparatively largest in this environment. The third environment (E3), shown in Fig. 4(c), is a confined space that gives the robot very little space to maneuver. The total volume of free space is comparatively smallest in E3. Note that, in all three environments, the robot is unable to explore the totality of the unknown space due to the arm's limited flexibility and the presence of obstacles that both prevent the arm from reaching certain configurations and obstruct the sensor's FOV.

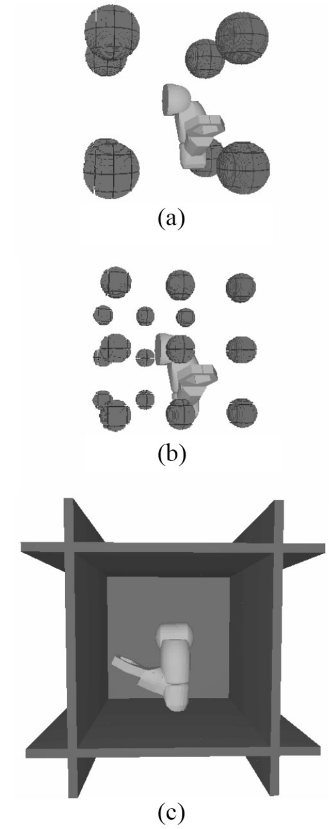

Fig. 4. Three 3-D environments used in the evaluation of the exploration algorithm on the simulated manipulator arm. (a) Environment E1. (b) Environment E2. (c) Environment E3.

TABLE I
Exploration Algorithms Used in the Evaluation

Exploration Algorithm	Path Planner	Cost Function	Utility Function
EXP-τ-RRT*	RRT*	int.-squared torque	weighted product
EXP-τ-RRT	RRT	int.-squared torque	weighted product
EXP-d-RRT*	RRT*	Euclidean distance	weighted product
EXP-τ-RRT*-v_k	RRT*	int.-squared torque	volume
RF	RRT	Euclidean distance	weighted sum
MER	PRM	–	entropy reduction
SET	RRT	Euclidean distance	volume

B. Results

1) Exploration Algorithms: The algorithms implemented for the evaluation are summarized in Table I. EXP-τ-RRT* is the algorithm presented in Section V. The following three algorithms are similar to EXP-τ-RRT*, but they each retain a unique exploration component. EXP-τ-RRT uses the RRT path planner. EXP-d-RRT* substitutes the cost given in (9) by the Euclidean distance. The algorithm EXP-τ-RRT*-v does not employ the utility function in (12). It performs a greedier exploration by choosing, at each iteration, the path that maximizes the volume v_k.

The following three algorithms are existing approaches described in Section IV, namely, RF, MER, and SET. They are used in our comparison because they can be implemented on robots that possess multiple degrees of freedom and can operate in a 3-D workspace.

The following parameters are chosen in the implementation.

1) In Section V, the parameters m and Δt respectively designate the number of points needed to calculate the sum in (9) and the small time interval between two consecutive points. Our

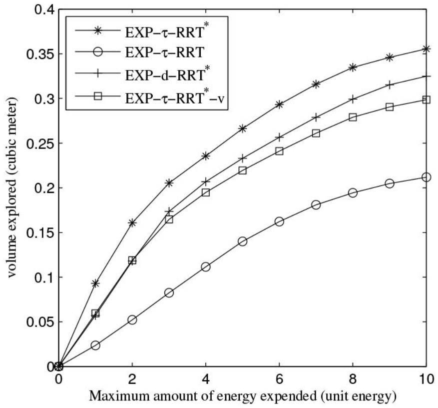

Fig. 5. Volume explored for values of $J^{\max}$ between 0 and 10 unit energy in Environment E1.

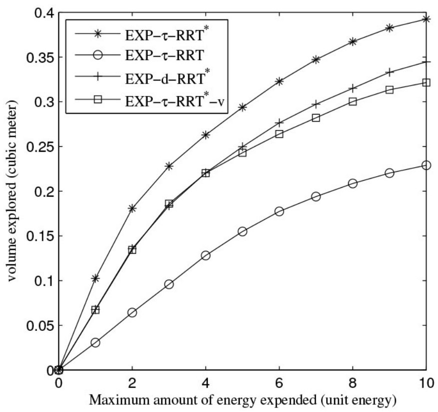

Fig. 6. Volume explored for values of $J^{\max}$ between 0 and 10 unit energy in Environment E2.

experience indicates that a small number of points suffices for obtaining good approximations. In our implementation, $m = 5$ and $\Delta t = 0.1s$.

2) The weighted product method uses a quantity α set to 0.5. Such value of α gives equal importance to optimizing information gain and cost.
3) The parameter K designates the number of configurations sampled by all the path planners listed in Table I. We choose $K = 3000$. We find that with a smaller K the environments tend to be incompletely explored.

2) Evaluation: The evaluation is performed based on the optimization problem defined in (7). Every exploration starts with identical initial conditions. To execute a path returned by an exploration algorithm, the robot arm is driven by torque computed by a composite inverse dynamics/PID controller, where the velocities and accelerations are obtained with a timing law of cubic splines.

The exploration halts if any of the following occurs.

1) The path planner cannot find a configuration whose utility function value is positive.
2) The amount of energy expended by the robot [computed via (2)] exceeds the upper bound $J^{\max}$.

Due to the fact that all of the exploration approaches are pseudo-random, we carry out 50 trials on every simulated environment. The comparison between the variants of Algorithm 1, in the environments E1, E2, and E3 is found in Figs. 5, 6, and 7, respectively. The mean of the total volume explored is shown for values of $J^{\max}$ between 0 and 10 unit energy. The comparison between Algorithm 1 and the

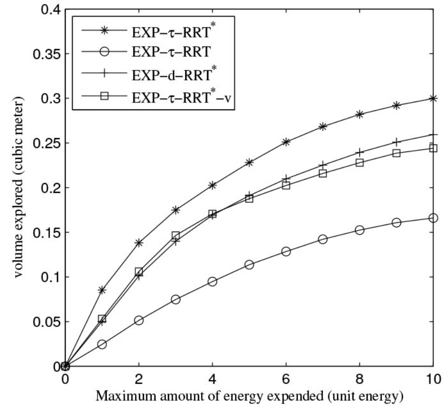

Fig. 7. Volume explored for values of $J^{\max}$ between 0 and 10 unit energy in Environment E3.

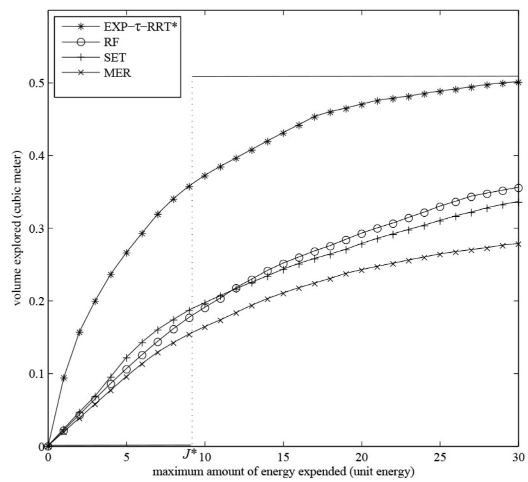

Fig. 8. Volume explored for values of $J^{\max}$ between 0 and 30 unit energy in Environment E1. The horizontal line shows the maximum volume that can be explored. The vertical dotted line shows an approximate minimum energy, denoted J^*, that must be used to explore the environment entirely.

remaining algorithms in Table I, in the environment E1, is found in Fig. 8. (Similar results were obtained for the Environments E2 and E3).

Fig. 8 also shows, as a reference, the maximum explorable volume and an approximate minimum energy, denoted J^*, used to explore the environment entirely. The value of J^* is found as follows. The environment is assumed to be known a priori. Using the method described by Englot and Hover [38], an approximate minimum of configurations is chosen such that the environment is totally explored by the union of their corresponding FOVs. Finally, the planner τ-RRT* is used to join the chosen configurations to form an exploration path.

3) Discussion: We observe that the relative efficiency of the algorithms is consistent among the three environments. The total volume explored is smaller in Environment E3 where the robot's view is the most obstructed. We find that EXP-τ-RRT* and EXP-d-RRT* perform better than the other two methods as they are both successful in reducing unnecessary movement of the arm (the movement of the arm during the exploration is shown in Fig. 9). However, the energy-based cost function offers an advantage over the Euclidean distance. Our results show a better performance of the RRT* path planner when compared to RRT, as EXP-τ-RRT is consistently less efficient than the other three algorithms. Under the same conditions, EXP-τ-RRT* shows an improvement of over 20% in all environments

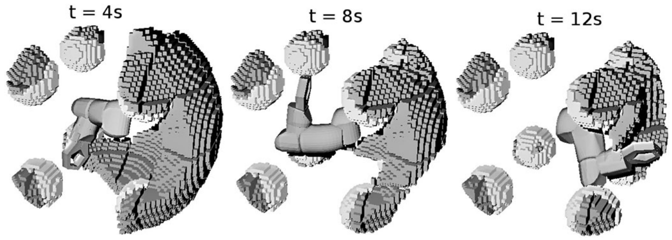

Fig. 9. Physical space of the robot arm during the exploration of E1 at 4, 8, and 12 s. The dark gray region corresponds to the unknown space, and the light gray region corresponds to the obstacle space.

when compared to EXP-τ-RRT*-v, which emphasizes the importance of using a utility function. We conclude that, based on the results, every component of Algorithm 1 contributes to a more efficient exploration.

The results also show that EXP-τ-RRT* performs better than other existing exploration approaches on average. Specifically, Fig. 8 shows that EXP-τ-RRT* has the largest mean value when compared to RF, MER, and SET. These algorithms do not attempt at reducing energy during the exploration process, which results in a less efficient exploration [with respect to the formulation in (7)].

Finally, the hypothesis (made in Section III) that $|\mathcal{W}_T^{\mathrm{u}}|$ converges to a single value for a large $J^{\max}$ is confirmed by the results in Fig. 8. Because Algorithm 1 is probabilistically complete, we can also deduce that, for environment E1, the explorable region's volume (i.e., $|\mathcal{W}'(\mathbf{q}_0)|$) is approximately equal 0.5 m^3.

VII. Conclusion

This technical correspondence formulates the EEP. It is expressed as a minimization of the volume explored. The energy function is used to form an additional constraint. This formulation of EEP allows a direct comparison of exploration algorithms. The energy constraint limit can be increased until the volume explored converges to a single value, for at least one of the algorithms used in the comparison.

A probabilistically complete approach EXP-τ-RRT* is presented. The algorithm typically chooses navigation paths that increase the robot's knowledge of its environment while expending minimal motor torques, thus reducing mechanical energy. It is compared in a simulated environment to other exploration approaches. The results confirm the validity of our formulation of EEP and show the importance of optimizing energy.

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