A Survey on Aerial Swarm Robotics

Soon-Jo Chung, *Senior Member, IEEE,* Aditya Paranjape, Philip Dames, *Member, IEEE,*
Shaojie Shen, *Member, IEEE,* and Vijay Kumar, *Fellow, IEEE*

***Abstract*—Aerial swarms are increasingly finding significant use for a variety of applications. A key enabling technology for swarms is the family of algorithms that allow the individual members of the swarm to communicate and allocate tasks amongst themselves, plan their trajectories, and coordinate their flight in such a way that the overall objectives of the swarm are achieved efficiently. These algorithms, often organized in a hierarchical fashion, endow the swarm with autonomy at every level and the role of a human operator can be reduced, in principle, to issuing high-level commands. This technology rests on clever and innovative application of theoretical tools from control and estimation, and it is the objective of this paper to review the state of the art in the manner in which these theoretical tools have been developed for, and applied to, aerial swarms. Aerial swarms differ from swarms of ground-based vehicles in two respects: they operate in a three dimensional (3-D) space, and the dynamics of individual vehicles adds an extra layer of complexity. We review dynamic modeling and conditions for stability and controllability that are essential in order to achieve cooperative flight and distributed sensing. The main sections of the paper focus on major results covering trajectory generation, task allocation, adversarial control, distributed sensing, monitoring, and mapping. Wherever possible, we indicate how the physics and subsystem technologies of aerial robots are brought to bear on these individual areas.**

***Index Terms*—Aerial robotics, distributed robot systems, networked robots.**

I. Introduction

Aerial robotics has become an area of intense research within the robotics and control community. Autonomous aerial robots can capitalize on the three-dimensional (3-D) airspace with aplomb, often times equipped with vertical take-off and landing capabilities using zero-emission distributed electric fans. Swarms of such aerial robots or autonomous Unmanned Aerial Vehicles (UAVs) are emerging as a disruptive technology to enable highly-reconfigurable, on-demand, distributed intelligent autonomous systems with high impact on many areas of science, technology, and society, including tracking, monitoring, and transporting systems. Autonomous aerial swarms are expected to deliver a greater capability than a single large vehicle, with significantly enhanced flexibility (adaptability, scalability, and maintainability) and robustness (reliability, survivability, and fault-tolerance) [1].

S.-J. Chung is with the Graduate Aerospace Laboratories of California Institute of Technology (GALCIT), Pasadena, CA 91125, USA (Email: sjchung@caltech.edu).

A. A. Paranjape is with the Department of Aeronautics, Imperial College London, South Kensington, London, United Kingdom (Email: a.paranjape@imperial.ac.uk).

P. Dames is with the Department of Mechanical Engineering, Temple University, Philadelphia, PA 19122, USA (Email: pdames@temple.edu).

S. Shen is with the Department of Electronic & Computer Engineering, Hong Kong University of Science and Technology, Hong Kong (Email: eeshaojie@ust.hk).

V. Kumar is with the School of Engineering & Applied Science, University of Pennsylvania, Philadelphia, PA 19104, USA (Email: kumar@seas.upenn.edu).

This survey article reflects on advances in aerial swarm robotics and recognizes that a number of technological gaps need to be bridged in order to achieve the aforementioned benefits of swarms of aerial robots through autonomous and safe operation. The papers included in this survey article represent the most important and promising ideas to address issues in modeling, control, planning, sensing, design, and implementation of aerial swarms, with an emphasis on enhanced flexibility, robustness, and autonomy.

Swarming aerial robots must autonomously operate in a complex 3-D world including urban canyons and an airspace that is getting increasingly crowded with drones and commercial airplanes. The success of aerial swarms flying in a 3-D world is predicated on the distributed and synergistic capabilities of controlling individual and collective motions of aerial robots with limited resources for on-board computation, power, communication, sensing, and actuation (the so-called size, weight and power, or SWaP, tradeoff). The goal is to provide a unified framework within which to analyze the three-way trade-off among computational efficiency for large-scale swarms, stability and robustness, and optimal system performance.

Compared to prior survey articles focused on robotic swarms [2], we emphasize swarms of aerial robots flying in a 3-D world. Other related survey papers include [3]. Put together, the papers reviewed here address challenges associated with transitioning from 2-D to 3-D with limited SWaP in areas such as swarm coordination or collaboration and distributed tracking and estimation. Our survey paper on aerial swarm robotics also addresses the challenges underlying autonomous aerial swarm systems that can seamlessly interact, or be integrated, with other types of robots such as ground vehicles. From a technological standpoint, the broader impacts of research in aerial swarm robotics include scalability and down-compatibility with 2-D robotic networks (e.g., ground robots) and other 3-D unmanned systems such as spacecraft swarms [4], [5] and underwater swarms [6]. The distinguishing characteristics of aerial swarm robotics are summarized as follows.

3-D Flow and Swarm Autonomy: Motion planning and control methods for aerial swarms rely on autonomously-generated 3-D traffic flows that do not have fixed edges or roads. Prior work on 2-D swarm robotics is not best suited to real-time flight control and swarm operation that takes into account higher-fidelity six-degrees-of-freedom (6-DOF) flight dynamic models, traffic variations, weather, and other time-varying operational conditions. In our scenario, flight duration and range could be shorter (e.g., the aerial swarm system that is restricted to a single urban environment). These aspects stand in stark contrast to those focused on 3-D air traffic flow control with much longer time horizon [7]–[9] as well as 2D road traffic flow theory, bipartite matching, and transport operation theory that assume fixed flight pathways and road/route topologies. Hence, this survey does not include air traffic control or management [10]. We explore methods of simultaneous 6-DOF trajectory generation while applying optimal swarm routing or control techniques. Furthermore, compared with existing air traffic control systems, where human operators perform real-time control of airport congestion and prevention of mid-air collision [7], there is a much-reduced level of human factors involved in an autonomous aerial swarm system.

Scalability Through Hierarchy and Multi-Modality: The quest for theoretically well-founded and computationally-efficient or scalable algorithms for large-scale swarm autonomy in complex environments can be realized by hierarchical architectures, for decentralized planning, reasoning, learning, and perception, that address scalability and information sharing and management in the presence of uncertainties. Hierarchical approaches are pervasive in both the machine learning and control fields for dealing with complexity and high dimensionality (e.g., hierarchical task networks (HTNs) [11], hierarchical tree or lattice networks employed in Sequential Game Theory [12], and singular perturbation theory [13], [14]). The complexity of aerial swarms can be reduced by exploiting hierarchical connections in spatial and temporal scales of large-scale aerial swarm networks. Other examples of hierarchically-combined dynamics include flight dynamics themselves. The outer loop (motion planning of swarms) will run an order of magnitude slower than the flight dynamics, while transient maneuvers of aerials swarms will be controlled at the same time scale as the rigid-body flight dynamics. Also, the inner-loop flight control that typically involves attitude dynamics is designed to run faster than the timescales of the rigid body dynamics of the aerial robot as well as the structural dynamics of the wing. We expand on aerial swarm algorithms and technologies that depend on hierarchical architectures. Further, due to the complexity and intrinsic randomness of the environment, cooperative estimation and planning algorithms must be integrated closely in conjunction with perception and reasoning of other vehicles, environmental conditions, and scientific/customer needs.

The organization of the present paper is shown in Fig. 1. In each section, we attempt to provide elementary working solutions of each subproblem. Further refinements of these tools lead us to the state of the art in the respective subject area. We review modeling of swarm dynamics and nonlinear stability tools, in particular for hierarchical decomposition in Sec. 2. We also review controllability issues for aerial swarms. In Sec. 3, we review optimal control, motion planning, task assignment, and other control algorithms. In Sec. 4, we discuss distributed sensing and estimation using aerial swarms, canonically for the purpose of tracking a common single target or multiple targets and distributed surveillance problems. Also, cooperative mapping algorithms using aerial vehicles are discussed. In Sec. 5, we review essential system-level and component technologies for aerial swarms. In Sec. 6, We conclude the paper with a discussion on open problems in the area of aerial swarms.

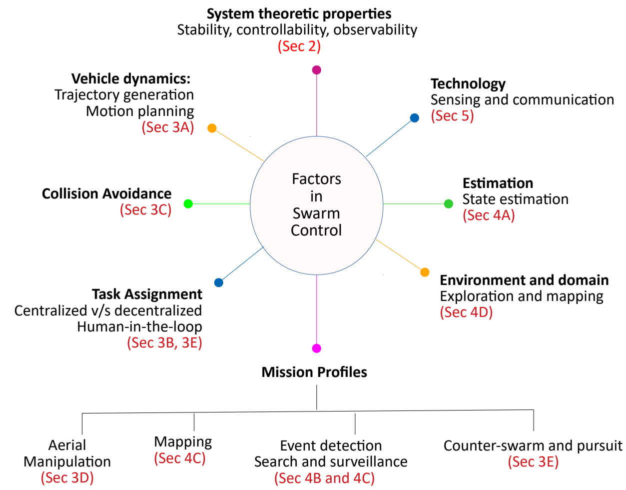

Fig. 1. Major themes in swarm control and the organization of the paper.

II. Models, Stability and Controllability of Swarms

A. Types of Multiagent Systems

A classification of multiagent systems is presented in Table I, based on the number of agents and their interaction. It has a direct bearing on how the systems are modeled: the choice of the governing equations, assumptions on the underlying connectivity, and the nature of the control inputs and information exchange.

In teams, individual behavior and strategies are designed to explicitly maximize a local objective and could see the agents competing against each other. In some cases, the locally optimum behavior guarantees on the global reward as well. This is the premise of game theoretic methods and auction algorithms [15], [16]. In auctions, for instance, maximizing the local benefit also maximizes the net global benefit (defined as the sum of individual benefits) and concurrently solves the dual pricing problem [15]. In contrast to teams, formations almost always consist of cooperative interactions and the relationship between the states of the agents is well-defined for objectives such as energy efficiency (e.g., flocks of birds in an aerodynamically optimum V-formation [17]). The concept of swarms generally refers to groups of similar agents that display emergent behavior arising from local interaction. The local interaction can be competitive or cooperative. Although a swarm typically implies large groups of agents, this survey article uses “swarm” to also include a smaller group (see Table I).

B. Models for Swarm Dynamical Systems

One of the earliest engineering models for flocking resulted out of the work of Reynolds [18] to generate a realistic visualization of flocks for computer graphics. Reynolds rules cover

TABLE I
CLASSIFICATION OF MULTI-AGENT SYSTEMS

Type	Scope	Size
Team	Typically small groups; each agent optimizes individual objectives in a cooperative or competitive manner	typically ≤ 10
Formation	Each agent is typically assigned a specific sub-task, role or placement	typically ≤ 10s
Swarm	Typically large groups of dispensable agents; global capability arises from emergent behavior	large

basic neighbor-to-neighbor interaction: a nonlinear function which governs the steady state separation between the agents and prevent collisions, and a velocity feedback term which tries to ensure that the velocity of each agent tracks the average of its neighbors. Reynolds' model is given as:

$$\ddot{\boldsymbol{x}}_i = \dot{\boldsymbol{v}}_i = \sum_{j \in \mathcal{N}_i} \left(k_s \nabla W(\boldsymbol{x}_j - \boldsymbol{x}_i) + k_a(\boldsymbol{v}_j - \boldsymbol{v}_i)\right) + \boldsymbol{f}_i \tag{1}$$

where $\boldsymbol{x}_i$ and $\boldsymbol{v}_i$ denote the position and the velocity of the i^{th} agent; $W(\boldsymbol{x}_j - \boldsymbol{x}_i)$ is a coupling function; $\mathcal{N}_i$ is the neighborhood of i^{th} agent; and $\boldsymbol{f}_i$ denotes an external influence on the agent, such as the influence of the leader or intruder.

Another early work [19] involved a flock in two dimensional space and discrete time using the following equations:

$$\begin{aligned} \boldsymbol{x}_i(t+1) &= \boldsymbol{x}_i(t) + \boldsymbol{v}_i(t)\Delta t \\ \theta_i(t+1) &= \frac{1}{\text{card}(\mathcal{N}_i)} \sum_{j \in \mathcal{N}_i} \theta_j(t) + \Delta\theta_i(t) \end{aligned} \tag{2}$$

where the noise $\Delta\theta_i(t)$ is normally distributed in the set $[-\eta, \eta]$. Importantly, the velocity $\boldsymbol{v}_i$ is assumed to have a constant magnitude for all i and t, with its heading given by $\theta_i(t)$. Despite the apparent simplicity of the model, it is able to capture the possibility of long-range order, as explained later in this section.

A generalized representation of the models in [18] and [19] can be obtained by using partial *difference* equations (PdEs) [20], [21]. The rules for obtaining PdEs permit a natural association with continuum PDEs, and consequently, ways for deriving flocking laws based on PDEs other than the wave equation used in [20].

A unified, *nonlinear* continuum model, as against models based on discretely defined agents on a graph, was proposed in [22]:

$$\begin{aligned} &\frac{\partial \boldsymbol{v}}{\partial t} + \underbrace{(\boldsymbol{v} \cdot \nabla)\boldsymbol{v}}_{\text{convection}} = \alpha\boldsymbol{v} - \beta\|\boldsymbol{v}\|^2\boldsymbol{v} - \nabla P(\rho) \\ &+ \underbrace{D_L \nabla(\nabla \cdot \boldsymbol{v}) + D_1 \nabla^2 \boldsymbol{v} + D_2(\boldsymbol{v} \cdot \nabla)^2 \boldsymbol{v}}_{\text{diffusion}} + \boldsymbol{f} \\ &\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \end{aligned} \tag{3}$$

This model was claimed to resemble that of bird flocks for 2 spatial dimensions, although the model itself is not constrained to any particular number of dimensions and could be applicable to 3 dimensional flocks as well. The constants β, $D_{\{\cdot\}}$ are all positive; the term $\alpha > 0$ corresponds to an ordered velocity state (steady flight speed $\|\boldsymbol{v}\| = \sqrt{\alpha/\beta}$), while $\alpha < 0$ gives rise to a disordered phase (e.g., a flock loitering around a fixed point). The pressure term $P = \sum_k \sigma_k(\rho - \rho_0)^k$, where σ_k's are constants and ρ_0 is the mean local density, replaces the potential-like term in Reynolds' model. Finally, $\boldsymbol{f}$ denotes disturbances, modeled as Gaussian noise.

In [19], it was seen that increasing the value of the noise (i.e., η) caused the flock to spontaneously choose an ordered state. The critical value of the noise was found to be correlated with the number of agents in the flock. This was conjectured to be due to the diffusive flow of information in the flock; i.e., due to agents interacting with a time-varying set of neighbors leading, in the long run, to diffusion of information throughout the flock. This conjecture was borne out in [22] for a two dimensional flock, wherein nonlinear convection terms were found to be responsible for stabilizing the ordered state across large length scales.

In the context of swarms, one is interested in the question of stability and convergence of the states of the individual agents. For such analysis, it is common to use a system of linear(ized) equations, the simplest of which is the system

$$\dot{\boldsymbol{x}}_i = \sum_{j \in \mathcal{N}_i} w_{ij}(\boldsymbol{x}_j - \boldsymbol{x}_i), \quad i = 1, \ldots, n \tag{4}$$

$$\Leftrightarrow \qquad \dot{\boldsymbol{x}} = -(\mathcal{L} \otimes \mathbf{I}_p)\boldsymbol{x}, \; \mathcal{L}_{ij} = \begin{cases} w_{ij} & \exists \text{ edge from node j} \\ & \text{to node i} \\ 0 & \text{otherwise} \end{cases}$$

The matrix $\mathcal{L}$ or $(\mathcal{L} \otimes \mathbf{I}_p)$ is called a Laplacian matrix and satisfies the $\mathcal{L}\mathbf{1}_n = 0$, where $\mathbf{1}_n \in \mathbb{R}^n$ is a vector of ones. It is evident that a constant $\mathcal{L}$ corresponds to a fixed communication topology; when the communication topology evolves with time, we use a time-varying $\mathcal{L}(t)$. This is identical to the diffusive coupling term one would find from (2).

For problems involving assignment or routing, it helps to model the environment as a collection of "functional bins," together with accessibility conditions which restrict the agents' transition between the bins. The end objective is to assign n agents to a set of m bins, where each bin can accommodate up to $p_i \geq 1$ ($m < n$; $i = \{1, 2, \ldots, m\}$) agents. For each agent i and a bin j, the accessible set E_{ij} implicitly accounts for the dynamics of the agent as well as the geometric constraints imposed by the environment. Such models have been used for swarm shape control with probabilistic transition maps between the bins [23] and quadrotor formation control with deterministic transition laws [24], [25].

C. Physics-Based Models for Robotic Agents

General linear systems similar to (4) can be constructed readily in a double integrator setting (e.g., attitude dynamics on SO(3) or rigid body motions on SE(3)), or by replacing the dynamics with a nonlinear version. Of particular interest here are swarm systems comprising Euler-Lagrange equations which appear routinely in robotic systems such as manipulators [26], [27], spacecraft or aircraft rigid body motions (SE(3)), often times with articulated wings [14], [28], [29], appendages, or manipulators, and attitude dynamics on SO(3) [30]–[32]:

$$M_i(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_i+C_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i+\boldsymbol{g}_i(\boldsymbol{q}_i)=\boldsymbol{\tau}_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_i,\boldsymbol{q}_d,\boldsymbol{q}_{j\in\mathcal{N}_i},\dot{\boldsymbol{q}}_{j\in\mathcal{N}_i}) \tag{5}$$

where $\boldsymbol{q}_i \in \mathbb{R}^p$ are the generalized states of the i^{th} agent; $\boldsymbol{q}_d(t)$ is the desired trajectory or a virtual leader for a target collective motion; and the external force/ torque $\boldsymbol{\tau}_i$ is the source of coupling with other neighbors. If a linear diffusive coupling is used, $\boldsymbol{\tau}_i$ would produce $\mathcal{L}$ similar to (4).

In [33], the full 6-DOF aircraft model is used with actuator time delays to compute optimal motion primitives and 3-D path planning for fast flight through a forest. It shows that a conventional 2-D Dubin's vehicle model, often times used for 2-D aircraft motion planning and swarm control, is not appropriate for aerial robots moving in 3-D. For the purpose of studying swarms of fixed- or flapping-wing aerial robot, it may suffice to model the aerial robot as a point-mass (mass m) with its velocity dynamics (speed V, climb angle γ, and heading χ) described by

$$\begin{aligned}
[\dot{x},\, \dot{y},\, \dot{h}] &= V[\cos\gamma\,\cos\chi,\, \cos\gamma\sin\chi,\, \sin\gamma] \\
m\dot{V} &= T\cos\alpha - D(V,\alpha) - mg\sin\gamma \\
mV\dot{\gamma} &= \frac{1}{mV}(L(V,\alpha)+T\sin\alpha)\cos\mu - W\cos\gamma \\
mV\dot{\chi} &= (L+T\sin\alpha)\frac{\sin\mu}{\cos\gamma}
\end{aligned} \tag{6}$$

where L, D, and T are the lift, drag, and thrust, respectively. In flapping-wing aerial robot, T is additionally a function of V and α. This model is accurate under the assumption that the rotational dynamics (α and μ) are stable and converge rapidly to the commanded value. The 3-D aerial robot model can be used effectively to reduce the computational burden on a motion planning system and generate trajectories that are optimal, stable, and safe (i.e., with collision avoidance) [14], [33]. In [34], model-based control laws were derived, together with a collision-avoiding system, for a swarm of parafoil-payload systems. A model similar to (6) was employed, and feedback about the position of the neighboring agents was used to command the desired value of the turn rate ($\dot{\chi}$ in (6)) of each agent.

Although the terms L, D, and T have been presented in the spirit of control inputs in (6), it is important to note that their values could be affected significantly in a swarm of aerial robots by flow induced by neighboring aircraft. When an aerial robot experience failures and is unable to hold its position accurately, it could affect have a detrimental effect on the efficiency of the formation due to the adverse disruption in the flow field experienced by the faulty aircraft's neighbors. This sort of physics-based interaction is almost unique to atmospheric flight vehicles.

D. Synchronization with Leader Following

For controlling swarms, it is useful at times to define a leader or a virtual leader (or desired trajectory) that the rest of swarm agents can follow (see Fig. 2). The motion of a

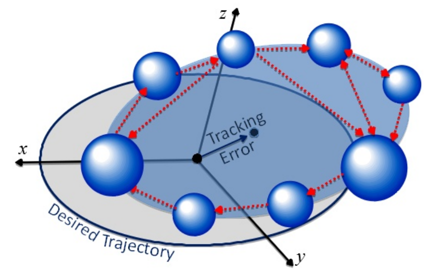

Fig. 2. A swarm of heterogeneous rigid bodies converging to the desired shape (ellipsoid), whose center can be viewed as a virtual leader for the swarm [30]. The angular separation between the vehicles is synchronized actively and shows smaller synchronization error than tracking errors. Dotted lines show diffusive couplings via communication or relative sensing.

leader can be given a priori or controlled directly by a separate dynamics. Alternatively, a desired trajectory (virtual leader) can be computed using optimal control or motion planning formulation (see Sec. 3). The remaining agents are controlled only indirectly through interaction between neighbors [35] or through interaction with a leader [36], [37]. In [27], the unified treatment of synchronization with neighboring agents and trajectory-tracking of a virtual leader or a desired collective behavior for highly nonlinear dynamics such as rigid bodies on SE(3) (e.g., aerial swarms) and robots with multi-DOF manipulators is presented. Based on time-scale separation, this unified framework integrates trajectory tracking with an exponentially-stabilizing consensus controller that synchronizes the relative motions of swarms faster than following a common leader or a desired trajectory, thereby yielding a smaller synchronization error than an uncoupled tracking control law in the presence of bounded disturbances and modeling errors [30] (see Fig. 2). This time-scale separation can be interpreted as a hierarchical connection of faster and slower dynamics as discussed in Sec. II-F. Other works followed the same problem formulation of synchronizing coupled nonlinear dynamical systems concurrently with trajectory tracking for various multi-robot/multi-vehicle applications. We can leverage concurrent synchronization of mixing multiple virtual leaders with many synchronized group to create a complex time-varying swarm comprised of numerous heterogeneous systems [27], [30], [32], [38]. One needs to determine *how many* (virtual) leaders need to be chosen, and *which* agents to nominate as leaders. This question is analogous to that of controllability, and the dual observability problem corresponds to sensor placement for distributed estimation.

E. Leader Selection and Sensor Placement

When the dynamics of the aerial swarm agents are identical, controllability from a given node (i.e., leader) depends on the topology of the graph *as well as* the individual edge weights. A system defined on a graph is said to be structurally controllable when it is controllable for *almost all* edge weights, and strongly structural controllability when it is controllable for *all* edge weights.

The existence of a rooted tree is necessary and sufficient for structural controllability with a single leader node [39]–[42]. Conditions and algorithms to determine whether a set of

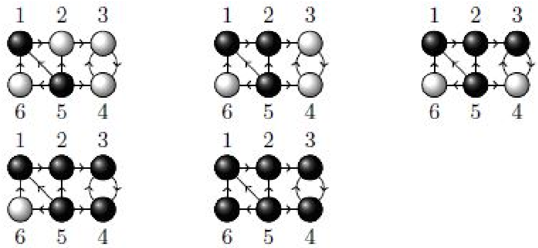

Fig. 3. Algorithm to determine whether a system can be controlled from a candidate set of leader nodes, from [45]. The candidate leader nodes are initially marked black, and the rest are white. If precisely one neighbor of a black node is white, then the white node is changed to black. This process is repeated until no color change is possible. The system is strongly controllable if all nodes are rendered black.

input nodes permit strong structural controllability have been derived in [43]–[45] (see Fig. 3 for example). Formulae linking the number of driver nodes needed for a large network and its aggregate properties (number of nodes, mean degree and the degree exponent) are presented in [46]. It was observed that driver nodes with the highest degree of controllability tend *not* to be nodes of the largest degree.

In practical problems, we are generally interested in controllability for a *given* set of edge weights, and especially for identical edge weights (i.e., the system is described by the Laplacian matrix $-\mathcal{L}$ in (4)). For this problem, there exist necessary conditions based on symmetry and equipartition [47]–[49]. While these conditions are not sufficient, a set of sufficient conditions have been presented for path and cycle graphs [50], and for a class of weakly-connected digraphs [51].

It must be noted that the selection of leaders need not be optimal just by the virtue of its controllability properties. Therefore, it is necessary to analyze the influence of the driver nodes on the actual control objectives [52]. If the control objective is to optimize an objective function, as shall be seen in (7), one can solve the problem of determining the input nodes computationally using techniques from submodular optimization [53]–[55]. Sub-modular optimization can also be viewed from a hierarchical organization standpoint emphasized in this paper.

F. Synchronization and Hierarchical Stability for Swarms

Consider (4) with diffusive couplings. It is well-known that the matrix $\mathcal{L}$ gives rise to a stable system under the following conditions on the underlying graph:

1) Undirected time-invariant graphs: the graph is connected [56].
2) Directed time-invariant graphs: consensus to the average value if and only if the graph is balanced and weakly connected [57]. Existence of a rooted tree guarantees consensus, though not necessarily to the average value [58].
3) Time-varying undirected/directed graphs: the time-varying graph satisfies a generalized strong connectivity condition ([59]; Propositions 1 and 2 in [58]).

The Laplacian matrix ($\mathcal{L}$) captures the effect of diffusive coupling terms on swarm or synchronization stability. The spectral characteristics of Laplacian matrices have been used to prove the stability of flocks obeying Reynolds rules [19], [59]–[61], stability under distance-based communication topology [62], and exponential stabilization in networked, nonlinear Euler-Lagrange systems [27], [28], [30], [63]. The effect of nonlinearities on the stability of networked systems through bifurcations was illustrated in [64]. Alternate methods for stability analysis include tools from renormalization groups [22] and the theory of normally hyperbolic invariant manifolds [65]. The Laplacian matrix defined above can be replaced by its variant, the *edge Laplacian*, to solve for stability as well as robustness and optimality [66].

The aforementioned conditions are conclusive in the absence of other dynamical terms like (4). However, for stability analysis of networked nonlinear systems that have both and the Laplacian matrix ($\mathcal{L}$ in (4)) and the nonlinear dynamical terms (e.g., convection terms of (3) or the Lagrangian form in (5)), the passivity of the input-output dynamics [67]–[69] are commonly used. Input-to-state stability (ISS) is also useful to study stability of swarm systems with bounded uncertainties [70], [71]. Contraction analysis [72] is used to study global exponential stability of multiple solution trajectories, and hence forms a basis of incremental stability analysis. Hence, contraction-based incremental stability represents an important departure from traditional passivity-based methods using Lyapunov functions that are concerned primarily with stability of equilibrium points.

Such an *exponentially-safe* and *robust* synchronization framework can also be used to study synchronization stability and robustness of a networked nonlinear dynamics connected by a synchronization controller or diffusive communication couplings [27], [73]. One major advantage of incremental stability in a synchronization framework [27], [30], [73] over the passivity formalism is that a hierarchically combined structure of dynamic systems, emphasized in this paper, can be handled more easily because of differential analysis of contraction analysis without using some implicit motion integral.

Further, it can be shown that contraction-based exponential incremental stability using a Riemannian metric possesses superior robustness and related to input-to-state stability (ISS), output passivity, and finite-gain L_p stability [30]. Many types of model uncertainty can be cast into a bounded perturbation term, including constant unknown time delays [27], [72] and errors arising from heterogeneous dynamics [27], [63]. Recently, incremental stability has been extended to synchronization stability of multiple Itō stochastic nonlinear differential equations [38], [74] with unbounded stochastic disturbances.

An extension of some of the aforementioned results arises in the form of event-triggered information exchange. Instead of exchanging communication continuously or over finite intervals of time, as in the previous cases, it is sufficient for stability to exchange information between neighboring agents at discrete *instants* of time. Conditions for stability in such cases have been found for single integrator dynamics on undirected graphs [75], consensus on balanced digraphs [76], convergence to a trajectory on time-varying graphs [77] as well as for synchronization of general nonlinear dynamics on balanced graphs [30], [73], [78]. These conditions typically depend on the underlying dynamics, and also help determine the conditions under which communication must be triggered.

III. Control of Swarms in 3-D World

Typical tasks for which swarms are suitable include distributed sensing, search and rescue [79], and imaging using sparse aperture techniques [1], [5]. These problems can be split into two distinct classes: one where the environment is to be explored (e.g., coverage, map building), and one where the environment is only to be traversed or exploited (e.g., crossing a field of obstacles). In order to effectively complete any of these tasks a swarm must be capable of planning paths for all team members to safely and reliably reach their final destinations. Not only does each individual robot need to avoid collisions with static and dynamic obstacles in the environment, but the individuals in the swarm must also avoid collisions with one another. Furthermore, in complex, obstacle-filled environments, the robots need to sequence their motions to avoid having one robot block the paths for others. For example, if the swarm needs to pass through a small bottleneck and the end goal for one of the agents is just through that bottleneck, then it must be the last one to pass through in order to not block the rest of the team [80].

A. Trajectory Generation and Motion Planning for Swarms

Approaches to trajectory generation may be classified on the basis of whether or not the trajectories are generated in conjunction with task allotment problem discussed in Sec. III-B. Trajectories generated independently of the task assignment algorithm can be thought of in the same light as traditional optimal motion planning or boundary value problems. Popular randomized algorithms, such as RRT, PRM, and RRT*, may not be effective for obtaining optimal and safe flight of multiple 6-DOF aerial robots; not only can they not effectively handle 6-DOF nonlinear dynamics, but they also use a finite set of primitives predicated on asymptotic optimality without using higher-fidelity dynamic models, which could preclude a large set of otherwise flyable trajectories. In contrast, the rapid advancement in computing capacity combined with algorithmic improvements has enabled the development of tools that are capable of solving constrained optimization problems in real-time, which can provide explicit or approximate solutions to an optimal control problem of the form

$$\sum_{j=1}^{N}\left(h\big(t_f^j, x^j(t_f^j)\big) \;+\int_{t_0^j}^{t_f^j} L\big(\gamma^j(t), u^j(t), \alpha^j(t), t\big)\,dt\right) \tag{7}$$

Subject to:

Valid goal & task assignment including terminal states

Robot dynamics, capabilities, and input constraints

State constraints (collision-free region, sensing restrictions)

where $\gamma^j(t)$ denotes the trajectory for robot j, $h(\cdot)$ denotes a terminal cost, $\alpha^j(\cdot)$ denotes a set of parameters of a mode of operation, and $L(\cdot)$ is the cost-to-go functional. The first constraint ensures that robots are assigned to valid goals or end at desired terminal states $(x^j(t_f^i))$ while the second constraint ensures that the trajectories obey both the kinematic and dynamic constraints of the robots and the input $(u^j(\cdot))$ constraints. The third constraints ensure that the optimal trajectories begin at the actual initial states while ensuring safety and other state-dependent constraints. Since the cost function is optimized in real-time over a finite-time horizon, often times recomputed using the current states of the robots as the initial conditions, (7) can be viewed as model predictive control (MPC) [25], [81]–[84].

Optimality in the multi-robot path planning problem (7) may be with respect to any number of different objectives, including integrated control effort, maximum single-robot travel distance, last arrival time, and total distance or time [85]. One must ensure that the resulting paths are kinematically or dynamically feasible for the robots to follow [86], [87]. Direct optimal control approaches [25], [82]–[84] cast the dynamics into equality constraints between the states in successive time steps for optimization (e.g., iterative linearization of dynamics in sequential convex programming [25], [84]). Alternatively, one can find a geometric path for each robot to reach its goal and then use these paths as inputs to a trajectory optimization step to make the paths dynamically feasible [87].

Another objective of trajectory design and motion planning is to enable the design of control laws for the robotic agents. One direct way to obtain control input values is to re-solve the trajectory generation problem in the MPC setting (7) and apply the new optimal control input value frequently. But the process can be computationally expensive and stability guarantees are challenging. Alternately, the control design can be separated from optimal trajectory design by treating the optimized state trajectory for each robot, obtained from (7), as a desired trajectory for the tracking controller [25], [30], [84], [88], [89]. This approach has the benefit of setting up the control design problem in the traditional input-tracking or model reference setting with guaranteed closed-loop stability. It is particularly suitable for robotic systems, such as aerial robots, whose physical models are complex but well-understood from the point of view of control design. Alternately, control laws designed without virtual leaders typically are a sum of terms that achieve, respectively, trajectory-following, coordination with neighbors, and collision-avoidance. As explained above, trajectory-following laws can be derived readily using a physical model of the robots. Terms for coordination and collision-avoidance require sensing and communication with other agents in the formation. Controllers capable to accommodating time-varying communication topologies have been derived and demonstrated for quadrotors [90] using modified temporal coordinates, for Dubin's vehicles using local potential functions [91], and for spacecraft [92].

Trajectory generation occasionally requires a hierarchical "model-based" approach when motion requirements stem from specific tasks that the swarm needs to perform, or from the necessity to deal with exigencies. For instance, cooperative construction [93], [94] requires formation-like motion as well as specialized low-level motions for stabilizing and manipulating objects. In such cases, physics-based models for manipulation can be solved to find a relative motion plan between the managing robots, while a global path plan can be constructed using any of the well-known approaches [93].

Specialized controllers may need to be designed to allow aerial robots flying in an energy-efficient formation to deal with actuator failures in individual aircraft and enable them to hold their formation [95]. These controllers benefit from aerodynamic models which help estimate the influence of neighboring aircraft on the controllability of a given aircraft. As pointed out in Sec. II-B, aerial swarms differ from ground-based robots in that the individual aircraft are coupled aerodynamically, due to the flow induced by one vehicle on its neighbors. The team may also generate trajectories that accounts for these aerodynamic effects and plans trajectories that minimize disturbance [96]. Alternately, the team can reconstruct the wake profile, as demonstrated in [97], although it requires that the aircraft perform cross-track motion to ensure stability of the estimator.

Looking at constraints beyond collision avoidance and dynamic feasibility, one key factor with UAVs is their limited battery life. To extend the mission life, a cooperative team of ground vehicles may be used as mobile recharging stations [98]. The UAVs then plan paths to ensure that they are able to accomplish their missions while maintaining power.

B. Simultaneous Planning with Distributed Assignment

In a homogeneous swarm of robots, it does not matter which agent completes a given task. This fact may be exploited to do simultaneous task assignment and trajectory planning for teams of 100's of agents in a centralized or decentralized formulation [80]. This decentralized formulation is sub-optimal compared to the centralized solution, but is still complete. For example, simultaneous optimal assignment and trajectory planning computes an optimal terminal point constraint of (7) for shape reconfiguration control [25], [84].

A special case of assignment is cooperative pursuit, wherein multiple pursuers seek a single target. A pursuit strategy and conditions for a successful pursuit in a bounded domain were determined in [99]. The conditions for a successful pursuit link the relative speeds of the pursuers and the evader, the turning radius of the pursuer (under the assuming an arbitrarily agile evader), and the total number of pursuers. More realistic, physics-based scenarios have been investigated in the context of missile interception, wherein multiple missiles are used to defend against one or more incoming (target) missiles. The target missiles are generally assumed to use a standard optimal guidance and evasion law, and part of the challenge lies in estimating the states of the missile and its guidance law. In fact, just the time delay involved in estimating the states can have a severely adverse effect on the pursuit [100]. In [101], cooperative estimation of the target states (compared to each missile using solely its own estimates) was shown to improve the likelihood of success significantly. Information sharing between the missiles can also be used to directly tune their navigation law, as demonstrated in [102], to achieve a synchronized hit on the target.

A scenario related to cooperative pursuit is that of multiple UAVs tracking a single target. From the point of view of trajectory generation, it is of interest to consider scenarios wherein the environment is populated with no-go areas and with terrain features that may occlude the target sporadically, such as a typical urban neighborhood. In order to facilitate the generation of trajectories which minimize occlusion, it is beneficial to develop adequate models of the sensors, such as gimbaled cameras, that are used to track the target. The constraints of the tracking system can then be added to the dynamic limitations of each UAV to generate guidance laws for the complete team of UAVs [103].

The simplest task assignment problem is the following static, symmetric problem: given a set of n agents, n bins, and a matrix of rewards $P \in \mathbb{R}^{n\times n}$ (or, equivalently, a matrix of costs $C \in \mathbb{R}^{n\times n}$) , where $P_{i,j}$ (resp. $C_{i,j}$) denotes the reward derived (resp. cost incurred) by agent i from being assigned to bin j and $P_{i,j} = -\infty$ (resp. $C_{i,j} = \infty$) denotes an infeasible assignment, determine the map $A : i \mapsto j = A(i)$ which assigns to each agent *a unique* bin while maximizing the collective reward $\sum_i P_{i,\,A(i)}$ (resp. minimizing the equivalent collective cost). Parallel or distributed algorithms to solve target assignment include many variants of distributed auction algorithms [16], [25], [104]–[106] and decentralized hierarchical strategies [107] that approximate true optimality of Kuhn's centralized Hungarian method.

An elementary auction algorithm is illustrated in Algorithm 1. This algorithm is centralized, and requires a central register where information about the bids and assignments is maintained. In contrast, distributed algorithms distribute computation as well as communication among the agents (rather than maintaining a central register). For instance, the algorithm in [25] adjusts the number of targets based on the number of agents available at a specific stage. This is accomplished through bidding rather than a consensus-like process, which is useful in large swarms with agents that may drop out spontaneously. This distributed target assignment can be solved simultaneously to provide goal states of real-time optimal trajectory generation, thereby effectively solving (7) [25], [80], [108]

Algorithm 1 Symmetric Auction-based Assignment

```
Given: Terminal bins to be filled; for each (agent, bin),
  the cost of occupancy; an initial price for each bin
while There is an unoccupied bin do
  for Each unassigned agent do
    Choose the least expensive (cost + price) bin
    Determine the total cost (cost + price) of the least
      and the second least expensive bin
    Bid = difference in the prices + random number
    Cost of the least expensive bin increased by the bid
    Assign the least expensive bin to the agent, and the
      previous occupant (if any) is unassigned
  end for
end while
End: Each agent has a unique bin assigned to it
```

An equivalent geometric problem involves partitioning a physical volume into portions that are then assigned to each agent inside the volume. A well-known result is that the optimal partition corresponds to the generation of Voronoi cells using a suitable metric function [109]. This approach was introduced in [109] for sensor coverage, and generalized in [110], [111] to cover learning (of the task distribution) and decentralized information sharing.

Assignment can be obtained as the solution to an optimal transport problem [112] when the transition between bins is modeled in a probabilistic framework through homogeneous Markov matrices. An improved approach has been proposed in [23] using time-inhomogeneous Markov chains which allow for the inclusion of feedback terms, thereby solving both bin-to-bin swarm shape control and stochastic target assignment.

C. Collision Avoidance and Collision-Free Motions

The problem of collision avoidance becomes particularly challenging in swarms because the "obstacles" encountered by a robot include other members of its swarm, and collision avoidance has to factor in the need to maximize the performance of the swarm (e.g., avoid increasing the time to complete an assignment). The most intuitive techniques for avoiding collisions are speed adjustment [113] and sequentially replanning the trajectories [84] without changing the assignment in an optimal control framework (7). In particular, mixed-integer linear programming has been successfully derived for optimal collision-free motions and applied to mobile robots, spacecraft, and UAVs [82], [83]. More recently, sequential convex programming (SCP) has been used to approximate collision-free regions by incrementally drawing hyperplanes and has been demonstrated in simulation and experiments on swarms [25], [84]. The conservatism of hyperplane-based convexification of collision-free regions has been relaxed by expanding convex spherical regions along graph-based primitive paths in [114]. An alternate approach to replanning just the trajectories involves reassigning the goals as shown in [80]. The reassignment is purely local and need not affect the criterion used for the assignment in the first place.

A more direct approach to collision avoidance in swarms involves the use of artificial potential fields [115]–[118] or barrier functions [119], [120]. It must be noted that Reynolds' model (1) also includes the gradient of a potential function. Potential fields are computationally easy to implement for the purpose of collision avoidance, but not necessarily for path planning. Furthermore, artificial potential fields directly couple the dynamics of the individual robots and this can adversely affect the stability of the swarm if the communication topology is not selected properly. Connectivity is not enough to guarantee stability in directed graphs (see Sec. II-F). An approach similar to potential fields involves using the gradient of a Lyapunov function which implicitly takes into account the possibility of collisions. Such control laws have been constructed using a differential game approach [121], [122] and simultaneously solve a greedy optimization problem. The difficulty lies in solving the optimal control problem in the presence of nonlinearities and local communication.

D. Aerial Manipulation

Aerial robotic swarms have the ability to transport objects in two ways, where each individual robot is capable of carrying an object or where multiple robots are required to lift a single object. In either scenario, the object may be suspended via cables attached to the robots [123]–[127] or may be rigidly attached to the robots [128]–[132]. UAVs that are rigidly attached to the objects use a variety of grippers, including friction-based [128], penetration-based [129], or magnetic [131].

In the first case, having a swarm of robots allows a large number of objects to be moved more quickly. This can be used for tasks such as package delivery [127], [132] and construction [128].

In the second case, small teams of robots may be used to cooperatively transport a single object [123]–[126], [129]–[131]. This task requires some type of communication between the robots. This is typically done in an explicit manner, but can also be done implicitly by sensing the internal forces [126]. The swarm also seeks to minimize these internal forces, as these represent wasted energy usage [130].

E. External Control of Aerial Swarms

External control of swarms refers to one of two situations:

1) The swarm is assigned objectives in real time by an external user, especially a human operator.
2) Some or all members of the swarm interact with an adversary or a hostile agent which, in turn, is within a human user's control.

At the simplest level, a human teleoperator sends motion commands to the swarm. In order to reduce the cognitive load on the operator, it is desirable to minimize the number of inputs that the operator must provide and manage. To this end, it is possible to control the bulk motion of the swarm by guiding a single virtual leader and controlling the size and shape of the swarm with respect to this virtual leader [133], [134]. The human could also issue a command in a *language* that the swarm is designed to understand. This is no different conceptually from the usual setting of an autonomous swarm, since it involves the human acting essentially outside the algorithmic loop. It has been explored in [135], [136]. The next level of sophistication involves the human issuing commands using natural language, while still staying outside the algorithmic loop that controls the swarm. Here, the challenge is one of inferring a specific command from the operator's verbiage [137]. The highest explored level of sophistication is using computer to infer human intent. Here, the human is very a much a part of the algorithmic loop: the algorithm that controls the swarm actively seeks input from the human about its performance. A framework for extending it to a team of robotic agents (including UAVs) was proposed in [138].

The concept of adversarial control addresses the case where there is no direct way of tapping into a swarm's command and control algorithm. An example of adversarial control is the family containment and herding strategies modeled after dolphins [21], sheep-dogs [139]–[144] and birds of prey used to herd a flock of birds [145]. In [145], the authors examined the use of a robotic UAV, possibly one built to resemble a bird of prey like a falcon, to herd flocks of birds away from sensitive areas like airports and solar farms. The UAV interacts with the flock by engaging birds located on the boundary of the flock. The herding algorithm make use of the flock's inherent tendency to maintain a cohesive structure to reduce the number of robotic agents required to achieve herding. The perturbation in the velocity of the birds on the boundary of the flock diffuses through the swarm and causes the flock to alter its heading and speed. It has been shown in [145] that a single robotic agent suffices to herd a flock of birds, while related work [142] suggests that the quality of the herding can be improved substantially by using multiple UAVs.

One particular problem of interest in the context of adversarial control is inferring the model underlying the swarm's motion. In [145], experimental data was used to identify a model for the response of a flock of birds to a UAV located within a certain range of the flock. The approach adopted in [145] works for flocks whose response to perturbations is based on a static, deterministic law. When the response takes a more strategic, dynamic form, it is necessary to use learning-based techniques which expressly account for this behavior [146], [147]. A filter-based technique lies mid-way between the two sets of aforementioned approaches. Consider the case of missiles where it is known that a target missile follows one from a well-defined set of navigation laws at all times. The exact law and its parameters are unknown. Such problems can be solved efficiently using a bank of filters to determine the most likely model, as demonstrated in [148].

IV. Aerial Distributed Sensing, Monitoring, and Cooperative Mapping

Distributed sensing is one of the main application areas of aerial robotic swarms. Swarms of aerial robots have the ability to simultaneously gather information from disjoint locations. They are also more robust to failures in sensing and actuation since there is some measure of redundancy in the system. Distributed sensing tasks can have three main foci: targets, space, and maps. With any focus, the robots need to have information about the area of interest and the objects within it to safely and successfully complete the task. However, the goal in each sub-task is different. In the first, the goal is to search for and/or track targets inside of an area of interest. In the second, the goal is to maximize some measure of sensor coverage or to ensure that all areas are eventually covered, possibly at a desired frequency. In the last, the goal is to build a map of the unknown or partially-known environment.

A. Target Search and Tracking

Target search and tracking is a canonical distributed sensing task. From an aerial robotics perspective, several key variants of this problem have been studied. The divisions between the variants occur along two main categories: static vs. dynamic targets and single- vs. multi-target. In the multi-target case, there are two sub-cases of a known vs. unknown number of targets. Note that the latter problem can be significantly more difficult: when the number of targets is known, then a detection (or lack thereof) not only gives the team information about what is in the field of view of the sensors, but *also* what is outside of the field of view. For example, if the team knows that there are 8 targets and that 4 of them are currently visible, then they know that there are 4 left to be found. In the case where the number of targets is unknown, then seeing 4 targets only tells the team that there are at least 4 targets.

1) Single, Dynamic Target: A team of robots has the ability to simultaneously view disjoint regions of an area of interest or to simultaneously view the same region from different perspectives. The former allows the team to more quickly gain global information while the latter allows the team to more quickly decrease uncertainty and to be robust to sensor errors. This problem can be written as a distributed estimation task [149], [150]. In a general discrete-time representation, the target's dynamics are given by:

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}_k(\boldsymbol{x}_k, \boldsymbol{w}_k, \Delta)\,,\ \forall k \in \mathbb{N}\,, \tag{8}$$

where $\boldsymbol{f}_k$ is a nonlinear, time-varying function of the target state $\boldsymbol{x}_k$, the independent and identically distributed (i.i.d.) process noise $\boldsymbol{w}_k$, and the discretization time step size Δ. A network of N heterogeneous sensing agents are simultaneously tracking (8). Let $\boldsymbol{y}_k^i$ denote the measurement taken by the i^{th} agent at the k^{th} time instant:

$$\boldsymbol{y}_k^i = \boldsymbol{h}_k^i(\boldsymbol{x}_k, \boldsymbol{v}_k^i),\ \forall i \in \mathcal{V} = \{1, \ldots, N\}\,,\ \forall k \in \mathbb{N}\,, \tag{9}$$

where $\boldsymbol{h}_k^i$ is a nonlinear time-varying function of the state $\boldsymbol{x}_k$ and the i.i.d. measurement noise $\boldsymbol{v}_k^i$. Then, agents are able to use the distributed Bayesian filtering proposed in [149], to track the state of the target (see Algorithm 2).

Algorithm 2 Distributed Bayesian Filtering Algorithm

1. Compute the local prior of the target(s)
2. Obtain local measurements and communicate with neighbors
3 Fuse probability distributions using Logarithmic Opinion Pool
5. Compute posteriors and iterate

This type of problem is found in a variety of settings, including tracking a radio-tagged animal with a team of UAVs [151] or seeking, tracking, and capture a hostile UAV [152]. Additionally, these UAV teams may collaborate with a team of ground robots and/or fixed sensors [151]. The problem of target tracking is further complicated when not only is the state of the target unknown, but also the state of each UAV [153].

2) Multiple Targets of Known Number: The simplest form of multi-target tracking is when the number of targets is known and the targets are stationary [151]. However, tracking dynamic targets has been of greater interest in the multi-target scenario. One common situation for a small team of UAVs is where the number of targets is larger than the number of robots. In this situation the team must decide whether to focus on tracking the largest number of targets or tracking individual targets with a high quality of tracking. This trade-off typically results in a decision about the elevation of the robot, where a high elevation results in a large sensor field-of-view but higher sensor noise [108], [154]. Furthermore, simultaneously planning trajectories for large teams can be computationally expensive and slow. This problem is typically mitigated through the use of approximation algorithms [108], [154] or anytime planning algorithms [155]. In order to cooperatively plan, the robots must be able to share information across the team. In the case where robots have limited communication range, line-of-sight visibility, and operate in a cluttered environment, it can be difficult to maintain connectivity across the team [156].

3) Multiple Targets of Unknown Number: As mentioned above, when the number of targets is unknown the search problem becomes much more difficult and the team must always explore the entire environment in order to complete the task. The standard method used to solve this task is to utilize a quadtree representation to adaptively refine the environment in areas that are likely to contain targets [157]–[159]. The main distinction in these works is that [157], [158] assume that each robot sees one and only one cell, which implicitly connects the elevation of the robots to the quadtree resolution (and the sensing quality). On the other hand, [159] allows the robots to see multiple cells and utilize the theory of random finite sets [160] to estimate the set of targets.

Tracking an unknown number of dynamic targets is even more difficult since, barring being able to see the entire environment at one time, there is no way for the team to be sure that they have seen every target. [161] considers the situation where the number of targets is unknown but constant. They focus on creating an efficient, camera-based tracking for collision avoidance within a large swarm of UAVs, which is run on-board UAVs in real-time. This is useful in a single-team situation to be robust to delays or failures in the communication and in situations where there are multiple, non-communicating teams in the same airspace.

Perhaps the most challenging target search and tracking problem is when the number of targets is unknown and dynamically changes over time, e.g., due to targets entering and leaving the area of interest. The tool most commonly used in this scenario is the PHD filter [160], which allows the team to simultaneously estimate the number of targets and the dynamic state of each target. This has been used by a small team of fixed-wing UAVs to track vehicles on roadways using an information-based technique [162] and by a large team of multi-rotor UAVs to track ground robots using a Voronoi-based controller [163].

B. Surveillance and Monitoring

Target tracking, as the name implies, takes a target-centric approach. The alternative is to take an area-centered approach, where the team of robots focuses on covering an area of interest with sensors. This is often called surveillance. If all areas of interest must be visited at some desired, or maximum, frequency, the problem is called persistent monitoring. Surveillance and monitoring have been the subject of a large body of literature, including many aerial swarm-specific approaches.

A surveillance or monitoring task is a tuple $(\mathcal{R}, \gamma, Q)$, where $\mathcal{R}$ is the robot model, γ are the curves followed by the robots, and Q is the set of points of interest [164]. Let $\phi(q,t)$ be the field at point q and time t, which often represents the time elapsed since the point q was last seen by some robot or some local measure of uncertainty about the environment. Then the goal is to find a set of trajectories γ that minimizes the cost

$$\gamma^* = \arg\min_{\gamma} \left(\max_{q \in Q} \left(\limsup_{t\to\infty} \phi(q,t) \right) \right) \tag{10}$$
$$\text{subject to} \quad \limsup_{t\to\infty} \phi(q,t) \text{ is finite } \forall q \in Q$$
$$\text{Robot capabilities}, \mathcal{R}$$

In general, the value of the field $\phi(q,t)$ increases over time and decreases only when a robot observes it. Due to the finite time horizon considered in this problem, it is computationally expensive to solve for robot trajectories, especially in the multi-UAV case.

1) Persistent Monitoring: Early works in multi-UAV persistent monitoring used a heuristic approach to extend a single-UAV solution to the situation with multiple UAVs [165], [166]. Other work focused on enabling real-time computation onboard the UAVs by using parameterized B-spline curves to define the set of feasible trajectories [167]. The authors later extended this work to more efficiently account for sensor and vehicle limitations [168]. Specifically, they address the fact that the sensor field of view and the turning radius of fixed-wing UAVs are typically of a comparable length scale, making it difficult to see all parts of the environment.

Another way to think about a monitoring problem is as a vehicle routing problem, where the UAVs must visit a desired set of locations [169].

2) Surveillance: A related task is surveillance, where the primary distinction from persistent monitoring is that there is no longer a hard requirement on the frequency that each area is visited. Instead, the goal is often to maximize some measure of coverage or information [170], [171]. The robots in the team communicate over a multi-hop network and solve the surveillance task in a distributed fashion.

In addition to maximizing coverage or information, the swarm can be tasked to monitor a spatio-temporal field over the environment. Such spatio-temporal fields are often found in environmental monitoring and precision agriculture tasks, where the field could be something like water temperature or nutrient concentration. One recent approach to this are Rapidly-exploring Random Cycles (RRCs) [172], which are better suited to surveillance and monitoring tasks where areas must be consistently revisited than their cousin, Rapidly-exploring Random Trees (RRTs) [173], which focus on single-use trajectories. Monitoring spatio-temporal fields is a challenging task and may often be better accomplished by using a heterogeneous team [174].

C. Cooperative Aerial Mapping

In contrast to surveillance and monitoring tasks, where the goal is to only to observe the environment, mapping is the process of acquiring a globally-consistent representation of an environment. Such representations can be sparse [175], semi-dense [176], or full dense [177]. While dense representations can be directly used for autonomous navigation [177] or geographical reference, sparse representations are often only used for state estimation [178] or collaborative control of robotic agents. Due to the fact that the environment is often only partially known or even totally unknown, mapping tasks are often coupled with localization issues, which turn them into the classic simultaneous localization and mapping (SLAM) setting. Admittedly, SLAM and its extension to distributed multi-robot SLAM are extensively studied areas. In this paper, we limit our discussion to only those that are most relevant to aerial robot swarms.

The technical contributions of collaborative mapping experiments are limited when they are conducted in simulation or in a lab setting. However, due to the high technical barrier of deploying multiple aerial robots in a real-world setting, a very small number of collaborative mapping systems are tested in realistic settings. Even for successful applications, the scale is limited to a few (less than ten) robots. Further discussion of these technical challenges follows in Sec. V.

Problems and current solutions for multi-robot mapping are reviewed in [179]. In the following, we categorize mapping solutions based on their sensing modalities and environment representation as either visual sparse, visual dense, or lidar-based solutions.

1) Visual Sparse Mapping: Visual sparse representation consists of points and lines, which are extracted and tracked from images. Points are usually augmented with descriptors for feature matching purposes. By matching 3D points and lines, robots can estimate their relative poses and fuse their local maps to maintain geometric consistency, and achieve effective cooperation in large-scale environments [180]. Robots may also maintain the position uncertainty of each point in the map for handling of dynamic objects [181]. Early work on vision-based collaborative SLAM for aerial robots was introduced in [175], in which a centralized ground station is used to collect data from multiple aerial robots. The data is used to perform sparse feature matching for robot localization, and to detect overlap in the sensor field of view of different robots. Recent results utilizing similar mapping framwork were presented in [182], [183].

Visual-inertial SLAM frameworks are often more suitable for aerial robot systems thanks to the guaranteed availability onboard IMUs. State-of-the-art visual-inertial SLAM frameworks are often able to process multi-session maps [184], [185], making them ideal for merging maps acquired by multiple robots into globally consistent representations. The global localization capability of these frameworks also enables drift-free pose estimation of multiple aerial robots in the same sparse visual map.

Real-world swarm systems typically have very strict constraints in communication bandwidth. To this end, researchers have been focusing on minimizing or limiting the amount of data required to perform decentralized mapping [186], [187]. Specifically, [187] proposes a decentralized SLAM framework based on decentralized place recognition and optimization algorithms. These algorithms scale linearly with respect to the size of the team, and build highly compact representations of the environment, resulting in very low bandwidth usage. This enables robots to navigate in environments where absolute positioning is not available, and where there is no central base station. Another approach to decrease bandwidth usage is to utilize object-based models rather than exchanging raw sensor measurements (e.g., point clouds or RGB-D data) [188].

2) Visual Dense Mapping: Dense mapping systems describe the environment using a dense collection of points or planes. Dense representations are very powerful for autonomous navigation in cluttered environments, but they also poses much higher requirements in terms of processing power and data storage. RGB-D cameras that provide both depth and color information for each image are often used for cooperative visual dense mapping [189], [190]. In these works, several robots send local maps to a cloud server to performs map merging and batch optimization, since the onboard computation was not far from enough for handling dense data. Recent works demonstrated real-time pose estimation for autonomous flight and cooperative dense mapping using onboard computation with two quadrotors equipped with RGB-D cameras [191], and with a heterogeneous team of a quadrotor and a ground robot [192].

3) Lidar-based Mapping: Lidar is another commonly used sensor for mapping applications. In [193], a small heterogeneous team of a quadrotor and a ground robot is used for cooperative mapping, where the actuation advantages of each agent can be utilized to ensure that the entire space is explored. Scan matching is used for merging maps from the two robots. An expectation maximization (EM) algorithm that utilizes lidar scan information was proposed in [194] for efficient identification of inliers in multi-robot loop closure. This significantly improves the trajectory accuracy over long-term navigation.

V. Technology for Swarming

A practical perspective is given to discuss the main hardware and software components necessary to support a swarm application in practice. Aerial robotic swarms have been studied extensively in simulations, but have not been used in full-scale experimental tests in real-world scenarios until recently. This is due to a confluence of factors. On-board sensing and computation has significantly improved to the point where it is possible to do real-time state estimation. At the same time, drone hardware has significantly improved in recent years with the explosive growth of the commercial drone market. In the US alone, the commercial market grew from \$40M in 2012 to nearly \$1B in 2017 [195] and the global UAV market is expected to surpass \$12B by 2021 [196]. This growth has lowered hardware costs enough to make large-scale swarms possible.

A. Platforms

Some of the first work on aerial robotic swarm hardware focused on providing an open-source hardware and software stack that did not require any external infrastructure [197]. This was meant as an educational tool to teach young scholars to work with hardware and to give them a testbed to implement their ideas. This initial swarm only had a handful of robots in it. Other indoor swarms utilize motion capture systems for localization [198], [199]. More recently, the field has focused on expanding the size of the swarm, with one of the the largest indoor swarms consisting of 49 CrazyFlie quadrotors simultaneously flying in a motion capture system [200]. The palm-sized CrazyFlie platform does not have sufficient on-board computation or sensing for state estimation, but is ideal for large-scale, indoor swarms.

Other researchers have focused on getting the swarm outside of the lab, including a swarm of 12 quadrotors working both indoors and outdoors without the need for any external infrastructure [201] and outdoor formation flight of 10 aerial robots [202].

The robots use Visual-Inertial Odometry (VIO) for state estimation, which allows them to navigate in challenging outdoor conditions, including at night and with wind. An even larger outdoor swarm of 50 fixed-wing UAVs was also recently demonstrated [203], with the goal of becoming a testbed to study adversarial swarm systems.

B. Vehicle Power Management

With any swarms, one of the key challenges is power management. For example, in [203] the full 50 robots in the swarm were all airborne for only 10 minutes out of the 60 minutes it took for all of the vehicles to be launched and land safely. Unlike fixed-wing UAVs, vertical take-off and landing UAVs, such as multi-rotors, are able to simultaneously take off and land but have a significantly shorter flight time.

Recharging or refueling robots can be done from static charging pads [204], [205] or on mobile charging pads (i.e., on top of ground vehicles) [98]. Health monitoring beyond fuel or battery levels is also important. For example, the operator of the team may also be interested in malfunctions, degradation, or failure of sensors, actuators, and other components [206].

C. State Estimation

Due to the inherently unstable dynamics of most aerial robot configurations, robust state estimation is essential for almost all aerial robot applications. This is the fundamental building block that enables transition from simulation or lab settings (with external motion capture systems) to real-world deployment. In the following, we categorize state estimation solutions as either based on external sensors or self-contained with on-board sensors.

1) State Estimation using External Sensors: External sensing options, such as real-time kinematic (RTK) GPS, optical motion capture systems, and ultra-wideband (UWB) solutions, have enabled impressive cooperative missions on aerial robots. It is well known that GPS, which provides absolute longitude and latitude information, is suitable for large scale outdoor environments. RTK GPS further achieves centimeter-level accuracy with the help of additional base stations. These GPS-based solutions have powered various commercial aerial swarm shows by Intel[1], EHang[2], and others. In indoor GPS-denied environments, optical motion capture systems enable millimeter-level position tracking utilizing multiple infrared cameras [207], [208]. Alternatively, UWB-based solutions offer a less expensive and more flexible, but less accurate, state estimate for large-scale indoor aerial swarms [209]. The major drawback of any of these systems is that they require the installation of fixed infrastructure, limiting the swarm to operate in a fixed airspace.

2) State Estimation using On-board Sensors: To enable swarms to operate in any environment, one must eliminate the need for external sensors for state estimation. Instead, the robots must rely on on-board sensors, such as camera, lidars, and inertial measurement units (IMUs). Cameras and lidars are exteroceptive sensors, relying on external features to provide incremental pose estimates [210]. On the other hand, IMUs are interoceptive sensors, providing high-frequency velocity, attitude feedback for real-time control purpose. A recent major breakthrough in this area is the use of visual-inertial odometry (VIO) [201] for real-time state estimation and feedback control. On-board camera sensors can also be used to localize other members of the swarm [211]–[215]. This can be used to enable distributed formation control without the need for any explicit communication between agents.

D. Communication Infrastructure

The communication infrastructure is another essential building block for real-world deployment of aerial swarm systems, as it enables exchange of state information, motion plans, and high-level swarm behaviors. Researchers often select short-range but low power consumption communication protocols, such as Bluetooth, ultra-wideband (UWB), or standard Wi-Fi, for building up the communication infrastructure. A detailed discussion of these protocols was presented in [216]. However, due to limited bandwidths, these protocols may not satisfy the communication requirement for large-scale swarms. Researchers are looking into possible alternates that demonstrates low-latency, high-reliability, and high bandwidth, such as URLLC [217].

Due to a limited selection of physical communication infrastructure components, current swarm realizations have very limited selection on communication topology. Centralized topologies with one ground station and multiple agents are used for most cases [200], [207], [208]. Decentralized communication topologies are still mostly in the realm of theoretical research [57], [59]. Very limited experimental results are presented in the literature [218], [219].

VI. Conclusion and Future Work

In the near future, our 3-D airspace will be shared by swarms of aerial robots, performing complex tasks that would be impossible for a single vehicle. We have reviewed work that could provide fundamental technological building blocks to realize the future from the algorithm, analysis, perception, and application standpoints. The research issues discussed in this survey paper span hierarchical integration of swarm synchronization control with safe trajectory optimization and assignment, and cooperative estimation and control with perception in the loop.

One aim is to put together a cohesive set of research goals and visions towards realizing long-term autonomy of aerial

[1]https://www.intel.com/content/www/us/en/technology-innovation/aerial-technology-light-show.html

[2]http://www.ehang.com/news/249.html

swarm systems. In addition, we emphasized the importance of the three-way trade-off among computational efficiency for large-scale swarms, stability and robustness, and optimal system performance. To do this, it is imperative to advance beyond methods that are currently being used in autonomous drones and general swarm robotics.

Toward long-term swarm autonomy: One important area of further study is developing learning and decision-making architectures that will endow swarm aerial robots with high levels of autonomy and flexibility. We argue that such architectures will ultimately lead to reduced risk and cost as well as long-term autonomous operations. To be successful, any such architecture must be capable of reasoning about the wide-ranging nature of uncertainties and modeling errors, ranging from known *unknowns* (e.g., sensor and actuator noise) to unknown *unknowns* (e.g., wind disturbance, hardware failures). All of these impact the safety and robustness of algorithms and system-level functions of swarm behaviors. Furthermore, computation and communication within a swarm must be fast enough to ensure stability under model changes at the various timescales and bandwidths within the system. These challenging conditions are due to the limited resources available on each robot combined with the complex interactions of 3-D swarm flight dynamics in complex environments. These issues are exacerbated by the complexity and dimensionality of interconnected aerial swarm dynamics involving tens to hundreds of heterogeneous vehicles.

For complex aerial swarm dynamical systems with highly uncertain environmental models, the role of high-level classification and decision making in flight in conjunction with low-level swarm control and estimation systems can be characterized mathematically through the properties of stability, convergence, and robustness. Various aspects of the swarm decision-making, control, and estimation should come in different frequencies and hierarchical levels to exploit scalability and computational efficiency. An example of such characterization on stability would be a mathematical theorem correlating desired models and parameters to be updated on-line as well as their update or learning rates, to functions of various system features, such as sampling rate, swarm control law update rate, bandwidth of dynamics and communication, dimensions of dynamic systems, and properties of environmental uncertainties. This should also provide a guideline as to gauge how efficient and effective a particular swarm algorithm or system-level architecture is to achieve autonomous aerial swarms. For example, distributed optimal planning (e.g., [25], [84]) requires communication of neighbors' optimal solutions up to a certain time horizon while a simultaneous target assignment algorithm would further worsen the required size of communicated information. It would be beneficial to combine such methods with on-lining adaptation methods that can forecast the neighbors' future behavior. Such on-line, learning-based prediction will effectively reduce communication requirements. The key idea is again combining formal mathematical analysis in hierarchical and multi-modal decomposition discussed earlier.

As swarms are deployed to a greater extent for aggressive autonomous missions, particularly by military entities, it will become necessary to create the means to exert some form of adversarial control on swarms. Such counter-swarm techniques can also be used for civilian purposes, such as maintaining law-and-order and herding birds and animals away from environmental hazards such as floods or wildfires. The work reported in Sec. III-E is a good starting point for this development. Key open questions include the type of maneuvers that need to be performed to rapidly assess an aerial swarm's internal dynamics; estimating its target location and intent; identifying the task and role assignment within a given swarm; and identifying the primary leader and sensing nodes. It is interesting to note the similarities with problems that have been studied in the context of social networks, which suggests that an adoption of the tools from that literature may provide early breakthroughs for counter-swarm development. The next level of questions would pertain to identifying ways of defeating such probing maneuvers, which is a direct analogue of the usual minimax paradigm for games. Even with the possibility of adopting well-established tools from the theory of social networks and games, an important and significant challenge at both levels is identifying the role of the aerial vehicle dynamics in enabling, and defeating, the probing operations. A cleverly executed set of maneuvers could help identify, and equally provide deceptive leads about, a swarm's intent, organization and capabilities.

In summary, many open problems and research issues in aerial swarm robotics involve characterization of the inter-dependencies between the properties of swarm vehicle dynamics, the properties of uncertainties, and different swarm learning/control methods employed, such that the performance of fully-autonomous aerial swarms can be sufficiently proven to excel in any complex environment.

Acknowledgment

The authors very much appreciate Frank Park's guidance and encouragement for pulling together this special issue. The authors thank Salar Rahili for his comments.

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Soon-Jo Chung (M'06–SM'12) received the B.S. degree (summa cum laude) from Korea Advanced Institute of Science and Technology, Daejeon, South Korea, in 1998; the S.M. degree in aeronautics and astronautics; and the Sc.D. degree in estimation and control from Massachusetts Institute of Technology, Cambridge, MA, USA, in 2002 and 2007, respectively. He is an Associate Professor of Aerospace and Bren Scholar and Jet Propulsion Laboratory Research Scientist in the California Institute of Technology. Dr. Chung was on the faculty of the University of Illinois at Urbana-Champaign (UIUC) during 2009-2016. His research focuses on spacecraft and aerial swarms and autonomous aerospace systems, and in particular, on the theory and application of complex nonlinear dynamics, control, estimation, guidance, and navigation of autonomous space and air vehicles. Dr. Chung is received the UIUC Engineering Deans Award for Excellence in Research, the Beckman Faculty Fellowship of the UIUC Center for Advanced Study, the U.S. Air Force Office of Scientific Research Young Investigator Award, the National Science Foundation Faculty Early Career Development Award, and three Best Conference Paper Awards from the IEEE, and the American Institute of Aeronautics and Astronautics. He is an Associate Editor of IEEE Transactions on Robotics and Journal of Guidance, Control, and Dynamics.

Aditya A. Paranjape received the B.Tech and M.Tech degrees in Aerospace Engineering from the Indian Institute of Technology (IIT) Bombay, Mumbai, India in 2007; and Ph.D. in Aerospace Engineering from the University of Illinois at Urbana-Champaign, Urbana, IL, USA, in 2011. He is currently Lecturer in Aerial Robotics at Imperial College London, London, United Kingdom. His research focuses on the dynamics and control of aircraft, spacecraft, infinite dimensional systems, and multi-agent systems.

Philip Dames received both his B.S. (summa cum laude) and M.S. degrees in Mechanical Engineering from Northwestern University in 2010 and his Ph.D. degree in Mechanical Engineering and Applied Mechanics from the University of Pennsylvania in 2015. From 2015–2016 he was a Postdoctoral Researcher in Electrical and Systems Engineering at the University of Pennsylvania. Since 2016, Philip has been an Assistant Professor of Mechanical Engineering at Temple University. His current research interests include the intersection of estimation, control, and communication to enable autonomy in large-scale, multi-agent systems.

Shaojie Shen received his B.Eng. degree in Electronic Engineering from the Hong Kong University of Science and Technology in 2009. He received his M.S. in Robotics and Ph.D. in Electrical and Systems Engineering in 2011 and 2014, respectively, all from the University of Pennsylvania. He joined the Department of Electronic and Computer Engineering at the Hong Kong University of Science and Technology in September 2014 as an Assistant Professor. His research interests are in the areas of robotics and unmanned aerial vehicles, with focus on state estimation, sensor fusion, computer vision, localization and mapping, and autonomous navigation in complex environments.

Vijay Kumar received the B.Tech. degree from the Indian Institute of Technology, Kanpur, India, and the Ph.D. degree from The Ohio State University, Columbus, OH, USA, in 1987. He is the Nemirovsky Family Dean of Penn Engineering with appointments in the Departments of Mechanical Engineering and Applied Mechanics, Computer and Information Science, and Electrical and Systems Engineering at the University of Pennsylvania. He has been on the Faculty in the Department of Mechanical Engineering and Applied Mechanics with a secondary appointment in the Department of Computer and Information Science at the University of Pennsylvania since 1987. He was the Deputy Dean for Research in the School of Engineering and Applied Science from 2000 to 2004. He was the Director of the General Robotics, Automation, Sensing and Perception Laboratory, a multidisciplinary robotics and perception laboratory, from 1998 to 2004. He was the Chairman of the Department of Mechanical Engineering and Applied Mechanics from 2005 to 2008. He was the Deputy Dean for Education in the School of Engineering and Applied Science from 2008 to 2012. He then served as the Assistant Director of Robotics and Cyber Physical Systems, White House Office of Science and Technology Policy (20122013). His research interests include robotics, specifically multirobot systems, and micro aerial vehicles. Dr. Kumar is a Fellow of the American Society of Mechanical Engineers (2003), a Fellow of the Institute of Electrical and Electronic Engineers (2005), and a member of the National Academy of Engineering (2013). He has been on the editorial boards of IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION, IEEE TRANSACTIONS ON AUTOMATION SCIENCE AND ENGINEERING, ASME Journal of Mechanical Design, ASME Journal of Mechanisms and Robotics, and Springer Tract in Advanced Robotics. He currently serves as the Editor of ASME Journal of Mechanisms and Robotics and as an Advisory Board Member of AAAS Science Robotics Journal. He received the 1991 National Science Foundation Presidential Young Investigator award, the 1996 Lindback Award for Distinguished Teaching (University of Pennsylvania), the 1997 Freudenstein Award for significant accomplishments in mechanisms and robotics, the 2012 ASME Mechanisms and Robotics Award, the 2012 IEEE Robotics and Automation Society Distinguished Service Award, the 2012 World Technology Network Award, and the 2014 Engelberger Robotics Award. He has won best paper awards at DARS 2002, ICRA 2004, ICRA 2011, RSS 2011, and RSS 2013, and has advised doctoral students who have won Best Student Paper Awards at ICRA 2008, RSS 2009, and DARS 2010. Biography text here.