



$$T_{wb} = \begin{bmatrix} \cos \phi & -\sin \phi & x \\ \sin \phi & \cos \phi & y \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{br} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{D}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{bf} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{D}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

(1) GET THE BODY TWIST IN INTO EACH WHEEL FRAME:

$$V_r = A_{T_{br}} V_b$$

$$V_f = A_{T_{bf}} V_b$$

$$V_r = \begin{bmatrix} 1 & 0 & 0 \\ \frac{D}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\phi}_b \\ \dot{x}_b \\ \dot{y}_b \end{bmatrix}$$

$$V_f = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{D}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\phi}_b \\ \dot{x}_b \\ \dot{y}_b \end{bmatrix}$$

$$V_r = \begin{bmatrix} \dot{\phi}_b \\ \frac{D}{2} \dot{\phi}_b + \dot{x}_b \\ \dot{y}_b \end{bmatrix} \quad (\text{Eq. 1})$$

$$V_f = \begin{bmatrix} \dot{\phi}_b \\ -\frac{D}{2} \dot{\phi}_b + \dot{x}_b \\ \dot{y}_b \end{bmatrix} \quad (\text{Eq. 2})$$

(2) PROPERTIES OF CONVENTIONAL WHEELS:

$$V_x = (U/r) \quad (\text{Eq. 3})$$

$$V_y = 0$$

(3) SOLVE FOR U FOR EACH WHEEL:

$$V_r = \begin{bmatrix} \dot{\phi}_r \\ U/r \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\phi}_b \\ \frac{D}{2} \dot{\phi}_b + \dot{x}_b \\ \dot{y}_b \end{bmatrix}$$

$$V_f = \begin{bmatrix} \dot{\phi}_f \\ U_f/r \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\phi}_b \\ -\frac{D}{2} \dot{\phi}_b + \dot{x}_b \\ \dot{y}_b \end{bmatrix}$$

$$U_r = \frac{1}{r} \left(\frac{D}{2} \dot{\phi}_b + \dot{x}_b \right)$$

$$U_f = \frac{1}{r} \left(-\frac{D}{2} \dot{\phi}_b + \dot{x}_b \right)$$

$$\begin{bmatrix} U_r \\ U_f \end{bmatrix} = \frac{1}{r} \begin{bmatrix} \frac{D}{2} & 1 \\ -\frac{D}{2} & 1 \end{bmatrix} \begin{bmatrix} \dot{\phi}_b \\ \dot{x}_b \\ \dot{y}_b \end{bmatrix} \quad (\text{Eq. 4})$$

$$u = H v_b \rightarrow v_b = H^+ u$$

IN ORDER TO FIND THE PSEUDOINVERSE OF H , USE THE FOLLOWING:

$$H^+ = H^T (H H^T)^{-1}$$

BECAUSE THE MATRIX IS FULL RANK W.R.T. ~~THE~~ ROWS.

$$H^+ = r \begin{bmatrix} \frac{D}{2} & -\frac{D}{2} \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \left(\begin{bmatrix} \frac{D}{2} & 1 & 0 \\ -\frac{D}{2} & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{D}{2} & -\frac{D}{2} \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \right)^{-1}$$

$$= r \left(\begin{bmatrix} \frac{D^2}{4} + 1 & -\frac{D^2}{4} + 1 \\ \frac{D^2}{4} + 1 & \frac{D^2}{4} + 1 \end{bmatrix} \right)^{-1}$$

$$= r \begin{bmatrix} \frac{1}{4} + \frac{1}{D^2} & \frac{1}{4} - \frac{1}{D^2} \\ \frac{1}{4} - \frac{1}{D^2} & \frac{1}{4} + \frac{1}{D^2} \end{bmatrix}$$

$$= r \begin{bmatrix} \frac{D}{2} \left(\frac{1}{4} + \frac{1}{D^2} \right) - \frac{D}{2} \left(\frac{1}{4} - \frac{1}{D^2} \right) & \frac{D}{2} \left(\frac{1}{4} - \frac{1}{D^2} \right) - \frac{D}{2} \left(\frac{1}{4} + \frac{1}{D^2} \right) \\ \left(\frac{1}{4} + \frac{1}{D^2} \right) + \left(\frac{1}{4} - \frac{1}{D^2} \right) & \left(\frac{1}{4} - \frac{1}{D^2} \right) + \left(\frac{1}{4} + \frac{1}{D^2} \right) \\ 0 & 0 \end{bmatrix}$$

$$= r \begin{bmatrix} \frac{1}{D} & -\frac{1}{D} \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \quad (\text{Eq. 5})$$