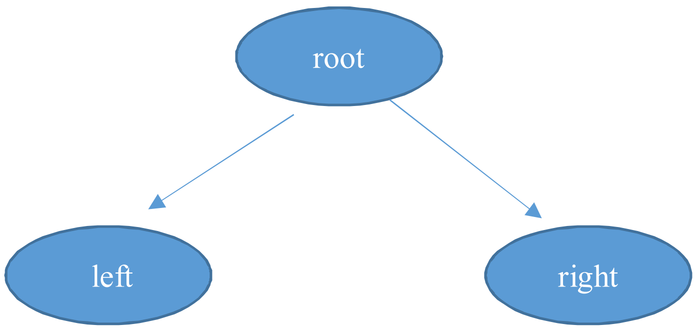
Heap

1. Keep the heap property(assuming it is all max heap)

Function: heapfy()

a. Algorithm:



1) If left > root, swap(left, root), or if right > root, swap(right, root)
2) Recursively call heapfy() on the swap the child node till end so the heap property is reserved.

b. Code

```
// n size of array, i is start index.
Void heapfy(int[] arr, int n, int i)
{
      int largest = i;
      int l = 2i+1;   // binary tree index relations
      int r = 2i+2;
      if(l<n && arr[l] > arr[largest])
         largest = i;
      if(r<n && arr[r]> arr[largest])
         largest = r;
     //swap
      If(largest != i)
      {
         swap(arr[i], arr[largest]);
         heapfy(arr, n, largest);
       }
}
```

c. Time complexity

h is the height of the tree

O(h) = O(log_2 n)

2. Build Heap

a. Algorithm

do heapfy() on all the nodes that have children, these nodes have index(n/2-1, 0) (based on binary tree property)

b. Code

```
void buildHeap(int[] arr, int n)
{
   for(int i=n/2  -1; i>=0; i--)
   {
      heapfy(arr, n, i);
   }
}
```

c. Time Complexity

Time = \sum _{h=0} ^ {log_2n} [\frac{n}{2^{h+1}}] O(h)

O(h) : time to call one heapfy()

[\frac{n}{2^{h+1}}]: number of nodes on the same tree level

\sum _{h=0} ^ {log_2^n} : sum over all the levels.

So time = O(n \sum_{h=0}^(lgn) h/(2^h))

= O(2n)

=O(n)

3. Heapsort

a. Algorithm

1) Build the heap
2) Start from the last element(index n-1),
3) swap it with the first element. Then the last element becomes sorted
4) Do heapfy on the unsorted elements
5) Move to the next unsorted element, in this case it is the last but two(index n-2), repeat 3) and 4) until all elements become sorted.

b. Code

```
Void heapsort(int arr[], int n)
// build heap
void buildHeap(arr, n);
for(int i = n-1; i>=0; i--)
{
   swap(arr[0], arr[i]);
   heapfy(arr, i, 0);
}
```

c. Time complexity

1) Build heap O(n)
2) Sort nlog_2 n

d. Comparison

1) Compare to quicksort

	QS	HS	comment
Average Speed	Faster		12nlogn vs 16 nlog n
Worst Speed		Faster	n^2 vs nlogn

2) Compare to mergesort

	MS	HS	comment
Space	O(n)	O(1)	MS needs additional space for merge
Stable sort	Yes		MS keeps order for same element
Better Cache performance	Yes		MS accesses the caches that are near to each other.

4. Binary heap time complexity analysis

a. Time complexity table

	Average	Worst
Search	O(n)	O(n)
Insert	O(1)	O(h) = O(long(n))
Delete	O(log(n))	O(long(n))
Peek	O(1)	O(1)

b. Insert

1) Add the element to the bottom level of the heap
2) Compare the added element with its parent
   If they are in correct order, stop
3) If not swap the element with its parent and return to the 2) by keeping comparing the parent

c. Insert average time complexity

Assuming a uniform distribution of numbers, which means for any element in the heap, it has a one-half chance of being greater than its parent. And it has one-fourth chance of being greater than its grandparent. So the expected number of swap during the insertion is
Probability of swapping with 1^{st} parent * number of swap +
Probability of swapping with 2^{st} parent * number of swap + …+
Probability of swapping with m^{st} parent

= ½ * 1 + ¼ * 2 + 1/8* 3 + ½^m * m

= \sum \frac{m}{2^m}

= 2 when m goes to \inf

Therefore the averaged time complexity is O(1)