1 Current Operator

Brief summary of conductance theory
In order to describe the conductivity, physicists have developed several models, starting from classical model, semiclassical model to quantum model. These model can be summarized as:
1) In classical model, the electrons are treated classically, and the movement is governed by the Newton's law. This model is good enough to explain the Ohm's law.
2) In semiclassical model, we consider the behavior of electron as a wave, and treat the movement of electrons as the propagation of the wavepacket, therefore the velocity of the electron is group velocity of the wave. This model utilize the particle-wave duality, and is capable of explaining the conduction in metal.
3) In quantum model, the velocity is the expectation value of the velocity operator given a wavefunction. We need to use this theory to derive the Hall conductance, and understand the topological behavior of Hall conductance.

Derivation of velocity operator
In quantum mechanics, the velocity operator is defined as

$$v = \frac{\partial H}{\partial p} = \frac{\partial H}{\hbar \partial k}$$

There exist several ways to understand this. First, we can recall the equation of motion in analytical mechanics. Given a Hamiltonian, the equation of motion is

$$v = \dot{q} = \frac{\partial H}{\partial p}$$

Wavefunction subject to adiabatic evolution

$$|\Psi(t)>= exp(-\frac{i}{\hbar}\int_{t_0}^{t} dt^{'} E_n(t^{'}))(|n> -i\hbar \sum_{n^{'} \neq n} |n^{'}> \frac{<n^{'}|\frac{\partial}{\partial t}|n>}{E_n - E_{n'}})$$

The expectation value of the velocity

$$\bar{v}(k,t) = \frac{\partial E_n(k)}{\hbar \partial k} - i \sum_{n^{'} \neq n} (<n|\frac{\partial H}{\partial k}|n^{'}> \frac{<n^{'}|\frac{\partial}{\partial t}|n>}{E_n - E_{n'}} - c.c.)$$

$$\bar{v}(k,t) = \frac{\partial E_n(k)}{\hbar \partial k} - i(<\frac{\partial n}{\partial k}|\frac{\partial n}{\partial t}> - <\frac{\partial n}{\partial t}|\frac{\partial n}{\partial k}>)$$

Where the second term is the Berry phase. The current operator is

$$j = -2e\frac{dk}{2\pi} f(k) v(k)$$