

1 Nonlinear Effect

Nonlinear effect means when a signal is passed into a device, the relationship between the output and input is not linear. This article aims to provide an explanation of the nonlinear process.

1.1 Harmonics

Suppose we have an input signal with a voltage

$$v_i(t) = V_{im} \cos(\omega_i t)$$

The output signal can be approximated by Taylor's expansion

$$\begin{aligned} v_o(t) &= a_1 V_{im} \cos(\omega_i t) + a_2 V_{im}^2 \cos^2(\omega_i t) + a_3 V_{im}^3 \cos^3(\omega_i t) \\ &= \frac{1}{2} a_2 V_{im}^2 + (a_1 V_{im} + \frac{3}{4} a_3 V_{im}^3) \cos(\omega_i t) + \frac{a_2}{2} V_{im}^2 \cos(2\omega_i t) \end{aligned}$$

All the terms with $\cos(n\omega_i t)$ are nonlinear terms. Therefore, an input signal with frequency ω has output with frequency $n\omega$, and this is called nonlinear effect. All the signal component with frequency $n\omega$ are called harmonics.

1.2 Gain compression

In the above derivation, the amplitude of $\cos \omega_i t$ is $a_1 V_{im} + \frac{3}{4} a_3 V_{im}^3$ is the gain to the input signal g_m . In most cases $a_3 < 0$, when v_{im} is sufficiently small, the first term dominates so the gain is equal to $\log(a_1)$. But when v_m is large enough, the second term is not negligible so the gain decreases from $\log(a_1)$. We define **1db compression point** as the input power when the gain is decreased by 1dB from $\log(a_1)$

1.3 Intermodulation

Suppose we have two input signals

$$V_i(t) = V_{1m} \cos(\omega_1 t) + V_{2m} \cos(\omega_2 t)$$

The third order output power term has two components

$$P_3 = \frac{3}{4} a_3 V_{1m}^2 V_{2m} \cos(2\omega_1 - \omega_2)t + \frac{3}{4} a_3 V_{1m} V_{2m}^2 \cos(2\omega_2 - \omega_1)t$$

if $V_{1m} = V_{2m} = V_m$

$$P_3 = \frac{3}{4} a_3 V_m^3 [\cos(2\omega_1 - \omega_2)t + \cos(2\omega_2 - \omega_1)t]$$

Since this power term has two frequency components, we call it **intermodulation power**, and we call P_3 as **third order intermodulation power**.

1.4 3rd order intercept point

The output power at base frequency

$$P_{o1} = \frac{1}{2}(a_1 V_m)^2 \equiv G_1 P_i$$

The output power at intermodulation frequency

$$P_{o3} = \frac{1}{2}\left(\frac{3}{4}a_3 V_m^3\right)^2 \equiv G_3 P_i^3$$

So

$$P_{o1} = 10\log G_1 + 10\log P_i$$

$$P_{o3} = 30\log G_3 + 30\log P_i$$

So both P_{o1} and P_{o3} has a linear relationship with P_i . The above equations represent two lines, but the slope of P_{o3} is 3 times of that of P_{o1} . The third-order intercept point(IP3) is defined as the intercept point of the above two lines(P_{o1} and P_{o3}). We can use the slope relation to find the intercept point.

$$\frac{IP_3 - P_{o3}}{IP_3 - P_{o1}} = 3$$

Therefore

$$IP_3 = \frac{1}{2}(3P_{o1} - P_{o3})$$