1 Convolutional Neural Network

1.1 Convolution Operation of Matrix

We denote the convolution operation as the following

$$Z = A * W + b$$

Where A, W, b are matrices with proper dimension. The star $*$ represent the convolution operation of the two matrix A and W. We define the matrix A with dimension $I^{in} \times J^{in}$. The matrix W has the dimension $P \times Q$ and we assume $P \leq I^{in}$, $Q \leq J^{in}$. The matrix Z has the dimension $I^{out} \times J^{out}$. And b is a scalar constant. The convolution operation is defined as

$$z_{ij} = \sum_{p=i}^{p+P} \sum_{q=j}^{q+Q} a_{pq} w_{pq} + b$$

An intuitive way of thinking the above operation is we put matrix W on top of matrix A and align the top left corner, meaning that a_{11} aligns with w_{11}. And for each overlapped element of matrix A and Z, we multiply them and get each product. Finally we sum all the product, add the bias b, then we get z_{11}. To get z_{12}, we still put W on top of A, but this time we align a_{12} with w_{11}. Then we follow the same procedure to calculate the product and summation. We keep going to slide W matrix along the matrix A, we can all the element of matrix Z.

1.2 Convolution Operation of Tensors

We denote the convolution operation as the following(same as above)

$$Z = A * W + b$$

Where A, W, b are tensors with proper dimension. The star $*$ represent the convolution operation of the two matrix A and W. We define the tensor A with dimension $I^{in} \times J^{in} \times K$. The tensor W has the dimension $P \times Q \times K \times N$. The tensor Z has the dimension $I^{out} \times J^{out} \times N$. And b is a vector of size N. The convolution operation is defined as

$$z_{ijn} = \sum_{p=i}^{p+P} \sum_{q=j}^{q+Q} \sum_{k=1}^{K} a_{pqk} w_{pqk}^{(n)} + b^{(n)}$$

Example Take A as a $2 \times 2 \times 3$ tensor, and W as a $2 \times 2 \times 3 \times 2$ tensor, then

$$\begin{aligned} z_{111} = {} & a_{111} w_{111}^{(1)} + a_{121} w_{121}^{(1)} + a_{211} w_{211}^{(1)} + a_{221} w_{221}^{(1)} \\ & + a_{112} w_{112}^{(1)} + a_{122} w_{122}^{(1)} + a_{212} w_{212}^{(1)} + a_{222} w_{222}^{(1)} \\ & + a_{113} w_{113}^{(1)} + a_{123} w_{123}^{(1)} + a_{213} w_{213}^{(1)} + a_{223} w_{223}^{(1)} \\ & + b^1 \end{aligned}$$

$$
\begin{aligned}
z_{112} = {} & a_{111}w_{111}^{(2)} + a_{121}w_{121}^{(2)} + a_{211}w_{211}^{(2)} + a_{221}w_{221}^{(2)} \\
& + a_{112}w_{112}^{(2)} + a_{122}w_{122}^{(2)} + a_{212}w_{212}^{(2)} + a_{222}w_{222}^{(2)} \\
& + a_{113}w_{113}^{(2)} + a_{123}w_{123}^{(2)} + a_{213}w_{213}^{(2)} + a_{223}w_{223}^{(2)} \\
& + b^{1}
\end{aligned}
$$