1 Current Operator

Derivation of velocity operator
Let us recall the semi-classical model where the movement of electrons is described as the movement of the wave-packet. And the velocity is given by the group velocity of the wave packet.

$$v_g = \frac{\partial \omega}{\partial k} = \frac{1}{\hbar}\frac{\partial E}{\partial k}$$

In analogy,

$$v = \frac{\partial H}{\partial p} = \frac{\partial H}{\hbar \partial k}$$

Wavefunction subject to adiabatic evolution

$$|\Psi(t)>= exp(-\frac{i}{\hbar}\int_{t_0}^{t} dt^{'} E_n(t^{'}))(|n> -i\hbar \sum_{n^{'}\neq n} |n^{'}> \frac{<n^{'}|\frac{\partial}{\partial t}|n>}{E_n - E_{n^{'}}})$$

The expectation value of the velocity

$$\bar{v}(q,t) = \frac{\partial E_n(q)}{\hbar \partial q} - i \sum_{n^{'}\neq n} (<n|\frac{\partial H}{\partial q}|n^{'}> \frac{<n^{'}|\frac{\partial}{\partial t}|n>}{E_n - E_{n^{'}}} - c.c.)$$

$$\bar{v}(q,t) = \frac{\partial E_n(q)}{\hbar \partial q} - i(<\frac{\partial n}{\partial q}|\frac{\partial n}{\partial t}> - <\frac{\partial n}{\partial t}|\frac{\partial n}{\partial q}>)$$

Where the second term is the Berry phase.