HierarchicalModel

April 17, 2017

```
In [49]: %matplotlib inline
         #posteior distribution of binomial data
         import numpy as np
         import scipy.stats as stats
         import matplotlib.pyplot as plt
         import matplotlib
         import astropy.io.fits as fits
         import scipy.special as special

         matplotlib.rc('xtick', labelsize=12)
         matplotlib.rc('ytick', labelsize=12)


         def loadData(filename):
             '''
             Read fits data

             '''
             tchfits = fits.open(filename)
             tabl = tchfits[1].data
             return tabl

         # star/galaxy ratio in a small field of sky
         filename = 'sdss_field.fits'
         sdss = loadData(filename)

         types = sdss.type
         J = 13
         #
         observs = np.zeros((J,2))
         for j in range(J):
             t0 = types[(j*50):np.min([(j*50+50),len(types)])]
             observs[j,:] = np.array([np.float(np.sum(t0==3)),len(t0)])
             print 'No.%(d0)d: N_star=%(d1).0f, N_all=%(d2)d\n' %\
                 {'d0':j,'d1':observs[j,0],'d2':observs[j,1]}
```

```
No.0: N_star=28, N_all=50
No.1: N_star=33, N_all=50

No.2: N_star=31, N_all=50

No.3: N_star=34, N_all=50

No.4: N_star=39, N_all=50

No.5: N_star=32, N_all=50

No.6: N_star=40, N_all=50

No.7: N_star=33, N_all=50

No.8: N_star=29, N_all=50

No.9: N_star=32, N_all=50

No.10: N_star=26, N_all=50

No.11: N_star=34, N_all=50

No.12: N_star=26, N_all=43
```

```
In [50]: #point estimate
         E_theta = np.mean(observs[:-1,0]/observs[:-1,1])
         D_theta = np.var(observs[:-1,0]/observs[:-1,1])
         print 'point estimates using the first 12 observations'
         print 'E(theta)=%(e).5f,\tD(theta)=%(v).5f' %\
             {'e':E_theta, 'v':D_theta}

         beta = (E_theta*(1-E_theta)/D_theta-1)*(1-E_theta)
         alpha = E_theta*beta/(1-E_theta)
         print '\nEstimates for the parameters of the prior density'
         print 'alpha=%(a).5f,\tbeta=%(b).5f' % {'a':alpha, 'b':beta}
```

```
point estimates using the first 12 observations
E(theta)=0.65167,        D(theta)=0.00603

Estimates for the parameters of the prior density
alpha=23.87784,        beta=12.76334
```

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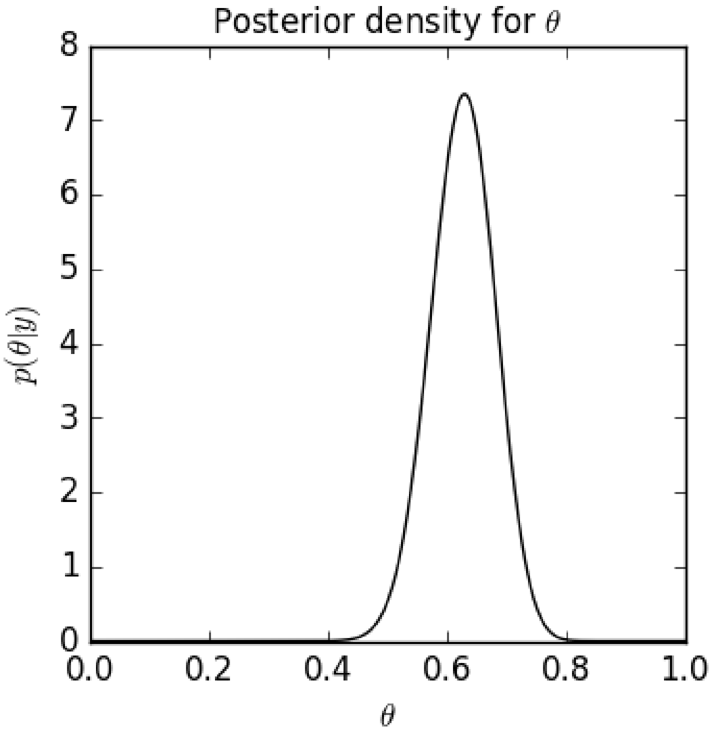
In [44]: y = observs[-1,0]
         n = observs[-1,1]
         alpha_post = alpha+y
```

```
beta_post = beta+n-y
E_theta_post = alpha_post/(alpha_post+beta_post)
D_theta_post = alpha_post*beta_post/\
    ((alpha_post+beta_post)**2*(alpha_post+beta_post+1))
print 'Parameters of te posterior density'
print'E(theta)=%(e).5f,\tstD(theta)=%(v).5f' % \
    {'e':E_theta_post, 'v':np.sqrt(D_theta_post)}
theta = np.arange(0,1,0.001)
p = stats.beta.pdf(theta,alpha_post, beta_post)

fig = plt.figure(figsize=[4,4])
ax = fig.add_subplot(111)
ax.plot(theta,p,'k-')
ax.set_xlabel(r'$\theta$',fontsize=12)
ax.set_ylabel(r'$p(\theta|y)$',fontsize=12)
ax.set_title(r'Posterior density for $\theta$',fontsize=12)
fig.show()
```

```
Parameters of te posterior density
E(theta)=0.62628,        stD(theta)=0.05387
```



```
In [61]: print np.log(alpha/beta),np.log(alpha+beta)
```

```
0.626373308021 3.60117272215
```

```
In [120]: #Bayesian model
          def posteriorln(x,y,n):
              alpha = x[0]
              beta = x[1]
              lnlikeli = 0.
              for j in range(len(y)):
                  lnlikeli += special.gammaln(alpha+beta)+\
                      special.gammaln(alpha+y[j])+\
                      special.gammaln(beta+n[j]-y[j])-\
                      special.gammaln(alpha)-special.gammaln(beta)-\
                      special.gammaln(alpha+beta+n[j])
              priorln = np.log(beta)-5./2.*np.log(alpha+beta)
              return lnlikeli+priorln+395.

          def draw_posterior(y,n,alphagrid,betagrid):
              p = np.zeros((len(alphagrid),len(betagrid)))
              for i in range(len(alphagrid)):
                  for j in range(len(betagrid)):
                      p[i,j] = posteriorln([alphagrid[i],betagrid[j]],y,n)
              return p



          alphagrid = np.arange(1,500,0.5)
          betagrid = np.arange(2,400,0.5)
          p = draw_posterior(observs[:-1,0],observs[:-1,1],\
                  alphagrid,betagrid)

          fig = plt.figure()
          ax = fig.add_subplot(111)
          amesh,bmesh = np.meshgrid(alphagrid,betagrid)
          ax.contour(amesh, bmesh, np.exp(p.T),20,\
                      extent=[alphagrid[0],\
                      alphagrid[-1],betagrid[0],betagrid[-1]])
          ax.set_xlabel(r'$\alpha$',fontsize=12)
          ax.set_ylabel(r'$\beta$',fontsize=12)

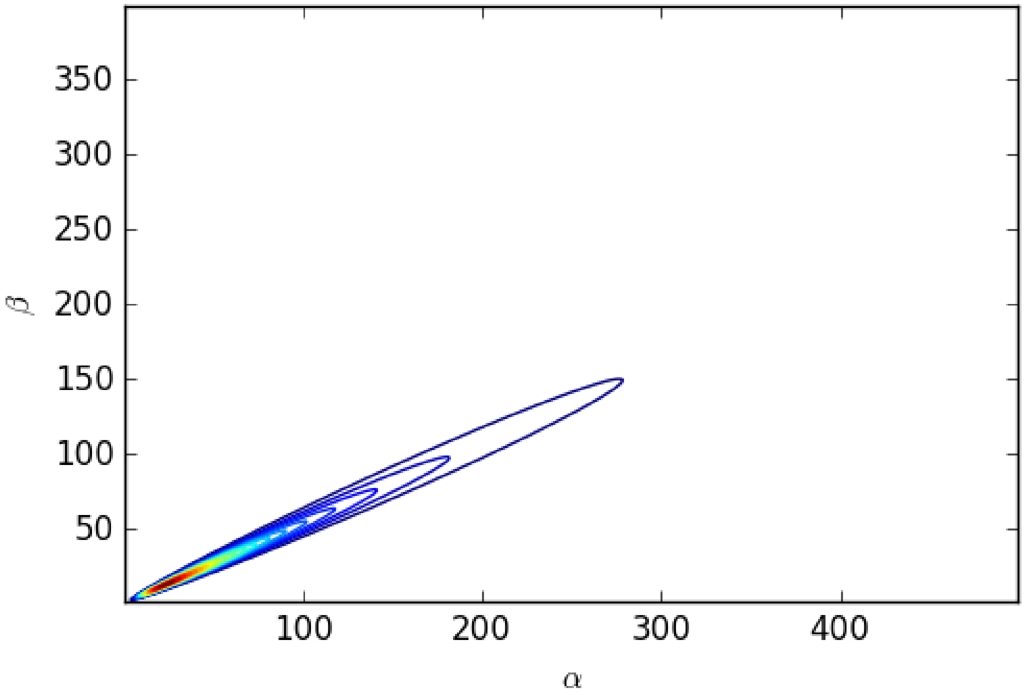
          fig.show()

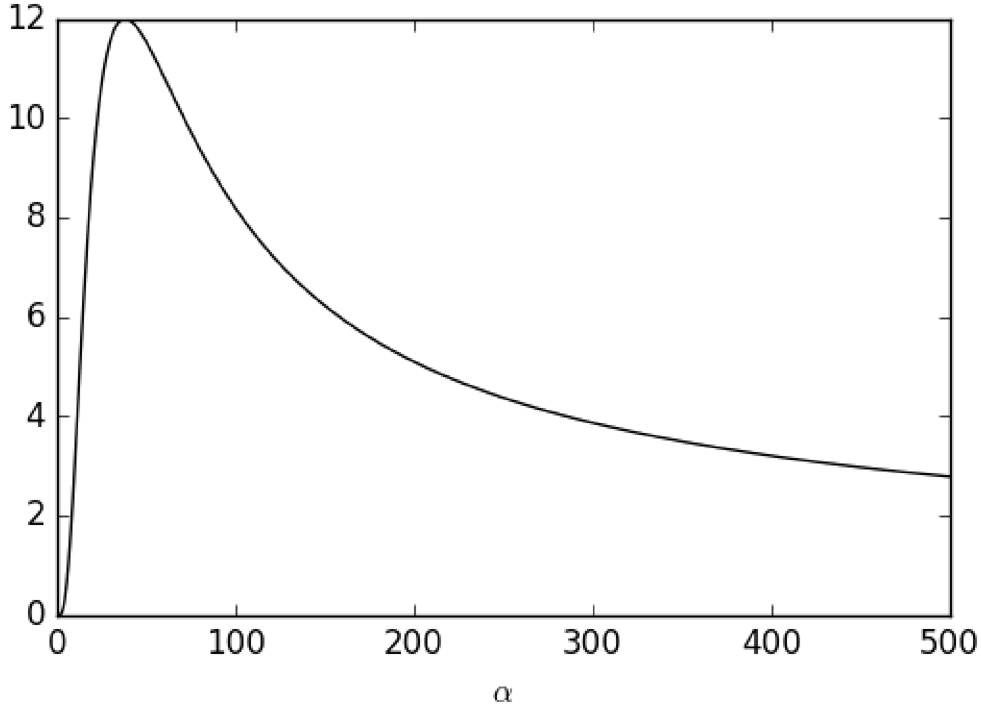
          fig = plt.figure()
          ax = fig.add_subplot(111)
          amesh,bmesh = np.meshgrid(alphagrid,betagrid)
          ax.plot(alphagrid, np.sum(np.exp(p), axis=1),'k-')
          ax.set_xlabel(r'$\alpha$',fontsize=12)
```

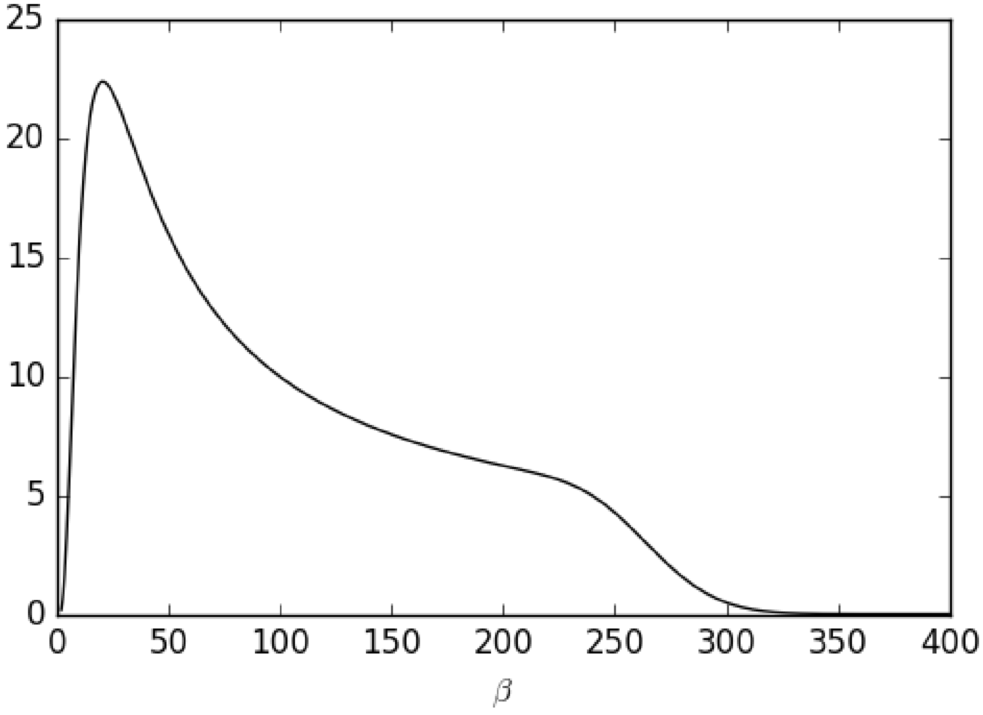
```
fig.show()

fig = plt.figure()
ax = fig.add_subplot(111)
amesh,bmesh = np.meshgrid(alphagrid,betagrid)
ax.plot(betagrid, np.sum(np.exp(p), axis=0),'k-')
ax.set_xlabel(r'$\beta$',fontsize=12)

fig.show()
```




12
10
8
6
4
2
0
0
100
200
300
400
500
α


25
20
15
10
5
0
0
50
100
150
200
250
300
350
400
β

```
In [93]: np.max(p)
         np.log(25+14)
```

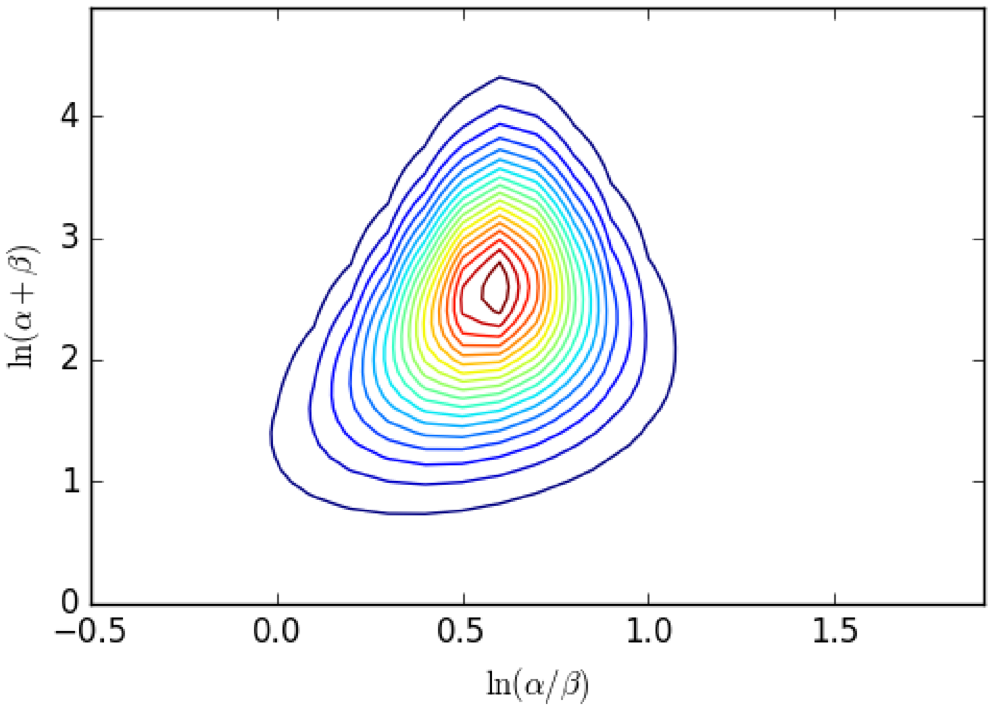
```
Out[93]: 3.6635616461296463
```

```
In [125]: def draw_posteriorln(y,n,agrid,bgrid):
              p = np.zeros((len(agrid),len(bgrid)))
              for i in range(len(agrid)):
                  for j in range(len(bgrid)):
                      alpha = np.exp(agrid[i]+bgrid[j])/(1+np.exp(agrid[i]))
                      beta = np.exp(bgrid[j])/(1+np.exp(agrid[i]))
                      #print alpha,beta
                      Jacob_detln = -np.log(alpha)-np.log(beta)
                      p[i,j] = posteriorln([alpha,beta],y,n)+Jacob_detln
              return p

          agrid = np.arange(-0.5,2,0.1)
          bgrid = np.arange(0,5,0.1)
          p2 = draw_posteriorln(observs[:-1,0],observs[:-1,1],\
                  agrid,bgrid)

          fig = plt.figure()
          ax = fig.add_subplot(111)
          amesh,bmesh = np.meshgrid(agrid,bgrid)
          ax.contour(amesh, bmesh, np.exp(p2.T-7.34),20,\
                      extent=[alphagrid[0],\
                      alphagrid[-1],betagrid[0],betagrid[-1]])
          ax.set_xlim([-0.5,2])
          ax.set_ylim([0,5])
          ax.set_xlabel(r'$\ln(\alpha/\beta)$',fontsize=12)
          ax.set_ylabel(r'$\ln(\alpha+\beta)$',fontsize=12)

          fig.show()
```



```
In [137]: #sample random values from p(ln(a/b),ln(a+b))
          N = 10000
          a0 = np.random.uniform(-0.5,2,size=N)
          b0 = np.random.uniform(0,5,size=N)
          p0 = np.random.uniform(0,0.0117,size=N)
          alpha0 = np.exp(a0+b0)/(1+np.exp(a0))
          beta0 = np.exp(b0)/(1+np.exp(a0))
          Jacob_detln = -np.log(alpha0)-np.log(beta0)
          p = np.exp([posteriorln([alpha0[i],beta0[i]],\
              observs[:-1,0],observs[:-1,1])+Jacob_detln[i] \
              for i in range(N)])
          ind = (p0<=p)
          a = a0[ind]
          b = b0[ind]
          alpha = alpha0[ind]
          beta = beta0[ind]

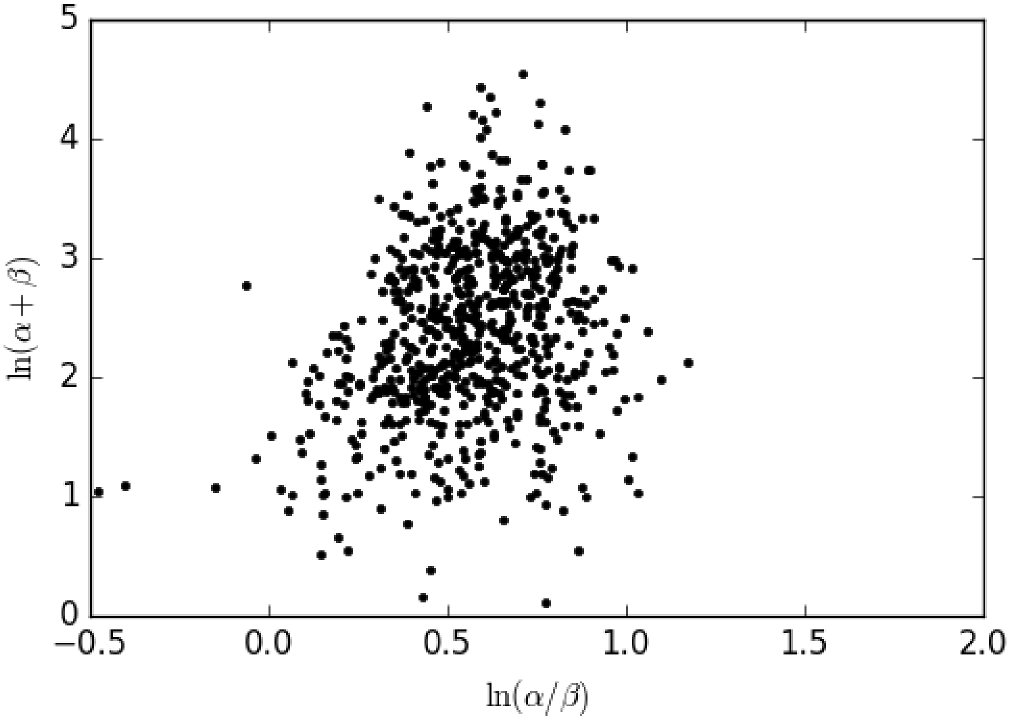
          print 'mean(alpha|y)=%(a).5f\tmean(beta|y)=%(b).5f' %\
              {'a': np.mean(alpha),'b': np.mean(beta)}
          fig = plt.figure()
          ax = fig.add_subplot(111)
          ax.plot(a,b,'k.')
          ax.set_xlim([-0.5,2])
          ax.set_ylim([0,5])
```

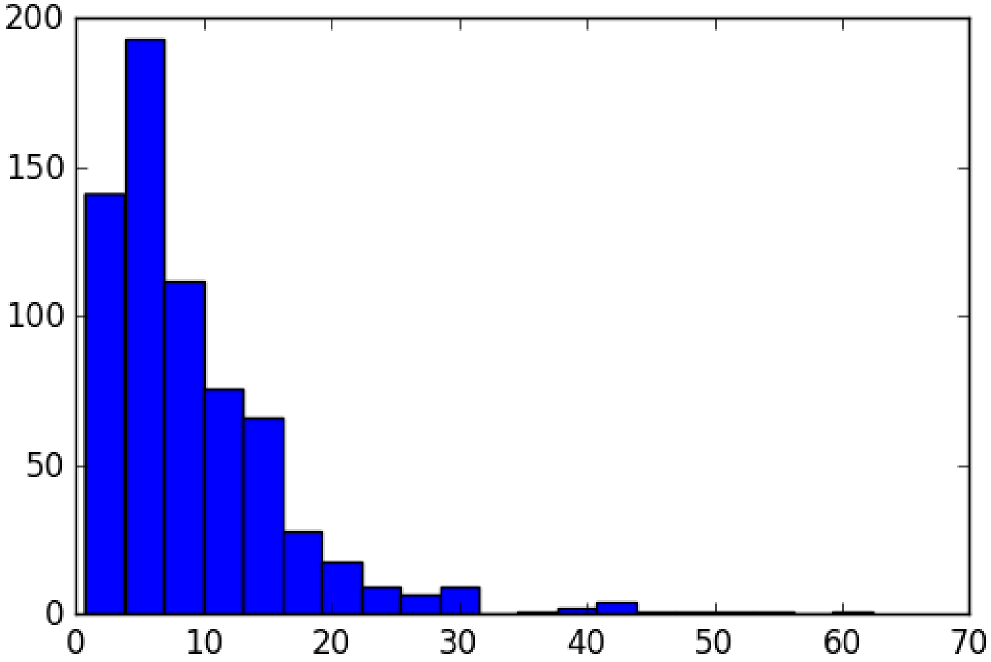
```
        ax.set_xlabel(r'$\ln(\alpha/\beta)$',fontsize=12)
        ax.set_ylabel(r'$\ln(\alpha+\beta)$',fontsize=12)
        fig.show()

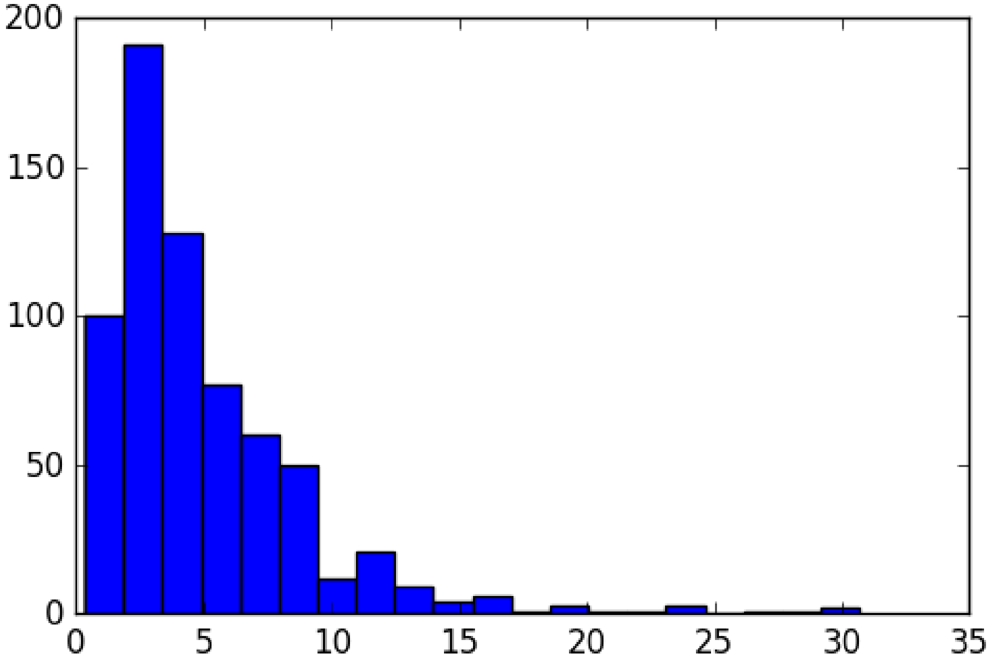
        fig = plt.figure()
        ax = fig.add_subplot(111)
        ax.hist(alpha,20)
        fig.show()

        fig = plt.figure()
        ax = fig.add_subplot(111)
        ax.hist(beta,20)
        fig.show()
```

```
mean(alpha|y)=9.17608        mean(beta|y)=5.06795
```





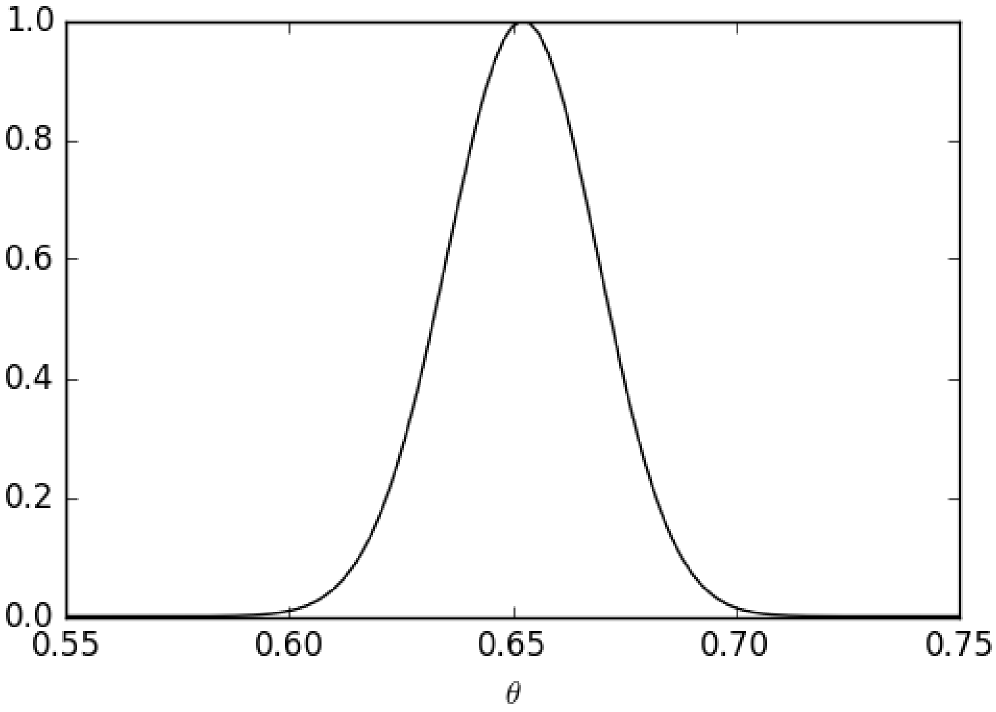


```
In [149]: #predict post density of theta
          theta = np.arange(0,1,0.002)
```

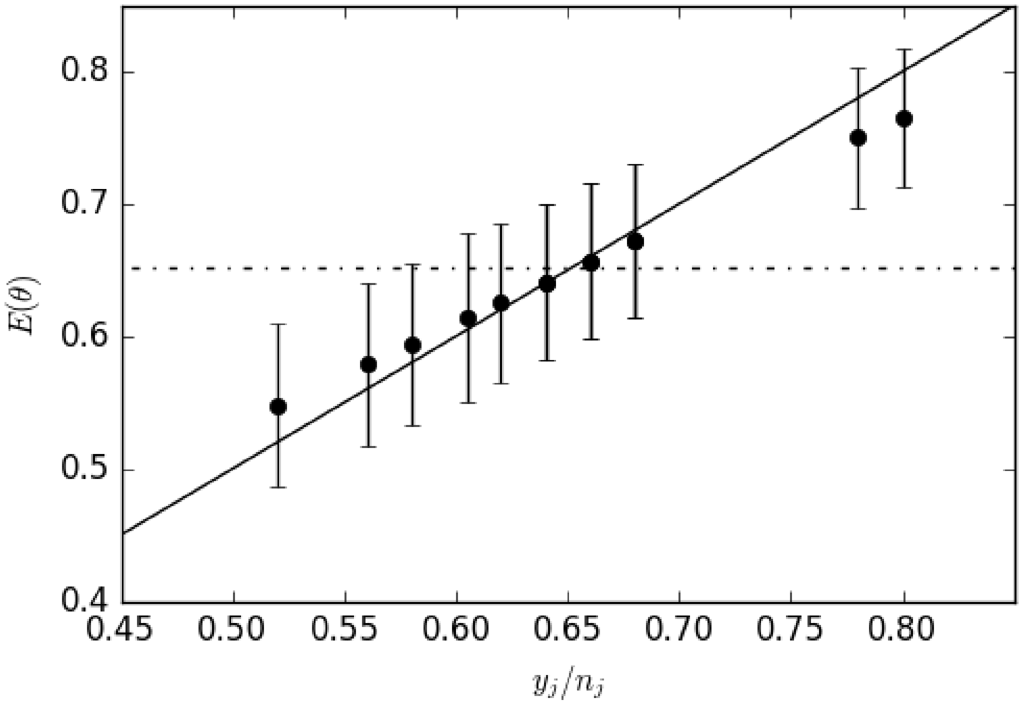
```python
pln = np.zeros(np.shape(theta))
for j in range(13):
    a = np.mean(alpha)+observs[j,0]
    b = np.mean(beta)+observs[j,1]-observs[j,0]
    pln = pln+np.log(stats.beta.pdf(theta,a,b))

p = np.exp(pln-np.max(pln))

mean_theta_post = np.sum(p*theta)/np.sum(p)
std_theta_post = np.sqrt(np.sum(p*(theta-\
    mean_theta_post)**2)/np.sum(p))
print 'mean(theta|alpha,beta,y)=%(p).5f' %\
    {'p':mean_theta_post}
print 'std(theta|alpha,beta,y)=%(p).5f' %\
    {'p':std_theta_post}
fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(theta,p,'k-')
ax.set_xlabel(r'$\theta$',fontsize=12)
ax.set_xlim([0.55,0.75])
fig.show()
```

```
mean(theta|alpha,beta,y)=0.65196
std(theta|alpha,beta,y)=0.01679
```

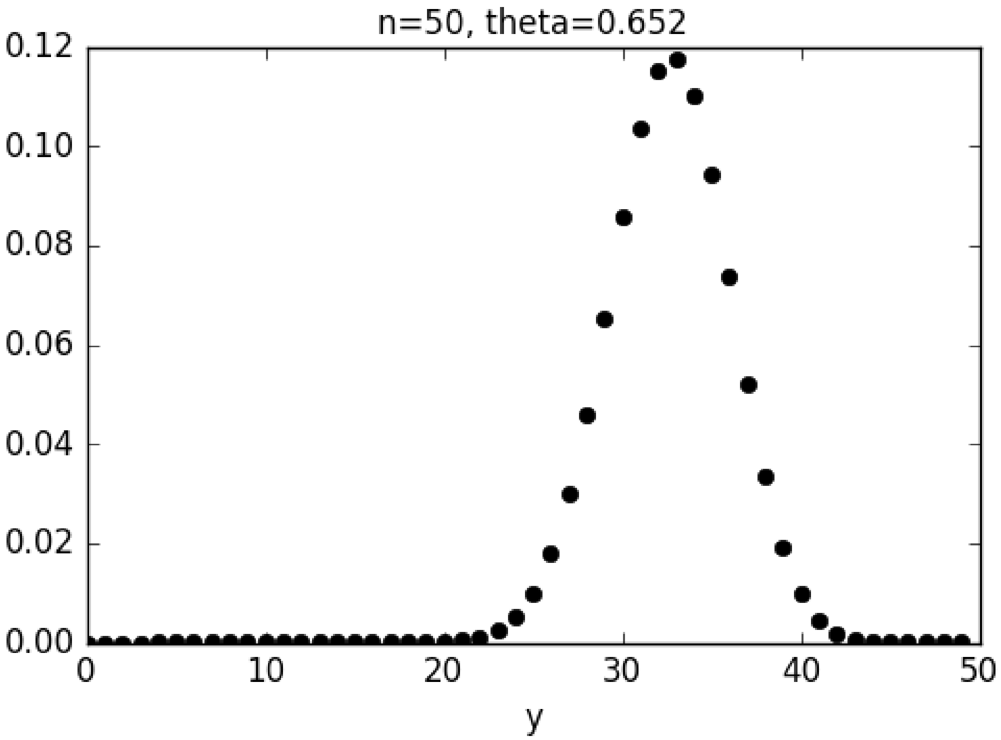
```
/Users/cliu/anaconda/lib/python2.7/site-packages/ipykernel/__main__.py:7: RuntimeWa
```



```
In [148]: #estimate theta_j from the given mean alpha and beta
          p_theta_post = np.zeros((13,2))
          for j in range(13):
              a = np.mean(alpha)+observs[j,0]
              b = np.mean(beta)+observs[j,1]-observs[j,0]
              p_theta_post[j,0] = a/(a+b)
              p_theta_post[j,1] =np.sqrt(a*b/((a+b)**2*(a+b+1)))


          fig = plt.figure()
          ax = fig.add_subplot(111)
          ax.errorbar(observs[:,0]/observs[:,1],p_theta_post[:,0],\
              yerr=p_theta_post[:,1],fmt='ko')
          ax.plot([0.45,0.85],[0.45,0.85],'-k')
          ax.plot([0,1],[mean_theta_post,mean_theta_post],'k-.')
          ax.set_xlabel(r'$y_j/n_j$',fontsize=12)
          ax.set_ylabel(r'$E(\theta)$',fontsize=12)
          ax.set_xlim([0.45,0.85])

          fig.show()
```



```
In [156]: #predict next y from posterior theta
          y = np.arange(0,50,1)
          p_y = stats.binom.pmf(y,50,mean_theta_post)

          fig = plt.figure()
          ax = fig.add_subplot(111)
          ax.plot(y,p_y,'ko')
          ax.set_xlabel('y',fontsize=12)
          ax.set_title('n=50, theta=%(t).3f' % {'t': mean_theta_post})

          fig.show()
```



```
In [ ]:
```