Information Asymmetry and Bidding Behavior in Common Value English Auction

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Abstract

In common value English auction, bidders may have different levels of private information toward the selling item. Due to the learning effect of the common value part, bidders with more precise information may be able to manipulate the auction outcome through strategic bidding behavior. However, English auctions are usually challenged by model incompleteness problem (Bikhchandani ., 2002). To overcomes the identification challenge, I utilize moment inequalities implied by the bidder's bidding history to estimate the value distribution and develop a structural econometric model to study the effect of information asymmetry on the agent's bidding behavior. The paper finds that the information premium mainly comes from the informed bidder's screening effect and is independent of the number of participants. Applying the data from Chinese Justice Auction, the estimation result shows that the noisy part in signal is very large.

Keywords: English auction, information asymmetry, multiple equilibria, moment inequalities

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1 Introduction

Regarding the impact of informational asymmetry on the agent's behavior, an auction, especially the common value auction, is an ideal laboratory for the research. Consider an English auction for selling object with common value. Each bidder privately observes a noisy signal of the item value and must decide when and how much to bid. In terms of information asymmetry, some bidders could receive more precise signal, named as the informed bidder, resulting in a higher expected payoff compared with their counterparts, i.e., the uninformed bidders. This net expected payoff between the two groups is called information premium or information rent in the literature. When making decisions, a bidder should consider her bid (if any) will be interpreted as good news about her private signal and therefore encourage further bidding by her rivals. Especially when the identity of the informed bidder is revealed to the public, other bidders will put higher weight on informed bidder's bidding behavior to update their evaluation. This learning process in common value English auction would make more aggressive bidding competition during the auction (Hernando-Veciana, 2009). On the other hand, a bidder would also realize that her rivals' signals must be relatively weak if her bid is the last submitted, i.e. she must anticipate the so-called winner's curse. The winner's curse will discourage further bidding activities. Therefore, the two offsetting effects jointly characterize the bidders' bidding behavior.

Studying the bidding behavior need to back out the value distribution first. But as emphasized in (Bikhchandani ., 2002), the equilibrium bidding strategy in the English auction involves multiple equilibria problem. This is because the bidding price only indicates the lower bound of bidder's evaluation on the selling item, one bidding price could result from many different bidding strategies, reflecting a wide range of possible private signals. On the other hand, Haile Tamer (2003) point out in English auction prices may rise in jumps of varying sizes, and active bidding by the rivals may discourage the bidder to make a bid close to her evaluation. Hence, it is difficult to observe every bidder's dropout prices, the highest possible bidding price under certain bidding strategy. The model incompleteness caused by such bidding mechanism means that we can only build a correspondence from bidding price to the bidder's private signal, mapping the private signal into a region of bidding prices.

In an ideal situation of an auction model, we bridge the bidding distribution with value distribution under one to one mapping. However, the multiple equilibrium bidding strategies imply that many different value distributions can map to one bidding distribution. And the model incompleteness further implies that many bidding distributions correspond to one value distribution. Therefore, a new challenge now becomes that many value distributions map to many bidding distributions, which requires a new identification and estimation approach to deal with. Back to the informational asymmetry effect on bidding behavior and value distribution, several key questions thus far remain inconclusive. how does the information rent redistribute among different agents? Is more information really good for the informed bidders? I propose to address above questions in the context of Chinese Justice Auctions.

Chinese Justice Auction origins from the fact that local courts in China have to deal with 600 billion RMB worth of confiscated assets and property rights every year. To speed up the disposal procedure and avoid corruption, the Supreme People's Court of China began to allow local courts to hold Online auctions to deal with confiscated assets in 2010. Since then, a large proportion of the confiscated assets have been disposed through Online platform. In 2015, there were more than 124,000 Online Justice Auctions held across the country. 84 percent of them were sold successfully. Analyzing the information asymmetry and agent's bidding behavior in Chinese Justice auctions is relevant for a number of reasons. First, the mechanism in the Chinese Justice auction satisfies all the key features of the common value English auction that we have discussed before. The auction data provides a ideal laboratory to investigate the new method for recovering value distribution in common value English auction. Second, in Chinese Justice Auction, there exists one typical type of bidder in the data called "priority bidder", which is defined as the person who has some connections to the selling item. Examples are like the tenants of the house for sale or the banks that lend the mortgage. Without loss of generality, the priority bidder in the data is equivalent to the informed bidder. The unique feature in this auction is that the priority bidders can choose to self-identify themselves or not. With the help of the auction data, we can study the bidder's bidding behavior under different information structure.

This paper aims at analyzing how bidders react to information asymmetry and estimating the value distribution from the data. This work departs from the previous analysis in many ways. First, the paper constructs a new model to analyze the bidding behavior in common value English auction. To overcome the multiple equilibria and incompleteness of the model, the new model in this paper combines Hong Shum (2003) and Haile Tamer (2003) to estimate the value distribution in common value English auction. Rather than selecting one equilibrium bidding strategy, I use the bidding history to construct lower and upper bounds on the bidders' private signals to encompass all the potential bidding strategies. Then, the parameters of the model can be recovered from moment inequality framework.

Second, the discussion of informational asymmetry will focus on how information premium is distributed across bidders. Because uninformed bidders will put more weights on the informed bidder's posting price, learning process will provide more precise information from the informed bidder. But the increased weighting part comes from other bidder's bidding activities as well as the bidder's own private signal. The reduced weights on the bidder's own private signal will lead to less precise information revelation from the learning process. This paper finds that the positive effect and negative effect on the learning process can be canceled out. But the existence of the informed bidder will screen out the low valued bidders and select high valued bidders entering into the auction. As a result, the information premium mainly comes from the screening effect not from the learning process. Similarly, this model predicts that uninformed bidder's aggressive bidding behavior comes from nothing but more high valued bidders entering into the auction. Regarding the interactions among informed and uninformed bidders, the simulation test find that informed bidder can significantly affect the auction outcome by strategically choosing certain bidding behavior. Hence, knowing more may be not good for the uninformed bidder.

This work relates to both theoretical and empirical auction literature. Asymmetric information in affiliated value auctions usually induces both competition-enhancing and competition-dampening two contrasting effects. Dionne . (2009) analyze how informational asymmetry among bidders affects the auction outcome. For the competition-enhancing effect, the Hernando-Veciana (2009) shows that better information on the common value induces more aggressive bidding by uninformed bidders in open than sealed auctions. On the other hand, the classical winner's curse would discourage uninformed bidder's participation, which is marked as competition-dampening effects. For instance, Milgrom Weber (1982) prove that the informed bidder earns strictly positive expected revenue while the uninformed bidders earn zero expected revenue. Empirical evidence in Hendricks Porter (1988) shows that a strong winner's curse will have a detrimental impact on drainage lease prices when better-informed bidders are present, discouraging uninformed bidder's bidding motivation. Another related strand of literature is the structural econometrics of auctions. Regarding the identification challenge in English auction, Haile Tamer (2003) explain why the English auction model could involve the incompleteness problem and propose a non-parametrically identification strategy on the distribution of valuation in independent private value (IPV) settings. Hong Shum (2003) estimate a asymmetric affiliated English auction with a trackable econometric model. Haile et al. (2018) consider the endogenous entry and propose a test for the common value in the first-price auction. Aradillas-López . (2013) also relax the IPV setting and investigate the identification of the ascending auctions with correlated private value. Regarding the test of information structure, Hendricks, Porter et al. (2018) use U.S. Offshore Oil and Gas Lease Auctions to do the competition and collusion test under first price auction. In this paper, to recover the bidder's private information, I construct the lower and upper bounds from the bidding price similar in Haile Tamer (2003). Because the lower bound and upper bounds construction involves the moment inequality, the estimation procedure follows Chernozhukov . (2007).

The rest of the paper is organized as follows. Section 2 builds the model and derives the basic results of the bidding behavior. Section 3 introduces the dataset and provides the empirical evidence for the informational asymmetry. Section 4 discusses identification strategies. Section 5 sets up the estimation procedure and describes the simulation test and empirical findings. Section 6 introduces the test for the information structure. Section 7 concludes.

2 Model

The paper considers a model of common value English auction[1]. Each auction τ is associated with observed characteristics $M_\tau \in \mathbb{M}$. M_τ includes the reservation price (γ), the minimum bidding increment (Δ), the item description, and etc. U_τ represents the features that bidders can observe but not the econometricians. The risk neutral bidder i values the object at $V_{i,\tau}$, but only receives a private and noisy signal $X_{i,\tau}$ of the valuation $V_{i,\tau}$. Let $V_\tau = (V_{1,\tau}, ..., V_{N_\tau,\tau})$, $X_\tau = (X_{1,\tau}, ..., X_{N_\tau,\tau})$ and $X_{-i,\tau} = X_\tau \backslash X_{i,\tau}$. The number of bidders entering auction τ is denoted by N_τ. In the biding stage, the realizations of (M_τ, N_τ) are common knowledge among bidders, as is the distribution of $(X_\tau, V_\tau)|(M_\tau, N_\tau)$. I make two main assumptions on this conditional distribution.

Assumption 1. *(i) For all $n \in supp\ N_\tau|(X_\tau, M_\tau)$, $F_{XV}(X_\tau, V_\tau|n,\ M_\tau)$ has a continuously differentiable joint density that is affiliated, exchangeable in the indices $i = 1, ..., n$, and positive the support of X and V;. (ii) For each bidder i, $\mathbb{E}[V_{i\tau}|X_{i\tau}, X_{-i\tau}; N_\tau, M_\tau]$ exists and is strictly increasing in $X_{i\tau}$.*

Because of the common value part, each bidder's expected valuation, $\mathbb{E}[V_{i\tau}|X_{i\tau}, X_{-i\tau}; N_\tau, M_\tau]$, depends on $X_{-i\tau}$. The auction is organized in the following mechanisms. An auction begins whenever a bidder starts to submit the first bid. The bidding price follows the reserve price plus integer multiple of the minimum bidding increment. In the model, the bidder whose bid is selected in period t is called the posting bidder. In the online Justice auction, bidders can submit their bid at any time. But their submission can be rejected if someone else submit the same or higher bid earlier. The auction ends until no one submits a new bid.

To simplify the mechanism while keeping the main features from the real justice auction. This model assumes that in each period t , all the remaining bidders will submit their

[1]Later I will introduce the endogenous entry for testing the information structure (Section 6)

bid directly to the auctioneer (like sealed bid auction). The auctioneer collects all the submissions and select highest bidding price as the posting bidder. If more than two bidders submit the same bid, the auctioneer randomly picks one of the bidders as the posting bidder, the corresponding bidding price will be the posting price at period t. The auctioneer ends current period after announcing the posting bidder and posting price. At the beginning of the next period, the bidding history is updated. Meanwhile, all bidders update their beliefs about their rivals' signals, make bidding decisions, and submit the new bids again. If there is no more new bid, the posting bidder wins the item and the auction ends (ending period is denoted by T). Without loss of generality, bidders are indexed by $i = 1, ...N_\tau$ where the ordering $1, ..., N_\tau$ indicates the order of latest bidding price of each bidder, so that bidder N's highest bidding price is the lowest among all the bidders, and bidder $i = 1$ wins the auction.

Because a typical common value English auction involves the learning process, every new bid will reveal some information to the public and other bidders can update their evolution from the new bidding price. Here I introduce the second assumption that regulates bidding behavior:

Assumption 2. *Bidders do not allow an opponent to win at a price they are willing to bid and they will never bid higher than* $\mathbb{E}[V_{i\tau}|X_{i\tau}, X_{-i\tau}; N_\tau, M_\tau]$

Assumption 2 is motivated by the rules in the auction model that it is a dominated strategy to let others win at price that is below their own evaluation of the item.[2] However, In this model, because the auctioneer only posts one bidder in each period, we do not know whether it is the case that all the other bidders have dropped out before that period or that there have been multiple submission but only one is selected. Hence, not just econometricians, even bidders themselves can not observe rivals' dropout prices. This is the major distinction from the previous button auction literature. If the dropout price is observable, remaining bidders can infer the private information possessed by the bidders who have dropped out (Milgrom Weber, 1982; Hong Shum, 2003) under certain selection

[2]This is "the essential feature of the English auction" (McAfee and McMillan, 1987): the ability of bidders to observe and respond to the current best bid with higher bids of their own.

rule (Bikhchandani ., 2002). But due to the unobservability of dropout price, we face the model incompleteness problem. Still, the bidding history provides a lower and upper bound on the corresponding bidder's private signal so that we can get partial identification of the value distribution. Fortunately, with the help of the learning process in the auction, we can construct a tighter bound for identification.

2.1 Equilibrium bidding strategies

Let us take the number of bidders N_τ as given and discuss equilibrium bidding function first. The bidding functions for bidder i in over bidding periods are defined as $\beta_i^t(X_i;\Omega_t)$,i.e. $\{\beta_i^1(X_i;\Omega_t),...,\beta_i^T(X_i;\Omega_T)\}$, where X_i denotes bidder i's private signal and Ω_t denotes the public information set at period t. Ω_t includes the bidding history up to period t, the observed characteristics M_τ, the unobserved features U_τ, the number of bidders N_τ and the existence of informed bidder. $\beta_i^t(X_i;\Omega_t)$ tells bidder i at which price she should stop bidding at period t. Within the interval between the posting price P_{t-1},, and $\beta_i^t(X_i;\Omega_t)$, bidder i can submit any price equal to $P_{t-1}+k\Delta, k\in\mathbb{N}_+$, where Δ represents the minimum bidding increment. $\beta_i^t(X_i;\Omega_t)$ actually reflects bidder i's real evaluation, which is called *pivotal bidding function* in this paper. The set of pivotal bidding functions $\left\{\beta_i^1(X_i;\Omega_t),..,\beta_i^T(X_i;\Omega_t)\right\}$ for bidders $i=1,...,N$ are common knowledge.

As the auction proceeds, we have the past bidding path $\{P_{t-1},...P_1\}$ of the corresponding bidders. Since the pivotal bidding functions are common knowledge, bidder i can use the the bidding history to infer the history dependent lower bound of private signals $\{\underline{X}_1^t,...,\underline{X}_N^t\}$ by inverting the bidding functions, i.e., $\underline{X}_j^t=(\beta_j^t)^{-1}(P_j^{t*};\Omega_t)$, where P_j^{t*} represents the highest bidding price of bidder j up to period t. In what follows, I focus on increasing bidding strategies (i.e. $\beta_i^t(X_i;\Omega_t)$ is increasing in X_i for $t=1,2,...,T$) as described in Hong Shum (2003); Haile Tamer (2003). Notice that the bidders have been sorted by their posting price, e.g., $i=1$ indicates the winner of the item. For any period t, the lower bounds of the bidders' private signals can be constructed by system equations of N conditional expectations:

$$\begin{aligned}
\mathbb{E}[V_1|\underline{X}_1^t, \underline{X}_2^t ..., \underline{X}_N^t; \Omega_t] &= P_1^{t*}, \\
\mathbb{E}[V_2|\underline{X}_1^t, \underline{X}_2^t ..., \underline{X}_N^t; \Omega_t] &= P_2^{t*}, \\
&\vdots \\
\mathbb{E}[V_N|\underline{X}_1^t, \underline{X}_2^t ..., \underline{X}_N^t; \Omega_t] &= P_N^{t*},
\end{aligned} \tag{1}$$

where $\underline{X}_1^t, \underline{X}_2^t ..., \underline{X}_N^t$ are the unknown variables and P_i^{t*}, $i = 1, ..., N$ represents the highest bidding price for bidder i up to period t. If bidder i has not submitted the bid, his highest posting price is set to reserve price $P_i^{t*} = \gamma$. The system of equations in (1) is justified and we can get the unique solutions for $\underline{X}_1^t, \underline{X}_2^t ..., \underline{X}_N^t$, which is summarized in the following lemma.

Lemma 1. *The solution of the N unknown variables in equations. (1) are unique and strictly increasing in $P^{t*} = \{P_1^{t*}, ..., P_N^{t*}\}$, for all possible realizations of the history dependent lower bounds of $\underline{X}_1^t, \underline{X}_2^t ..., \underline{X}_N^t$.*

Lemma 1 relates the existence of a monotonic equilibrium to the nonlinear system of equations (1). With the help of Lemma 1, we can back out the history dependent lower bounds, $\{\underline{X}_1^t, \underline{X}_2^t ..., \underline{X}_N^t\}$, of private signals for each bidder. Regarding the upper bounds, we introduce a high level assumption:

Assumption 3. *when making decisions, bidder expects all her rivals will dropout and she will win in the next period.*

Whenever submitting a new bid, bidders will reveal part of their private signals to the public. A rational bidder will choose to keep the information revelation frequency as low as possible. Thus, bidder chooses to bid only when she must bid, for instance, a situation that she expects all her rivals will dropout. Otherwise, the winner would be the current posting bidder. Then we apply this assumption to construct the system of equations to recover the upper bound for the rivials' private signals of bidder i at period t:

$$\begin{aligned}
\mathbb{E}[V_1|\overline{X}_1^t,\overline{X}_2^t...,\overline{X}_N^t;\Omega_t] &= P_i^t+\Delta,\\
\mathbb{E}[V_i|\overline{X}_1^t,\overline{X}_2^t...,\overline{X}_N^t;\Omega_t] &= P_i^t+\Delta,\\
&\vdots\\
\mathbb{E}[V_N|\overline{X}_1^t,\overline{X}_2^t...,\overline{X}_N^t;\Omega_t] &= P_i^t+\Delta,
\end{aligned} \tag{2}$$

where $\overline{X}_1^t,\overline{X}_2^t...,\overline{X}_N^t$ excluding $\overline{X}_i^t$ are the unknown upper bounds for bidder i's rivals, and $P_i^t+\Delta$ represents the bidder i's next period bidding price. Similarly, we get the lemma 2.

Lemma 2. *The solution of the N unknown variables in equations. (2) are unique and strictly increasing in $P_i^t+\Delta$, for all possible realizations of the upper bounds of $\overline{X}_1^t,\overline{X}_2^t...,\overline{X}_N^t$.*

In the symmetric case, it is not hard to see the history dependent upper bound are equal across bidders, i.e., $\overline{X}_i^t=\overline{X}_j^t,\ i,j\in\{1,2,...,N\}$. The following proposition states that the above three assumptions plus two lemmas are sufficient to ensure the existence of equilibrium bidding strategies.

Proposition 1. *For auction τ with N_τ number of bidders, given assumptions 1,2,3 and lemma 1,2, there exists pivotal bidding functions in period t,*

$$\beta_i^t(X_i,\Omega_t)=\mathbb{E}[V_i|X_i,\underline{X}_j^t\le X_j\le\overline{X}_j^t,\ j=1,...,N,j\neq i;\Omega_t], \tag{3}$$

for the bidders $i=1,...N$, where lower bound, $\underline{X}_j^t$, upper bound, $\overline{X}_j^t$, and public information, Ω_t, are defined above. Moreover, there exists a Bayesian Nash Equilibrium in strictly increasing bidding strategies when

$$\beta_i^t(X_i,\Omega_t)\ge P_{t-1}+\Delta,\ i=1,2,...,N,$$

where posting price, P_{t-1}, and minimal bidding increment, Δ, are defined above.

Proof. In the appendix □

Unlike an unique bidding strategy in button auction model, **Proposition 1** defines an equilibrium bidding rules that allow bidders to have multiple bidding strategies, but all the bidding strategies should satisfy the pivotal bidding functions. This provides a convenient way to deal with the multiple equilibrium bidding strategies in English auction model.

2.1.1 An illustrative example

In auction, a structural econometric model would utilize the mapping between unobserved private signals and observed bidding history. But characterizing the conditional joint distribution of $F_{XV}(X_\tau, V_\tau | N_\tau, M_\tau)$ in English auction are usually involved in heavy computational burden, which comes from the recursive computation of the learning process. For example, let us assume four bidders (A, B, C, D) attend the auction and assume by $t = 4$, bidding prices are $\left\{P_A^1, P_C^2, P_D^3, P_A^4\right\}$. At the beginning of $t = 5$, all bidders have to calculate the lower bounds as well as upper bounds of the signals of their rivals from the price history $\{P_A^4, P_B^0, P_C^2, P_D^3\}$, where bidder B's bidding price (P_B^0) equals to the reserve price. We get $\left\{\underline{X}_A, \underline{X}_B, \underline{X}_C, \underline{X}_D\right\}$ and $\left\{\overline{X}_A, \overline{X}_B, \overline{X}_C, \overline{X}_D\right\}$ by solving the system of equations. Then, bidder i will use the lower and upper bounds to calculate pivotal bidding function (3). By comparing the updated conditional expected value with the current bidding price, bidder i then decides whether to bid or not in this period. If the auctioneer selects another bidder (e.g., B) to be the posting bidder, we would use new bidding path $\left\{P_A^4, P_B^5, P_C^2, P_D^3\right\}$ and calculate the bounds again in the next period. Such complicated computation occurs in every period. Therefore, I first take the parametric approach that helps to reduce the computational burden and understand which channels affect the bidding behavior. Particularly, The model assumes that bidders' valuations are log-normally distributed to keep closed-form expressions which are easy to invert.

2.2 parametric model setup

The value of the object to bidder i ,V_i, is assumed to take a multiplicative form $V_i = A_i \times V$, where A_i is a private value for bidders and V is a common value component unknown to all bidders. I assume that V and A_i are independently and follow *log normal distribution*. Let $v \equiv \ln V$, and $a_i \equiv \ln A_i$:

$$
\begin{aligned}
v &= \mu_v + \epsilon_v \sim N(\mu_v, \sigma_v^2), \\
a_i &= \mu_a + \epsilon_a \sim N(\mu_a, \sigma_a^2), \\
v_i &= a_i + v. \qquad (4)
\end{aligned}
$$

Each bidder is assumed to have a single noisy signal of the value of the object, $X_i = V_i \times E_i$, where $E_i = V \cdot \exp\{\sigma_\epsilon \xi_i\}$ and ξ_i is an (unobserved) error term that has a normal distribution with mean 0 and variance 1.Given such information structure, if we let $v_i \equiv \ln V_i$ and $x_i \equiv \ln X_i$, conditioning on M, the joint distribution of $(V_i, X_i, i = 1..., N) = \exp(v_i, x_i, i = 1, .., N)$ is fully characterized by $\{\mu_v, \mu_a, \sigma_v, \sigma_a, \sigma_\epsilon\}$. These key parameters are denoted by θ in the model.

The above discussion of information structure assumes away the informational asymmetry. If there is an informed bidder, $i = I$, her signal is assumed to be more precise compared to others:

$$
\xi_i = 0, \; if \;\; i = I, \; and \; \xi_i > 0 \; if \;\; i \neq I. \qquad (5)
$$

(5) implies that only the informed bidder I is able to observe her own valuation directly, i.e. $x_I = v_I$. In the rest of the paper, the model assumes that there exists at most one informed bidder in the auction. Still, I is unable to observe the common value part, as a_i and v are not individually distinguishable in equation (4). Because of log-normal assumption, the system of equations in (1),(2) and (3) is log-linear in the signals, allowing us to derive the equilibrium bidding functions for each period in closed form. However, the signals recovered from system of equations in period t only represents bidders' beliefs in period t. This is why we call it history dependent private signals. Let us begin with the system of equations (1) that defines the equilibrium inverse bidding strategies (lower bounds) for N

bidders in period t:

$$P_1^{t*} = \mathbb{E}[V_1|\underline{X}_1^t = (\beta_1^t)^{-1}(P_1^{t*}),\ \underline{X}_2^t = (\beta_2^t)^{-1}(P_2^{t*}), ..., \underline{X}_N^t = (\beta_N^t)^{-1}(P_N^{t*}); \theta],$$
$$P_2^{t*} = \mathbb{E}[V_1|\underline{X}_1^t = (\beta_1^t)^{-1}(P_1^{t*}),\ \underline{X}_2^t = (\beta_2^t)^{-1}(P_2^{t*}), ..., \underline{X}_N^t = (\beta_N^t)^{-1}(P_N^{t*}); \theta],$$
$$\vdots\vdots$$
$$P_N^{t*} = \mathbb{E}[V_1|\underline{X}_1^t = (\beta_1^t)^{-1}(P_1^{t*}),\ \underline{X}_2^t = (\beta_2^t)^{-1}(P_2^{t*}), ..., \underline{X}_N^t = (\beta_N^t)^{-1}(P_N^{t*}); \theta],$$

where $(\beta_i^t)^{-1}(\underline{X})$ represents the inverse mapping from the lower-bound signal to the bidding price but saves the Ω_t for notational convenience. Given the log-normal assumption, the conditional expectation of V_i take the form:

$$\mathbb{E}[V_i|X_1, ..., X_N; \theta, \Omega_t] = \exp\left(\mathbb{E}(v_i|x_1, .., x_N; \theta, \Omega_t) + \frac{1}{2}Var(v_i|x_1, ..., x_N; \theta, \Omega_t)\right), i = 1, ..., N. \tag{6}$$

Furthermore, we denote the mean and variance–covariance matrix of $(v_i, x_1, ...x_N)$ by $\mu_i \equiv (\mu_i, \mu^*)$ and $\Sigma_i \equiv \begin{bmatrix} \sigma_i^2 & \sigma_i^{*'} \\ \sigma_i^* & \Sigma^* \end{bmatrix}$. Then, the conditional mean and variance of jointly normal random variables for the history dependent lower bound private signals are:

$$\mathbb{E}[v_i|\underline{x} \equiv (\underline{x}_1^t, ..., \underline{x}_N^t)'; \theta, \Omega_t] = (\mu_i - \mu^{*\prime}\Sigma^{*-1}\sigma_i^*) + (\underline{x}^t)'\Sigma^{*-1}\sigma_i^*, \tag{7}$$

and

$$Var(v_i|\underline{x}^t; \theta, \Omega_t) = \sigma_i^2 - \sigma_i^{*\prime}\Sigma^{*-1}\sigma_i^*. \tag{8}$$

By plugging (7) and (8) into (6), we see that the conditional expectation function in (7) are log-linear in $\underline{x}$. After obtaining the lower bounds at period t, we do the same operation for the upper bound:

$$\mathbb{E}[V_1|\overline{X}_1^t = (\beta_1^t)^{-1}(P_i^t + \Delta), \overline{X}_2^t = (\beta_2^t)^{-1}(P_i^t + \Delta), ..., \overline{X}_N^t = (\beta_N^t)^{-1}(P_i^t + \Delta); \theta, \Omega_t] = P_i^t + \Delta,$$
$$\mathbb{E}[V_2|\overline{X}_1^t = (\beta_1^t)^{-1}(P_i^t + \Delta), \overline{X}_2^t = (\beta_2^t)^{-1}(P_i^t + \Delta), ..., \overline{X}_N^t = (\beta_N^t)^{-1}(P_i^t + \Delta); \theta, \Omega_t] = P_i^t + \Delta,$$
$$\vdots\vdots$$
$$\mathbb{E}[V_N|\overline{X}_1^t = (\beta_1^t)^{-1}(P_i^t + \Delta), \overline{X}_2^t = (\beta_2^t)^{-1}(P_i^t + \Delta), ..., \overline{X}_N^t = (\beta_N^t)^{-1}(P_i^t + \Delta); \theta, \Omega_t] = P_i^t + \Delta,$$

where $\bar{\beta}_i^t(\overline{X})$ represents the mapping from the upper-bound signal to the bidding price. And we get $\overline{x}^t$ as well. Now we can finally recover the pivotal bidding function (3) by plugging the log form lower bound, $\underline{x}^t$, and upper bound, $\overline{x}^t$, into the equations:

$$\mathbb{E}[V_i|X_1,\bar{X}_j^t \geq X_j \geq \underline{X}_j^t, j \neq i, j = 1,2,...N;\theta] = \exp\begin{pmatrix} \mathbb{E}(v_i|x_i,\overline{x}_j \geq x_j \geq \underline{x}_j^t, j \neq i, j = 1,2,...N;\theta)+ \\ \frac{1}{2}Var(v_i|x_i,\overline{x}_j^t \geq x_j \geq \underline{x}_j^t, j \neq i, j = 1,2,...N;\theta) \end{pmatrix}, \ i =$$

So far, we get the pivotal bidding functions under symmetric case. When an identified informed bidder attend the auction, the learning process has a little different from the previous discussion. Since informed bidder knows exactly her evaluation of the objective, she does not need to consider others' private signals. The bidding function for the informed bidder is:

$$\exp(\mathbb{E}[v_I|x_I;\Theta]) \geq P_I^t.$$

Uninformed bidders learn from each other through the correlation across the private signals: $\sigma_i^{*\prime}\Sigma^{*-1}$. Signal recovered from the informed bidder has higher weight and is more valuable to the uninformed bidders. Hence, the pivotal function of the uninformed bidders becomes:

$$\begin{aligned} \mathbb{E}[V_i|X_i,\underline{X}_j^t \leq X_j \leq \overline{X}_j^t,\ j = 1,...,N,\ j \neq i, I; & \\ \underline{X}_I^t \leq X_I \leq \overline{X}_I^t;\Omega_t\ ;\theta] & \geq P_i^t, \end{aligned}$$

where $\underline{X}_i^t$ and $\overline{X}_i^t,\ i \in 1,...,N,I$ represent the history dependent vectors of lower and upper-bound signals respectively.

2.3 Information Premium Discussion

Based on the previous construction, the model has several new features. First of all, under the same conditional expected value, the higher the lower bounds of rivals' private signals are, the lower the bidder i's private signal will be. Because the increased lower bounds of the rivals' signals lead to a higher learning effect, bidder i still gets higher evaluation of the item even if her own private signal is not high.

Conclusion 1. In pivotal functions ($\mathbb{E}[V_i|X_i, \underline{X}_j \leq X_j \leq \overline{X},\ j=1,...,N, j \neq i; \Omega_t, N; \theta]$), X_i is non-increasing with respect to $\underline{X}_j$.

Besides above mentioned learning effects, the information asymmetry will introduce another channel of learning process. If an informed bidder enters the auction, other uninformed bidders know the informed bidder has more precise private signal to the value of the item. They will put more weight to the bid submitted by the informed bidders. Such difference between an informed bidder and uninformed bidders reflects the information premium. There are two ways to formulate the measurement of information premium. First, the net expected evaluation of uninformed bidders with respect to the existence of the informed bidder is defined as ϖ_1 :

$$
\begin{aligned}
\varpi_1 = & \mathbb{E}_t[V_i|X_i; \overline{X}_I \geq X_I \geq \underline{X}_i;\ \overline{X}_j \geq X_j \geq \underline{X}_j,\ j \neq i, I, j=1,2,...N; \Omega_t; \theta] \\
& - \mathbb{E}_t[V_i|X_i; \overline{X}_j \geq X_j \geq \underline{X}_j,\ j \neq i, j=1,2,...N; \Omega_t; \theta]\ . \qquad (9)
\end{aligned}
$$

The first term in (9) indicates the conditional expectation of the uninformed bidder i with an informed Ias one of his/her rivals. While the second term indicates the same situation without the informed bidder I. This premium will also depend on the bidding history and the number of bidders. In particular, if more bidders attend the auction, the influence of the informed bidder will be diluted. As more rivals attend the auction, bidder i will have more information sources to update the information. This metric tells us how sensitive an uninformed bidder reacts to the existence of informed bidder.

From the simulated results in figure 1, we see that as the lower bound of the informed opponent increases, bidder i's updated expected value is rising quickly. but when the updating process only comes from the uninformed bidder, the marginal increment of information value is decreasing. As the price raises higher, bidder i may be afraid that the higher bidding price of the uninformed bidders may come from a noisy or private signal rather than a higher common value. In addition, figure 2 shows the number of bidders on will attenuate informed bidder's influence. As we can see, when more and more bidders attend the auction, the impact of the informed bidder on the updating process is decreasing.

Another measurement of the information premium can be defined as the net difference between the winning bid of informed bidder and uninformed bidder, which is defined by:

$$\varpi_2 = b(I = 1; N_\tau, M_\tau) - b(i = 1; N_\tau, M_\tau).$$

where $b(i; N_\tau, M_\tau)$ indicates the bidder i's highest posting price[3] under N_τ participants and M_τ observable characteristics in auction τ. The metric ϖ_2 directly measures the information premium paid from the informed bidder when she wins the item. As the uninformed bidders will put more weights on informed bidder's private information, the uninformed bidder will correspondingly reduce the weight from other uninformed rivals as well as his own private signals. Dionne . (2009) claims that ϖ_2 is independent of the number of participants N_τ. But, instead of rigors proof, they employ the simulation test to support the argument. In this paper, I find that that the positive effect from the informed bidder exactly offsets the negative effect of the decreased weighting from other uninformed rivals. Thus, as the number of bidders increases, it does not affect the information premium.

Proposition 2. *The information premium defined by*

$$\varpi_2 = b(I = 1; N_\tau, M_\tau) - b(i = 1; N_\tau, M_\tau).$$

is independent of the number of participants N_τ.

Proof. In the appendix C. □

Regarding the common value auction, the most commonly used metrics of the information premium are the winner's curse and loser's curse. In the literature, the winner's curse refers

[3] i represents the rank order of the bidders in auction τ.

to the that fact that bidder$i$ realizes that others have lower private signals:

$$\mathbb{E}[V_i|X_i = X_i^*, X_j < X_i^*, j \neq i; \theta] - \mathbb{E}[V_i|X_i = X_i^*; \theta].$$

the net difference is negative, meaning that winning brings bad news (Krishna, 2009). Moreover, the larger the x_i or N_τ , the worse the curse will be. On the other side, the common value auction may also induces the loser's curse : early dropout may also be bad news (Pesendorfer Swinkels, 1997). Similar to the winner's curse, the information premium for the loser's curse can be expressed as:

$$\mathbb{E}[V_i|X_i = X_i^*; \theta] - \mathbb{E}[V_i|X_i = X_i^*, X_j > X_i^*,\ j \neq i; \theta].$$

As bidder i's private signal X_i^* increases, the marginal premium is increasing as well. The winner's curse and loser's curse are like mirror image in which the private signal, x_i, determines the relative strength of the two offsetting effects. Under the symmetric value distribution settings, winner's curse is equivalent to loser's curse when $X_i^* = \mu_x$. When $X_i^* > \mu_x$, winner's curse effect dominates, vice versa.

If we consider the learning process, history dependent information premium can be defined as:

$$\begin{cases} \mathbb{E}[V_i|X_i = X_i^*; X_i^* \geq X_j \geq \underline{X}_j^t,\ j \neq i; \theta] - \mathbb{E}[V_i|X_i; \overline{X}_j^t \geq X_j \geq \underline{X}_j^t,\ j \neq i; \theta], & X_i^* > \underline{X}_j^t \\ \mathbb{E}[V_i|X_i = X_i^*; \overline{X}_j^t \geq X_j \geq \underline{X}_j^t,\ j \neq i; \theta] - \mathbb{E}[V_i|X_i; \overline{X}_j^t \geq X_j \geq X_i^*,\ j \neq i; \theta]. & X_i^* < \underline{X}_j^t \end{cases}$$

As the lower bound $\underline{X}_j$ increases, the curse will be alleviated, which is explained in the appendix. If the lower bounds of other bidders are high, the item tends to have higher common value, alleviating the curse of winning the item. Information revelation usually reduces the information premium of the high valued bidder. As the price gradually rises up, a (uninformed) bidder will realize that someone may have a higher signal and start following the high valued bidder. This will induce more aggressive competition that reduces the expected revenue of winning the auction.

3 Data Statistics and Empirical Evidence

3.1 Chinese Justice Online Auction

The primary objective of the Chinese Justice Auction is to process confiscated properties transparently and efficiently. In general, the courts need to deal with 600 billion RMB or more worth of assets and property rights every year. The item for selling includes furniture, car, house, land, and etc. The auction is organized as a typical online English auction. Before the auction begins, the auctioneer (local court) will publish the announcement Online 15 days earlier. If it is a housing auction, the auctioneer will publish the announcement one month earlier. The announcement includes important information such as evaluation value, reservation price, the minimum bidding increment, location, the existence of the priority bidder, usage information, photos, documentation, plus the third-party evaluation report. Anyone who wants to attend can register the E-form application with the required margin in the pre-auction period[4]. The margin will be refunded if the bidder fails to win the auction. If the item fails to be sold out at first time, the item has extra chance to organize a new auction again. So each item will have two chances to hold a normal auction. In the first-time auction, the reservation price should be no less than 80% (70% since 2017) of the evaluation rice of the item. In the second-time auction, the reserve price can be no less than 80% of the first-time auction's reserve price. If the item can not be sold successfully after the second-time auction, the item will turn to other disposal methods.

During the auction, anyone who submits the bid successfully at the current bidding period is called posting bidder. And her bid is called posting price, the temporary highest bidding price. Whoever wants to win the item has to submit a bid no less than the posting price plus the integral multiple of minimum bidding increment. The number of bidders is publicly known at the beginning of the auction. However, the bidder's identity is anonymous. In the data, there is one typical type of bidder called "priority bidder", who, by definition, has the correlation with the selling property. For example, the priority bidder could be the tenant

[4]The required margin should be in the range of 5% to 20% of the item value. Usually, the local court sets the required margin as 10% of the item value.

of the house for selling, or the banks that lend the mortgage. Many auctions will identify the bid submitted by the priority bidders. Some priority bidders will bid very actively, and finally win the auction. But some priority bidders bid very inactively. The heterogeneous priority bidder's bidding behavior is one of the distinctive features in the data, which will be discussed in detail later. After the auction, the website will publish the winning bidder's information if the auction is sold successfully. In this research, I focus on first-time housing auction and the data time ranges from 2015Q1 to 2019Q3. For the time being, I select auction data from 7 cities in eastern aera of China with 10000 successful first-time real estate auctions to run the test.

3.2 Data Statistics

The online platform provides detailed information for each auction: the description of the item, the date for the auction, the winner of the auction, the reservation price, the evaluation price, the location of the item if the item was a real estate, the existence of a priority agent, the local court who organized the auction, the number of bidders in the auction, the length of bidding period. More importantly, we can observe the bidding history per auction.

The data sample excludes one bidder auction in which the bidder wins the auction at the reservation price. From the summary statistics in table 1, we see that only 239 auctions have the priority bidders, which is a very small portion of the total data sample. The reserve ratio, which is defined by the reservation price over the evaluation price, ranges from 0.7 to 1, satisfying the reservation price regulation on the first-time auction. Regarding the number of participants, nearly 80% of the successful sold first-time auctions have no more than 8 bidders. The normalized winning price, defined as $\frac{win\ price}{evaluation\ price}$, has relatively narrow range: 80% auctions has the value less than 1.10. But the highest winning price is around 2.5 times than the evaluation price. This implies a relatively large variance across the auctions. Another key variable is the bidding increments. Larger increment will increase the bidding cost, reduce the bidding activity and depress the information revelation process. Notice that although the bidding increment is large in terms of absolute value, compared to the total values of the item, the minimum bidding ladder is, in average, less than 0.4% of the reserve price (ladder ratio in the table). This implies that minimum bidding increment is not a problem in the analysis of this open ascending auction. From figure 3, we see that there exist a fat tail in the auction. And the winning bid is positively correlated with the number of bidders. Because the existence of priority bidder introduces information asymmetry problem, I will compare the bidding outcomes in detail between the two subsets grouped by the priority bidder in the next part.

3.3 Priority Bidders and Information Asymmetry

The priority bidder in the auction refers to the one who has the correlation with the selling property. In this paper, the priority bidder is assumed to be the informed bidder. Since the priority bidder could be, for example, either the tenant of the house or the bank that lends the mortgage, there exists large heterogeneity across different types of bidders. Being a priority bidder, the agent wins the item whenever there is a tie during the bidding stage. While others need to add minimum bidding requirement to bid against posting bidder. To become a priority bidder, a bidder needs to fulfill an E-application form claiming the priority bidder privilege. If the informed bidder wants to hide her identity, she can also choose not to apply. Because the being a priority bidder is not a mandatory requirement, for those auctions without priority bidder, we are not sure about whether there exists an informed bidder or not. Compared with the low benefits of being a priority bidder, if the priority bidder really wants the item, it is better to hide her identity. However, in the auction, we actually observe many priority bid actively and win the item. Since revealing the identity will induce more aggressive bidding competition, there must be alternative incentive that urge the informed bidder to reveal their identity.

Regarding the existence of priority bidder, the data displays many distinct features of the bidding outcomes in figure 5. First, auctions with priority bidder will have higher winning price while fewer participants. Conceptually, the identity of the informed bidder will have two effects on uninformed bidder's decision. On the one hand, it reveals that the item has a higher common value, encouraging uninformed bidders to participate. On the other hand, the existence of informed bidder indicates the item has higher value in ex ante, discouraging bidder's without deep pocket to participant. Second, auctions with the priority bidder have higher bidding frequency, where the bidding frequency is defined as the total number of bids submitted within an auction. The number indicates that bidders tend to behave more aggressively in the auctions with the priority bidder. To specify the effect of priority bidder on the winning price, number of bidders and bidding frequency, I conduct the following econometric analysis:

$$y_\tau = \delta 1_\tau\{priority\} + \phi Z_\tau + \epsilon_\tau,$$

where the y_τ indicates the dependent variables such as winning index, the number of bidders or the bidding frequency. Notice that I define the winning price index as the net increase from reserve price, $\frac{win\ price}{evaluation\ pricce}$. The indicator $1_\tau\{priority\}$ represents whether the current auction τ has the priority bidder or not. The Z_τ represents the control variables, such as the reserve price, the location fixed effect, the housing area, and etc. The error term ϵ_τ is independent of other covariates. The reduced form regression (controlling year and location effects) shows that the identity of priority has a significant positive effect on the winning price. The effect on the number of bidders is negative but insignificant. For the winning price, the existence of priority bidder will drive the winning bid up 4.4% relative to the reserve price. Meanwhile, one more bidder attending the auction will also drive the winning price up to 3% relative to the reserve price. In addition, the priority bidder's effect can be mitigated by encouraging more bidder's attending the auction. Both the number of bidders and the existence of priority bidder tell the same story. The existence of the priority bidder indicates the item must have higher value to attract the informed bidder. Similarly, more bidders attending the same auction implies people have a consistent belief about the value of the item.

The existence of the priority bidder not only attracts more bidders attending the auction but also increases the entry threshold for other bidders, resulting in less but high valued bidders attendance. All the above discussion is under the assumption that all priority bidder would like to win the item. However, we observe that not all priority bidders bid actively in the auction. Only 68 out of 239 auctions with priority bidders shows the active bidding outcomes, where priority bidders are the winner or the second highest bidder of the auction. The divergent bidding behavior across priority bidders may imply that priority bidder have *some strategic behaviors* under different situation. Later in the simulation test, we will see more clearly how the priority bidder affect the auction outcome.

4 Identification

4.1 Identification strategy

As pointed out in (Bikhchandani ., 2002), even we can observe all the bidders' dropout price in open ascending auction, there still exists multiple bidding equilibria issue. The equilibrium bidding strategy proposed by Milgrom Weber (1982) is just a special case. Moreover, the model assumes that each bidder's dropout price is unobservable and sellers randomly pick one of the bidder as the posting bidder each time. Hence, not only one bidding price may result from multiple bidding equilibria, but also one typical equilibrium bidding strategy can lead to multiple bidding path because of the random posting price selection mechanism by the seller.

The identification challenge for the set of distribution parameters, θ, comes from the above mentioned many to many correspondence between value distribution and price distribution. Although there is no moment equality for us to point identify the parameters, we can still get partial identification by the moment inequalities. Similar to Haile Tamer (2003), the bidder's highest posting price helps to construct the lower bound of the dropout price and the winning bid helps to provide the upper bounds for the losers. If we rank the bidder based on highest posting price, we have the following conditional moment inequalities:

$$\begin{aligned} \mathring{P}_i^{t_i} &\leq \beta(X_i, \Omega_t; \theta) \leq P_{win} + \Delta \quad i \geq 2, \\ \mathring{P}_{win} &\leq \beta(X_i, \Omega_T; \theta) \qquad\qquad i = 1, \end{aligned} \tag{10}$$

where $\mathring{P}_i^t$ indicates the highest bidding price of bidder i at period t, $\Omega_t$indicates information needed up to t, θ indicates the set of parameters that we care about, and Δ represents the minimum bidding increment. The conditional moment inequalities in (10) provide the classical moment conditions for the identification. In the following part, I will discuss how to utilize the bidding history to help identify the key parameters.

Because everyone values the common value part, the higher the common value is, the higher value of the item will be. On the seller side, to maximize the revenue, the seller has the incentive to set a higher reserve price for the item with higher common value. Clearly, higher reserve price is positively correlated with a higher common value realization. Hence, the mean value of the common value part is correlated with the reserve price, i.e., $\mu_x(\gamma, Z)$, where Z represents other control variables. If we further divide the bidding behavior into the within auction variation and across auction variation, we can capture the common value part from the differences of the two variations. This is because bidders within the same auction receive the identical common value shocks. The within auction variation only correlates with private value part and noisy part, whereas the across auction variation (i.e., winning price) also includes the common value part. Therefore, common value part can be identified by the interaction of two variations.

However, since the noisy part affect not only the within auction variation but also the across auction variation, both the within auction variation and across auction variation can not help us distinguish the private part and noisy part. Fortunately, we have the informed bidder (priority bidder) shown up in the auction. Because our model assumes that the informed bidder knows his value precisely. In theory, the winning bid variation of the informed bidder only contains the common value part and private value part. By comparing winning bid variation between informed and uninformed bidders, we are able to isolate the noisy part, helping us to get more precise estimation for the key parameters θ. The identified feature of the moment inequality model is the set of parameter values that obey these restrictions for X and represents the set of auction models that are consistent with the empirical evidence. Here the identified set is defined as:

Definition 1. Let Θ_I be such that

$$\Theta_I = \{\theta \in \Theta,\ s.t. \text{ inequalities 10 are satisfied at } \theta\ \forall X\ a.s.\}.$$

We say that Θ_I is the identified set. Regarding the model setup, the set for identification contains $\{\mu_\nu,\ \sigma^2_\nu,\ \sigma^2_\alpha,\ \sigma^2_\epsilon\}$, where I further restrict mean value of the private part,μ_a, and noisy part,μ_ϵ to zero.

5 Estimation

5.1 Estimation Setup

The estimation problem for θ is based on the moment inequality (10). The objective function can be expressed as:

$$Q(\theta) = \frac{1}{T}\sum_{\tau=1}^{T}\Big[\Big|\Big|\{\boldsymbol{p}_\tau^{lower} - \mathbb{E}_{\boldsymbol{x}}\boldsymbol{\beta}(\boldsymbol{x},\Omega_\tau;\theta)\}_+\Big|\Big| + \Big|\Big|\{\boldsymbol{p}_\tau^{upper} - \mathbb{E}_{\boldsymbol{x}}\boldsymbol{\beta}(\boldsymbol{x},\Omega_\tau;\theta)\}_-\Big|\Big|\Big],$$

where $\{A\}_+ = [a_1 1(a_1 \geq 0), .., a_N 1(a_N \geq 0)]$ and $\{A\}_- = [a_1 1(a_1 \leq 0), .., a_N 1(a_N \leq 0)]$, and where $||\cdot||$ represents the Euclidean norm, T represents the number of auctions, $\boldsymbol{p}_\tau^{lower}$ and $\boldsymbol{p}_\tau^{upper}$ represent the vector of lower and upper bounds (log form) respectively for each bidder in auction τ, and $\boldsymbol{\beta}(\boldsymbol{x},\Omega_\tau;\theta) = [\beta_1(x_1,\Omega_\tau;\theta),\ \beta_2(x_2,\Omega_\tau;\theta), ...\beta_N(x_N,\Omega_\tau;\theta)]$ represent the vectorized pivotal functions. It is straightforward to see that $Q(\theta) \geq 0$ for all $\theta \in \Theta$ and that $Q(\theta) = 0$ if and only if $\theta \in \Theta_I$, the identified set in Definition 1. Following Chernozhukov . (2007) (CHT hereafter), I construct the consistent estimation for the identified set, which contains parameters that cannot be rejected as the truth. The major difficulty in estimation is calculating the multivariate integral. To estimate Θ_I, I fist take the sample analog of $Q(\theta)$ and estimated set $\hat{\Theta}_I$:

$$\hat{\Theta}_I = \{\theta \in \Theta | nQ_n(\theta) \leq \nu_n\},$$

where $\nu_n \to \infty$ and $\frac{\nu_n}{n} \to 0$ and the sample analog of the first part objective function is

$$Q_n^1(\theta) = \frac{1}{T}\sum_{\tau=1}^{T}\sum_{i=1}^{N_\tau}\left\{\left|\left|\boldsymbol{p}_{i,\tau}^{lower} - m_i^\tau(\Omega_\tau,\theta)\right|\right|_+ + \left|\left|\boldsymbol{p}_{i,\tau}^{upper} - m_i^\tau(\Omega_\tau,\theta)\right|\right|_-\right\}.$$

$$m_i^\tau(\Omega_\tau,\theta) = \frac{\frac{1}{S}\sum_{s=1}^{S}\beta_i(x_i^s,\Omega;\theta)\mathbf{1}(\boldsymbol{x}\in\mathscr{T}_\tau(\theta))}{\frac{1}{S}\sum_{s=1}^{S}\mathbf{1}(\boldsymbol{x}\in\mathscr{T}_\tau(\theta))}$$

where $||a||_+$ represents Euclidean norm of $a\cdot\mathbf{1}(a\geq 0)$, $||a||_-$ represents Euclidean norm of $a\cdot\mathbf{1}(a\leq 0)$ and $P_{i,\tau}^{lower}$, $P_{i,\tau}^{upper}$ represent the lower and upper bounds respectively for bidder i in auction τ. Because we use the ordered bidding prices to construct the moment conditions, the observed posting price order restricts the support of (simulated) private signals $x_1,...,x_N$ to a region $\mathscr{T}_\tau(\theta)\in\mathbb{R}^N$ given, where $\mathscr{T}_\tau(\theta)$ represents the order information that simulated private signals should satisfy. Because the dropout price is unobserved, the restriction is expressed by lower bound of signal x:

$$\beta_{i+1}^{-1}(P_{i,\tau}^{lower},\Omega;\theta)\geq\beta_{i+1}^{-1}(P_{i+1,\tau}^{lower},\Omega;\theta),\ i=1,2,...N_\tau-1 \tag{11}$$

$$\mathscr{T}_\tau(\theta)=\{\underline{x}_1,..,\underline{x}_N : \text{(11) is satisfied}\}$$

The $m_i^\tau(\Omega_\tau,\theta)$ represents the simulated first moments of pivotal function conditional on the bidding history Ω_τ and parameter set θ. However, such simulation is ill-behaved due to the non-smoothness of the indicator functions. To bypass the problem, I borrow the independent probit kernel-smoother method in Hong Shum (2003) and reformulate the $m_i^\tau(\Omega_\tau,\theta)$ as:

$$\bar{m}_i^\tau(\Omega_\tau,\theta) = \frac{\left[\frac{1}{S}\sum_{s=1}^{S}\beta_i^\tau(\mathbf{x}_s,\Omega_\tau;\theta)\prod_{t_k}\prod_{i\leq j<k}\phi\left(\frac{\beta_j^{\tau,t_k}(\mathbf{x}_s,\Omega;\theta)-\beta_k^{\tau,t_k}(\mathbf{x}_s,\Omega;\theta)}{h}\right)\right]}{\left[\frac{1}{S}\sum_{s=1}^{S}\prod_{t_k}\prod_{i\leq j<k}\phi\left(\frac{\beta_j^{\tau,t_k}(\mathbf{x}_s,\Omega;\theta)-\beta_k^{\tau,t_k}(\mathbf{x}_s,\Omega;\theta)}{h}\right)\right]}, i=\{1,...,N\}$$

where t_k indicates the period that bidder k posts her highest bidding price, $\beta_k^{\tau,t_k}(\mathbf{x}_s,\Omega;\theta)$ indicates the pivotal function of bidder k in her highest bidding period, and $\beta_j^{\tau,t_k}(\mathbf{x}_s,\Omega;\theta)$, $i\leq j<k$ indicates the pivotal function of bidder j at period t_k in auction τ. Remember that the bidder is ranked by their highest bidding price, where $i=1$ indicates the winner of

the auction and $j < k$ implies that bidder j's highest posting price is higher than bidder i's. I denote the denominator of the $\bar{m}_i^\tau(\Omega_\tau, \theta)$ as $\tilde{P}_{\mathcal{T}_\tau}(\theta)$. Notice that i can be informed and uninformed bidder. And the sample analog can be transformed to be:

$$Q_n(\theta) = \frac{1}{T}\sum_{\tau=1}^{T}\sum_{i=1}^{N_\tau}\Big\{\big|\big|\tilde{P}_{\mathcal{T}_\tau}(\theta)P_{i,\tau}^{lower} - \tilde{P}_{\mathcal{T}_\tau}(\theta)\cdot m_i^\tau(\Omega_\tau, \theta)||_+ + ||\tilde{P}_{\mathcal{T}_\tau}(\theta)P_{i,\tau}^{upper} - m_i^\tau(\Omega_\tau, \theta)\tilde{P}_{\mathcal{T}_\tau}(\theta)||_-\Big\}$$

The model supported by the estimated parameters should be consistent with the empirical evidence. The inference construction refers to the CHT's Hausdorff-consistent estimator of the set Θ_I. The key is to find the confidence set (region) C_n such that $\lim_{n\to\infty} P(\theta_I \in C_n) \geq \alpha$ for a pre-specified $\alpha \in (0,1)$ for any $\theta_i \in \Theta_I$. This confidence region is based on the principle of collecting all of the parameters that cannot be rejected, which is constructed as follows:

$$C_n(c) = \{\theta \in \Theta : \ n(Q_n(\theta) - \min_t Q_n(t)) \leq c\}.$$

The detailed construction procedure follows CHT.

5.2 Empirical Results

The parameter set of value distribution waiting for estimation consists of $\theta_I = \{\mu_v,\ r\ , \sigma_v^2,\ \sigma_\alpha^2,\ \sigma_\epsilon^2\}$, where r represent the coefficient of the control variable, reserve price. The mean value of private signal is $E[X_i] = E[V_i] = \mu_v + r\log(\gamma)$. Because there are more auctions without the priority bidder in the data, I assume the the auctions without the priority bidder have the symmetric information structure first and conduct the estimation procedure. The estimation results are shown in table 5.

From the estimation results, we see that each bidder indeed has private value for the item, although σ_α^2 is very small compared to the common value part. The common value part actually determines the learning effect among the participants. But based on the estimation results, we see that the learning effect is largely blocked by the noisy part. The huge variance in noisy part indicates if someone owns information advantages, he/she will have much larger information advantage. Moreover, from the model, we know that If σ_ϵ^2 is large, uninformed bidders will put much higher weight on the bid submitted by the informed bidder. If the informed bidder bids actively in the auction, every other uninformed bidders would learn that the item should worth high and everyone will bid actively. However, if the informed bidder bids very inactively or never show up during the bidding stage, uninformed bidders would expect the item probably has much less value. Because the uninformed bidder puts much higher weight on the informed bidder's bid, even if his/her own private signal may be high enough, the uninformed bidder would dropout quickly. And the auction revenue could be seriously affected by the informed bidder's bidding behavior. In the next part we will employ counterfactual experiment to test for the impact of priority bidder's behavior on winning price.

5.3 Simulation Test

To evaluate the performance of the model, I apply Monte Carlo Simulation to do the Experiment. I generate the simulated auction data under the model assumptions. To generate the data, we have to specify those key value distribution parameters first. The mean and variance for the signal v_i, x_i follow the table setup. Notice that a_i and ϵ_i are independent with each other. Based on the parameters, we generate random private signals x_i for each bidder. For each auction, I also randomize the bidding ladder,Δ. At the very beginning, each bidder only knows their own private signal x_i. Once someone starts to bid, bidders can obtain more and more price information from the bidding process. They will combine their own private signals and others' signals recovered from the bidding activity to update their conditional expectation. After simulating S auctions, we can calculate the moment conditions and distributions that we are interested in. To simulate the data, I let the number of participants range from 2 to 10 and draw $S = 5000$ replications with randomized parameter set[5]. The preliminary results are shown in figure 6.

[5] Based on the fixed parameter value in table 3, I add random shock $\epsilon_\mu \sim U(-0.075, 0.075)$ to the mean of common value μ_v and $\epsilon_{\sigma 1}, \epsilon_{\sigma 2}, \epsilon_{\sigma 3} \sim U(-0.05, 0.05)$ to the σ_v^2, σ_α^2, σ_ϵ^2 in every replication. This ensures that my results are not dependent on a specific parameter set.

As predicted from the model, the presence of the informed bidder will induce more aggressive bidding, resulting in a higher winning bid and more frequent bidding submission. To see the learning effect, I calculate the bidding ladder difference between the highest posting position (winning bid) and the rest. Since the distance between the second highest and the highest is always 1, I drop out the second highest result and calculate the average bidding ladder difference in figure 6(c). In general, if the bidders can learn from each other, given other things equal, higher learning effect will induce lower ladder distance. However, when an informed bidder enters the auction, two more effects will be introduced. Because the presence of the informed bidder brings a higher valued signal to the item, the low valued bidders realize that it is not possible for them to win the item. They will stop bidding quickly. While for those high valued bidders, the presence of informed bidder will give them more confidence to bid aggressively, resulting in a higher winning bid. In the meanwhile, the bidding ladder distance between lower valued and higher valued bidders will enlarge as well. Therefore, we see a fat tail under the informed bidder in figure 6(c) and the left shifted density crest under the informed bidder in figure 6(d).

Regarding the strategic behavior of the informed bidder, I further control the level of bidding activity for the informed bidder under different situations (figure 7). The experiment procedure is described in the appendix. The counterfactual analysis illustrates three scenarios. First, in normal case, the auctioneer randomly picks one of the bid submitted by all the candidate bidders (including the informed bidder's bid). Second, in active case, the auctioneer deliberately chooses the bid submitted by the informed bidder every other period. Third, in inactive case, the auctioneer randomly picks one of the bid submitted by all but the informed bidder. From figure 6, we see that under the inactive case, the wining price distribution has significantly left shift pattern, which implies by strategically depress their bidding activities, the informed bidders would have an big impact on the bidding outcomes.

6 Conclusion

In common value English auction, recovering the value distribution is non-trivial because mapping from the value distribution to the bidding distribution is no longer one to one. Due to the multiple equilibria issue and incompleteness of econometric structure, we face the many to many correspondence issue. The new challenge requires a new method for the value distribution estimation. On the other hand, the learning process and winner's curse also makes it hard to trace the bidding behavior under information asymmetry. The redistribution effect of information premium still remains inconclusive. Moreover, policy analysis such as auction revenue and efficiency is quite sensitive to whether the information structure is symmetric or not. Thus, testing the asymmetric information is a pre-request for policy analysis.

This paper aims to address those issues. The Chinese Justice auction data offers an ideal laboratory to analyze the problems. the paper builds a structural auction model that utilizes the bidding history to construct the upper and lower bound to disentangle the estimation difficulty. To test the asymmetric information, we need the endogenous entry decisions that bridge the variations of number of participants with different information structure. from the model, we see that the existence of the informed bidder will screen out the low valued bidder and select high valued bidder enter into the auction. As a result, the information premium mainly comes from the screening effect not from the learning process. This further leads to the fact that information premium is independent of number of participants in the auction. From the estimation result, the private signal has quite large noisy part which reduces the auction revenue. Also the unproportionally large noisy private signal indicates the informed bidder could have much higher information advantages in the auction. Through the simulation test, informed bidder can significant affect the auction outcome by strategically choosing her bidding behavior. The framework developed in this paper can have broader application. Any auctioneer who holds an open ascending auction format wants to analyze the information structure and the corresponding equilibrium bidding outcome can make use of this model. Indeed, the model is just simple abstraction from the real world that aims to capture the key influence of information structure. A lot of details are not included, which is waited for further research.

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Appendix A. Proofs

Proof of Proposition 1. Similar to the proof of Proposition 1 in Hong and Shum (2003), we show that if all bidders $j \neq i$ follow their own equilibrium strategies $\beta_j^t(\cdot)$, bidder i's best response is to play $\beta_i^t(\cdot)$ because this guarantees that bidder i will win the auction if and only if her expected net payoff is positive conditional on winning.

Given the current bidding price of bidder i, P_i^t, if bidder i wins the auction in period t when all remaining bidders simultaneously exit at a price of P_i^t, her ex-post valuation is:

$$\mathbb{E}[V_i|X_i, \underline{X}_j \leq X_j \leq \overline{X},\ j = 1, ..., N, j \neq i; \Omega_t].$$

Since this conditional expectation is increasing in X_i (from Assumption 1), bidder i makes a positive expected profit from winning in period t by staying actively in the auction at a price of P if and only if $X_i \geq (\beta_i^t)^{-1}(P_i^t; \Omega_t) \Leftrightarrow \beta_i^t(X_i; \Omega_t) \geq P_i^t$ (Assumption 2). In other words, $\beta_i^t(X_i, \Omega_t)$ specifies the price below which bidder i makes a positive expected profit by staying in the auction and above which bidder i makes a negative expected profit by staying in the auction. Therefore, for every realization of X_i, $\beta_i^t(X_i; \Omega_t)$ specifies a best-response dropout price for bidder i in period t.

Proof of Proposition 3. Suppose that there exists a bidder k whose private signal is X_k. Given the entry threshold$\underline{X}_\tau^*$ at auction τ, bidder k has the following choices:

1. $X_k < \underline{X}_\tau^*$, bidder k decides to enter.
2. $X_k < \underline{X}_\tau^*$, bidder k decides not to enter.
3. $X_k \geq \underline{X}_\tau^*$, bidder k decides to enter.
4. $X_k \geq \underline{X}_\tau^*$, bidder k decides not to enter.

We have to show 1 and 4 are dominated by 2 and 3. Let us focus on 1 and 2 first. Under the scenario $X_k \geq \underline{X}^*$, if bidder k decides to enter, one of her rivals has higher conditional expectation towards the item. The incentive constraints is violated:

$$\sum_{n}^{\infty} P(N_\tau = n|M_\tau) \left\{ \begin{array}{c} \mathbb{E}[V_i|X_i,\ X_j, j=1,2,...,n, j\neq i] \\ -\mathbb{E}[V_j|X_i,\ X_j, j=1,2,...,n, j\neq i] \end{array} \right\} < 0$$

$$\rightarrow$$

$$\mathbb{E}[V_i|X_i,\ X_j, j=1,2,...,n, j\neq i] < \mathbb{E}[V_j|X_i,\ X_j, j=1,2,...,n, j\neq i].$$

The ex-ante payoff expectation for bidder k attending the auction is non-positive. While if he/she chooses not to enter, bidder k receive zero payoffs, which dominate the enter decision. Similarly, when $X_k \geq \underline{X}_\tau^*$, the payoff for bidder who decides to enter is non-negative. However, if the bidder k decides not to enter, she can only receive negative payoffs. Therefore, The best response of the entry decision for any potential bidder is to enter auction τ whenever $X_k \geq \underline{X}_\tau^*$.

Explanation of Curse Relief. The conjuncture is equivalent to show as $\underline{X}$ increases,

$$\frac{d}{d\underline{X}} \left\{ \mathbb{E}[V_i|X_i; x_i \geq X_j \geq \underline{X}_j,\ j\neq i] - \mathbb{E}[V_i|X_i; X_j \geq \underline{X}_j,\ j\neq i] \right\} < 0.$$

Due to the property of the conditional expectation and truncated normal distribution, we learn that

$$\frac{d}{d\underline{X}} \left\{ \mathbb{E}[V_i|X_i; x_i \geq X_j \geq \underline{X}_j,\ j\neq i] - \mathbb{E}[V_i|X_i; X_j \geq \underline{X}_j,\ j\neq i] \right\} \propto \frac{d}{d\underline{X}} R \left\{ \frac{\phi(\underline{X}_j) - \phi(x_i)}{\Phi(x_i) - \Phi(\underline{X}_j)} - \frac{\phi(\underline{X}_j)}{1-\Phi(\underline{X}_j)},\ j\neq i \right\}$$

Without loss of generality, I assume that $\alpha = \underline{X}_j$ and $\beta = x_i$. Then we can get

$$\frac{\phi(\alpha)-\phi(\beta)}{\Phi(\beta)-\Phi(\alpha)} - \frac{\phi(\alpha)}{1-\Phi(\alpha)} = \frac{\phi(\alpha)[1-\Phi(\beta)] - \phi(\beta)[1-\Phi(\alpha)]}{(1-\Phi(\alpha))(\Phi(\beta)-\Phi(\alpha))} \equiv A(\alpha)$$

It is not hard to see that

$$\frac{d}{d\alpha}A(\alpha) = \frac{\phi(\alpha)[-\alpha(1-\Phi(\beta)) + \phi(\beta) + (1-\Phi(\alpha))]}{(1-\Phi(\alpha))^2(\Phi(\beta)-\Phi(\alpha))^2},$$

where $\phi'(\alpha) = -\alpha\phi(\alpha)$. As $\alpha \uparrow$, $-\alpha(1-\Phi(\beta)) + \phi(\beta) + (1-\Phi(\alpha)) < 0$, which indicates the alleviation of the curse.

Algorithm for Priority bidder's strategic behavior counterfactual experiment.

- Draw private signal X based on estimated parameter $\hat{\theta}$
- Generate the identity of informed bidder
- Increase high bid by Δ each period
- Identify the set of bidders willing to bid
- Choose one candidate bid as posting price at each period t
  - normal: no restriction
  - active: choose informed first
  - inactive: never choose informed

Appendix B. Graphs and Tables

Table 1: Summary Statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
reserve ratio	10,035	0.777	0.105	1	0.7	0.8	1
winning premium	10,035	1.006	0.206	0.700	0.859	1.107	1.945
number of bidder	10,035	5.437	3.485	1	3	8	15
priority bidder	239	0.024	-	-	-	-	-
bidding spread	8,644	111.218	416.633	0.000	0.688	82	17,236
reserve price(1000)	10,035	3,048	4,350	69	790	3,436	45,210
ladder ratio	10,035	0.005	0.004	0.00000	0.002	0.006	0.073

Table 2: Reduced Econometric Analysis

	Dependent variable:			
	win_eval	num_bidder	len_history	bid_spread
	(1)	(2)	(3)	(4)
priority bidder	0.041***	−0.159	4.363	4.446
	(0.010)	(0.221)	(3.492)	(29.459)
number of bidders	0.031***		5.562***	18.913***
	(0.0005)		(0.176)	(1.453)
reserve price	1.143***	−4.484***	11.396	45.990
	(0.021)	(0.491)	(7.798)	(65.487)
price / m^2	−0.00000***	−0.00000*	0.00005***	0.00004
	(0.00000)	(0.00000)	(0.00001)	(0.0001)
Constant	−0.094***	7.173***	8.386	−45.041
	(0.022)	(0.510)	(8.157)	(68.257)
Observations	9,041	9,041	9,041	8,643
R^2	0.547	0.094	0.246	0.052
Adjusted R^2	0.546	0.092	0.245	0.050
Residual Std. Error	0.138	3.171	50.164	406.076
	(df=9024)	(df=9026)	(df=9024)	(df=8626)
F Statistic	681.485***	66.780***	184.322***	29.510***
	(df = 16; 9024)	(df = 14; 9026)	(df = 16; 9024)	(df = 16; 8626)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3: Estimation Results

	μ_v	γ	σ_v^2	σ_a^2	σ_ϵ^2
Estimated	0.0080	1.2929	0.1609	0.00135	0.1226
lower	0.00524	1.163	0.1279	0.00129	0.1063
upper	0.0109	1.400	0.1906	0.00144	0.1336

Table 4: Information Parmeter for Simulation

name	moment	parameter	value
common value v	mean	μ_v	-0.35
	variance	σ_v	0.2
private value a	mean	$\bar{a}$	0
	variance	σ_a^2	0.15
noise	mean ϵ		0
	variance	σ_ϵ^2	0.1
control coefficient r			0

Appendix C Proof for the proposition.2

I will show that the sum of the coefficients in front of different bidders' private signals will be identical with and without the existence of informed bidder. The proof evolves complicated algebra matrix operation. The information premium is defined as the difference of winning bid under informed and uninformed case:

$$\varpi_2 = b(i = 2|I = 1; N, M) - b(UI = 2|I = \emptyset; N, M),$$

where the first term represents the winning bid (second highest bid) from the informed bidder and the second term represents the uninformed bidder's winning bid. From the property of conditional expectation of multivariate normal distribution, the difference only relates to the coefficient matrix of the private signals $\mathbf{X} = (X_1, X_2, ..., X_N)$:

$$b(\cdot; N, M) = \mathbb{E}[v_i|x_i, x_{-i}] = (\mu_i - \mu^{*\prime}\Sigma^{*-1}\sigma_i^*) + \boldsymbol{x}' \underbrace{\Sigma^{*-1}\sigma_i^*}_{\text{coefficient matrix}},$$

where $\mu_i \equiv (\mu_i, \mu^*)$ and $\Sigma_i \equiv \begin{bmatrix} \sigma_i^2 & \sigma_i^{*'} \\ \sigma_i^* & \Sigma^* \end{bmatrix}$. The problem now transforms into comparing $\mathbb{E}[\boldsymbol{x}'\Sigma^{*-1}\sigma_i^*|informed]$ and $\mathbb{E}[\boldsymbol{x}'\Sigma^{*-1}\sigma_i^*|uninformed]$. In the rest of analysis, we will calculate the coefficient matrix of all bidders' private signals $\mathbf{X}$. The calculation can be decomposed into three steps: first, compute the inverse matrix and decompose the $\Sigma^{*-1}\sigma_i^*$ into learning effect and private evaluation: bidder i's coefficient, informed rival's coefficient and uninformed rival's coefficients; second, recover the winner's private signal x_1 from the system of equations; third compare the difference between with and without the informed bidder.

Step 1: Compute the inverse matrix and decompose the $\Sigma^{*-1}\sigma_i^*$ The Σ^* is symmetric and positive definite, we can do the eigenvalue decomposition, where

$$\Sigma_1^* = \begin{bmatrix} \sigma_v^2+\sigma_a^2 & \sigma_v^2 & \cdots & \sigma_v^2 \\ \sigma_v^2 & \sigma_v^2+\sigma_a^2+\sigma_\epsilon^2 & \cdots & \sigma_v^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_v^2 & \sigma_v^2 & \cdots & \sigma_v^2+\sigma_a^2+\sigma_\epsilon^2 \end{bmatrix}, \Sigma_0^* = \begin{bmatrix} \sigma_v^2+\sigma_a^2+\sigma_\epsilon^2 & \sigma_v^2 & \cdots & \sigma_v^2 \\ \sigma_v^2 & \sigma_v^2+\sigma_a^2+\sigma_\epsilon^2 & \cdots & \sigma_v^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_v^2 & \sigma_v^2 & \cdots & \sigma_v^2+\sigma_a^2+ \end{bmatrix}$$

Based on eigenvalue decomposition, we can first derive the eigenvalue:

$$\begin{aligned} |\Sigma_0^* - \lambda I| &= \begin{vmatrix} n\sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda & \sigma_v^2 & \cdots & \sigma_v^2 \\ n\sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda & \sigma_v^2+\sigma_a^2+\sigma_\epsilon^2 & \cdots & \sigma_v^2 \\ \vdots & \vdots & \ddots & \vdots \\ n\sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda & \sigma_v^2 & \cdots & \sigma_v^2+\sigma_a^2+\sigma_\epsilon^2 \end{vmatrix}, \\ &= \begin{vmatrix} 1 & 0 & \cdots & 0 \\ 1 & \sigma_a^2+\sigma_\epsilon^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & \sigma_a^2+\sigma_\epsilon^2 \end{vmatrix} (n\sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda), \\ &= (n\sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda)[\sigma_a^2+\sigma_\epsilon^2-\lambda]^{n-1} = 0. \end{aligned}$$

The corresponding eigenvector is also easy to calculate. For $\lambda = \sigma_a^2 + \sigma_\epsilon^2$, we have $\Sigma_0^* - \lambda I = \mathbf{1}_{n\times n}$. For $\lambda = n\sigma_v^2 + \sigma_a^2 + \sigma_\epsilon^2$, there is no restriction for the eigenvector. Without loss of generality, we can write the eigenvector as:

$$V = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix}.$$

To normalize the eigenvector as $V \cdot V^T = 1 = V \cdot V^{-1}$, we should define $\bar{V} = \frac{1}{\sqrt{n}} V$ and we can write down the decomposition as

$$\Sigma_0^* = \bar{V} \Lambda \bar{V}^T = \bar{V} \Lambda \bar{V}^{-1}.$$

Thus, $(\Sigma_1^*)^{-1} = \bar{V} \Lambda^{-1} \bar{V}^T$.Let $\lambda^* = n\sigma_v^2 + \sigma_a^2 + \sigma_\epsilon^2$ and $\lambda_0 = \sigma_a^2 + \sigma_\epsilon^2$, we can express the $(\Sigma_1^*)^{-1}$ as

$$\frac{1}{n} \begin{bmatrix} \frac{2}{\lambda} + \frac{1}{\lambda^*} & \frac{1}{\lambda^*} - \frac{1}{\lambda} & \cdots & \frac{1}{\lambda^*} - \frac{1}{\lambda} \\ \frac{1}{\lambda^*} - \frac{1}{\lambda} & \frac{2}{\lambda} + \frac{1}{\lambda^*} & \cdots & \frac{1}{\lambda^*} - \frac{1}{\lambda} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\lambda^*} - \frac{1}{\lambda} & \frac{1}{\lambda^*} - \frac{1}{\lambda} & \cdots & \frac{2}{\lambda} + \frac{1}{\lambda^*} \end{bmatrix}.$$

When multiplying the $\Gamma = \begin{bmatrix} \sigma_v^2 & \sigma_v^2 + \sigma_a^2 & \cdots & \sigma_v^2 \end{bmatrix}$, we can get $(\Sigma_1^*)^{-1}\Gamma$ as :

$$\frac{1}{n} \begin{bmatrix} \frac{n\sigma_v^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda} \\ \frac{n\sigma_v^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} + \frac{n-1\sigma_a^2}{\lambda} \\ \frac{n\sigma_v^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda} \\ \vdots \\ \frac{n\sigma_v^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\sigma_v^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda_0} \\ \frac{\sigma_v^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda_0} \\ \frac{\sigma_v^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda_0} \\ \vdots \\ \frac{\sigma_v^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda_0} \end{bmatrix}}_{\text{learning effect}} + \underbrace{\begin{bmatrix} 0 \\ \frac{\sigma_2^2}{\lambda_0} \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\text{private effet}} \tag{12}$$

Regarding the informed bidder's case, the derivation of the $(\Sigma_1^*)^{-1}\Gamma$ is a little complicated:

$$
\begin{aligned}
|\Sigma_0^* - \lambda I| &= \begin{vmatrix} n\sigma_v^2+\sigma_a^2-\lambda & \sigma_v^2 & \cdots & \sigma_v^2 \\ n\sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda & \sigma_v^2+\sigma_a^2+\sigma_\epsilon^2 & -\lambda\cdots & \sigma_v^2 \\ \vdots & \vdots & \ddots & \vdots \\ n\sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda & \sigma_v^2 & \cdots & \sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda \end{vmatrix}, \\
&= \begin{vmatrix} 1 & \sigma_v^2 & \cdots & \sigma_v^2 \\ \frac{n\sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda}{n\sigma_v^2+\sigma_a^2-\lambda} & \sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda & \cdots & \sigma_v^2 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{n\sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda}{n\sigma_v^2+\sigma_a^2-\lambda} & \sigma_v^2 & \cdots & \sigma_v^2+\sigma_a^2+\sigma_\epsilon^2-\lambda \end{vmatrix} (n\sigma_v^2+\sigma_a^2-\lambda), \\
&= \begin{vmatrix} 1 & 0 & \cdots & 0 \\ 1+\frac{\sigma_\epsilon^2}{n\sigma_v^2+\sigma_a^2-\lambda} & \sigma_a^2+\sigma_\epsilon^2-\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda}-\lambda & \cdots & -\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda} \\ \vdots & \vdots & \ddots & \vdots \\ 1+\frac{\sigma_\epsilon^2}{n\sigma_v^2+\sigma_a^2-\lambda} & -\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda} & \cdots & \sigma_a^2+\sigma_\epsilon^2-\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda}-\lambda \end{vmatrix} (n\sigma_v^2+\sigma_a^2-\lambda), \\
&= \begin{vmatrix} \sigma_a^2+\sigma_\epsilon^2-(n-1)\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda}-\lambda & -\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda} & \cdots & -\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda} \\ \sigma_a^2+\sigma_\epsilon^2-(n-1)\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda} & \sigma_a^2+\sigma_\epsilon^2-\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda}-\lambda & \cdots & -\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_a^2+\sigma_\epsilon^2-(n-1)\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda} & -\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda} & \cdots & \sigma_a^2+\sigma_\epsilon^2-\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda}-\lambda \end{vmatrix}_{(n-1)\times(n-1} \\
&= \begin{vmatrix} 1 & 0 & \cdots & 0 \\ 1 & \sigma_a^2+\sigma_\epsilon^2-\lambda & \cdots & 0 \\ 1 & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & \sigma_a^2+\sigma_\epsilon^2-\lambda \end{vmatrix}_{(n-1)\times(n-1)} (\sigma_a^2+\sigma_\epsilon^2-(n-1)\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda})(n\sigma_v^2+\sigma_a^2-\lambda) \\
&= (\sigma_a^2+\sigma_\epsilon^2-(n-1)\frac{\sigma_\epsilon^2\sigma_v^2}{n\sigma_v^2+\sigma_a^2-\lambda})(n\sigma_v^2+\sigma_a^2-\lambda)[\sigma_a^2+\sigma_\epsilon^2-\lambda]^{n-2} = 0 \\
&= \left\{(\sigma_a^2+\sigma_\epsilon^2)(n\sigma_v^2+\sigma_a^2-\lambda)-(n-1)\sigma_\epsilon^2\sigma_v^2\right\}[\sigma_a^2+\sigma_\epsilon^2-\lambda]^{n-2} \\
&= \left[(\lambda-\sigma_a^2)(\lambda-n\sigma_v^2-\sigma_a^2-\sigma_\epsilon^2)+\sigma_v^2\sigma_\epsilon^2\right][\sigma_a^2+\sigma_\epsilon^2-\lambda]^{n-2} = 0
\end{aligned}
$$

Thus the eigenvalues for the Σ_1^* are:

$$\lambda_0 = \sigma_a^2 + \sigma_\epsilon^2,\ \lambda_1, \lambda_2 = \frac{A \mp \sqrt{A^2 - 4D}}{2},$$

$$where$$

$$A = n\sigma_v^2 + 2\sigma_a^2 + \sigma_\epsilon^2$$
$$D = n\sigma_v^2\sigma_a^2 + \sigma_a^2 + \sigma_\epsilon^2\sigma_a^2 + \sigma_v^2\sigma_\epsilon^2$$

The eigenvectors are also complicated, for $\lambda_0 = \sigma_a^2 + \sigma_\epsilon^2$, we get the similar vector as before, $v = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \end{bmatrix}^T$. However, regarding λ_1, λ_2, the vectors has the following properties: $\begin{bmatrix} Y & X & X & \cdots & X \end{bmatrix}^T$. And X and Y should satisfy:

$$(n-1)\sigma_v^2 X + (\sigma_v^2 - \frac{n\sigma_v^2 + \sigma_\epsilon^2 \mp \sqrt{A^2 - 4D}}{2}) = 0,$$
$$\sigma_v^2 Y + (n-1)\sigma_v^2 X + (\frac{\sigma_a^2}{2} - \frac{n\sigma_v^2 \mp \sqrt{A^2 - 4D}}{2})X = 0.$$

Therefore, $\frac{X_1}{Y_1} = \frac{(n\sigma_v^2 + \sigma_\epsilon^2 - \sqrt{A^2 - 4D})}{(n\sigma_v^2 - \sigma_\epsilon^2 - \sqrt{A^2 - 4D})}$ for λ_1 and $\frac{X_2}{Y_2} = \frac{n\sigma_v^2 + \sigma_\epsilon^2 + \sqrt{A^2 - 4D}}{n\sigma_v^2 - \sigma_\epsilon^2 + \sqrt{A^2 - 4D}}$ for λ_2. Now we can construct the eigenvectors for x:

$$\begin{bmatrix} aY_1 & b_1 & \cdots & 0 & cY_2 \\ aX_1 & b_2 & b_1 & 0 & cX_2 \\ aX_1 & 0 & b_2 & 0 & cX_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ aX_1 & 0 & 0 & b_2 & cX_2 \end{bmatrix}$$

And the coefficient matrix $(\Sigma_1^*)^{-1}\Gamma$ will be:

$$\begin{bmatrix} a^2Y_1(Y_1 + 2X_1)\frac{\sigma_v^2}{\lambda_1} + c^2Y_2(Y_2 + 2X_2^2)\frac{\sigma_v^2}{\lambda_2} + a^2X_1Y_1\frac{\sigma_a^2}{\lambda_1} + c^2X_2Y_2\frac{\sigma_a^2}{\lambda_2} \\ a^2(X_1Y_1 + 2X_1^2)\frac{\sigma_v^2}{\lambda_1} + c^2(X_2Y_2 + 2X_2^2)\frac{\sigma_v^2}{\lambda_2} + a^2X_1^2\frac{\sigma_a^2}{\lambda_1} + c^2X_2^2\frac{\sigma_a^2}{\lambda_2} \\ a^2(X_1Y_1 + 2X_1^2)\frac{\sigma_v^2}{\lambda_1} + c^2(X_2Y_2 + 2X_2^2)\frac{\sigma_v^2}{\lambda_2} + a^2X_1^2\frac{\sigma_a^2}{\lambda_1} + c^2X_2^2\frac{\sigma_a^2}{\lambda_2} \\ \vdots \end{bmatrix} + \begin{bmatrix} 0 \\ b^2z_1^2\frac{\sigma_a^2}{\lambda_0} \\ -b^2z_1^2\frac{\sigma_a^2}{\lambda_0} \\ \vdots \end{bmatrix}. \tag{13}$$

Step 2: Recover the winner's private signal x_1 Since we use the second highest bidder's bidding price to calculate the information premium, bidder $i = 2$ can observe all the the rivals' private signals except for the highest one. To recover the expected private signal for the highest bidder, we need to build a system of equations :

$$E[V_1|X_1, X_2, X_{j\geq 2}] = P,$$
$$E[V_2|X_2, X_1, X_{j\geq 2}] = P.$$

Because there is only one unknown X_1, we can solve the X_1 by combing the above equations. And the above system equations can help us build the connection between the informed and uninformed bid. And we will show that the information premium is independent of number of bidders. For the symmetric case(without informed bidder), we can finally get $X_1 = X_2$. And the conditional expectation of the second highest bidder becomes

$$\begin{aligned}E[V_2|X_2, X_1, X_{j\geq 3}] &= R_1X_1 + R_2X_2 + R_3X_3 ... \\ &= R^*(X_1 + X_2) + \tilde{R}\sum_{J\geq 3}^{N} X_j.\end{aligned}$$

For the asymmetric (with informed bidder), we will get that

$$\begin{aligned}X_1 &= \frac{R_2}{1-R_1}X_2 + \frac{R_3}{1-R_1}X_3 + ... + \frac{R_N}{1-R_1}X_N \\ E[V_2|X_2, X_1, X_{j\geq 3}] &= R_1X_1 + R_2X_2 + \sum_{j\geq 3}^{N} R_jX_N \\ &= R_1[\frac{R_2}{1-R_1}X_2 + \sum_{j\geq 3}^{N}\frac{R_3}{1-R_1}X_3] + R_2X_2 + \sum_{j\geq 3}^{N} R_jX_N \\ &= \frac{R_2}{1-R_1}X_2 + \sum_{j\geq 3}^{N}\frac{R_j}{1-R_1}X_j.\end{aligned}$$

Step 3: Compare the difference If, without loss of generality, we set $X_i = 1$, the information premium can be converted to the difference between the coefficient matrix $\mathbf{1} \cdot (\Sigma_1^*)^{-1}\Gamma - \mathbf{1} \cdot (\Sigma_0^*)^{-1}\Gamma$. The next task is to find the connection between (12)and (13). To solve the above a, b, c and $(X_1, Y_1), (X_2, Y_2)$,(Z_1, Z_2), I apply the guess and verify method. I normalize the Y_1 and Y_2 as 1, $Z_1 = -Z_2 = 1$. And I find a, b, c and X_1, X_2 in eigenvector matrix that have $VV^T = VV^{-1} = 1$ also satisfy:

$$a^2\lambda_2 + c^2\lambda_1 = (n-1)\sigma_v^2 + \sigma_a^2 + \sigma_\epsilon^2, \tag{14}$$

$$a^2 X_1 \lambda_2 + c^2 X_2 \lambda_1 = -\sigma_v^2, \tag{15}$$

$$a^2 X_1^2 \lambda_2 + c^2 X_2^2 \lambda_1 = \frac{\sigma_v^2 + \sigma_a^2}{2}, \tag{16}$$

$$b^2 = \frac{1}{n-1}. \tag{17}$$

Then based on (14),(15),16 and 17, we can solve the adjustment coefficient $1 - R_1$ as

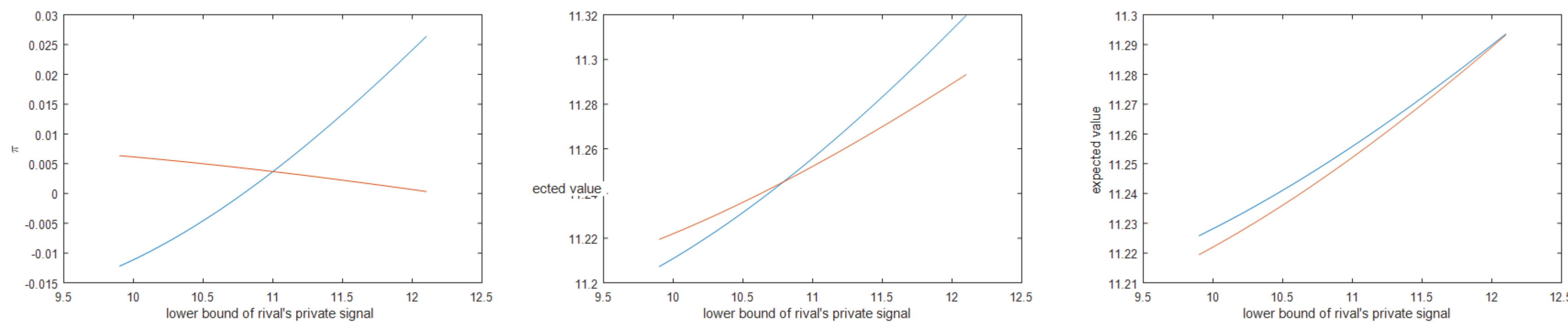
$$\begin{aligned} 1 - R_1 &= 1 - \left[a^2(1 + 2X_1)\frac{\sigma_v^2}{\lambda_1} + c^2(1 + 2X_2^2)\frac{\sigma_v^2}{\lambda_2} + a^2 X_1 \frac{\sigma_a^2}{\lambda_1} + c^2 X_2 \frac{\sigma_a^2}{\lambda_2} \right] \\ &= \sigma_v^2 \sigma_\epsilon^2. \end{aligned}$$

And correspondingly,

$$\begin{aligned} \left(\frac{1}{1-R_1}\right)\left[a^2(X_1 + 2X_1^2)\frac{\sigma_v^2}{\lambda_1} + c^2(X_2 + 2X_2^2)\frac{\sigma_v^2}{\lambda_2} \right] &\Longleftrightarrow \frac{\sigma_v^2}{\lambda^*}\frac{n}{n}, \\ \left(\frac{1}{1-R_1}\right)\left[a^2 X_1^2 \frac{\sigma_a^2}{\lambda_1} + c^2 X_2^2 \frac{\sigma_a^2}{\lambda_2} \right] &\Longleftrightarrow \frac{\sigma_a^2}{\lambda^*}\frac{1}{n}, \\ \left(\frac{1}{1-R_1}\right) b^2 \frac{\sigma_a^2}{\lambda_0} &\Longleftrightarrow \frac{\sigma_a^2}{\lambda_0}\frac{1}{n}. \end{aligned}$$

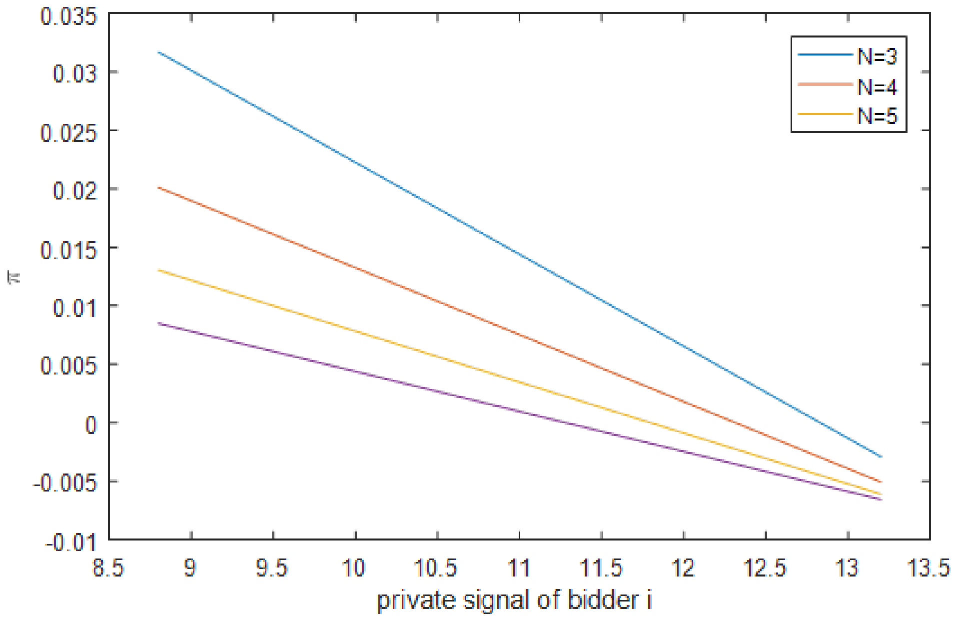
Therefore, $\mathbf{1} \cdot (\Sigma_1^*)^{-1}\Gamma - \mathbf{1} \cdot (\Sigma_0^*)^{-1}\Gamma$ equals to a constant number which is independent of number of bidders!

Figure 1: Information Premium and Variations of lower bounds



Remark. For all the figures, The settings are $N = 5$, $x_i = 10.95$. The information parameters are $\{\mu_v = 11, r = 1, \sigma_v^2 = 0.8; \sigma_a^2 = 1.2; \sigma_\epsilon^2 = 0.8\}$. The first graph on the left indicates how the changes of information premium respond to the lower bound of rival's private signal. The blue line indicates the case when the informed opponent changes her lower bound. And the red line indicates the case when the uninformed opponent changes the lower bound. The second graph shows how the conditional expected value varies when the informed opponent changes the lower bound. And the third graph shows the situation when the uninformed opponent changes her lower bound.

Figure 2: Information Premium and Number of bidders



Remark. The lower bound of the rivals are fixed at $X = 11$. The information parameters are $\{\mu_v = 11, r = 1, \sigma_v^2 = 0.8; \sigma_a^2 = 1.2; \sigma_\epsilon^2 = 0.8\}$.

Figure 3: Information Premium Independent of the Number of Bidders

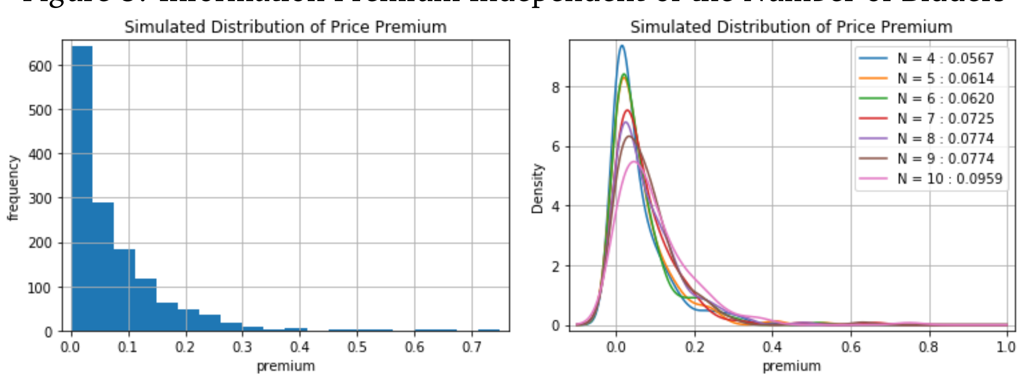
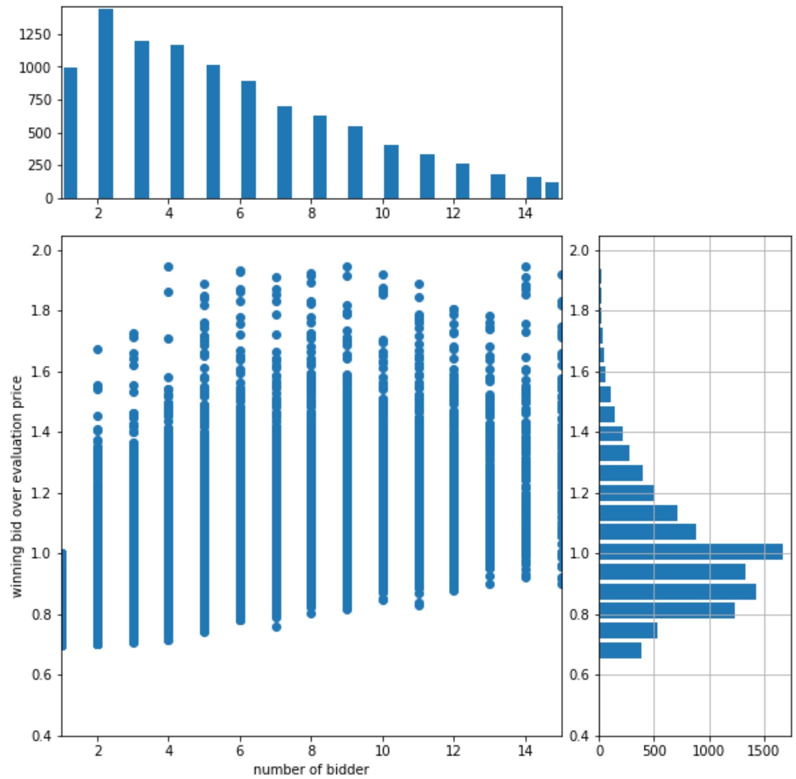


Figure 4: The relation between Winning bid and Number of bidders



Remark. The y axis is the ratio of winning bid over reserve price, measuring the total bidding ladders from reservation price.

Figure 5: Bidding Outcomes of the Auction with / without Priority Bidder

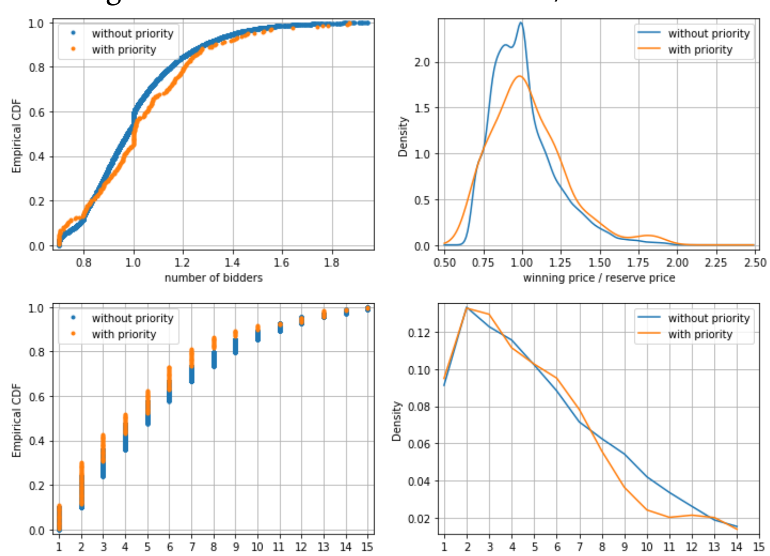
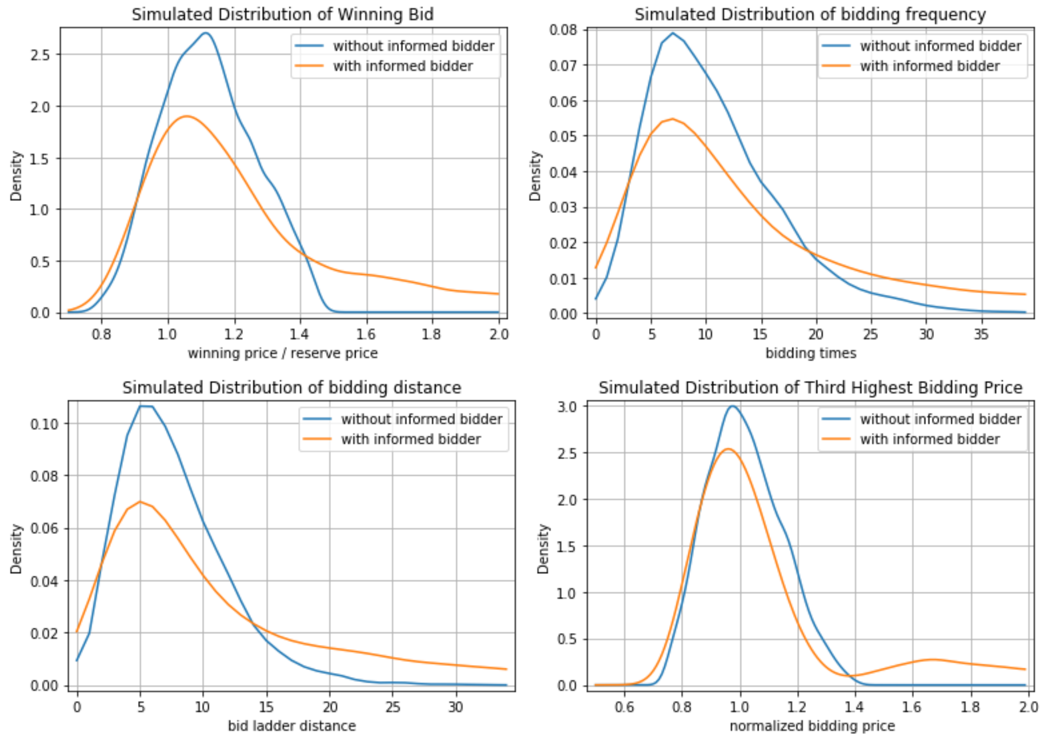


Figure 6: Simulation Results



Remark. Figures from left to right and top to bottom are denoted (a), (b), (c), (d).

Figure 7: Counterfactual analysis of priority bidder's behavior