Exercise

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1. Set that are numerically equivalent to N are called countable. Can you show $Z \times Z$ is countable as well ? (hint: define a function from N to $Z \times Z$)

2. Using the Chain Rule to calculate dz/dt for each of the following functions:

$$z = f(x, y) = \sqrt{x^2 + y^3},\ x = x(t) = e^{2t},\ y = y(t) = e^{-t}$$

3. Please do the following calculation

   (a) Let $X \sim N(0, 1)$ ($\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$), Find $\int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$

   (b) Find the eigenvalue and eigenvector of $\begin{bmatrix} 5 & 2 \\ 1 & 0 \end{bmatrix}$

4. Check the "Dixit-Stiglitz" style CES functions and production functions.

5. Suppose a consumer consumes two goods, x and y and has utility function $u(x, y) = x^{0.5} \cdot y^{0.5}$. He has a budget of \$500 . The price of x is $P_x = 20$ and the price of y is $P_y = 25$. Find his optimal consumption bundle using the Lagrange method. What if the price of y increase to $P_y = 40$?

6. Check the logit function and extreme distribution. Can you see the connection between extreme distribution and logit function?
   https://www.itl.nist.gov/div898/handbook/eda/section3/eda366g.htm