

Final Project Presentation

Prove the equivalence of CFG and PDA

Part I: convert any CFG to PDA using computer program

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Context-Free Grammar:

$S \rightarrow 0S0 \mid 1S1 \mid \varepsilon$

Example:

$w=011110$

Context-Free Grammar:

$S \rightarrow 0S0 \mid 1S1 \mid \varepsilon$

Example:

$w=011110$

Input for the program

%S // Start Symbol

%S-> 0%S0 | 1%S1 | ~ // Generation Rules, ~ represents ε for simplicity

Push-Down Automata generated by the program

Start state: q_START

$\delta(q_START, \epsilon, \epsilon) = \delta(q_S_0, \{ACCEPT\})$

$\delta(q_S_0, 0, \epsilon) = \delta(q_S_1, \epsilon)$

$\delta(q_S_1, \epsilon, \epsilon) = \delta(q_S_0, S_2)$

$\delta(q_S_0, 1, \epsilon) = \delta(q_S_3, \epsilon)$

$\delta(q_S_3, \epsilon, \epsilon) = \delta(q_S_0, S_4)$

$\delta(q_S_0, \epsilon, \epsilon) = \delta(q_S_5, \epsilon)$

$\delta(q_S_2, 0, S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_2, 0, S_4) = \delta(q_S_4, \epsilon)$

$\delta(q_S_4, 1, S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_4, 1, S_4) = \delta(q_S_4, \epsilon)$

$\delta(q_S_5, \epsilon, S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_5, \epsilon, S_4) = \delta(q_S_4, \epsilon)$

$\delta(q_S_2, 0, \{ACCEPT\}) = \delta(q_END, \epsilon)$

$\delta(q_S_4, 1, \{ACCEPT\}) = \delta(q_END, \epsilon)$

$\delta(q_S_5, \epsilon, \{ACCEPT\}) = \delta(q_END, \epsilon)$

Accept by empty stack only

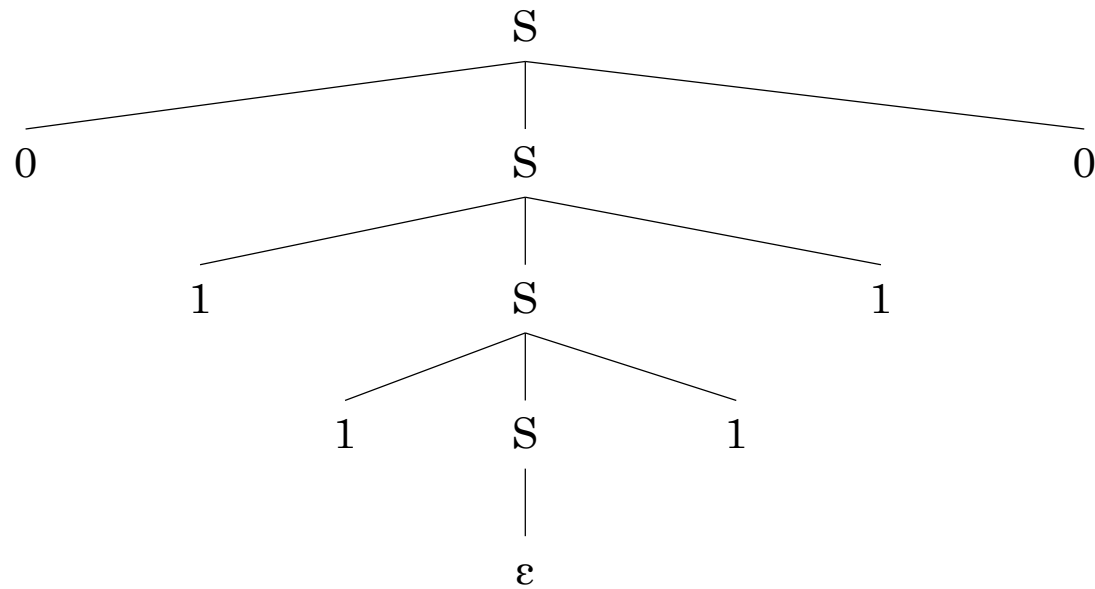
Context-Free Grammar:

$S \rightarrow 0S0 \mid 1S1 \mid \varepsilon$

Example:

$w=011110$

Parse Tree:



Push-Down Automata generated by the program

Start state: q_START

$\delta(q_START, \epsilon, \epsilon) = \delta(q_S_0, \{ACCEPT\})$

$\delta(q_S_0, 0, \epsilon) = \delta(q_S_1, \epsilon)$

$\delta(q_S_1, \epsilon, \epsilon) = \delta(q_S_0, S_2)$

$\delta(q_S_2, 0, \epsilon) = \delta(q_S_3, \epsilon)$

$\delta(q_S_0, 1, \epsilon) = \delta(q_S_4, \epsilon)$

$\delta(q_S_4, \epsilon, \epsilon) = \delta(q_S_0, S_5)$

$\delta(q_S_5, 1, \epsilon) = \delta(q_S_6, \epsilon)$

$\delta(q_S_0, \epsilon, \epsilon) = \delta(q_S_7, \epsilon)$

$\delta(q_S_3, \epsilon, S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_3, \epsilon, S_5) = \delta(q_S_5, \epsilon)$

$\delta(q_S_6, \epsilon, S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_6, \epsilon, S_5) = \delta(q_S_5, \epsilon)$

$\delta(q_S_7, \epsilon, S_2) = \delta(q_S_2, \epsilon)$

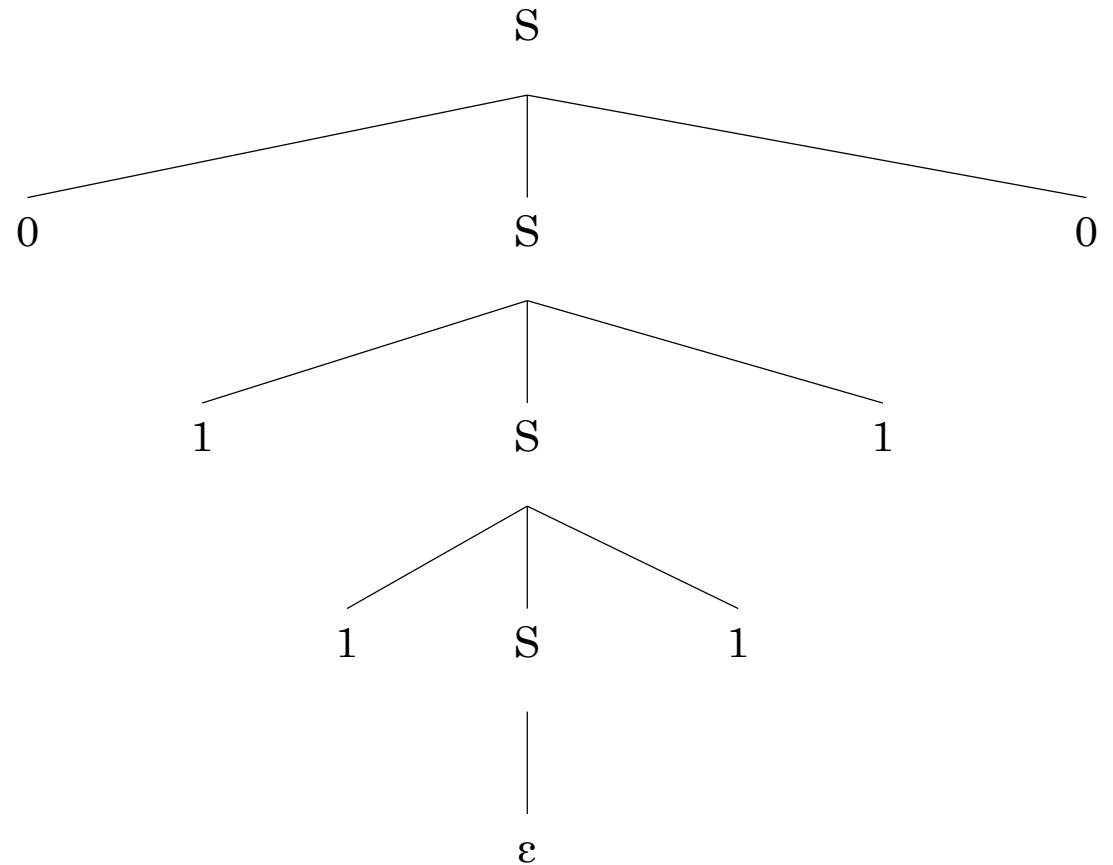
$\delta(q_S_7, \epsilon, S_5) = \delta(q_S_5, \epsilon)$

$\delta(q_S_3, \epsilon, \{ACCEPT\}) = \delta(q_END, \epsilon)$

$\delta(q_S_6, \epsilon, \{ACCEPT\}) = \delta(q_END, \epsilon)$

$\delta(q_S_7, \epsilon, \{ACCEPT\}) = \delta(q_END, \epsilon)$

Accept by empty stack only



Push-Down Automata generated by the program

Start state: q_START

$\delta(q_START, \epsilon, \epsilon) = \delta(q_S_0, \{ACCEPT\})$

$\delta(q_S_0, 0, \epsilon) = \delta(q_S_1, \epsilon)$

$\delta(q_S_1, \epsilon, \epsilon) = \delta(q_S_0, \%S_2)$

$\delta(q_S_2, 0, \epsilon) = \delta(q_S_3, \epsilon)$

$\delta(q_S_0, 1, \epsilon) = \delta(q_S_4, \epsilon)$

$\delta(q_S_4, \epsilon, \epsilon) = \delta(q_S_0, \%S_5)$

$\delta(q_S_5, 1, \epsilon) = \delta(q_S_6, \epsilon)$

$\delta(q_S_0, \epsilon, \epsilon) = \delta(q_S_7, \epsilon)$

$\delta(q_S_3, \epsilon, \%S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_3, \epsilon, \%S_5) = \delta(q_S_5, \epsilon)$

$\delta(q_S_6, \epsilon, \%S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_6, \epsilon, \%S_5) = \delta(q_S_5, \epsilon)$

$\delta(q_S_7, \epsilon, \%S_2) = \delta(q_S_2, \epsilon)$

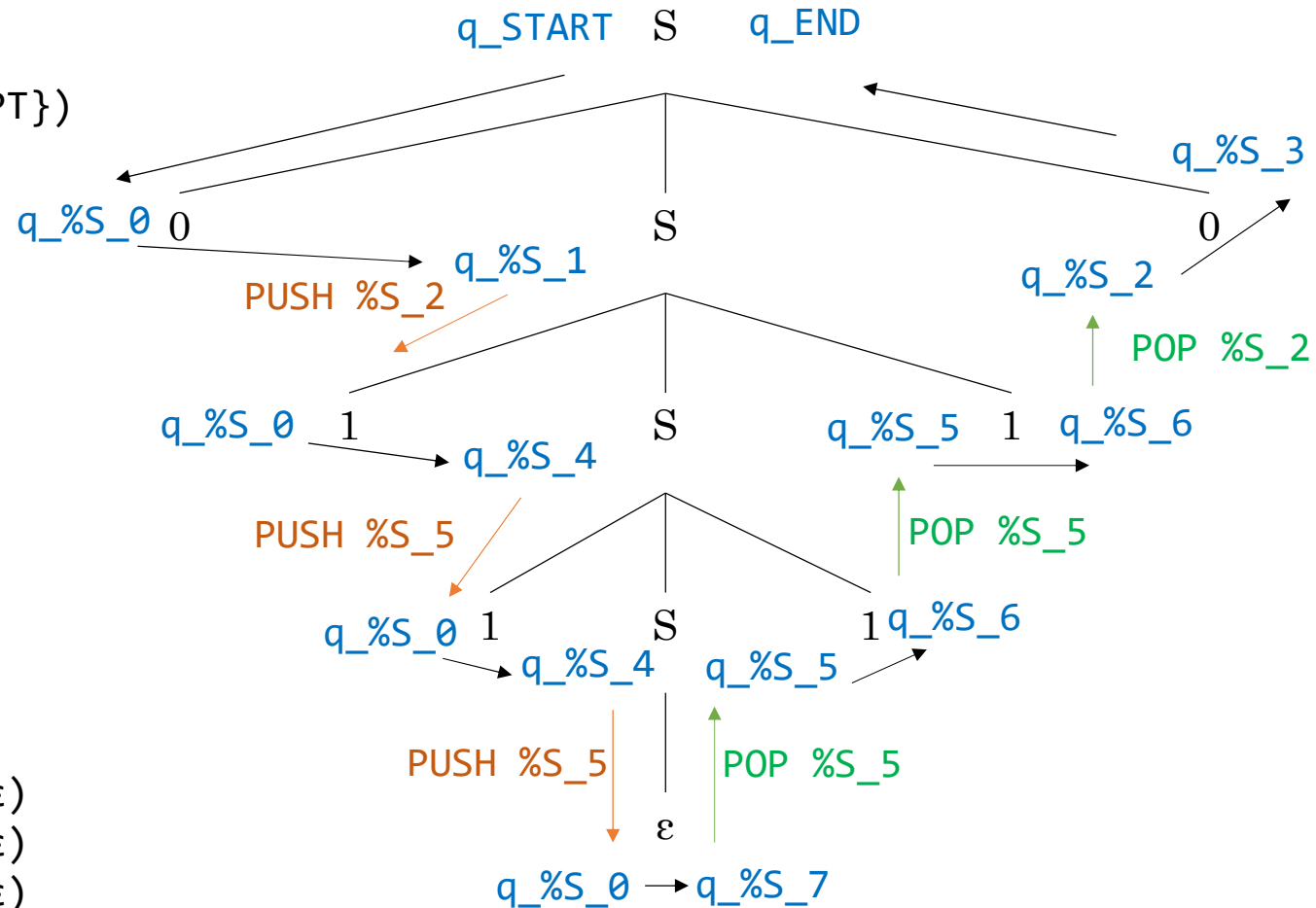
$\delta(q_S_7, \epsilon, \%S_5) = \delta(q_S_5, \epsilon)$

$\delta(q_S_3, \epsilon, \{ACCEPT\}) = \delta(q_END, \epsilon)$

$\delta(q_S_6, \epsilon, \{ACCEPT\}) = \delta(q_END, \epsilon)$

$\delta(q_S_7, \epsilon, \{ACCEPT\}) = \delta(q_END, \epsilon)$

Accept by empty stack only



Push-Down Automata generated by the program (improved)

Start state: q_START

$\delta(q_START, \epsilon, \epsilon) = \delta(q_S_0, \{ACCEPT\})$

$\delta(q_S_0, 0, \epsilon) = \delta(q_S_1, \epsilon)$

$\delta(q_S_1, \epsilon, \epsilon) = \delta(q_S_0, \%S_2)$

$\delta(q_S_0, 1, \epsilon) = \delta(q_S_3, \epsilon)$

$\delta(q_S_3, \epsilon, \epsilon) = \delta(q_S_0, \%S_4)$

$\delta(q_S_0, \epsilon, \epsilon) = \delta(q_S_5, \epsilon)$

$\delta(q_S_2, 0, \%S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_2, 0, \%S_4) = \delta(q_S_4, \epsilon)$

$\delta(q_S_4, 1, \%S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_4, 1, \%S_4) = \delta(q_S_4, \epsilon)$

$\delta(q_S_5, \epsilon, \%S_2) = \delta(q_S_2, \epsilon)$

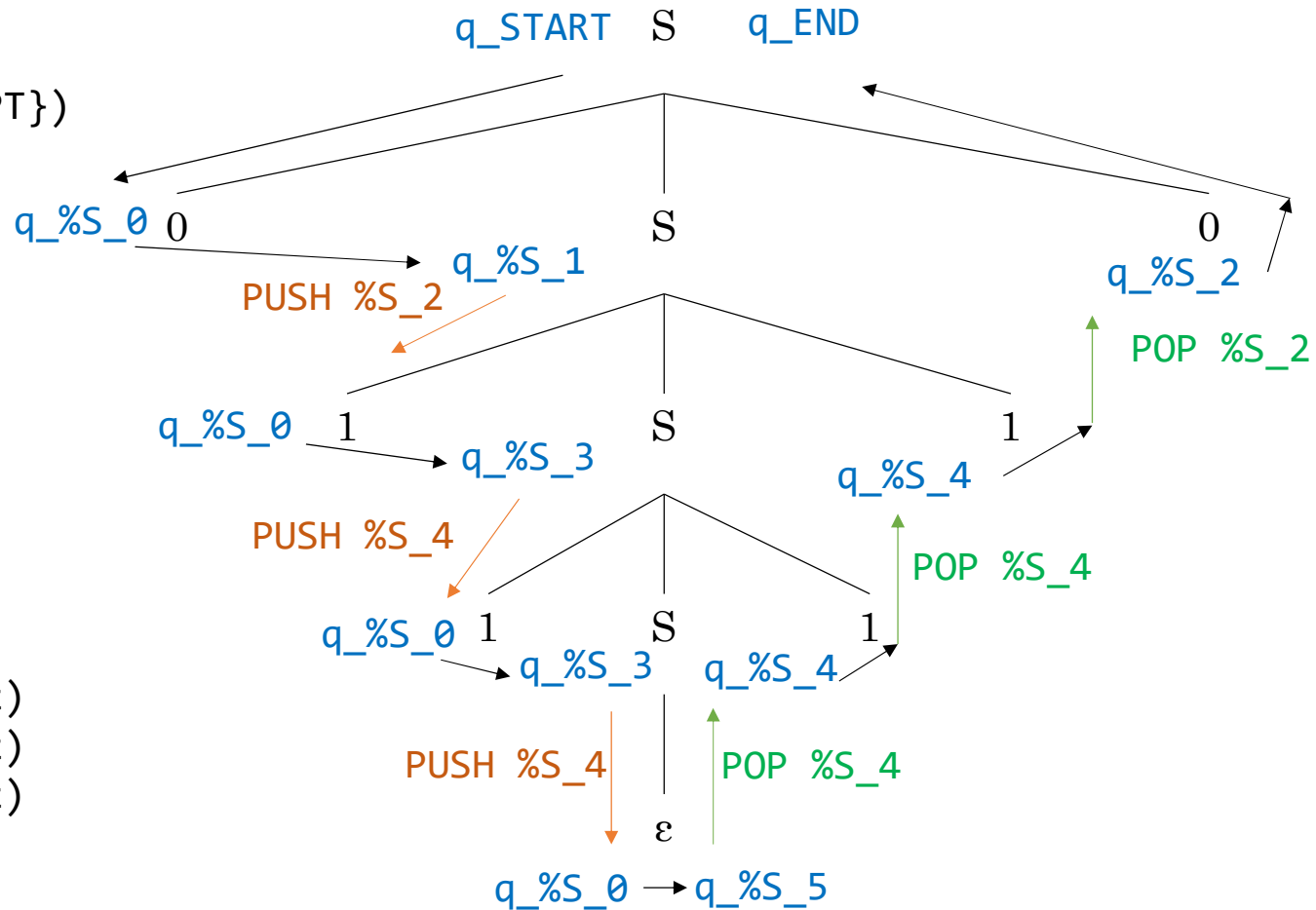
$\delta(q_S_5, \epsilon, \%S_4) = \delta(q_S_4, \epsilon)$

$\delta(q_S_2, 0, \{ACCEPT\}) = \delta(q_END, \epsilon)$

$\delta(q_S_4, 1, \{ACCEPT\}) = \delta(q_END, \epsilon)$

$\delta(q_S_5, \epsilon, \{ACCEPT\}) = \delta(q_END, \epsilon)$

Accept by empty stack only



Push-Down Automata generated by the program (further improved but not implemented)

Start state: q_START

$\delta(q_START, \epsilon, \epsilon) = \delta(q_S_0, \{ACCEPT\})$

$\delta(q_S_0, 0, \epsilon) = \delta(q_S_0, \%S_2)$

$\delta(q_S_0, 1, \epsilon) = \delta(q_S_0, \%S_4)$

$\delta(q_S_2, 0, \%S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_2, 0, \%S_4) = \delta(q_S_4, \epsilon)$

$\delta(q_S_4, 1, \%S_2) = \delta(q_S_2, \epsilon)$

$\delta(q_S_4, 1, \%S_4) = \delta(q_S_4, \epsilon)$

$\delta(q_S_0, \epsilon, \%S_2) = \delta(q_S_2, \epsilon)$

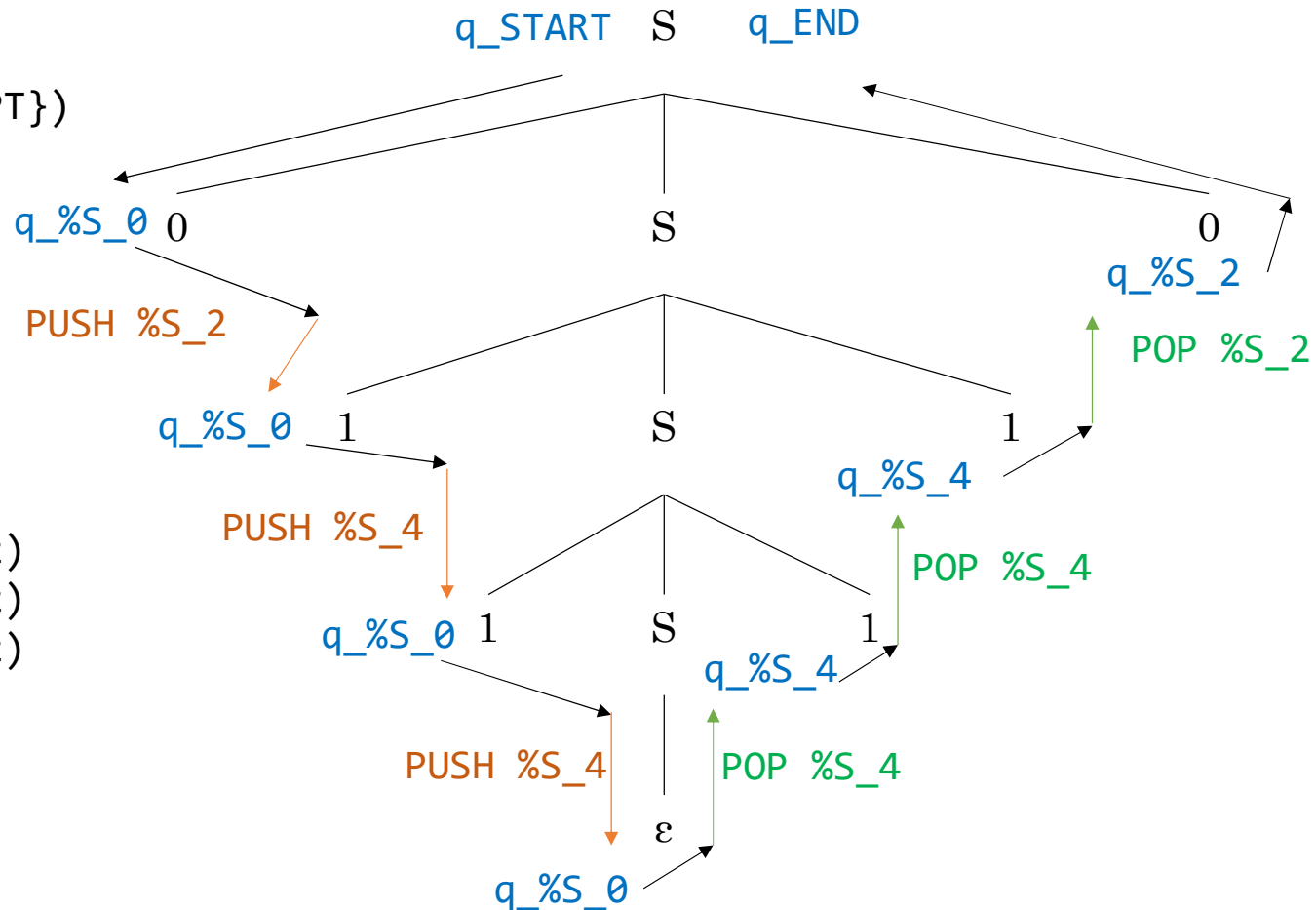
$\delta(q_S_0, \epsilon, \%S_4) = \delta(q_S_4, \epsilon)$

$\delta(q_S_2, 0, \{ACCEPT\}) = \delta(q_END, \epsilon)$

$\delta(q_S_4, 1, \{ACCEPT\}) = \delta(q_END, \epsilon)$

$\delta(q_S_0, \epsilon, \{ACCEPT\}) = \delta(q_END, \epsilon)$

Accept by empty stack only



Program Generated (with states renamed) vs Human Written

Start state: q_START

$\delta(q_0, \epsilon, \epsilon) = \delta(q_1, X)$

$\delta(q_1, 0, \epsilon) = \delta(q_1, Y)$

$\delta(q_1, 1, \epsilon) = \delta(q_1, Z)$

$\delta(q_2, 0, Y) = \delta(q_2, \epsilon)$

$\delta(q_2, 0, Z) = \delta(q_3, \epsilon)$

$\delta(q_3, 1, Y) = \delta(q_2, \epsilon)$

$\delta(q_3, 1, Z) = \delta(q_3, \epsilon)$

$\delta(q_1, \epsilon, Y) = \delta(q_2, \epsilon)$

$\delta(q_1, \epsilon, Z) = \delta(q_3, \epsilon)$

$\delta(q_2, 0, X) = \delta(q_4, \epsilon)$

$\delta(q_3, 1, X) = \delta(q_4, \epsilon)$

$\delta(q_1, \epsilon, X) = \delta(q_4, \epsilon)$

Accept by empty stack only

12 Transition Rules

(Can be generated by
further improved program)

Start state: q_0

$\delta(q_0, \epsilon, \epsilon) = \delta(q_1, X)$

$\delta(q_1, 0, \epsilon) = \delta(q_1, A)$

$\delta(q_1, 1, \epsilon) = \delta(q_1, B)$

$\delta(q_1, 0, A) = \delta(q_2, \epsilon)$

$\delta(q_1, 1, B) = \delta(q_2, \epsilon)$

$\delta(q_2, 0, A) = \delta(q_2, \epsilon)$

$\delta(q_2, 1, B) = \delta(q_2, \epsilon)$

$\delta(q_2, \epsilon, X) = \delta(q_3, \epsilon, \epsilon)$

8 Transition Rules

(Written by myself)

Start state: q_0

Start stack: X_0

$\delta(q_1, 0, X_0) = \delta(q_1, A)$

$\delta(q_1, 1, X_0) = \delta(q_1, B)$

$\delta(q_1, 0, A) = \delta(q_1, AA)$

$\delta(q_1, 1, B) = \delta(q_1, BB)$

$\delta(q_1, 0, A) = \delta(q_2, \epsilon)$

$\delta(q_1, 1, B) = \delta(q_2, \epsilon)$

$\delta(q_1, 0, B) = \delta(q_1, AB)$

$\delta(q_1, 1, A) = \delta(q_1, BA)$

$\delta(q_2, 0, A) = \delta(q_2, \epsilon)$

$\delta(q_2, 1, B) = \delta(q_2, \epsilon)$

10 Transition Rules

(Modified from week 6
material example)

Push-Down Automata generated by the program (more examples: balanced parentheses)

Context-Free Grammar:

$B \rightarrow BB \mid (B) \mid \varepsilon$

Example:

$w = (())()$

Start state: q_START

$\delta(q_START, \varepsilon, \varepsilon) = \delta(q_B_0, \{ACCEPT\})$

$\delta(q_B_0, \varepsilon, \varepsilon) = \delta(q_B_0, \%B_1)$

$\delta(q_B_1, \varepsilon, \varepsilon) = \delta(q_B_0, \%B_2)$

$\delta(q_B_0, (, \varepsilon) = \delta(q_B_3, \varepsilon)$

$\delta(q_B_3, \varepsilon, \varepsilon) = \delta(q_B_0, \%B_4)$

$\delta(q_B_0, \varepsilon, \varepsilon) = \delta(q_B_5, \varepsilon)$

$\delta(q_B_2, \varepsilon, \%B_1) = \delta(q_B_1, \varepsilon)$

$\delta(q_B_2, \varepsilon, \%B_2) = \delta(q_B_2, \varepsilon)$

$\delta(q_B_2, \varepsilon, \%B_4) = \delta(q_B_4, \varepsilon)$

$\delta(q_B_4,), \%B_1) = \delta(q_B_1, \varepsilon)$

$\delta(q_B_4,), \%B_2) = \delta(q_B_2, \varepsilon)$

$\delta(q_B_4,), \%B_4) = \delta(q_B_4, \varepsilon)$

$\delta(q_B_5, \varepsilon, \%B_1) = \delta(q_B_1, \varepsilon)$

$\delta(q_B_5, \varepsilon, \%B_2) = \delta(q_B_2, \varepsilon)$

$\delta(q_B_5, \varepsilon, \%B_4) = \delta(q_B_4, \varepsilon)$

$\delta(q_B_2, \varepsilon, \{ACCEPT\}) = \delta(q_END, \varepsilon)$

$\delta(q_B_4,), \{ACCEPT\}) = \delta(q_END, \varepsilon)$

$\delta(q_B_5, \varepsilon, \{ACCEPT\}) = \delta(q_END, \varepsilon)$

Accept by empty stack only

Push-Down Automata generated by the program (more examples: two a's for each b)

Context-Free Grammar:

$S \rightarrow SaSaSbS \mid$
 $SaSbSaS \mid$
 $SbSaSaS \mid \epsilon$

Example:

w = aabbbaaaba

Start state: q_{START}

$\delta(q_{START}, \epsilon, \epsilon) = \delta(q_{S_0}, \{ACCEPT\})$
 $\delta(q_{S_0}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_1)$
 $\delta(q_{S_1}, a, \epsilon) = \delta(q_{S_2}, \epsilon)$
 $\delta(q_{S_2}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_3)$
 $\delta(q_{S_3}, a, \epsilon) = \delta(q_{S_4}, \epsilon)$
 $\delta(q_{S_4}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_5)$
 $\delta(q_{S_5}, b, \epsilon) = \delta(q_{S_6}, \epsilon)$
 $\delta(q_{S_6}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_7)$
 $\delta(q_{S_0}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_8)$
 $\delta(q_{S_8}, a, \epsilon) = \delta(q_{S_9}, \epsilon)$
 $\delta(q_{S_9}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_10)$
 $\delta(q_{S_10}, a, \epsilon) = \delta(q_{S_11}, \epsilon)$
 $\delta(q_{S_11}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_12)$
 $\delta(q_{S_12}, b, \epsilon) = \delta(q_{S_13}, \epsilon)$
 $\delta(q_{S_13}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_14)$

$\delta(q_{S_0}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_15)$
 $\delta(q_{S_15}, a, \epsilon) = \delta(q_{S_16}, \epsilon)$
 $\delta(q_{S_16}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_17)$
 $\delta(q_{S_17}, a, \epsilon) = \delta(q_{S_18}, \epsilon)$
 $\delta(q_{S_18}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_19)$
 $\delta(q_{S_19}, b, \epsilon) = \delta(q_{S_20}, \epsilon)$
 $\delta(q_{S_20}, \epsilon, \epsilon) = \delta(q_{S_0}, \%S_21)$
 $\delta(q_{S_0}, \epsilon, \epsilon) = \delta(q_{S_22}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_1) = \delta(q_{S_1}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_3) = \delta(q_{S_3}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_5) = \delta(q_{S_5}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_7) = \delta(q_{S_7}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_8) = \delta(q_{S_8}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_10) = \delta(q_{S_10}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_12) = \delta(q_{S_12}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_14) = \delta(q_{S_14}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_15) = \delta(q_{S_15}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_17) = \delta(q_{S_17}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_19) = \delta(q_{S_19}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \%S_21) = \delta(q_{S_21}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_1) = \delta(q_{S_1}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_3) = \delta(q_{S_3}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_5) = \delta(q_{S_5}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_7) = \delta(q_{S_7}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_8) = \delta(q_{S_8}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_10) = \delta(q_{S_10}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_12) = \delta(q_{S_12}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_14) = \delta(q_{S_14}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_15) = \delta(q_{S_15}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_17) = \delta(q_{S_17}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \%S_19) = \delta(q_{S_19}, \epsilon)$

$\delta(q_{S_14}, \epsilon, \%S_21) = \delta(q_{S_21}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_1) = \delta(q_{S_1}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_3) = \delta(q_{S_3}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_5) = \delta(q_{S_5}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_7) = \delta(q_{S_7}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_8) = \delta(q_{S_8}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_10) = \delta(q_{S_10}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_12) = \delta(q_{S_12}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_14) = \delta(q_{S_14}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_15) = \delta(q_{S_15}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_17) = \delta(q_{S_17}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_19) = \delta(q_{S_19}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \%S_21) = \delta(q_{S_21}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_1) = \delta(q_{S_1}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_3) = \delta(q_{S_3}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_5) = \delta(q_{S_5}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_7) = \delta(q_{S_7}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_8) = \delta(q_{S_8}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_10) = \delta(q_{S_10}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_12) = \delta(q_{S_12}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_14) = \delta(q_{S_14}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_15) = \delta(q_{S_15}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_17) = \delta(q_{S_17}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_19) = \delta(q_{S_19}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \%S_21) = \delta(q_{S_21}, \epsilon)$
 $\delta(q_{S_7}, \epsilon, \{ACCEPT\}) = \delta(q_{END}, \epsilon)$
 $\delta(q_{S_14}, \epsilon, \{ACCEPT\}) = \delta(q_{END}, \epsilon)$
 $\delta(q_{S_21}, \epsilon, \{ACCEPT\}) = \delta(q_{END}, \epsilon)$
 $\delta(q_{S_22}, \epsilon, \{ACCEPT\}) = \delta(q_{END}, \epsilon)$
 Accept by empty stack only

Reference:

Chinese University of Hong Kong - <https://www.cse.cuhk.edu.hk/~siuon/csci3130-f18/slides/lec11.pdf>

University of Notre Dame - <https://www3.nd.edu/~dchiang/teaching/theory/2016/notes/week06/week06b.pdf>

GitHub Repository:

<https://github.com/guoyizhou01/CS5252FinalProject>

Questions?