Chapter 1

Search and Unemployment Insurance

In a equilibrium setup: time period $t = 0, 1, 2, \ldots$, one consumption good. A continum of agents

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \phi(a_t)] \tag{1.1}$$

where $\beta \in (0, 1)$, $u' > 0$, $u'' < 0$. And $\phi' > 0, \phi'' > 0$. $a_t \in [0, +\infty)$ is the agent's effort.

Search:

- Any agent, when unemployed, making effort a_t in period t, finds a job with prob $\pi(a_t) \in [0, 1]$, $\forall\, a_t$. Assume $\pi' > 0, \pi'' < 0$.
- Unemployed agent didn't generate income.
- Jobs are identical, all pay a constant $y = 1$ units of the good in each period.
- Once employed, the agent has in each period an exogenous prob $\delta \in (0, 1)$ to be terminated in which case he goes back to labor mkt, unemployed.
- There is a government in the model who runs an umemployment insurance policy which has two dimensions:
  - b: unemployment insurance benefits paid per period to any unemployed worker
  - τ: income tax per period on each employed worker

Question: what is the optimal (b, τ)?

Consider an symmetric steady-state(stationary) equlibrium of the model in which

1. stationary: all "equilibrium objects" are time invariant.
2. symmetric: all unemployed workers take the same action in search.

Describing a-symmetric stationary equilibrium taking b and τ as given.

Defn: A S-S equilibrium of the model is a vector $\{a^*, E, U\}$, where

$$\begin{aligned} a^* &= \text{equilibrium search effort of the unemployed} \\ E &= \text{measure of employed workers at the begining (end) of the peroid} \\ U &= \text{measure of the unemployed workers} \ldots, E + U = 1 \end{aligned}$$

such that

1. a^* solves the unemployment worker's problem

$$V_u = \max_{a\in[0,\infty)} \Big\{\pi(a)\big[u(y-\tau)+\beta[\delta V_u + (1-\delta)V_e]\big] + (1-\pi(a))\big[u(b)+\beta V_u\big] - \phi(a)\Big\} \tag{1.2}$$

$$V_e = u(y-\tau) + \beta\big\{\delta V_u + (1-\delta)V_e\big\} \tag{1.3}$$

2. E_t = const. in equlibrium.

$$E_{t+1} = E_t(1-\delta) + (1-E_t)\pi(a^*) = E_t. \tag{1.4}$$

$$E(1-\delta) + (1-E)\pi(a^*) = E. \tag{1.5}$$

$$\delta E = \pi(a^*)(1-E). \tag{1.6}$$

The left side of eq. (1.6) is flow out of employment, and the right side is flow into the employment.

Rewrite as

$$V_u = \max_{a\in[0,\infty)} \Big\{\pi(a)V_e + (1-\pi(a))\big[u(b)+\beta V_u\big]\Big\} \tag{1.7}$$

$$1-\beta(1-\delta)V_e = u(y-\tau) + \beta\delta V_u \tag{1.8}$$

$$\delta E = \pi(a^*)(1-E) \tag{1.9}$$

FOC for eq. (1.7)

$$\pi'(a)\big[V_e - \beta V_u - u(b)\big] = \phi'(a) \tag{1.10}$$

$$V_e - \beta V_u - u(b) = \frac{\phi'(a)}{\pi'(a)} \tag{1.11}$$

Plug eq. (1.11) into eq. (1.7)

$$V_u = \pi(a^*)\big[V_e - \beta V_u - u(b)\big] + u(b) + \beta V_u - \phi(a^*) \tag{1.12}$$

$$= \pi(a^*)\frac{\phi'(a^*)}{\pi'(a^*)} + u(b) + \beta V_u - \phi(a^*) \tag{1.13}$$

Then we have

$$(1-\beta)V_u - u(b) = \pi(a^*)\frac{\phi'(a^*)}{\pi'(a^*)} - \phi(a^*) \tag{1.14}$$