Business Cycles and the Asset Structure of Foreign Trade

(IER, 1995)

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Contents

Current Section

Motivation

1. **Reason**: The extent of international financial integration is important for international business cycles.
   - Individuals' incentive to smooth consumption in response to fluctuations in income via financial markets.
   - The extent to which a country can trade on world financial markets will determine the extent to which its citizens can insure themselves against nation-specific components of business-cycle risk.
2. **Goal**: To provide a detailed analysis of the channels through which international financial linkages affect international business cycles.

Current Section

Preferences

- Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}[C_t^{\theta} L_t^{1-\theta}]^{1-\sigma}, \quad \text{home country;} \tag{1}$$

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}[(C_t^{*})^{\theta} (L_t^{*})^{1-\theta}]^{1-\sigma}, \quad \text{foreign country.} \tag{2}$$

- Time Constraints:

$$1 - L_t - N_t \geq 0, \quad \text{home country;} \tag{3}$$

$$1 - L_t^{*} - N_t^{*} \geq 0, \quad \text{foreign country.} \tag{4}$$

Technology

- Production Function:

$$Y_t = A_t K_t^{1-\alpha} (X_t N_t)^{\alpha}, \quad \text{home country;} \tag{5}$$
$$Y_t^* = A_t^* (K_t^*)^{1-\alpha_*} (X_t^* N_t^*)^{\alpha_*}, \quad \text{foreign country.} \tag{6}$$

where X_t and X_t^* are purely labor-augmenting technical change in the home and foreign countries, and grow at a common, constant gross rate: $\gamma = X_{t+1}/X_t = X_{t+1}^*/X_t^*$.

- Capital Accumulations:

$$K_{t+1} = (1-\delta) K_t + \phi(I_t/K_t) K_t, \quad \text{home country;} \tag{7}$$
$$K_{t+1}^* = (1-\delta) K_t^* + \phi(I_t^*/K_t^*) K_t^*, \quad \text{foreign country.} \tag{8}$$

Complete Markets

- Social Planner's Problem

$$
\begin{aligned}
\max \mathcal{L} \quad = \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \{[\pi u(c_t, L_t) + (1-\pi) u(c_t^*, L_t^*)] \\
& + \pi w_t (1 - L_t - N_t) + (1-\pi) w_t^* (1 - L_t^* - N_t^*) \\
& + \pi \lambda_t [(1-\delta) k_t - (\gamma k_{t+1} - \phi(i_t / k_t) k_t)] \\
& + (1-\pi) \lambda_t^* [(1-\delta) k_t^* - (\gamma k_{t+1}^* - \phi(i_t^* / k_t^*) k_t^*)] \\
& + p_t [\pi (A_t F(k_t, N_t) - c_t - i_t) \\
& + (1-\pi)(A_t^* F(k_t^*, N_t^*) - c_t^* - i_t^*)]\}
\end{aligned}
$$

- where $\tilde{\beta} \equiv \beta \gamma^{\theta(1-\sigma)}$, the multipliers are interpreted as:
  - w_t, w_t^*: wage rate
  - λ_t, λ_t^*: price of existing capital
  - p_t: price of the final good(price of new capital)

Complete Markets

FOCs:

$$c_t : p_t = u_1(c_t, L_t) \tag{9}$$

$$L_t : w_t = u_2(c_t, L_t) \tag{10}$$

$$N_t : w_t = p_t A_t F_2(k_t, N_t) \tag{11}$$

$$i_t : p_t = \lambda_t \phi'(i_t / k_t) \tag{12}$$

$$k_{t+1} : \gamma\lambda_t = E_t \mu(\frac{i_{t+1}}{k_{t+1}})\tilde{\beta}\lambda_{t+1} + \tilde{\beta} E_t p_{t+1} A_{t+1} F_1(k_{t+1}, N_{t+1}) \tag{13}$$

$$w_t : 1 - L_t - N_t = 0 \tag{14}$$

$$\lambda_t : \gamma k_{t+1} = (1-\delta)k_t + \phi(i_t / k_t)k_t \tag{15}$$

$$p_t : \pi[y_t - c_t - i_t] + (1-\pi)[y_t^* - c_t^* - i_t^*] = 0 \tag{16}$$

where $\mu(z) \equiv [\phi(z) - z\phi_1(z) + (1-\delta)]$.

Small Open Economy: Partial Equilibrium

- The economy is too small to affect the world interest rate. Flow budget constraint: . $(P_t^B \equiv (1+r_t)^{-1})$

$$\gamma P_t^B b_{t+1} + c_t + i_t \leq y_t + b_t \tag{17}$$

$$\begin{aligned} \max \mathcal{L} \quad = \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \{ u(c_t, L_t) + w_t(1 - L_t - N_t) \\ & + \lambda_t [(1-\delta)k_t - (\gamma k_{t+1} - \phi(i_t/k_t)/k_t)] \\ & + p_t(y_t + b_t - \gamma P_t^B b_{t+1} - c_t - i_t) \} \end{aligned}$$

- Additional FOCs:

$$b_{t+1} : \tilde{\beta} E_t p_{t+1} - \gamma p_t P_t^B = 0 \tag{18}$$

$$p_t : b_t + A_t F(k_t, N_t) - c_t - i_t - \gamma P_t^B b_{t+1} = 0 \tag{19}$$

- Transversality Conditions:

$$\lim_{t \to \infty} \tilde{\beta}^t p_t b_{t+1} = 0 \tag{20}$$

General Equilibrium with Restricted Asset Markets

- Each country can trade a noncontingent real bond with residents of the other country. The interest rate adjusts to clear the bond market. Bond-market clearing condition:

$$\pi b_t + (1-\pi) b_t^* = 0 \tag{21}$$

- Aggregate financial asset accumulation satisfies:

$$\begin{aligned} &\gamma \pi P_t^B b_{t+1} + \gamma (1-\pi) P_t^B b_{t+1}^* \leq \\ &\qquad \pi [b_t + y_t - c_t - i_t] + (1-\pi)[b_t^* + y_t^* - c_t^* - i_t^*] \end{aligned} \tag{22}$$

which implies

$$\pi (A_t F(k_t, N_t) - i_t - c_t) + (1-\pi)(A_t^* F(k_t^*, N_t^*) - c_t^* - i_t^*) \geq 0 \tag{23}$$

General Equilibrium with Restricted Asset Markets

- FOCs:

$$b_{t+1} : \tilde{\beta} E_t p_{t+1} - \gamma p_t P_t^B = 0 \quad (24)$$
$$b_{t+1}^* : \tilde{\beta} E_t p_{t+1}^* - \gamma p_t^* P_t^B = 0 \quad (25)$$
$$p_t : b_t + A_t F(k_t, N_t) - c_t - i_t - \gamma P_t^B b_{t+1} = 0 \quad (26)$$
$$p_t : b_t^* + A_t^* F(k_t^*, N_t^*) - c_t^* - i_t^* - \gamma P_t^B b_{t+1}^* = 0 \quad (27)$$

- The interest rate is endogenously determined:

$$P_t^B = \tilde{\beta} E_t(p_{t+1}/\gamma p_t) = \tilde{\beta} E_t(p_{t+1}^*/\gamma p_t^*)$$

Current Section

Business Cycle Statistics

TABLE 1
BUSINESS CYCLE STATISTICS FOR 8 OECD COUNTRIES

	Relative volatility				Contemporaneous correlation	
Country	σ_c/σ_y	σ_i/σ_y	$\sigma_{nx/y}$	ρ_y	$\rho(c, y)$	$\rho(i, y)$
Australia	0.69	2.17	1.46	0.67	0.62	0.55
Canada	0.88	2.83	0.83	0.79	0.72	0.62
France	0.89	1.92	0.81	0.79	0.58	0.45
Germany	0.70	3.40	0.88	0.71	0.64	0.80
Italy	0.82	2.49	1.76	0.78	0.70	0.80
Japan	1.12	2.31	0.93	0.74	0.47	0.60
Switzerland	0.77	2.88	1.50	0.70	0.74	0.73
United States	0.67	3.00	0.41	0.84	0.88	0.90

	Correlation with same U.S. variable		Additional labor market statistics for the U.S.	
Country	output	consumption		
Australia	0.24	0.11	σ_N/σ_y:	0.84
Canada	0.77	0.65	$\sigma_{\text{prod}}/\sigma_y$:	0.57
France	0.50	0.28	$\rho(N, y)$:	0.83
Germany	0.44	0.45	$\rho(\text{prod}, y)$:	0.54
Italy	0.47	0.23	$\rho(\text{prod}, N)$:	−0.04
Japan	0.42	0.41		
Switzerland	0.28	0.22		
United States	1.00	1.00		

Calibration

CALIBRATED PARAMETER VALUES

σ	2	coefficient of relative risk aversion
α	0.58	labor share
β		is set so that the steady state annual real interest rate is 6.5%
γ	1.004	the average quarterly gross growth rate of the economy
δ	0.025	quarterly depreciation rate
η	15	the elasticity of the investment-capital ratio with respect to Tobin's Q

BKK's Productivity Shocks Process

$$\begin{bmatrix} \log A_t \\ \log A_t^* \end{bmatrix} = \begin{bmatrix} \rho & \nu \\ \nu^* & \rho^* \end{bmatrix} \begin{bmatrix} \log A_{t-1} \\ \log A_{t-1}^* \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix}$$

$$\begin{matrix} \text{U.S.:} \\ \text{Canada:} \end{matrix} \begin{bmatrix} \log A_t \\ \log A_t^* \end{bmatrix} = \begin{bmatrix} 0.796 & 0.131 \\ (0.079) & (0.052) \\ 0.000 & 0.989 \\ (0.093) & (0.060) \end{bmatrix} \begin{bmatrix} \log A_{t-1} \\ \log A_{t-1}^* \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix}; \rho(\epsilon_t, \epsilon_t^*) = 0.434$$

$$\begin{matrix} \text{U.S.:} \\ \text{Europe:} \end{matrix} \begin{bmatrix} \log A_t \\ \log A_t^* \end{bmatrix} = \begin{bmatrix} 0.904 & 0.052 \\ (0.074) & (0.041) \\ 0.149 & 0.908 \\ (0.064) & (0.036) \end{bmatrix} \begin{bmatrix} \log A_{t-1} \\ \log A_{t-1}^* \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix}; \rho(\epsilon_t, \epsilon_t^*) = 0.258.$$

- Shocks to productivity are highly persistent, and there is some evidence of transmission of shocks from one country to another.
- The innovations to productivity are positively correlated across countries.

Unit Root Test

H_0: The Solow residuals follow a random walk without spillovers, but with possibly correlated innovations. $\Rightarrow$ Fail to reject.

TABLE 2
STATISTICAL PROPERTIES OF INTERNATIONAL SOLOW RESIDUALS

Panel A: Park and Choi J(p, q) test for unit root

The null hypothesis is a unit root: the hypothesis is rejected if the test statistic is smaller than the critical value.

Measure of Solow Residual (time period)			Test statistic			
			$J(1,2)$	$J(1,3)$	$J(1,4)$	$J(1,5)$
United States	(1965:3–1988:3)		0.124	0.645	0.699	0.745
Canada	(1965:3–1988:3)		0.343	1.346	1.948	3.461
United States	(1970:2–1986:4)		0.010	0.255	0.275	0.309
Europe	(1970:2–1986:4)		0.740	0.946	0.967	1.179
critical values:		1%	8.6*e*-5	0.011	0.055	0.123
		5%	0.002	0.055	0.160	0.295
		10%	0.009	0.120	0.290	0.452

Cointegration Tests

TABLE 2

STATISTICAL PROPERTIES OF INTERNATIONAL SOLOW RESIDUALS

Panel B: Tests for cointegration

We used Park's canonical cointegrating regression to estimate α_1 such that $\log A_t - \alpha \log A_t^* = \epsilon_t$, a stationary random variable. Next, we used Park's $H(p,q)$ test for stochastic cointegration; p-values are given in the table below. In each case, the United States is the unstarred variable (i.e., α is the coefficient on Canada and Europe).

			p-values			
	$\hat{\alpha}$	$se(\hat{\alpha})$	$H(1,2)$	$H(1,3)$	$H(1,4)$	$H(1,5)$
United States–Canada	0.580	0.061	0.313	0.523	0.707	0.462
United States–Europe	0.603	0.041	0.046	0.039	0.082	0.145

Panel C: Estimates of stochastic processes for Solow residuals

Δ denotes the first difference of the log of a variable, i.e., $\Delta A_t \equiv \log A_t - \log A_{t-1}$; as before the United States is the unstarred country. Standard errors are in parentheses.

United States–Canada:

$$\Delta A_t = \underset{(0.001)}{0.003} + \underset{(0.117)}{0.113}\,\Delta A_{t-1} + \underset{(0.101)}{0.048}\,\Delta A_{t-1}^* - \underset{(0.052)}{0.074}\,(A_{t-1} - A_{t-1}^*) + u_t$$

$$\Delta A_t^* = \underset{(0.001)}{0.005} + \underset{(0.131)}{0.283}\,\Delta A_{t-1} + \underset{(0.112)}{0.035}\,\Delta A_{t-1}^* + \underset{(0.058)}{0.021}\,(A_{t-1} - A_{t-1}^*) + u_t^*$$

$$\hat{\sigma}_u^2 = 8.38e^{-3}; \qquad \hat{\sigma}_{u^*}^2 = 9.34e^{-3}; \qquad \hat{\rho}(u, u^*) = 0.392.$$

United States–Europe: (error-correction term omitted due to lack of cointegration)

$$\Delta A_t = \underset{(0.001)}{0.002} + \underset{(0.126)}{0.003}\,\Delta A_{t-1} + \underset{(0.134)}{0.193}\,\Delta A_{t-1}^* + u_t$$

$$\Delta A_t^* = \underset{(0.001)}{0.005} + \underset{(0.110)}{0.196}\,\Delta A_{t-1} - \underset{(0.117)}{0.076}\,\Delta A_{t-1}^* + u_t^*$$

$$\hat{\sigma}_u^2 = 9.07e^{-3}; \qquad \hat{\sigma}_{u^*}^2 = 7.95e^{-3}; \qquad \hat{\rho}(u, u^*) = 0.228.$$

Trend Stationary Productivity with Spillovers

$$\rho = \rho^* = 0.906, \nu = \nu^* = 0.088, \rho(\epsilon_t, \epsilon_t^*) = 0.258.$$

TABLE 3
TREND STATIONARY SHOCKS

(1) Results for complete markets economy
(2) Results for economy trading noncontingent bonds and goods only

	Standard deviation		Relative standard deviation		Persistence		Correlation w/y, lag 0			Other correlations	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)		(1)	(2)
Output	2.01	1.99	1.00	1.00	0.75	0.76	1.00	1.00	y, y^*	0.04	0.06
Consumption	0.97	0.98	0.48	0.49	0.81	0.81	0.82	0.84	c, c^*	0.95	0.92
Investment	3.72	3.55	1.85	1.79	0.73	0.74	0.98	0.97	i, i^*	0.02	0.12
Labor	1.07	1.02	0.53	0.51	0.73	0.72	0.91	0.91	N, N^*	−0.70	−0.67
Wage	1.13	1.14	0.56	0.57	0.80	0.80	0.92	0.93	w, w^*	0.75	0.72
Net exports	0.57	0.59	0.29	0.30	0.80	0.80	0.65	0.65	s, i	0.95	0.94
Bonds	0.00	3.22	0.00	1.62	0.00	0.98	0.00	0.23	w, N	0.66	0.69

Random-Walk Productivity without Spillovers

$\rho = \rho^* = 1, \nu = \nu^* = 0.$

TABLE 4
UNIT ROOT IN PRODUCTIVITY

(1) Results for complete markets economy
(2) Results for economy trading noncontingent bonds and goods only

	Standard deviation		Relative standard deviation		Persistence		Correlation w/y, lag 0			Other correlations	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)		(1)	(2)
Output	2.58	1.59	1.00	1.00	0.87	0.82	1.00	1.00	y, y^*	−0.41	0.54
Consumption	1.03	1.67	0.40	1.05	0.82	0.80	0.72	0.85	c, c^*	0.89	−0.28
Investment	11.84	4.74	4.60	2.98	0.77	0.78	0.71	0.74	i, i^*	−0.92	−0.50
Labor	1.58	0.71	0.61	0.45	0.89	0.78	0.93	0.19	N, N^*	−0.91	−0.56
Wage	1.25	1.62	0.48	1.02	0.83	0.80	0.89	0.90	w, w^*	0.50	−0.11
Net exports	2.39	1.61	0.93	1.01	0.81	0.77	−0.18	−0.28	s, i	0.74	0.04
Bonds	0.00	8.18	0.00	5.13	0.00	0.98	0.00	0.35	w, N	0.66	−0.25

Cross-Country Correlation v.s. Persistence

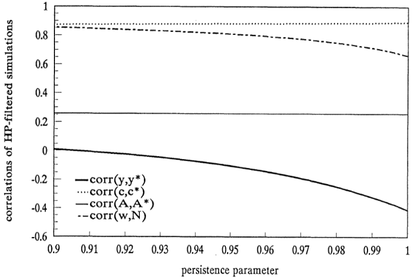


FIGURE 1A

COMPLETE MARKETS MODEL WITHOUT SPILLOVERS

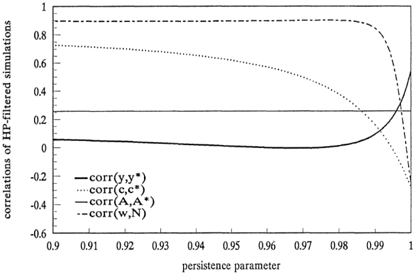
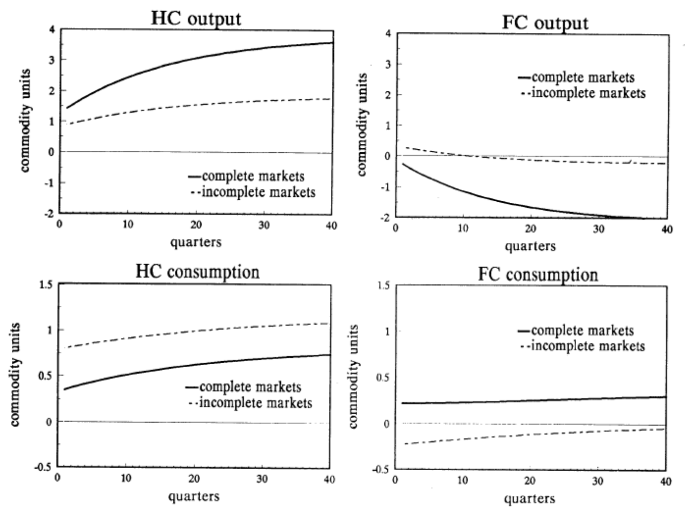


FIGURE 1B

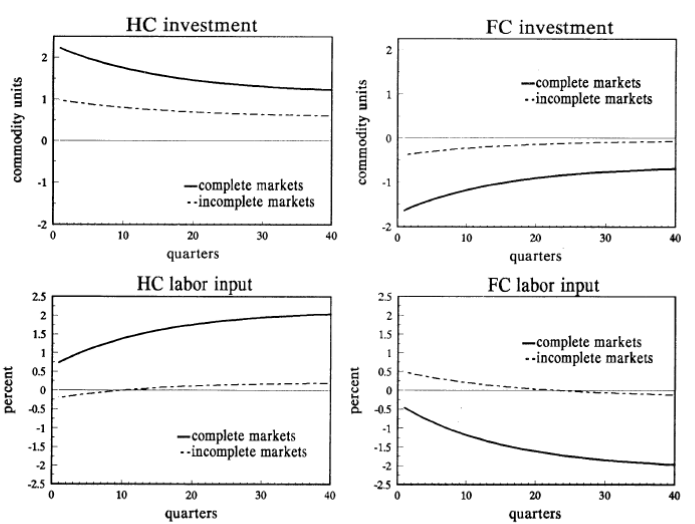
INCOMPLETE MARKETS MODEL WITHOUT SPILLOVERS

Current Section

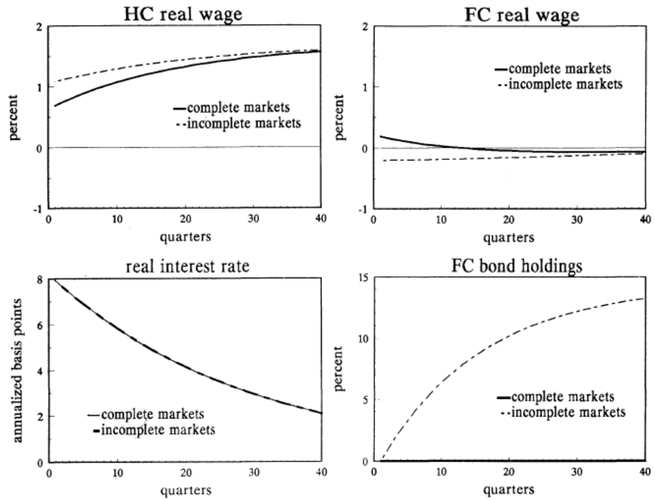
Random Walk Productivity



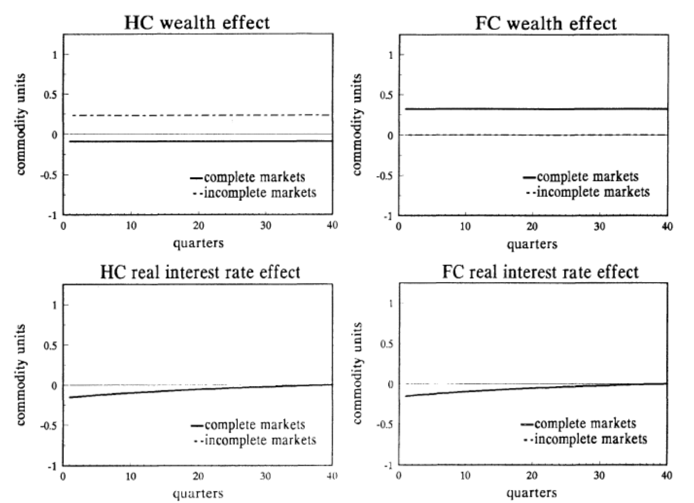
Random Walk Productivity II



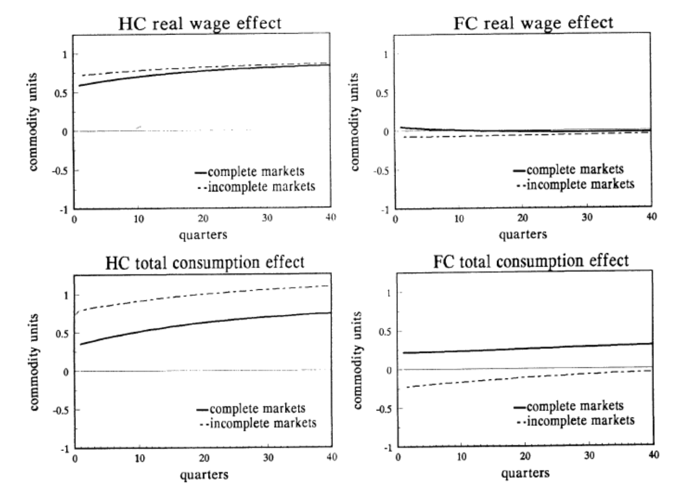
Random Walk Productivity III



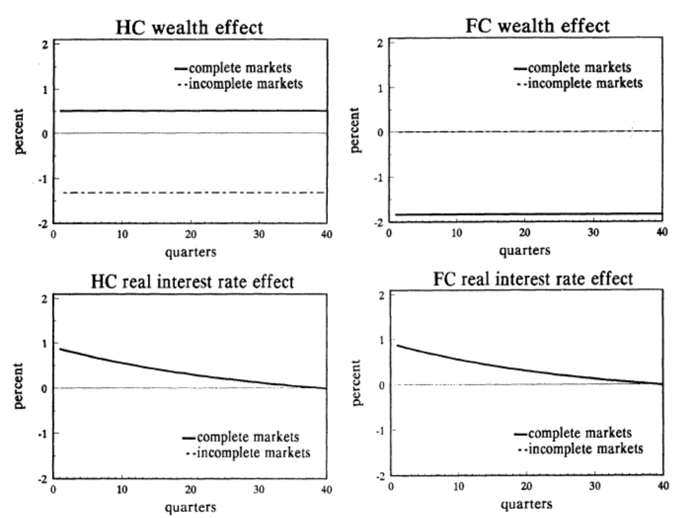
Hicksian Decomposition of Consumption Response



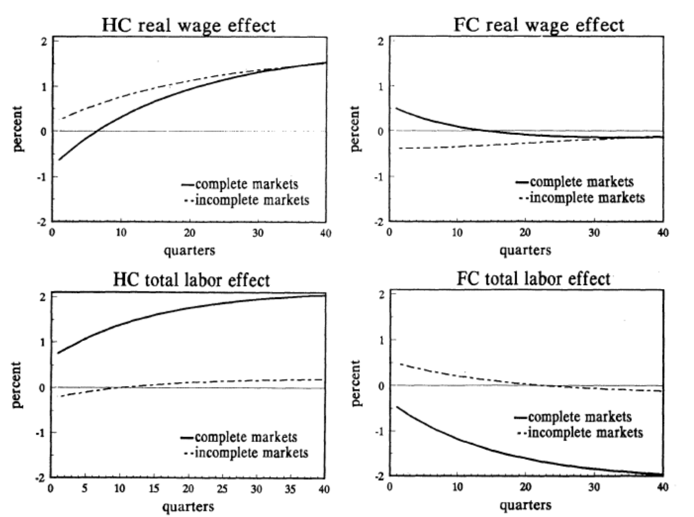
Hicksian Decomposition of Consumption Response II



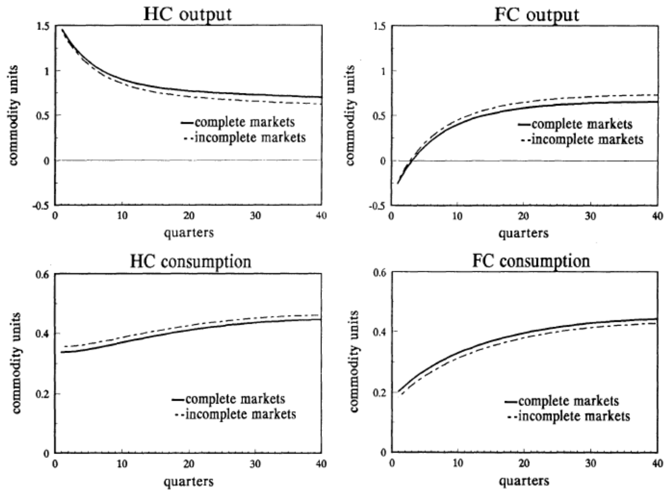
Hicksian Decomposition of Labor Response



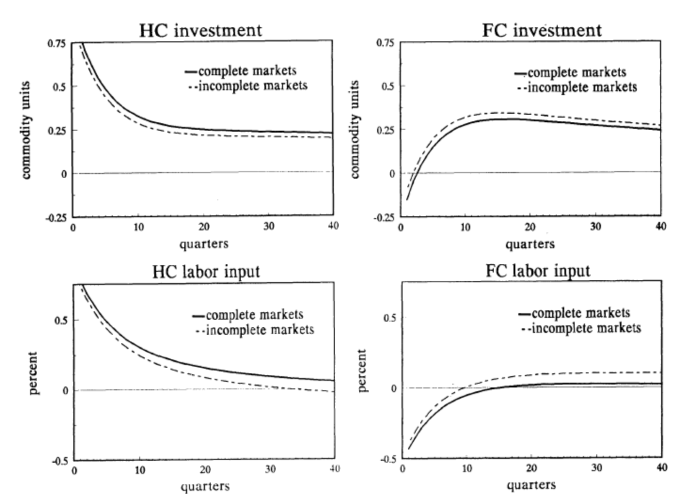
Hicksian Decomposition of Labor Response II



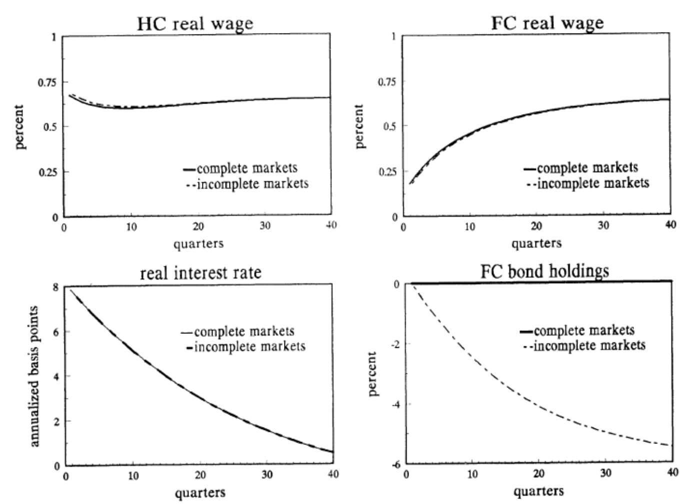
Trend-Stationary Shocks with Spillovers



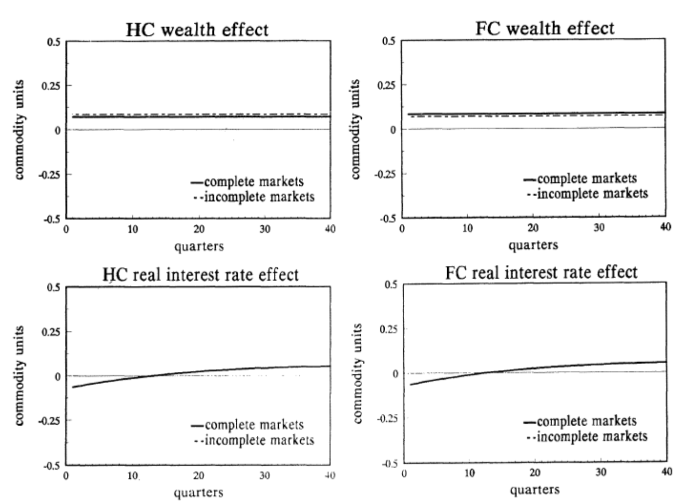
Trend-Stationary Shocks with Spillovers II



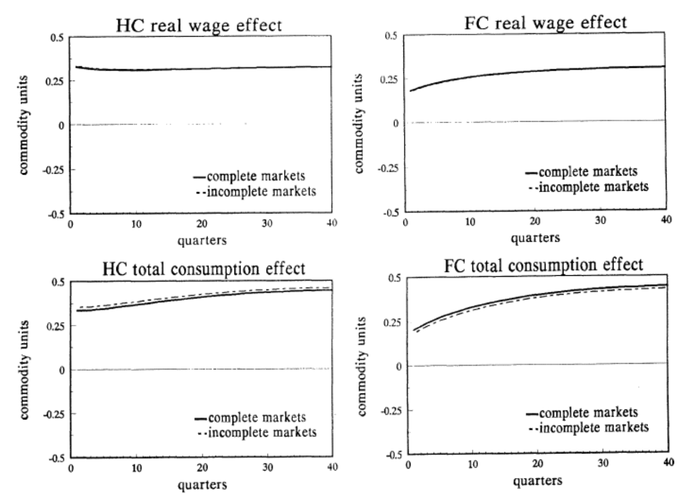
Trend-Stationary Shocks with Spillovers III



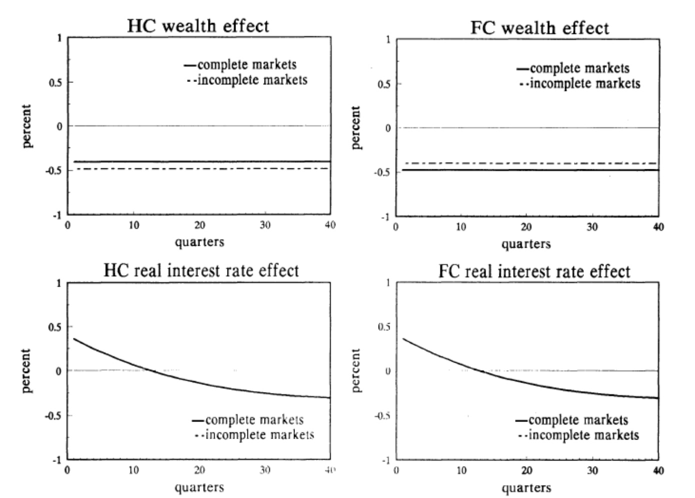
Hicksian Decomposition of Consumption Response



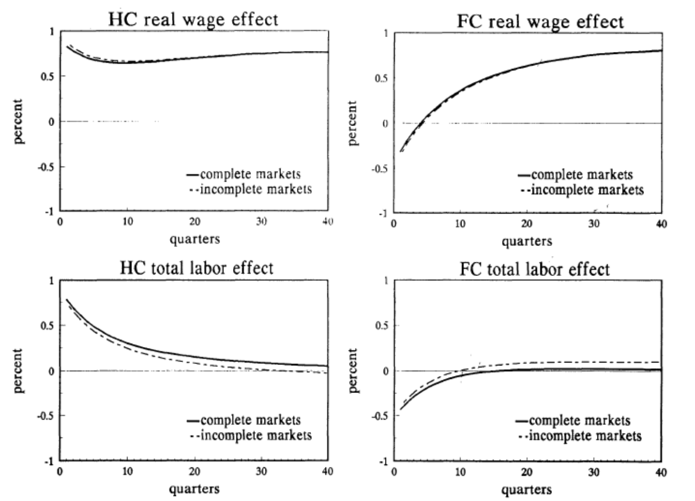
Hicksian Decomposition of Consumption Response II



Hicksian Decomposition of Labor Response



Hicksian Decomposition of Labor Response II



Current Section

Crucial Findings

1. Incomplete Markets v.s. Complete Markets model
   - Trend stationary international productivity process with substantial international ”spillovers” of productivity shocks, indistinguishable.
   - Productivity in each country follows a random walk without spillovers but with correlated innovations, quite different.
2. Major differences in the macroeconomic response to shocks under the alternative asset structures are due almost entirely to differential wealth effects.
3. Incomplete markets with random-walk productivity specification can generate results which are closer to data.