Artificial Intelligence

Heuristic Search

Andrea Torsello

Search in AI

Blind search strategies

- Breadth First
- Depth First
- Iterative Deepening

Analyzed

- Completeness
- Optimality
- Time Complexity
- Space Complexity

Optimality evaluated only in terms of number of steps

Uniform Cost Search

Uniform Cost is a **blind search** algorithm that is **optimal** according to any specified **path length** function

Can be obtained from the generic search stab function by selecting from ***L*** the node with the *minimal path length* from the *root*

- Can be implemented with a heap or any efficient priority queue implementation

Let ***L*** be a list of **visited** but not **expanded** nodes

1) Initialize ***L*** with the initial state
2) If ***L*** is empty, **FAIL**, else *extract* from ***L*** node ***n*** with minimum path length from the root
3) If ***n*** is a goal node, **SUCCEED** and return the path from the initial state to ***n***
4) Remove ***n*** from ***L*** and *insert* all the children of ***n***
5) Goto 2)

Time/Space complexity

Uniform cost is guided by path length rather than depth, so its time complexity cannot easily be characterized in terms of *b* and *d*

Rather, let *C** be the path length of the optimal solution and *ε* the minimum cost of any action.

Then the worst-case time and space complexity is

$$O(b^{1+\lfloor C^*/\epsilon \rfloor})$$

Which can be much greater than b^d

- Uniform cost can explore large trees of small steps before exploring paths with large and potentially useful steps.

When all steps costs are equal, uniform cost is similar to breadth first.

Heuristics

The assumption behind **blind search** is that we have no way of telling whether a particular search direction is likely to lead us to the goal or not

The key idea behind **informed** or **heuristic search** algorithms is to exploit a task-specific measure of goodness to try to either reach the goal more quickly or find a more desirable goal state.

Heuristic: From the Greek for “find”, “discover”.

> Heuristics are criteria, methods, or principles for deciding which among several alternative courses of action promises to be the most effective in order to achieve some goal”.
>
> Judea Pearl “ Heuristics” 1984

Problem knowledge comes in the form of an **heuristic function** *h'(n)*

- *h'(n)* assigns to each node *n* an estimate of the optimal path length from *n* to a goal node
- We assume that *h'* is non-negative and that *h'(n)=0* iff *n* is a goal node.

Heuristics for the game of 8

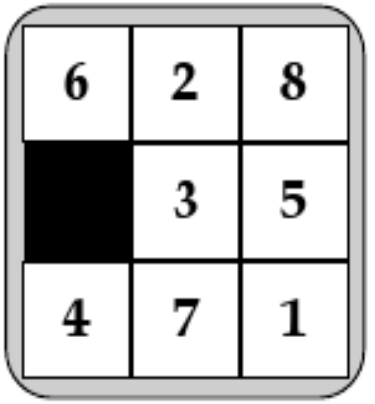
$h'_1(n)$ = number of misplaced tiles

$h'_2(n)$ = sum of Manhattan distance* of each tile

Example:

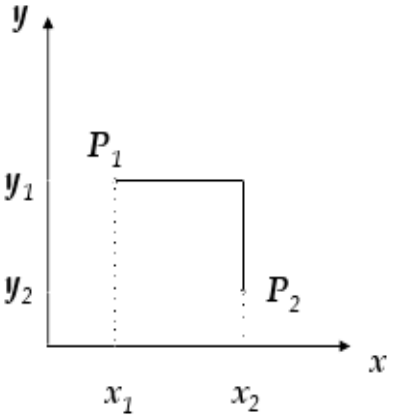
$h'_1(n) = 7$

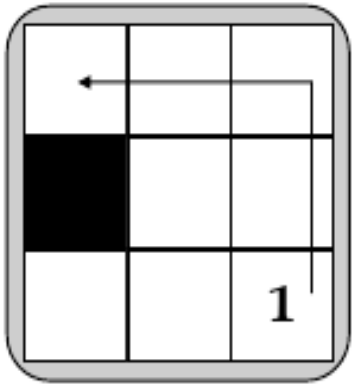
$h'_2(n) = 4+0+2+3+1+3+1+3=17$



* Given two planar points of coordinate $P_1=(x_1,y_1)$ and $P_2=(x_2,y_2)$, the Manhattan distance is defined as

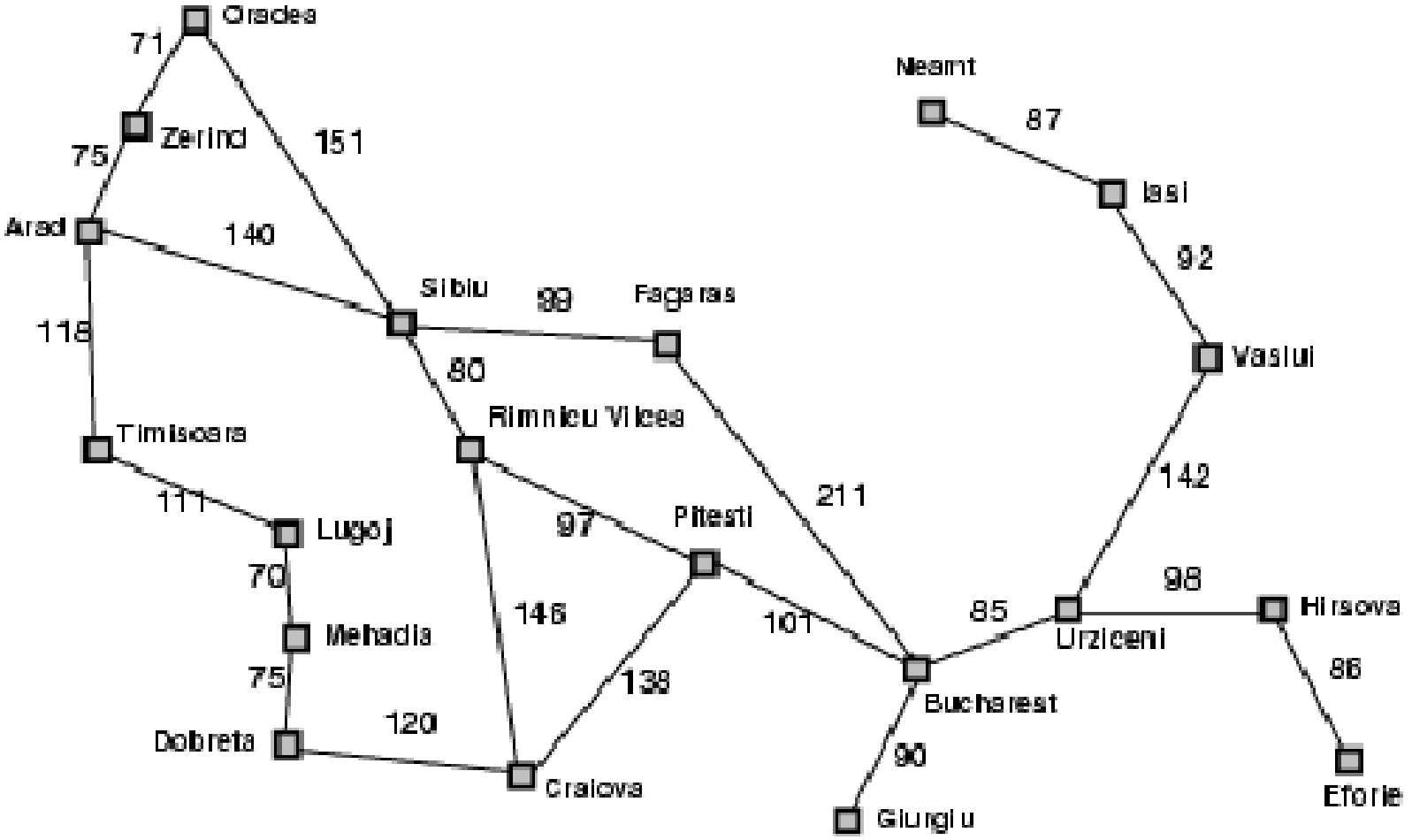
$$D(P_1, P_2) = |x2 - x1| + |y2 - y1|$$





Travel Planning

h'(n) = straight-line distance between *n* and goal node



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Best First (Greedy) Search

The idea behind best first search is to always explore the most promising path first.

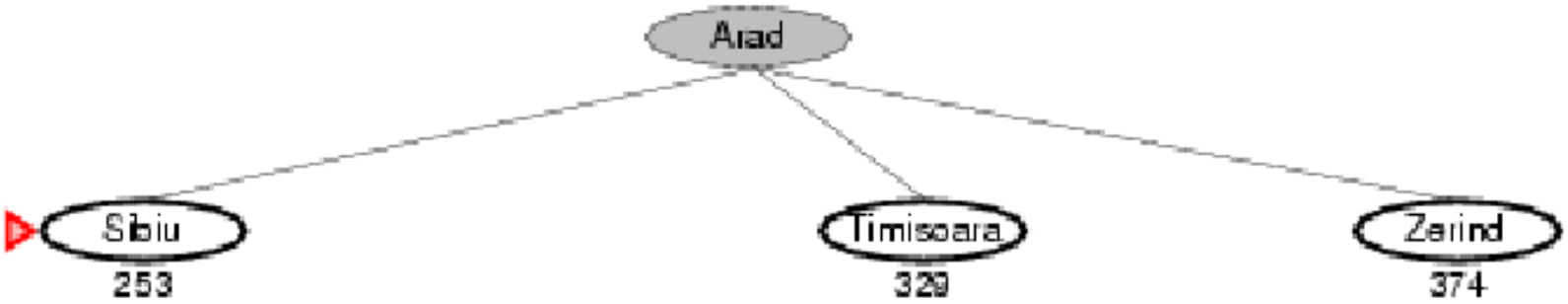
Can be obtained from the generic search stab function by selecting from ***L*** the node with the minimal value of the *expected distance to goal* (*value of heuristic function*)

Let ***L*** be a list of **visited** but not **expanded** nodes

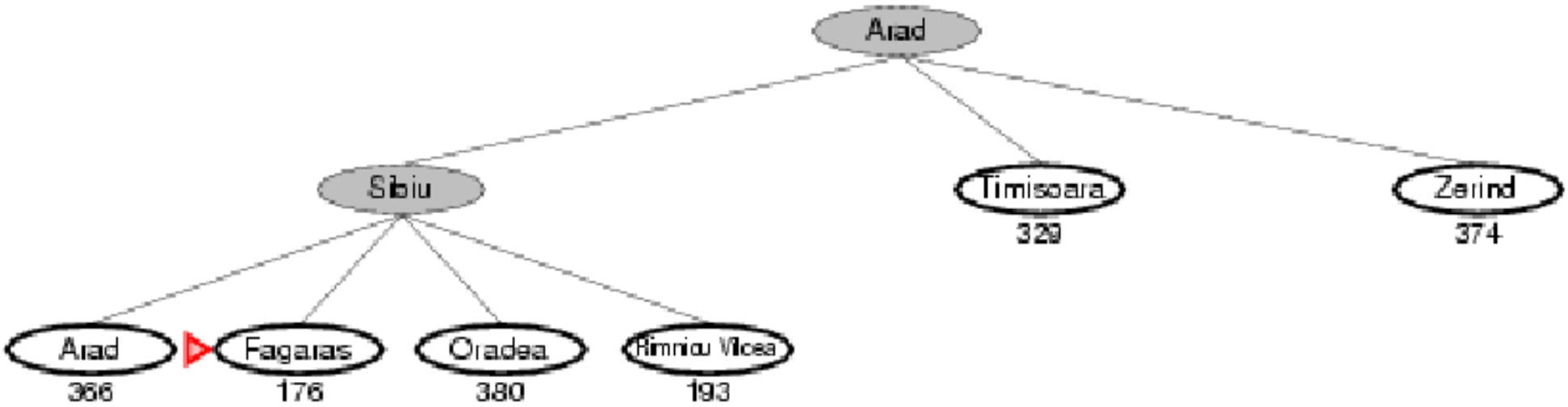
1) Initialize ***L*** with the initial state
2) If ***L*** is empty, **FAIL**, else *extract* from ***L*** node ***n*** with minimum value of *h'(**n**)*
3) If ***n*** is a goal node, **SUCCEED** and return the path from the initial state to ***n***
4) Remove ***n*** from ***L*** and *insert* all the children of ***n***
5) Goto 2)

Best First Search Example

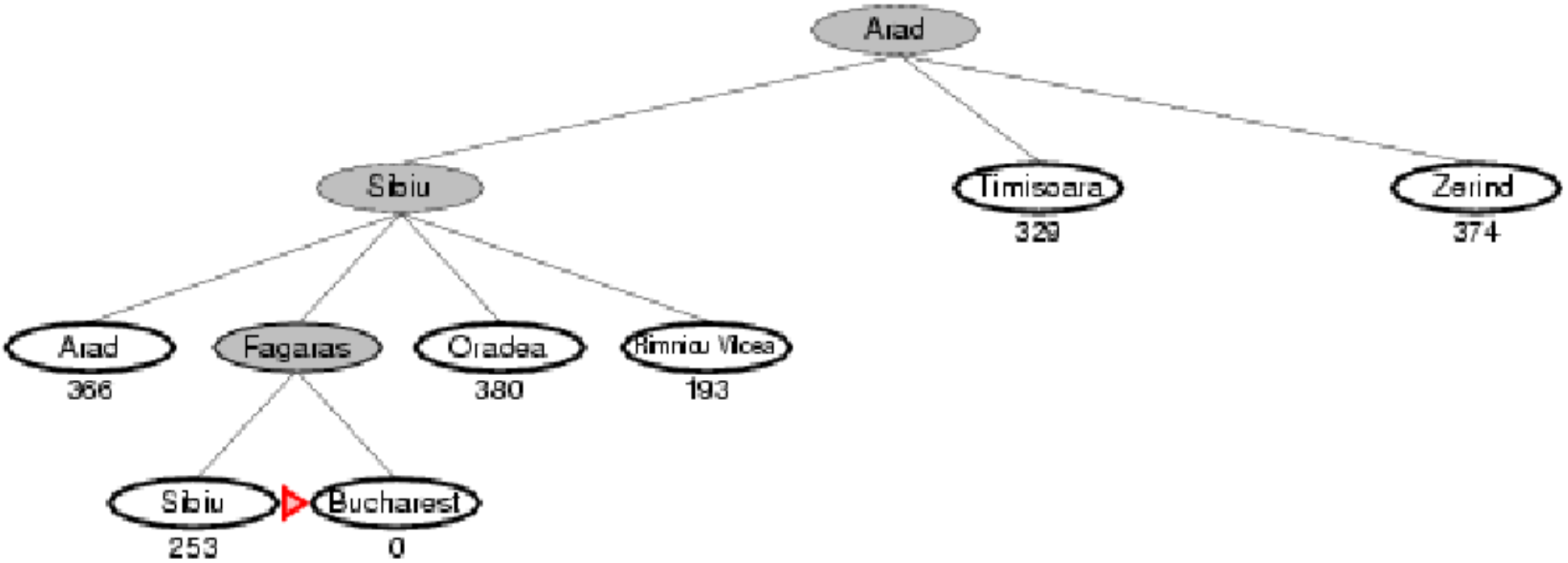
Best First Search Example



Best First Search Example



Best First Search Example



Analysis of Best First

Complete? No – can get stuck in loops, e.g., Iasi -> Neamt -> Iasi -> Neamt

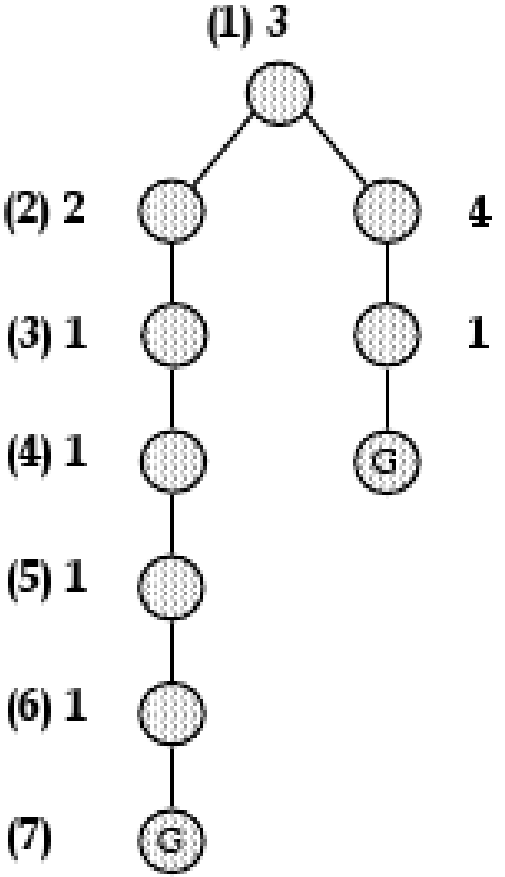
Time? O(bm), but a good heuristic can give dramatic improvement

Space? O(bm) -- keeps all nodes in memory

Optimal? No

Example of non optimality

- The number in parentheses represent the order of expansion
- The other the value of the heuristic function *h'*



A*

A* algorithm mixes the optimality of uniform cost with the heuristic search of best first

A* realizes a best first search with evaluation function

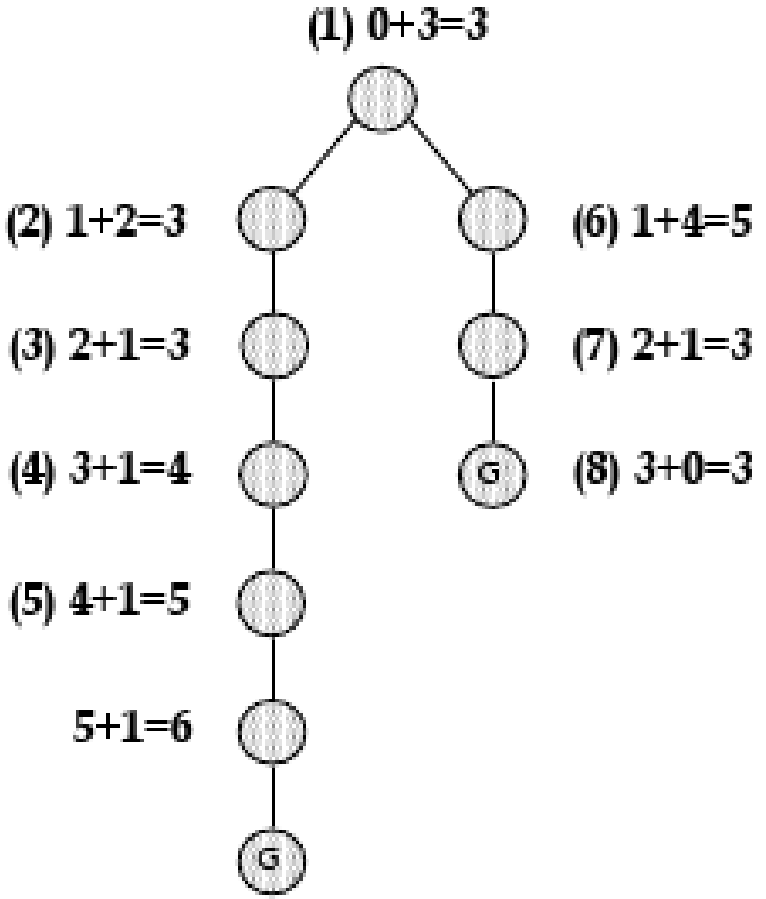
$$f(n) = g(n) + h'(n)$$

with

- *g(n)* is the path length from the root to ***n***
- *h'(n)* is the heuristic prediction of the cost from ***n*** to the goal

Let ***L*** be a list of **visited** but not **expanded** nodes

1) Initialize ***L*** with the initial state
2) If ***L*** is empty, **FAIL**, else *extract* from ***L*** node ***n*** with minimum value of *f(**n**)=g(**n**)+h'(**n**)*
3) If ***n*** is a goal node, **SUCCEED** and return the path from the initial state to ***n***
4) Remove ***n*** from ***L*** and *insert* all the children of ***n***
5) Goto 2)



A* search example

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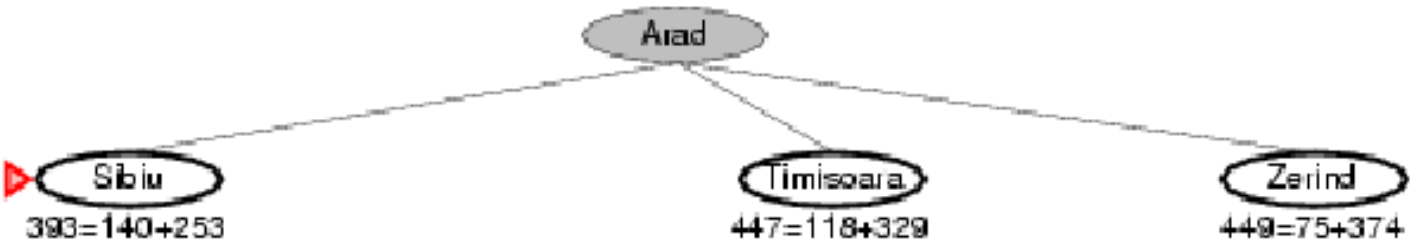
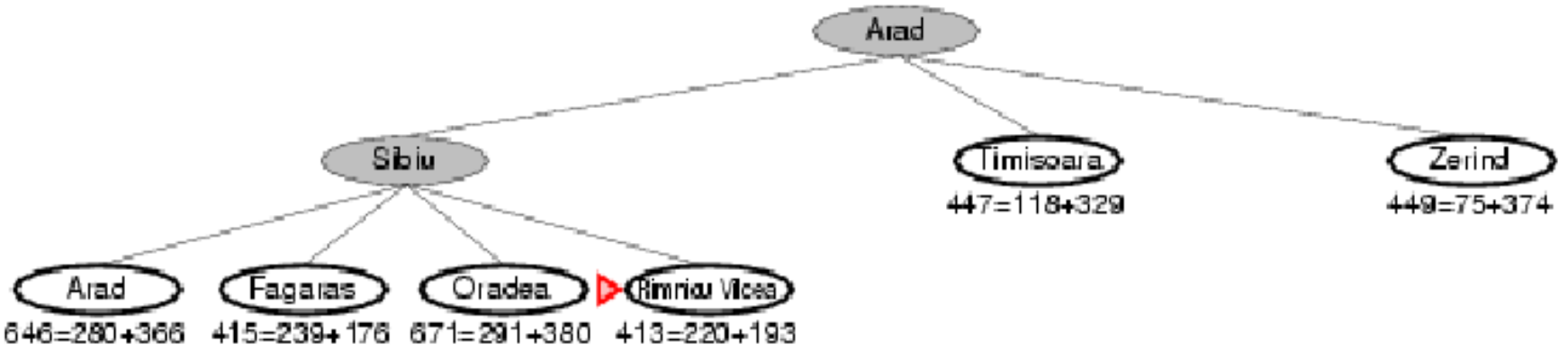
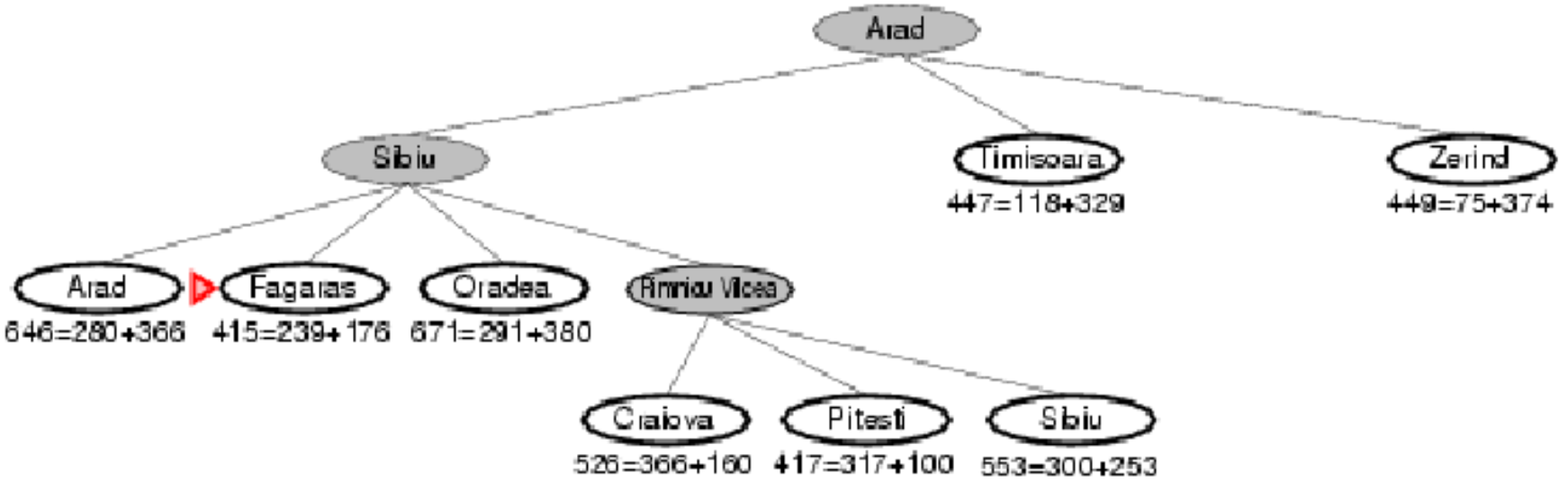


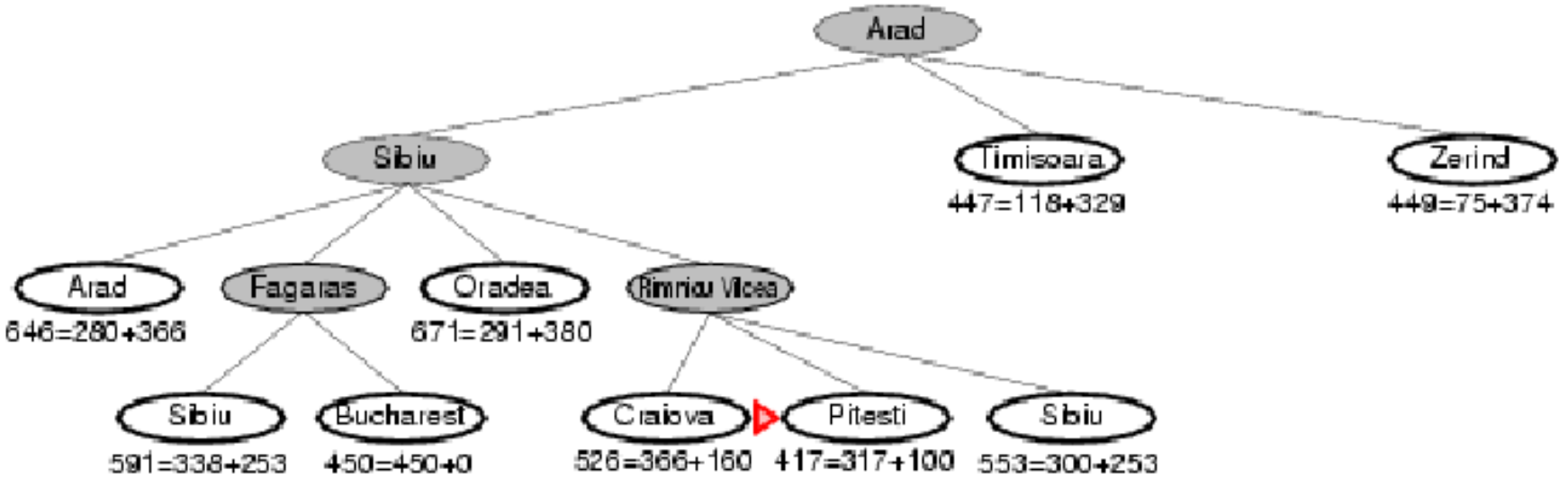
A* search example



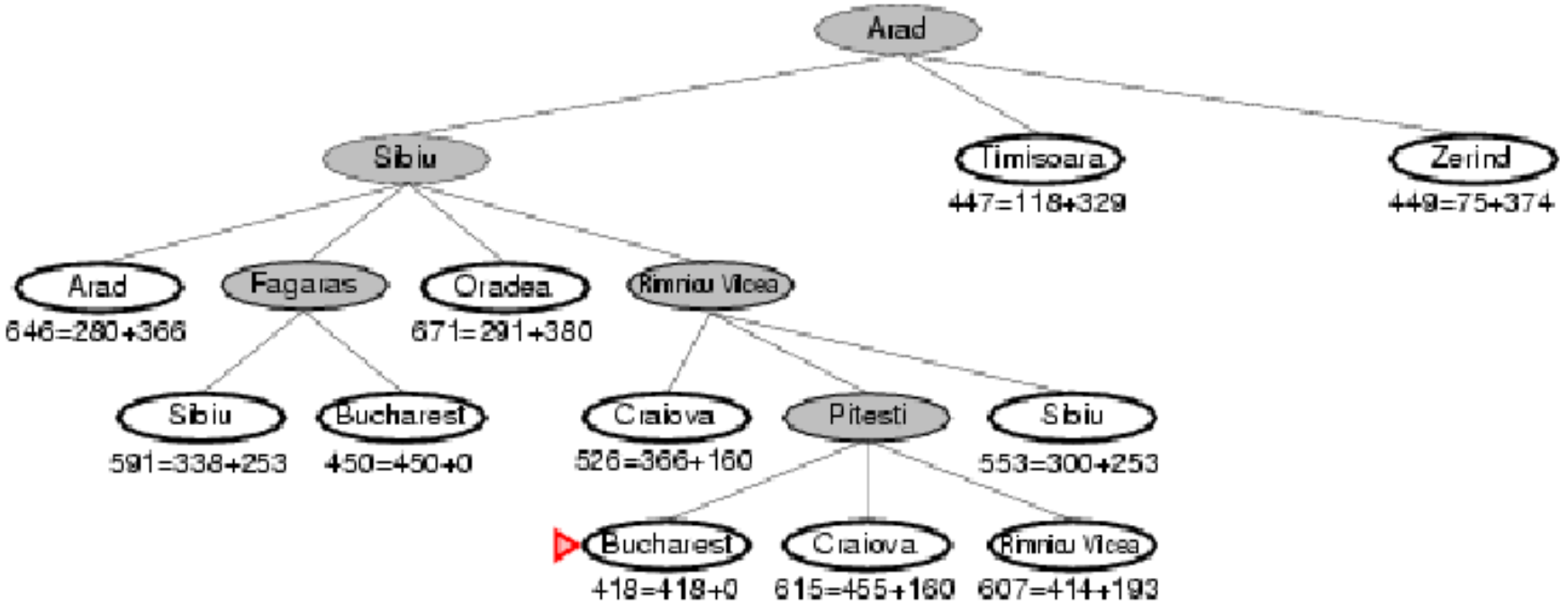
A* search example



A* search example



A* search example



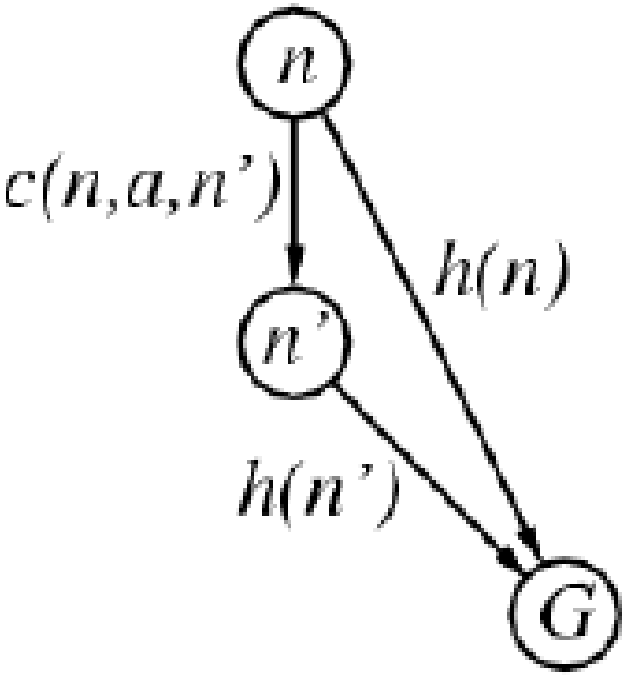
Consistent heuristics

A heuristic is consistent if for every node ***n***, every successor ***n'*** of ***n*** generated by any action *a*, with cost *c(**n**,a,**n'**)*

$$h'(n) \leq c(n, a, n') + h'(n')$$

If *h'* is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h'(n') \\ &= g(n) + c(n, a, n') + h'(n') \\ &\geq g(n) + h'(n) \\ &= f(n) \end{aligned}$$



i.e., *f(**n**)* is non-decreasing along any path

Corollary: *h'(**n**)* ≤ *h(**n**)*, where *h(**n**)* is the real (unknown) distance from the goal

Optimality of A*

Theorem: If function h' is consistent, then A* is optimal

Proof: Let ***s*** be an optimal goal node and ***x*** a non-optimal goal.
Let $\boldsymbol{n}_0\ \boldsymbol{n}_1\ \ldots\ \boldsymbol{n}_k = \boldsymbol{s}$ be the path from the root to ***s***. For all $i=0\ldots k$, we have $f(\boldsymbol{n}_i) < f(\boldsymbol{x})$.
In fact:

$$\begin{aligned} f(n_i) &= g(n_i) + h'(n_i) \leq g(n_i) + h(n_i) = g(s) \\ &< g(x) = g(x) + h'(x) = f(x) \end{aligned}$$

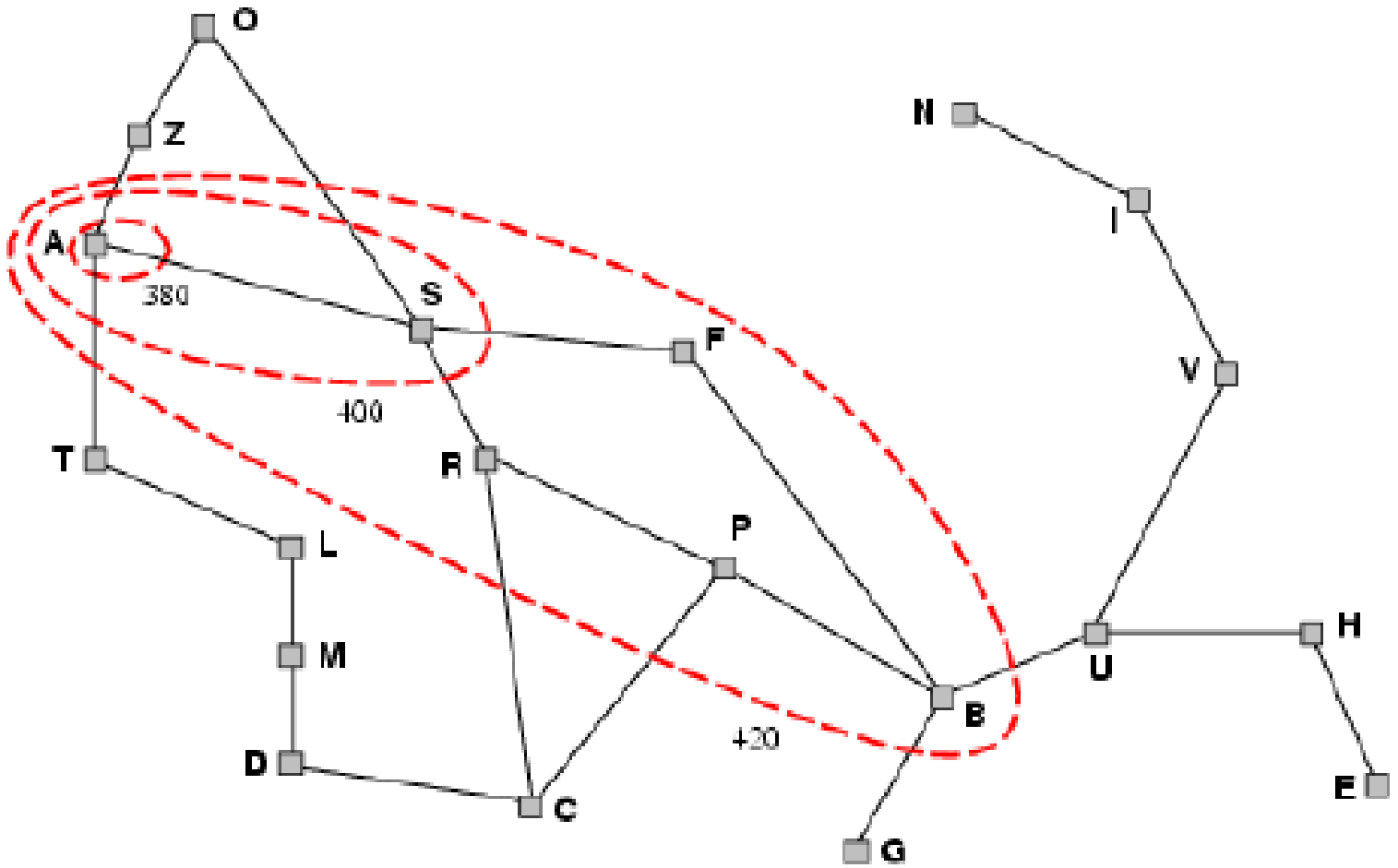
But then ***s*** must be expanded before ***x***.

Optimality of A*

A* expands nodes in order of increasing f value

Gradually adds "f-contours" of nodes

Contour i has all nodes with f=fi, where fi < fi+1



Properties of A*

Complete?	Yes (unless there are infinitely many nodes with $f \leq f(G)$)
Time?	Exponential
Space?	Exponential (Keeps all nodes in memory)
Optimal?	Yes

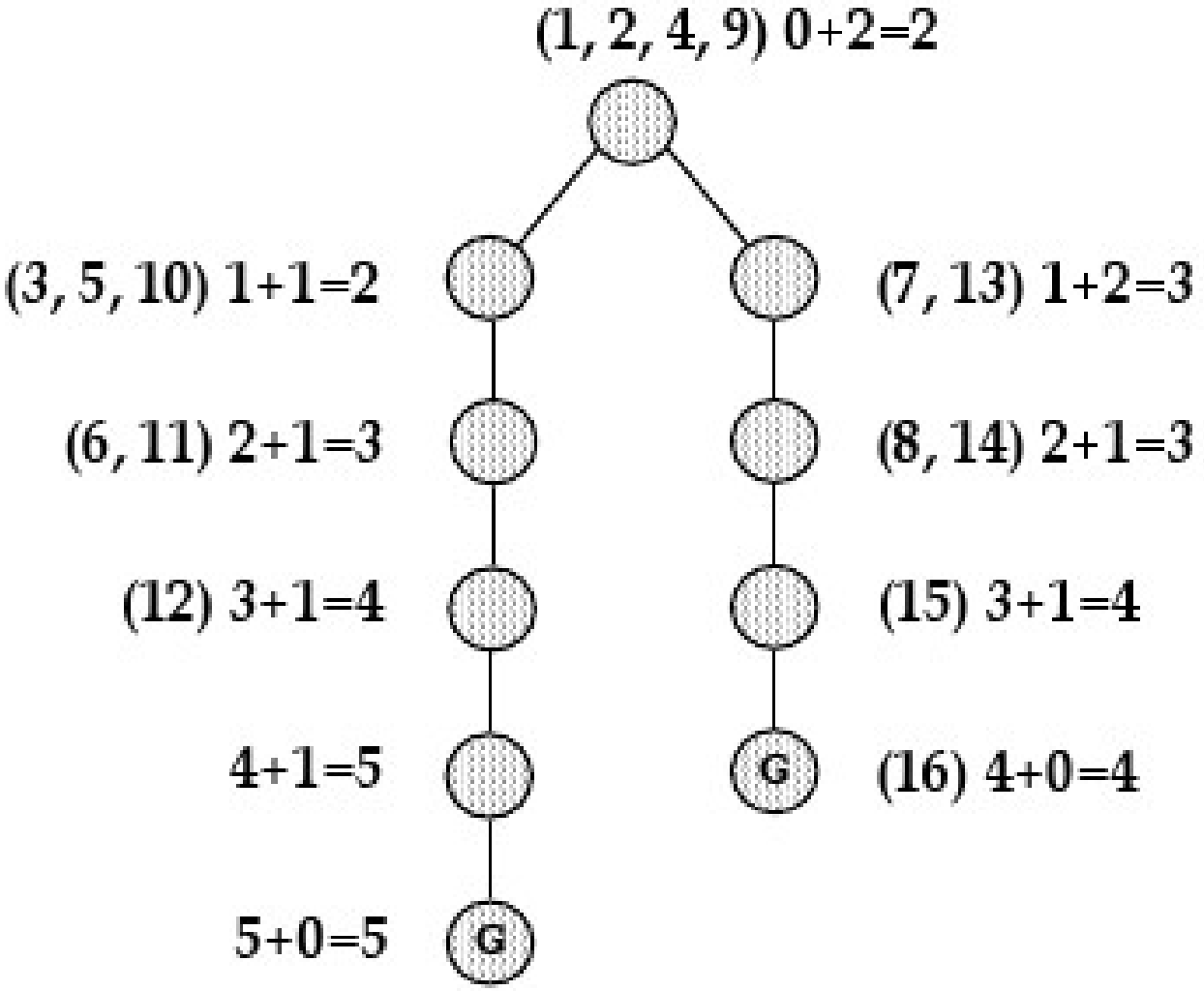
A* is an Heuristic counterpart of breadth first. We can improve the space requirements in a way similar to iterative deepening

Iterative Deepening A* (IDA*)

Let L be the list of visited but not expanded node, and ***C*** the maximum depth

1) Let ***C***=0
2) Initialize ***L*** to the initial state (only)
3) If ***L*** is empty increase ***C*** and goto 2),
   else *extract* a node ***n*** from the *front* of L
4) If ***n*** is a goal node, **SUCCEED** and return the path from the initial state to ***n***
5) Remove ***n*** from ***L***. If the level is smaller than ***C***, *insert* at the *front* of ***L*** all the children ***n***' of ***n*** with $f(\boldsymbol{n}') \leq \boldsymbol{C}$
6) Goto 3)

IDA* Example


(1, 2, 4, 9) 0+2=2
(3, 5, 10) 1+1=2
(7, 13) 1+2=3
(6, 11) 2+1=3
(8, 14) 2+1=3
(12) 3+1=4
(15) 3+1=4
4+1=5
G
(16) 4+0=4
5+0=5
G