Support Vector Machines and Multiple Kernel Learning

(lets do it large scale with shogun)

Sören Sonnenburg

Shogun Toolbox Data Days 2013, Juli 12, Berlin

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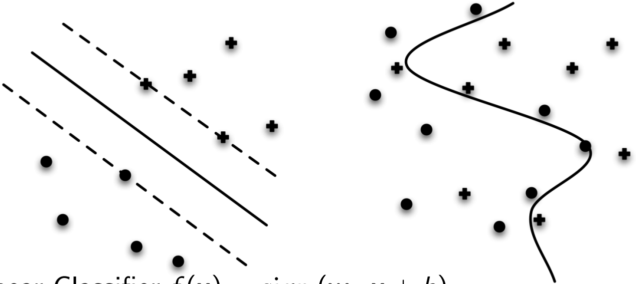
Outline

Classification

Given training examples $(\mathbf{x}_i, y_i)_{i=1}^N \in (\mathcal{X}, \{-1, +1\})^N$



- Linear Classifier $f(\mathbf{x}) = \operatorname{sign}(\mathbf{w} \cdot \mathbf{x} + b)$
- Kernel Machine (e.g. Support Vector Machine), learn weighting $\boldsymbol{\alpha} \in \mathbb{R}^N$ on training examples in kernel feature space $\Phi(\mathbf{x})$

$$f(\mathbf{x}) = \operatorname{sign}\left(\sum_{i=1}^{N} y_i \alpha_i k(\mathbf{x}, \mathbf{x}_i) + b\right),$$

where Kernel $k(\mathbf{x}, \mathbf{x}') = \Phi(\mathbf{x}) \cdot \Phi(\mathbf{x}')$

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Classification using Support Vector Machines

Via kernel: non-linear discrimination in input space

- learn weighting $\boldsymbol{\alpha} \in \mathbb{R}^N$ on training examples $(\mathbf{x}_i, y_i)_{i=1}^N$ in kernel feature space $\Phi(\mathbf{x})$

$$f(\mathbf{x}) = \operatorname{sign}\left(\sum_{i=1}^{N} y_i \alpha_i \mathrm{k}(\mathbf{x}, \mathbf{x}_i) + b\right)$$

- where Kernel $k(\mathbf{x}, \mathbf{x}') = \Phi(\mathbf{x}) \cdot \Phi(\mathbf{x}')$
- still linear classifier $f(\mathbf{x}) = \operatorname{sign}(\mathbf{w} \cdot \mathbf{x} + b)$ in kernel feature space, with weighting $\mathbf{w} = \sum_{i=1}^N y_i \alpha_i \Phi(\mathbf{x}_i)$ and examples $\mathbf{x} \mapsto \Phi(\mathbf{x})$

SVMs rock!

- Kernels - flexible!
- In many applications SVMs define the state-of-the-art!

Classification using Support Vector Machines

Via **kernel: non-linear** discrimination in input space

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Support Vector Machines in Shogun I

Implementations supporting Kernels

- LibSVM, LibSVMOneClass, MulticlassLibSVM, LibSVR (sequential minimal optimization)
- GNPPSVM (ℓ_2-SVM)
- GPBTSVM (chunking algorithm based SVM)
- GMNPSVM (true multi-class SVM)
- LaRank (true multi-class SVM)
- MPDSVM (minimal primal-dual SVM)
- SVMLight, SVMLightOneClass, SVRLight (chunking based, thread-parallel non-GPL)
- ScatterSVM (multi-class approximation)

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Kernels

Popular Kernels

- Gaussian, Polynomial, String Kernels
- Custom Kernel (bring your own)

Kernel Normalizers

- SqrtDiagKernelNormalizer (points on hypersphere with $R = 1$)
- ZeroMeanCenterKernelNormalizer (centered in feature space)
- AvgDiagKernelNormalizer, more `http://bit.ly/133aG2V`

Most comprehensive Kernel/SVM Framework

- Over 60 kernels available
- `http://bit.ly/15zlCWY`
- normalizers can be attached to kernels

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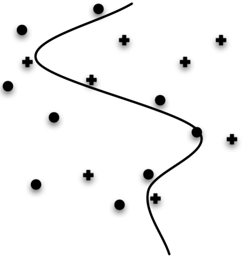
Kernel based Support Vector Machines



- SVMs learn weights $\boldsymbol{\alpha} \in \mathbb{R}^m$ over training examples in kernel feature space $\Phi : \mathbf{x} \mapsto \mathbb{R}^n$

- Decision function $f(\mathbf{x}) = \mathrm{sign}\left(\sum_{i=1}^{m} y_i \alpha_i \mathrm{k}(\mathbf{x}, \mathbf{x}_i) + b\right)$,
  with kernel $k(\mathbf{x}, \mathbf{x}') = \langle \Phi(\mathbf{x}), \Phi(\mathbf{x}') \rangle$

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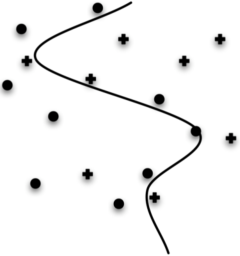

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Kernel based Support Vector Machines are a Dead End

The Curse of Support Vectors

To compute output on all m examples $\mathbf{x}_1, \ldots, \mathbf{x}_m$:

$$\forall j = 1, \ldots, m : \quad \sum_{i=1}^{m_s} \alpha_i y_i \, \mathrm{k}(\mathbf{x}_i, \mathbf{x}_j) + b$$

Computational effort:

- All $\mathcal{O}(m_s m T)$, (T time to compute the kernel)
- Effort Scales linearly with $m_s = \mathcal{O}(m) := \#\mathrm{SVs}$

$\Rightarrow$ **SVM's in bigO are not faster than standard k-NN.**

$\Rightarrow$ **Kernel Machines are just not large-scale!**

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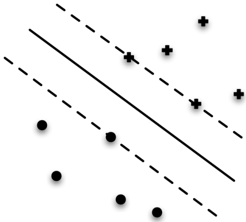
What about Linear SVMs ?



- **Linear** Support Vector Machines learn weights $\mathbf{w} \in \mathbb{R}^n$
- Decision function $\mathbf{f}(\mathbf{x}) = \langle \mathbf{w}, \mathbf{x} \rangle + b$

Recent Progress in Linear SVM solvers

- SGD (Bottou 2007), SGD-QN (Bordes et al., 2009)
- SVM^{perf} (Joachims 2006), liblinear (Fan et al. 2008)
- BMRM (Teo et.al. 2007), OCAS (Franc, Sonnenburg 2009)

$\Rightarrow$ **Linear training Effort** $\mathcal{O}(m)$

$\Rightarrow$ **Computing Outputs Linear Effort** $\mathcal{O}(nm)$

. . . already linear time but just linear

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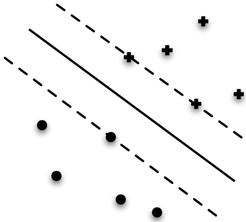

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Support Vector Machines in Shogun II

Linear SVMs

- OnlineLibLinear, LibLinear, MulticlassLibLinear (dual coordinate descent)
- NewtonSVM (newton primal svm)
- OnlineSVMSGD, SVMSGD (stochastic gradient descent)
- SGDQN (stochastic gradient descent with quasi-newton steps)
- SVMLin
- SVMOcas, MulticlassOCAS (cutting plane/bundle-method)

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Shogun's computational framework for linear SVMs

AIM: Development of a large scale learning framework for SVMs

> *"Algorithm [linear SVM solver] improvements do not improve the order of test error convergence. They can simply improve constant factors and therefore* **compete evenly with the implementation improvements***. Time spent refining the implementation is time well spent."*

from: Bordes, Bottou, Gallinari: SQD-QN: Careful Quasi-Newton Stochastic Gradient Descent. JMLR 2009.

Towards a computational framework for linear SVMs

Linear SVM solvers like liblinear, SGD, BMRM, OCAS only require two operations to access data:

(i) dot product between feature vector and the vector **w**:
$r \leftarrow \langle \mathbf{x}, \mathbf{w} \rangle$ **DOT**

(ii) multiplication with scalar $\alpha \in \Re$ and addition to vector $\mathbf{v} \in \Re^n$
$\mathbf{v} \leftarrow \alpha \mathbf{x} + \mathbf{v}$ **ADD**

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COFFIN

COFFIN really is just two simple ideas:

On demand compute. . .

1. Features $\Phi(\mathbf{x})$ (only non-zero dims)
   - Non-Linearity Possible
   - Examples: Low Degree Polynomial Kernel, Spectrum Kernel, Weighted Degree Kernel
   - On-the-fly (de)compression
2. Virtual Examples
   - Incorporating Invariances possible
   - Examples: Image translation, rotation, etc

Needs efficient data structure for w!

- . . ., dense, sorted array, trees, hashes
- fast only when $|\Phi_{\neq 0}(\mathbf{z})| \sim dim(\mathbf{z})$

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Data Structures

Effort of **ADD** and **DOT** for $\mathbf{z}$ and memory requirement of $\mathbf{w}$.

	Dense	Sorted Array	Tree
Add	$\mathcal{O}(\vert\Phi_{\neq 0}(\mathbf{z})\vert)$	$\mathcal{O}(\vert\mathbf{w}\vert_{\neq 0}) + \vert\Phi_{\neq 0}(\mathbf{z})\vert)$	$\mathcal{O}(\vert\Phi_{\neq 0}(\mathbf{z})\vert)$ to $\mathcal{O}(K\vert\Phi_{\neq 0}(\mathbf{z})\vert)$
Dot	$\mathcal{O}(\vert\Phi_{\neq 0}(\mathbf{z})\vert)$	$\mathcal{O}(\vert\mathbf{w}\vert_{\neq 0}) + \vert\Phi_{\neq 0}(\mathbf{z})\vert)$	$\mathcal{O}(\Phi_{\neq 0}(\mathbf{z})\vert)$ to $\mathcal{O}(K\vert\Phi_{\neq 0}(\mathbf{z})\vert)$
Mem	$\mathcal{O}(n)$	$\mathcal{O}(\sum_{i=1}^m \vert\Phi_{\neq 0}(\mathbf{z}_i)\vert)$	$\mathcal{O}(\sum_{i=1}^m \vert\Phi_{\neq 0}(\mathbf{z}_i)\vert)$

Sparse data structures have huge overhead!

Hashing to the Rescue (Shi et al (2009))

- Always use *dense* $\mathbf{w}$ with "compressed index"
- Hash function $h(J) \mapsto 1, \ldots, 2^\gamma$,
- $(\widehat{\Phi}(\mathbf{z}))_j = \sum_{i \in J; h(i)=j} (\Phi(\mathbf{z}))_i$

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Data Structures

Effort of **ADD** and **DOT** for $\mathbf{z}$ and memory requirement of $\mathbf{w}$.

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COFFIN is implemented in Shogun's DotFeatures

DotFeatures

- Dense-,SparseFeatures
- PolyFeatures, BinnedDotFeatures
- CombinedDotFeatures
- ExplicitSpecFeatures, ImplicitWeightedSpecFeatures
- HashedDocDotFeatures

You can

- Stack together features (dense, sparse,...)
- Provides abstraction to data representation

Use it!

Implemented for all linear SVMs

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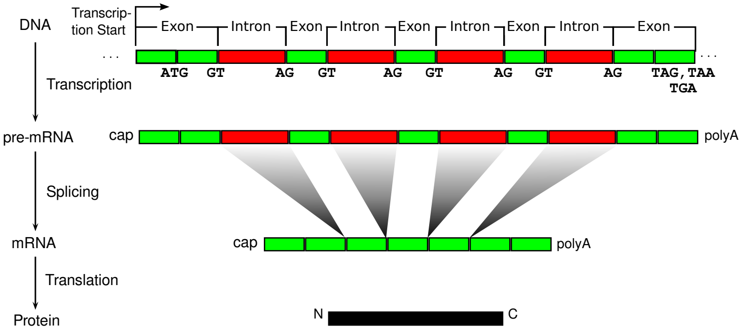
Splice Site Predictions



Application to Human Acceptor Splice Site Prediction

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Splice Site Prediction

Discriminate true signal positions against all other positions

$\approx$ **150 nucleotides window around dimer**

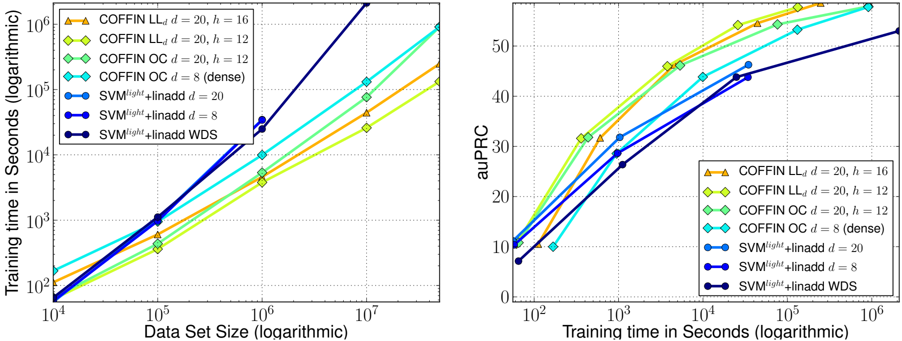
```
CT...GTCGTA...GAAGCTAGGAGCGC...ACGCGT...GA
```

- True sites: fixed window around a true site
- Decoy sites: all other consensus sites

```
AAACAAATAAGTAACTAATCTTTTAGGAAGAACGTTTCAACCATTTTGAG
AAGATTAAAAAAAAACAAATTTTTAGCATTACAGATATAATAATCTAATT
CACTCCCCAAATCAACGATATTTTAGTTCACTAACACATCCGTCTGTGCC
TTAATTTCACTTCCACATACTTCCAGATCATCAATCTCCAAAACCAACAC
TTGTTTTAATATTCAATTTTTTACAGTAAGTTGCCAATTCAATGTTCCAC
TACCTAATTATGAAATTAAAATTCAGTGTGCTGATGGAAACGGAGAAGTC
```

- 50 million training examples
- COFFIN with kernels: weighted spectrum and weighted degree (explicit and hashed representation) $\approx 200,000,000$ dims

Applications



It's fast and works!

- Factor 47 faster on $10 \cdot 10^6$ examples than linadd
- New state-of-the-art results auPRC 58.57% vs. 53.01%

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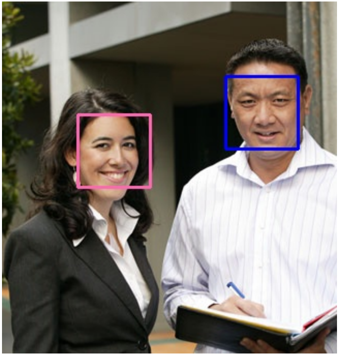
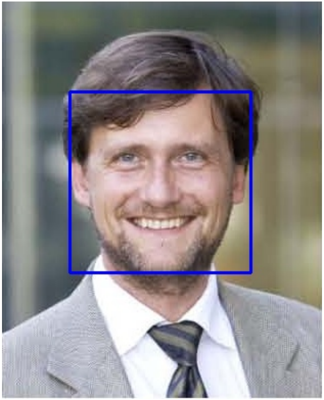
Gender Classification

Distinguish Females from Males solely based on Faces



- learn COFFIN on labelled faces
- virtual examples: translation, rotation, scale
- train $\approx$ 5 million sample (that would require 50GB) on Vojtechs notebook

Gender Classification I



It's fast and works - again!

- auROC 95.44%
- (vs. auROC 89.57% without VE)

Klaus-Robert Müller is a male!

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Gender Classification II

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Gender Classification III

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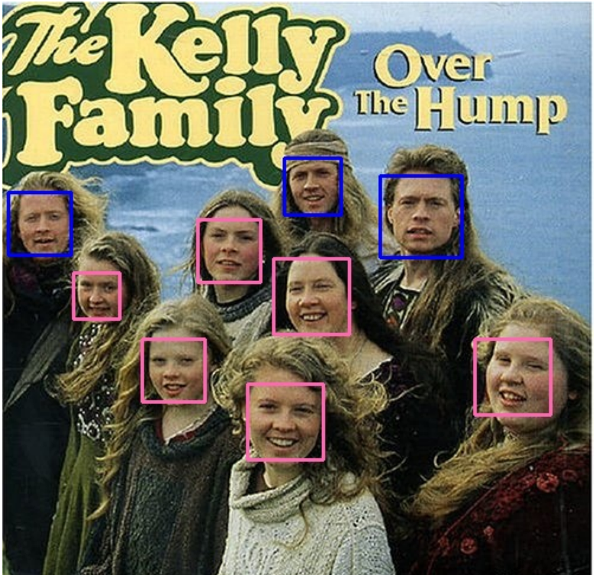
Gender Classification IV



Summary Support Vector Machines

Kernel SVMs

- Allows non-linear and complex models
- Applicable to mid-size datasets (1e6)
- General and often state-of-the art detectors

Linear SVMs

- Allows non-linearity with COFFIN/DotFeatures
- Applicable to huge datasets
- General and often state-of-the art detectors

Datasets, Scripts, Efficient implementation

- Data and Scripts `http://sonnenburgs.de/soeren/coffin`
- Implementation `http://www.shogun-toolbox.org`
- More machine learning software `http://mloss.org`

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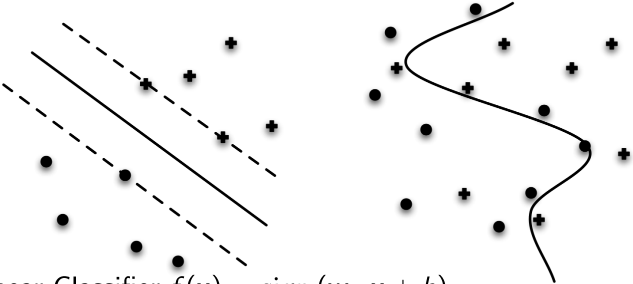
Multiple Kernel Learning

Given training examples $(\mathbf{x}_i, y_i)_{i=1}^N \in (\mathcal{X}, \{-1,+1\})^N$



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Classification using Kernel Machines I

Single Kernel

- Kernel Machine (e.g. Support Vector Machine)
  - learn weighting $\boldsymbol{\alpha} \in \mathbb{R}^N$ on training examples $(\mathbf{x}_i, y_i)_{i=1}^N$ in kernel feature space $\Phi(\mathbf{x})$

$$f(\mathbf{x}) = \operatorname{sign}\left(\sum_{i=1}^{N} y_i \alpha_i \mathrm{k}(\mathbf{x}, \mathbf{x}_i) + b\right)$$

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via kernel: non-linear discrimination in input space

Classification using Kernel Machines II

Multiple Kernels

- Linear combination of kernels $k(\mathbf{x}, \mathbf{x}') = \sum_{j=1}^{M} \beta_j \, k_j(\mathbf{x}, \mathbf{x}')$, $\beta_j \geq 0$. Learn $\boldsymbol{\alpha}$ *and* $\boldsymbol{\beta}$. Resulting classifier:

$$f(\mathbf{x}) = \operatorname{sign}\left(\sum_{j=1}^{M} \beta_j \sum_{i=1}^{N} y_i \alpha_i k_j(\mathbf{x}, \mathbf{x}_i) + b \right)$$

Learn weighting over training examples α and kernels β

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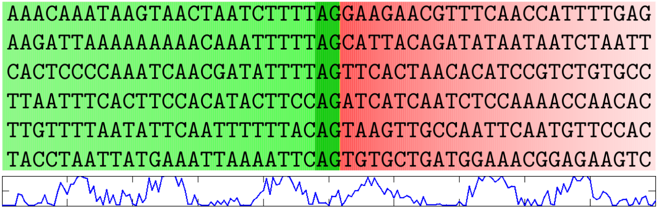
Combining Heterogeneous Data

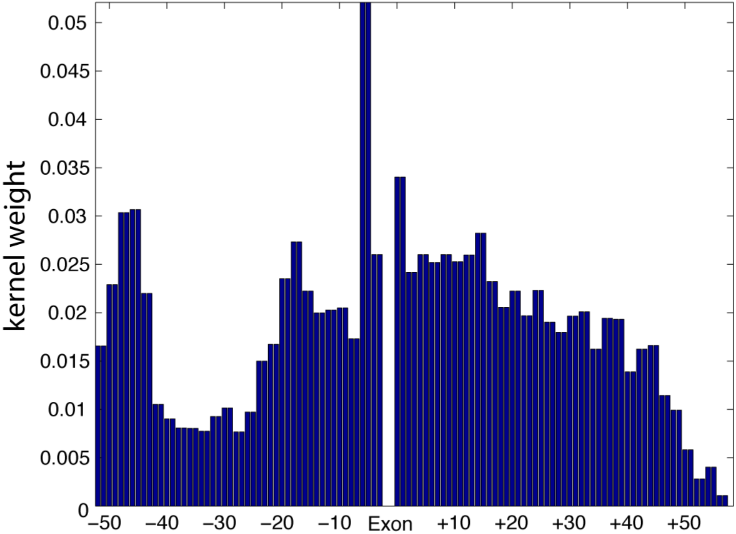
- Consider data from different domains: e.g Bioinformatics features: DNA-strings, binding energies, conservation, structure,...



$$k(\mathbf{x}, \mathbf{x}') =$$
$$\beta_1 \, k_{dna}(\mathbf{x}_{dna}, \mathbf{x}'_{dna}) + \beta_2 \, k_{nrg}(\mathbf{x}_{nrg}, \mathbf{x}'_{nrg}) + \beta_3 \, k_{3d}(\mathbf{x}_{3d}, \mathbf{x}'_{3d}) + \cdots$$

Interpretability

- Bioinformatics: One weight per position in sequence



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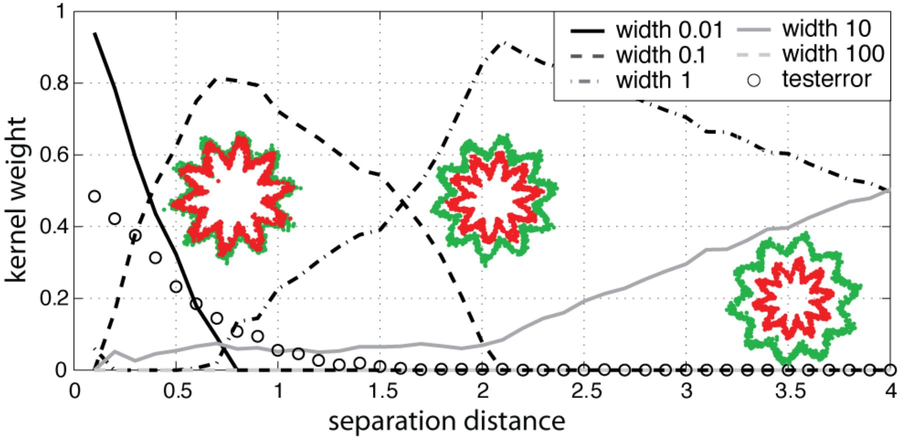
Automated Model Selection



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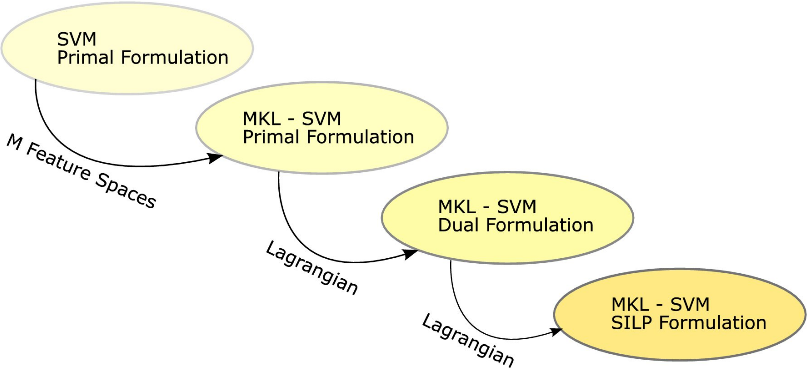
Derivation



For details see Sonnenburg, Rätsch, Schäfer, Schölkopf 2006

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SVM Primal Formulation

$$
\begin{array}{rl}
\min & \frac{1}{2}\|\mathbf{w}\|_2^2 + C\sum_{i=1}^{N}\xi_n \\
\text{w.r.t.} & \mathbf{w} \in \mathbb{R}^D, \boldsymbol{\xi} \in \mathbb{R}_+^N, b \in \mathbb{R} \\
\text{s.t.} & y_i\left(\mathbf{w}^\top \Phi(\mathbf{x}_i) + b\right) \geq 1 - \xi_i, \forall i = 1, \ldots, N
\end{array}
$$

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MKL Primal Formulation

$$
\begin{aligned}
\min \quad & \frac{1}{2}\left(\sum_{j=1}^{M} \beta_j \|\mathbf{w}_j\|_2\right)^2 + C\sum_{i=1}^{N} \xi_n \\
\text{w.r.t.} \quad & \mathbf{w} = (\mathbf{w}_1, \ldots, \mathbf{w}_M),\ \mathbf{w}_j \in \mathbb{R}^{D_j}, \quad \forall j = 1 \ldots M \\
& \boldsymbol{\beta} \in \mathbb{R}_+^M,\ \boldsymbol{\xi} \in \mathbb{R}_+^N,\ b \in \mathbb{R} \\
\text{s.t.} \quad & y_i\left(\sum_{j=1}^{M} \beta_j \mathbf{w}_j{}^{\mathsf{T}} \Phi_j(\mathbf{x}_i) + b\right) \geq 1 - \xi_i,\ \forall i = 1, \ldots, N \\
& \sum_{j=1}^{M} \beta_j = 1
\end{aligned}
$$

Properties: equivalent to SVM for $M = 1$; solution sparse in "blocks"; each block j corresponds to one kernel

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MKL Dual Formulation

Bach, Lanckriet, Jordan 2004:

$$
\begin{array}{rl}
\min & \gamma - \sum_{i=1}^{N} \alpha_i \\
\text{w.r.t.} & \gamma \in \mathbb{R}, \boldsymbol{\alpha} \in \mathbb{R}^N \\
\text{s.t.} & 0 \leq \boldsymbol{\alpha} \leq C, \sum_{i=1}^{N} \alpha_i y_i = 0 \\
& \frac{1}{2} \sum_{r=1}^{N} \sum_{s=1}^{N} \alpha_r \alpha_s y_r y_s K_j(\mathbf{x}_r, \mathbf{x}_s) - \gamma \leq 0, \ \forall j = 1, \ldots, M
\end{array}
$$

Properties: equivalent to SVM for $M = 1$

The Semi-Infinite Linear Program I

$$
\begin{array}{rl}
\max & \theta \\
\text{w.r.t.} & \theta \in \mathbb{R}, \boldsymbol{\beta} \in \mathbb{R}^M_+ \text{ with } \sum_{j=1}^{M} \beta_j = 1 \\
\text{s.t.} & \sum_{j=1}^{M} \beta_j \left(\underbrace{\frac{1}{2} \sum_{r=1}^{N} \sum_{s=1}^{N} \alpha_r \alpha_s y_r y_s K_j(\mathbf{x}_r, \mathbf{x}_s)}_{=:S_j(\boldsymbol{\alpha})} - \sum_{i=1}^{N} \alpha_i \right) \geq \theta \\
& \text{for all } \boldsymbol{\alpha} \text{ with } 0 \leq \boldsymbol{\alpha} \leq C \text{ and } \sum_{i=1}^{N} y_i \alpha_i = 0
\end{array}
$$

Properties:

- linear, optimize over a convex combination of $\boldsymbol{\beta}$
- infinitely many constraints, one for each $0 \leq \boldsymbol{\alpha} \leq C$
- can use standard SVM to identify most violated constraints

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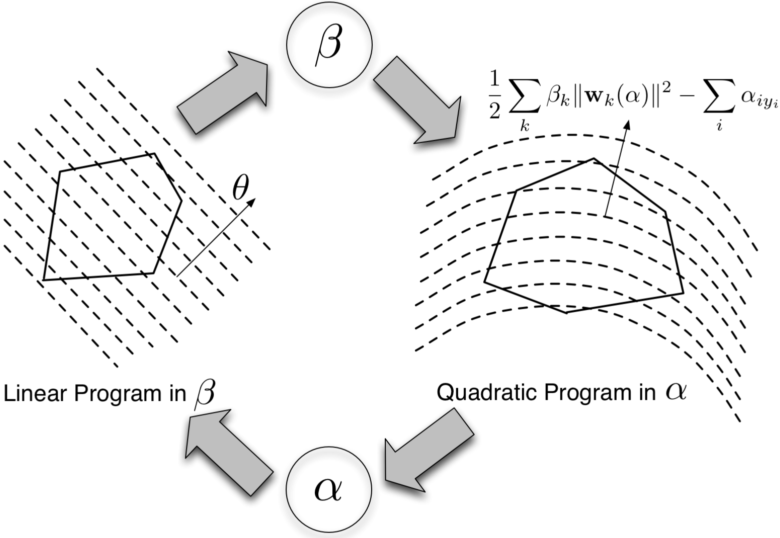
The Semi-Infinite Linear Program II

$$
\begin{array}{rl}
\max & \theta \\
\text{w.r.t.} & \theta \in \mathbb{R}, \boldsymbol{\beta} \in \mathbb{R}^M_+ \text{ with } \sum_{j=1}^{M} \beta_j = 1 \\
\text{s.t.} & \sum_{j=1}^{M} \beta_j \left(\frac{1}{2} S_j(\boldsymbol{\alpha}) - \sum_{i=1}^{N} \alpha_i \right) \geq \theta \\
 & \text{for all } \boldsymbol{\alpha} \text{ with } 0 \leq \boldsymbol{\alpha} \leq C \text{ and } \sum_{i=1}^{N} y_i \alpha_i = 0
\end{array}
$$

Solving the SILP:

- Column Generation
  - fast, but no known convergence rate
- Use Boosting like techniques: Arc-GV or AdaBoost*
  - known convergence rate $\mathcal{O}(\log(M)/\varepsilon^2)$
- Chunking like algorithm
  - consider suboptimal SVM solutions: empirically 3-5 times faster

Solving the SILP: Column Generation I



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Solving the SILP: Column Generation II

$$
\begin{array}{rl}
\max & \theta \\
\text{w.r.t.} & \theta \in \mathbb{R}, \boldsymbol{\beta} \in \mathbb{R}_+^M \text{ with } \sum_{j=1}^{M} \beta_j = 1 \\
\text{s.t.} & \sum_{j=1}^{M} \beta_j \left(\frac{1}{2} S_j(\boldsymbol{\alpha}) - \sum_{i=1}^{N} \alpha_i \right) \geq \theta \\
& \text{for all } \boldsymbol{\alpha} \text{ with } 0 \leq \boldsymbol{\alpha} \leq C \text{ and } \sum_{i=1}^{N} y_i \alpha_i = 0
\end{array}
$$

- iteratively find most violated constraints, solve linear program with current constraints, . . . , till convergence to the global optimum

$$
\sum_{j=1}^{M} \beta_j \left(\frac{1}{2} S_j(\boldsymbol{\alpha}) - \sum_{i=1}^{N} \alpha_i \right) = \frac{1}{2} \sum_{r=1}^{N} \sum_{s=1}^{N} \alpha_r \alpha_s y_r y_s \sum_{j=1}^{M} \beta_j k_j(\mathbf{x}_r, \mathbf{x}_s) - \sum_{i=1}^{N} \alpha_i,
$$

- solved by taking set of most violated constraints into account
- most violated constraints given by SVM solution for fixed β

Regression

Primal Formulation:

$$\min \quad \frac{1}{2}\left(\sum_{j=1}^{M} \beta_j \|\mathbf{w}_j\|\right)^2 + C\sum_{i=1}^{N}(\xi_i + \xi_i^*)$$

$$\text{w.r.t.} \quad \mathbf{w} = (\mathbf{w}_1, \ldots, \mathbf{w}_M), \mathbf{w}_j \in \mathbb{R}^{D_j}, \quad \forall j = 1 \ldots M$$

$$\boldsymbol{\beta} \in \mathbb{R}_+^M, \ \boldsymbol{\xi} \in \mathbb{R}^N, \ \boldsymbol{\xi}^* \in \mathbb{R}_+^N, \ b \in \mathbb{R}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \beta_j \mathbf{w}_j{}^{\mathsf{T}} \Phi_j(\mathbf{x}_i) + b \leq y_i + \varepsilon + \xi_i, \forall i = 1 \ldots N$$

$$\sum_{j=1}^{M} \beta_j \mathbf{w}_j{}^{\mathsf{T}} \Phi_j(x_i) + b \geq y_i - \varepsilon - \xi_i^*, \forall i = 1 \ldots N$$

$$\sum_{j=1}^{M} \beta_j = 1$$

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One Class

Primal Formulation:

$$\min \quad \frac{1}{2}\left(\sum_{j=1}^{M} \beta_j \|\mathbf{w}_j\|_2\right)^2 + C\sum_{i=1}^{N}\xi_i - \rho$$

$$\text{w.r.t.} \quad \mathbf{w} = (\mathbf{w}_1, \ldots, \mathbf{w}_M),\ \mathbf{w}_j \in \mathbb{R}^{D_j}, \quad \forall j = 1 \ldots M$$

$$\boldsymbol{\beta} \in \mathbb{R}_+^M,\ \boldsymbol{\xi} \in \mathbb{R}_+^N$$

$$\text{s.t.} \quad y_i\left(\sum_{j=1}^{M} \beta_j \mathbf{w}_j{}^{\mathsf{T}} \Phi_j(\mathbf{x}_i)\right) \geq \rho - \xi_i, \forall i = 1, \ldots, N$$

$$\sum_{j=1}^{M} \beta_j = 1$$

Generalized for arbitrary strictly convex differentiable loss functions (Sonnenburg, Rätsch, Schäfer, Schölkopf 2006)

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Multiclass

Primal Formulation (Zien, Ong 2007):

$$\begin{aligned}
\min \quad & \frac{1}{2}\left(\sum_{j=1}^{M} \beta_j \|\mathbf{w}_j\|_2\right)^2 + C\sum_{i=1}^{N}\xi_n \\
\text{w.r.t.} \quad & \mathbf{w} = (\mathbf{w}_1, \ldots, \mathbf{w}_M),\ \mathbf{w}_j \in \mathbb{R}^{k_j}, \quad \forall j = 1 \ldots M \\
& \boldsymbol{\beta} \in \mathbb{R}_+^M,\ \mathbf{s} \in \mathbb{R}^{N\times c},\ \boldsymbol{\xi} \in \mathbb{R}_+^N,\ b \in \mathbb{R} \\
\text{s.t.} \quad & \xi_i = \max_{u \neq y_i} s_{iu},\ s_{iu} \geq 0, \\
& \sum_{j=1}^{M} \beta_j \mathbf{w}_j^{\mathsf{T}} \left(\Phi_j(\mathbf{x}_i, y_i) - \Phi_j(\mathbf{x}_i, u)\right) + b_{y_i} - b_u \geq 1 - s_{iu}, \\
& \quad \forall i = 1 \ldots N,\ \forall u = 1 \ldots c
\end{aligned}$$

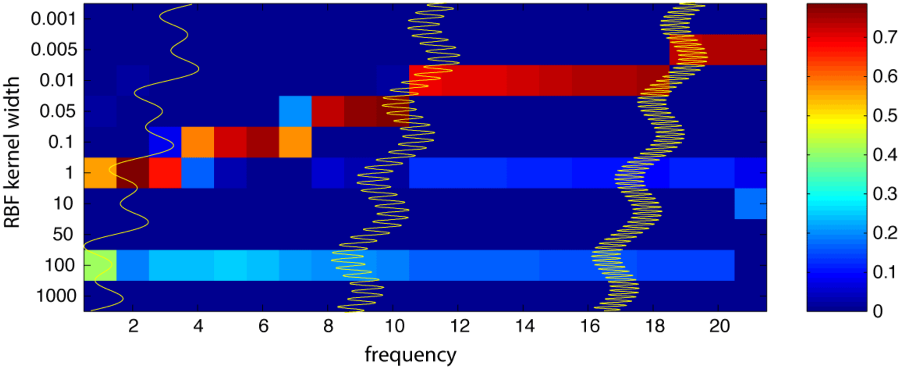
Further Extension

Extension

- ℓ_p norm MKL
- faster algorithms
- convergence bounds
- and many more

$\vdots$

Automated Model Selection - Regression



- $f(x) = \sin(ax) + \sin(bx) + cx$ for varying a
- Support Vector Regression with 10 RBF-Kernels of different width

Knowledge discovery

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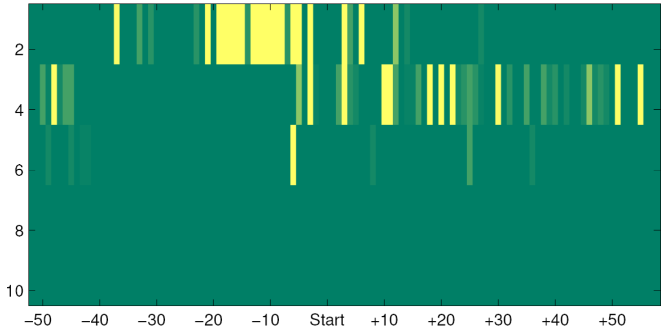
Feature Extraction



- Support Vector Classification on Bioinformatics problem, distinguish “splice sites” form “fake sites” (aligned DNA sequences)
- One weight β_j per position and per sub-sequence length
- Displayed: Learned weights of 500 kernels

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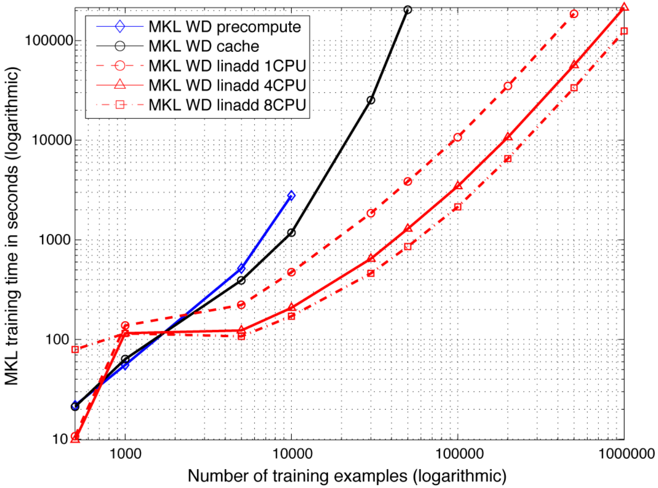
A word about speed.



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Summary and Outlook

MKL learns convex combination of kernels

$\Rightarrow$ allows (to some extend) for automated model selection

$\Rightarrow$ allows for interpreting SVM result

$\Rightarrow$ matches prior knowledge on real-world bioinformatics problem

- **Simple:** iterative wrapper algorithm around single kernel SVM
- **General:** same technique applicable to a wide range of problems (1-class, 2-class, Multiclass, Regression, ...)
- **Fast:** suitable for large scale problems ($> 100,000$ examples)

Download free source `http://www.shogun-toolbox.org`**.**