

# Data-Driven Chance-Constrained Stochastic Unit Commitment Under Wind Power Uncertainty

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**Abstract**—Rapid integration of cheap, clean but highly intermittent wind energy into power systems brings challenges to ISOs to maintain the system reliability. Stochastic Programs (SP) may result in biased and unreliable unit commitment (UC) and economic dispatch (ED) decisions by fixing the probability distribution of wind output. Robust Optimization (RO) approaches sacrifice system's cost-effectiveness in exchange of reliable UC and ED schedules. In this paper, we develop a data-driven chance-constrained two-stage stochastic UC model to bridge the gap between SP and RO. Without any particular assumption of wind output distribution, the data-driven chance constraint limits the worst-case chance of load imbalance to be no more than a specified tolerance, by taking advantage of historical data. We apply Column-and-Constraints Generation to solve our model. By experiments, we show the effectiveness of our model and the value of data.

**Index Terms**—Data-driven, Chance Constraint, Stochastic Unit Commitment, Wind Uncertainty

## NOMENCLATURE

### A. Parameters

$SU_i^b$	Start-up cost of generation unit $i$ at bus $b$ .
$SD_i^b$	Shut-down cost of generation unit $i$ at bus $b$ .
$MU_i^b$	Minimum up-time of generation unit $i$ at bus $b$ .
$MD_i^b$	Minimum down-time of generation unit $i$ at bus $b$ .
$CL_i^b$	Lower limit of generation capacity for generation unit $i$ at bus $b$ .
$CU_i^b$	Upper limit of generation capacity for generation unit $i$ at bus $b$ .
$\bar{R}_i^b$	Ramp-up rate limit of generation unit $i$ at bus $b$ .
$\underline{R}_i^b$	Ramp-down rate limit of generation unit $i$ at bus $b$ .
$L_{ij}$	Flow capacity of the transmission line $(i, j)$ connecting bus $i$ and bus $j$ .
$F_{ij}^b$	Flow distribution factor for the transmission line connecting bus $i$ and bus $j$ , based on the net injection at bus $b$ .
$d_t^b$	Demand at bus $b$ in time $t$ .
$w_t^b(\xi)$	Wind power output at bus $b$ in time $t$ for scenario $\xi$ .
$\delta$	Load imbalance tolerance in the chance constraint.
$\epsilon$	Risk level of energy imbalance.
$F_i(\cdot)$	Fuel cost of generation unit $i$ .
$\pi_t$	Penalty cost per unit of energy imbalance in time $t$ .

### B. Variables

$y_{it}^b$	Binary variable indicating whether generation unit $i$ is on ( $= 1$ ) or off ( $= 0$ ) in time $t$ .
$u_{it}^b$	Binary variable indicating whether generation unit $i$ is started up ( $= 1$ ) or not ( $= 0$ ) in time $t$ .
$v_{it}^b$	Binary variable indicating whether generation unit $i$ is shut down ( $= 1$ ) or not ( $= 0$ ) in time $t$ .
$g_{it}^b$	Amount of electricity generated by generation unit $i$ at bus $b$ in time $t$ .
$s_t(\xi)$	Amount of energy imbalance (shortage or oversupply) in time $t$ corresponding to scenario $\xi$ .

## I. INTRODUCTION

THE US government long-term financial incentives, such as tax credit programs and retirement plans, to promote investments in clean and cheap renewable energy (e.g. wind power) and to replace pollutant thermal plants with clean ones, have led to rapid renewable energy installments. Due to these stimulus plans, it is predicted that wind power will reach 20% of total energy generation across the country in 2030 [1]. However, unpredictable and intermittent nature of renewable energy affects the power system stability and reliability. In this regard, system operators (ISOs/RTOs) have been traditionally utilizing excessive online reserves to prepare for the case that the actual wind power output is much lower than its predicted level. References [2] and [3] investigate the amount of required reserve for power systems with large amount of installed wind capacity. They show that the more wind capacity is integrated with the system, the higher reserve capacity should be scheduled. With the increasing penetration of wind energy into the grid, purely increase the ancillary service deployment is not practical.

Recently, stochastic programming (SP) approaches have been widely applied to address the uncertainties in UC and ED. In [4], a stochastic linear programming model addressing security-constrained unit commitment with large amount of wind power capacity is presented. References [5] and [6] are extensions of [4] considering uncertain demand and equipment outages. In addition, two-stage stochastic models are commonly used to address wind power uncertainty, which typically consider UC in the first stage before the wind power output is known, and ED in the second stage after the wind power

output is realized [7], [8], [9]. Furthermore, in order to reduce load shedding and renewable energy curtailment, risk-averse stochastic unit commitment models integrating chance constraints and expected value constraints have been successfully developed [10], [11], [12]. However, for SP, the probability distribution of the unknown parameters is usually assumed known; in practice, the distribution information is usually unknown and the UC decisions obtained with the inaccurate distribution assumption may affect the system reliability and cost efficiency.

In addition, robust optimization (RO) approaches have been successfully developed for UC and ED under uncertainty [13], [14], [15]. In RO, the optimal schedules are obtained by considering the worst-case scenario of the unknown parameter which varies within an uncertainty set. In fact, this approach acquires system robustness by sacrificing the cost effectiveness of the system. Moreover, RO does not utilize the historical data to a large extent because uncertainty set construction needs limited information about the random parameter.

To address the reliability issues of SP and over conservatism of RO, data-driven and distributionally robust optimization approaches have recently been developed [16], [17], [18]. In these approaches, the unknown probability distribution of the random parameter is allowed to vary in a confidence set, which is constructed by learning from a given historical data set. Though data-driven approaches are still risk-averse approaches since they consider the worst-case distribution in the confidence set, the conservatism is generally less than that in RO. Moreover, as the size of data increases, the conservatism of this approach decreases. These approaches have recently been applied to several operational problems under uncertainty in power systems [19], [20], [21].

In order to take advantage of considerable amount of wind output historical data available for ISOs/RTOs, in this paper, we propose a data-driven chance-constrained stochastic UC (DDCHC) under wind uncertainty. We formulate it as a two-stage model, in the first stage consists of the traditional stochastic unit commitment problem and data-driven chance constraint that is used to restrict the probability of power imbalance, and in the second stage, the penalty cost due to energy imbalance is considered for the case that the actual power output differs from the one committed. To conclude the contributions, (1) We develop a data-driven chance-constrained two-stage stochastic model under wind uncertainty which utilizes available historical data to obtain reliable but cost efficient generation schedules. (2) We show by experiments that our approach solutions are more reliable than the traditional chance-constrained UC (CCUC) solutions. Also, although our approach is risk-averse, its conservatism decreases as the size of data increases.

The remaining parts of this paper are organized as follows: In section II, we describe the mathematical formulation of the model, the confidence set construction, and the chance constraint reformulation. In section III, we discuss the solution methodology and efficient algorithm to solve the problem. In section IV, we show our numerical experiments and results to verify the effectiveness of our model. Finally, in section V, we conclude this study.

## II. MATHEMATICAL FORMULATION

### A. Chance-Constrained Two-stage Formulation

The above-mentioned chance-constrained two-stage UC can be formulated as follows:

$$\min \sum_t \sum_b \sum_i (SU_i^b u_{it}^b + SD_i^b v_{it}^b + F_i(g_{it}^b)) + E[Q(y, u, v, g, \xi)] \quad (1)$$

$$s.t. \quad -y_{i(t-1)}^b + y_{it}^b - y_{ik}^b \leq 0, \quad \forall t, \forall b, \forall i, \forall k : 1 \leq k - (t-1) \leq MU_i^b \quad (2)$$

$$y_{i(t-1)}^b - y_{it}^b + y_{ik}^b \leq 1, \quad \forall t, \forall b, \forall i, \forall k : 1 \leq k - (t-1) \leq MD_i^b \quad (3)$$

$$-y_{i(t-1)}^b + y_{it}^b - u_{it}^b \leq 0, \quad \forall t, \forall b, \forall i, \quad (4)$$

$$y_{i(t-1)}^b - y_{it}^b - v_{it}^b \leq 0, \quad \forall t, \forall b, \forall i, \quad (5)$$

$$CL_i^b y_{it}^b \leq g_{it}^b \leq CU_i^b y_{it}^b, \quad \forall t, \forall b, \forall i, \quad (6)$$

$$g_{it}^b - g_{i(t-1)}^b \leq (2 - y_{i(t-1)}^b - y_{it}^b) CL_i^b + (1 + y_{i(t-1)}^b - y_{it}^b) \bar{R}_i^b, \quad \forall t, \forall b, \forall i, \quad (7)$$

$$g_{i(t-1)}^b - g_{it}^b \leq (2 - y_{i(t-1)}^b - y_{it}^b) CL_i^b + (1 - y_{i(t-1)}^b + y_{it}^b) \bar{R}_i^b, \quad \forall t, \forall b, \forall i, \quad (8)$$

$$-L_{ij} \leq \sum_b F_{ij}^b(w_t^b(\xi) + \sum_r g_{rt}^b - d_t^b) \leq L_{ij}, \quad \forall t, \forall (i, j), \quad (9)$$

$$Pr(-\delta \leq \sum_b \sum_i g_{it}^b + \sum_b w_t^b(\xi) - \sum_b d_t^b \leq \delta) \geq 1 - \epsilon, \quad \forall t, \quad (10)$$

$$y_{it}^b, u_{it}^b, v_{it}^b \in \{0, 1\}, g_{it}^b \geq 0, \quad \forall i, \forall b, \forall t, \quad (11)$$

where  $Q(y, u, v, g, \xi)$  is

$$\min \sum_t \pi_t s_t(\xi) \quad (12)$$

$$s.t. \quad s_t(\xi) \geq \sum_b \sum_i g_{it}^b + \sum_b w_t^b(\xi) - \sum_b d_t^b, \quad \forall t, \quad (13)$$

$$s_t(\xi) \geq \sum_b d_t^b - \sum_b \sum_i g_{it}^b - \sum_b w_t^b(\xi), \quad \forall t, \quad (14)$$

where, the first-stage objective function (1) consists of startup, shutdown and fuel costs. Constraints (2) and (3) force the minimum up-time and minimum down-time limits, respectively. Constraints (4) and (5) represent start-up and shutdown status constraints, respectively. Constraints (6) limit the generation capacity lower and upper bounds. Constraints (7) and (8) represent the ramping constraints. Constraints (9) restrict the transmission line capacity limits. Chance constraints (10) ensure the chance that load imbalance violates a specified tolerance level is no more than a predefined risk level. Constraints (13) and (14) calculate the energy shortage or oversupply for each time period, respectively. In addition, we approximate the quadratic generation cost  $F_i(\cdot)$  using a J-piece piecewise linear function as below:

$$\phi_{it}^b \geq \alpha_{it}^{jb} y_{it}^b + \rho_{it}^{jb} g_{it}^b, \quad \forall t, \forall b, \forall i, j = 1, \dots, J. \quad (15)$$

### B. Confidence Set Construction

As discussed above, we allow the wind output distribution  $P$  to be unknown and ambiguous. We assume that  $P$  belongs to a confidence set  $\mathbb{D}$  with a specific confidence level  $\beta$  (for example 95%). By utilizing the data information (i.e., historical data of wind output), we construct a distribution-based confidence set  $\mathbb{D} = \{P \in \Omega : d(P, \hat{P}) \leq \varphi\}$ , where  $\Omega$  is the set of all distributions,  $\hat{P}$  represents the reference distribution,  $d(P, \hat{P})$  is the probability distance between  $P$  and  $\hat{P}$ , and  $\varphi$  represents the tolerance level. We obtain the reference distribution by drawing a histogram using historical data. By partitioning the sample space  $\mathcal{W}$  into  $N$  bins, such that  $\mathcal{W} = \bigcup_{n=1}^N B_n$ , and then counting the frequency of samples in each bin, i.e.,  $S_n$ , we can obtain the reference distribution  $\hat{P} = (\hat{p}_1, \hat{p}_2, \dots, \hat{p}_N)$ , in which  $\hat{p}_n = S_n/S, \forall n$ , and  $S$  is the total amount of data. To measure  $d(P, \hat{P})$ , in this study, we use  $L_\infty$  norm. Accordingly, we can define confidence set  $\mathbb{D}$  as follows:

$$\mathbb{D} = \{P \in \mathbb{R}_+^N \mid \max_{1 \leq n \leq N} |p^n - \hat{p}^n| \leq \varphi\}. \quad (16)$$

According to [22], given  $S$  historical data points and  $N$  bins, the convergence rate between ambiguous distribution  $P$  and reference distribution  $\hat{P}$  is as below:

$$Pr\{\|P - \hat{P}\|_\infty \leq \varphi\} \geq 1 - 2N \exp(-2S\varphi). \quad (17)$$

If the confidence level (the right-hand side of (17)) is equal to  $\beta$ , then, we have

$$\varphi = (1/2S) \log(2N/(1 - \beta)). \quad (18)$$

From (18) we observe, as the number of historical observations increases to infinity, the value of  $\varphi$  goes to zero and  $\hat{P}$  converges to  $P$  accordingly.

### C. Data-Driven Chance Constraint and Its Reformulation

Since the true distribution of wind output is ambiguous within  $\mathbb{D}$ , we restrict that chance constraint (10) should be satisfied under the worst-case distribution in  $\mathbb{D}$ . Therefore, we can formulate the data-driven chance constraint as follows:

$$\begin{aligned} \min_{P \in \mathbb{D}} \quad & Pr(-\delta \leq \sum_b \sum_i g_{it}^b + \sum_b w_t^b(\xi) - \sum_b d_t^b \leq \delta) \\ & \geq 1 - \epsilon, \quad \forall t. \end{aligned} \quad (19)$$

Inequality (19) can be further reformulated as

$$\begin{aligned} \min_{P \in \mathbb{D}} \quad & \sum_{n=1}^N p_t^n \cdot \mathbf{1}_{[-\delta, \delta]}(\sum_b \sum_i g_{it}^b + \sum_b w_t^b(\xi^n) - \sum_b d_t^b) \\ & \geq 1 - \epsilon, \quad \forall t, \end{aligned} \quad (20)$$

where,  $\mathbf{1}_{[-\delta, \delta]}(\cdot)$  is an indicator function that equals to 1 if  $-\delta \leq \sum_b \sum_i g_{it}^b + \sum_b w_t^b(\xi) - \sum_b d_t^b \leq \delta$  and 0 otherwise, and  $\xi^n$  is the central point of bin  $n$ . Then, we let binary variables  $z^n = \mathbf{1}_{[-\delta, \delta]}(\sum_b \sum_i g_{it}^b + \sum_b w_t^b(\xi^n) - \sum_b d_t^b)$ ; and use big  $M$  method to reformulate (20) as the following MILP

model:

$$\begin{aligned} -\delta - (1 - z_t^n)M &\leq \sum_b \sum_i g_{it}^b + \sum_b w_t^b(\xi) \\ -\sum_b d_t^b &\leq (1 - z_t^n)M + \delta, \quad \forall t, \forall n, \end{aligned} \quad (21)$$

$$\min_p \sum_{n=1}^N p_t^n z_t^n \geq 1 - \epsilon, \quad \forall t, \quad (22)$$

$$\varphi + \hat{p}_t^n \geq p_t^n \geq -\varphi + \hat{p}_t^n, \quad \forall t, \forall n, \quad (23)$$

$$\sum_{n=1}^N p_t^n = 1, \quad \forall t, \quad (24)$$

$$p_t^n \geq 0, \quad \forall t, \forall n, \quad (25)$$

where constraints (23) and (24) are reformulation of the confidence set  $\mathbb{D}$ . To eliminate the minimization operation in constraint (22), we dualize (22) to (25) to get the following formulation:

$$\max \sum_{n=1}^N \lambda_t^n (-\varphi + \hat{p}_t^n) - \tau_t^n (\varphi + \hat{p}_t^n) + \eta_t, \quad (26)$$

$$s.t. \quad \eta_t + \lambda_t^n - \tau_t^n \leq z_t^n, \quad \forall t, \forall n, \quad (27)$$

$$\lambda_t^n, \tau_t^n \geq 0, \eta_t \text{ free}, \quad \forall t, \forall n, \quad (28)$$

where  $\tau_t^n$ ,  $\lambda_t^n$  and  $\eta_t$  are dual variables of constraints (23) and (24). Thus, constraint (22) to (25) can be reformulated as:

$$\sum_{n=1}^N \lambda_t^n (-\varphi + \hat{p}_t^n) - \tau_t^n (\varphi + \hat{p}_t^n) + \eta_t \geq 1 - \epsilon, \quad (29)$$

$$\text{Constraints (27) - (28)}. \quad (30)$$

### D. Objective Reformulation

Based on the above-defined confidence set for the unknown wind output distribution, we formulate the data-driven chance-constrained two-stage UC model. In the second stage, we consider the the worst-case imbalance penalty cost associated with the worst-case wind output distribution in  $\mathbb{D}$ . Since different scenarios  $\xi^n$  are independent, we can interchange the summation and the minimization operations. Hence, we can reformulate the second-stage objective function to its data-driven formulation:

$$\begin{aligned} \max_{P \in \mathbb{D}} \quad & E_P[Q(y, u, v, g, \xi)] \\ & = \max_{P \in \mathbb{D}} \min_s \sum_{n=1}^N \sum_t p_t^n \pi_t s_t(\xi^n). \end{aligned} \quad (31)$$

## III. SOLUTION METHODOLOGY

In this paper, we apply the Column-and-Constraint generation method [23], in a decomposition framework. We have the following master problem (MP):

$$\min_{y, u, v, \phi, g, z, \lambda, \tau, \eta} (SU_i^b u_{it}^b + SD_i^b v_{it}^b + \phi_{it}^b) + \vartheta$$

$$s.t. \quad \text{Constraints (2) - (8) and (15)}$$

$$\text{Constraints (9), (13), (14) and (21), } \forall n,$$

$$\text{Constraints (29), (27) and (28),}$$

$$\text{Optimality cuts,}$$

where  $\vartheta$  is the second-stage optimal objective value. As for the subproblem (SP), we initially dualize the second-stage problem, i.e., constraints (13) and (14). Then, we formulate SP as following:

$$\omega(g) = \max_{\mu, \gamma, p} \sum_{n=1}^N \left( \left( \sum_b \sum_i g_{it}^b + \sum_b w_t^b(\xi) - \sum_b d_t^b \right) \mu_t^n + \left( \sum_b d_t^b - \sum_b \sum_i g_{it}^b - \sum_b w_t^b(\xi) \right) \gamma_t^n \right) \quad (32)$$

$$s.t. \quad \mu_t^n \leq \pi_t p_t^n, \quad \forall t, \forall n, \quad (33)$$

$$\gamma_t^n \leq \pi_t p_t^n, \quad \forall t, \forall n, \quad (34)$$

$$-\varphi + \hat{p}_t^n \leq p_t^n \leq \varphi + \hat{p}_t^n, \quad \forall t, \forall n, \quad (35)$$

$$\sum_{n=1}^N p_t^n = 1, \quad \forall t, \quad (36)$$

$$\mu_t^n, \gamma_t^n, \hat{p}_t^n \geq 0, \quad \forall t, \forall n, \quad (37)$$

where constraints (35) to (36) represent the confidence set  $\mathbb{D}$ .  $\hat{p}_t^n$  and  $p_t^n$  denote the reference distribution and the true distribution of wind output at time  $t$  for scenario  $n$ , respectively. Also,  $\mu_t^n$  and  $\gamma_t^n$  are dual variables of constraints (13) and (14), respectively. We briefly describe the solution algorithm in the following steps:

1. Initialization. Set  $k = 1$ ,  $\vartheta = -\infty$
2. Solve MP and get the first-stage decision variables.
3. Fix the generation level  $g$  and solve SP to obtain  $\omega(g)$ .
4. If  $\omega(g) \leq \vartheta$ , stop and output the first-stage decisions. Otherwise, set  $k = k + 1$ . Generate and add the following cut (38) to MP and go to step 2:

$$\vartheta \geq \sum_{n=1}^N \sum_t p_t^n \pi_t s_t(\xi^n). \quad (38)$$

Notice here, since our model allows energy imbalance, the first-stage solutions are always feasible and no feasibility check is required in step 3.

#### IV. CASE STUDY

In order to test the effectiveness of our proposed approach, we conduct experiments on a modified IEEE 118-bus system (available at <http://motor.ece.iit.edu/data>). To generate the historical data set of wind output, we assume the uncertain wind power follows a multivariate normal distribution with mean equal to the forecasted value and variance equal to 0.3 of the mean. We set the number of bins to be 5. Then, using Monte Carlo simulation, we generate a historical data set for each wind farm in each time period. In addition, in the piecewise linear cost function, we set the number of pieces to be 5. We use C++ and CPLEX 12.6 to implement our model on a computer with Intel(R) Xeon(R) 3.2 and 8 GB memory.

1) *Effects of the Historical Data*: First, we show how the number of historical data points can effect the conservatism of the proposed approach. We set the confidence level  $\beta$  to be 99% and test the performance of the proposed DDCHC for different size of data ranging from 10 to 10000. We report the performance of the proposed approach in Table I. We also report the result of CCUC for a set of 50000 historical data, as

the benchmark. From Table I and Fig. 1 we can observe that, as the size of data gets larger, both the value of  $\varphi$  and DDCHC objective value decrease. That is because, with larger data sets,  $\mathbb{D}$  gets smaller and DDCHC becomes less conservative. Theoretically, as the size of historical data goes to infinity, the confidence set  $\mathbb{D}$  shrinks (reference distribution converges to true distribution) and eventually DDCHC becomes risk neutral.

TABLE I  
EFFECTS OF HISTORICAL DATA ON TOTAL COST

# of data	DDCHC			CCUC	
	$\varphi$	Obj(\$m)	T(s)	Obj(\$m)	T(s)
10	0.34539	2.0886	12	1.5611	10
50	0.06908	1.6655	12	1.5611	10
100	0.03454	1.6130	11	1.5611	10
500	0.00691	1.5714	11	1.5611	10
1000	0.00345	1.5669	10	1.5611	10
5000	0.00069	1.5621	10	1.5611	10
10000	0.00035	1.5616	10	1.5611	10

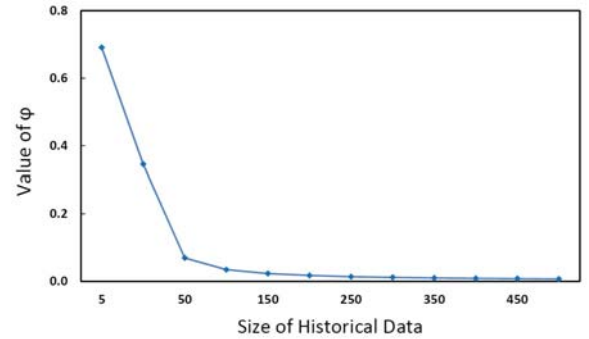


Fig. 1. Effects of the size of historical data on the value of  $\varphi$

2) *Effects of the Confidence level*: In this section, we show by experiments that how the confidence level  $\beta$  affects the conservatism of DDCHC. We set the number of historical data to be 100 and test DDCHC performance over a range of confidence level between 0.5 to 0.99. Table II and Fig. 2 illustrate that as the value of  $\beta$  increases, both the value of  $\varphi$  and DDCHC objective value increase. In fact, larger  $\varphi$  means higher chance that  $\mathbb{D}$  should include the true wind output distribution, and therefore  $\mathbb{D}$  is larger. Hence, DDCHC becomes more conservative as the value of  $\beta$  increases.

TABLE II  
EFFECTS OF THE CONFIDENCE LEVEL ON TOTAL COST

$\beta$	DDCHC	
	$\varphi$	Obj(\$m)
0.5	0.01498	1.5829
0.6	0.01609	1.5845
0.7	0.01753	1.5865
0.8	0.01956	1.5895
0.9	0.02303	1.5929
0.95	0.02649	1.6013
0.99	0.03454	1.6130

3) *Comparison with Traditional Chance-Constrained UC*: We compare the performance of the proposed DDCHC with that of CCUC in terms of the system reliability. For different size of historical data sets from 10 to 100, we first solve



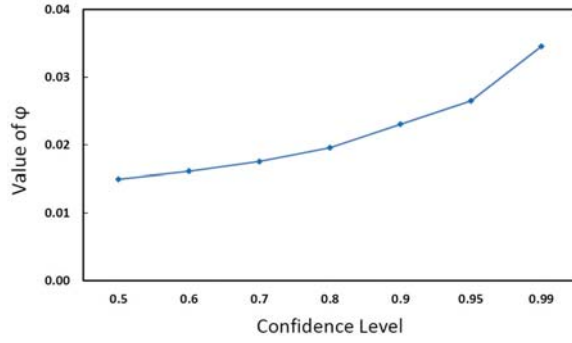


Fig. 2. Effects of the confidence level on the value of  $\phi$

the modified 118-bus system by both DDCHC and CCUC. Then, we fix the optimal thermal generation levels obtained by DDCHC and CCUC and solve the second stage for random instances of wind output under randomly simulated distribution scenarios. We report the operational cost (OC), total cost (TC) and the computational time in Table III. We observe that DDCHC results in higher operational costs but less total cost in comparison with CCUC. This is because, DDCHC is conservative, compared to CCUC, and schedules more generators such that they can accommodate the worst-case wind output distribution (to avoid large energy imbalances) for the whole next day. Therefore, DDCHC is a risk-averse approach. In fact, especially in case of unexpected realization of wind output, compared to CCUC operational decisions, DDCHC generation schedules and generation levels lead to less energy and load imbalances.

TABLE III  
DDCHC VERSUS CCUC

# of Data	DDCHC			CCUC		
	OC (\$m)	TC (\$m)	T(s)	OC (\$m)	TC (\$m)	T(s)
10	1.1949	2.6937	12	1.1911	2.7678	9
20	1.1946	2.6872	12	1.1913	2.7603	9
30	1.1943	2.6807	12	1.1917	2.7555	9
50	1.1935	2.6723	12	1.1920	2.7494	9
100	1.1925	2.6644	11	1.1922	2.7413	9

## V. CONCLUSION

In this study, we presented a data-driven chance-constrained stochastic unit commitment, in which the chance constraint controls the level of energy imbalance. Unlike traditional chance constraint which assumes that the unknown parameter follows a certain probability distribution, our proposed approach learns from a given historical data set to construct a confidence set for the unknown wind output distribution. Our model is a two-stage model which deals with the UC and ED decisions in the first stage and considers the worst-case energy imbalance cost associated with the worst-case wind output distribution in the second stage. By numerical experiments, we show our approach leads to more reliable and robust generation schedules than those of the traditional chance-constrained UC. We also show that as the number of historical data goes to infinity, the conservatism of the proposed approach vanishes eventually.

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