

Stochastic Minimization of Imbalance Cost for a Virtual Power Plant in Electricity Markets

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Abstract— In this paper, we investigate the optimal real-time operation strategy for a virtual power plant (VPP) to reduce its imbalance cost when participating in a competitive electricity market for energy. To capture the intrinsically stochastic nature of renewable power generation and market prices, the problem is formulated as a stochastic program. The decisions in the balancing market include actual renewable power usage and flexible demand management. Our model ensures that a certain portion of the renewable power generation will be utilized. A Lyapunov optimization based algorithm, which does not require the knowledge of probability distributions of the related random processes, is proposed to solve the problem. Both analytical and numerical results are presented to illustrate the effectiveness of the proposed algorithm.

I. INTRODUCTION

The issue of global warming and the limited supply of carbon-intensive fossil fuels have called for utilization of clean renewable energy sources (RESs) such as solar, wind, and geothermal. Many governments around the world have set up renewable portfolio standards (RPSs) to promote the production of energy from RESs. As of today, renewable energy producers mainly enroll in feed-in tariff (FIT) programs, in which they receive guaranteed grid access and favorable regulated FITs. However, this approach may not be sustainable since FITs have a fixed expiry date, mostly 12-15 years, after which the renewable energy producer becomes a non-subsidized producer that needs to participate in electricity markets [1]. Moreover, the effectiveness of FITs on reducing the carbon emission is in doubt now, as illustrated by the German electricity market [2].

Due to the stochastic variations of energy produced from non-dispatchable renewable distributed generators (RDGs) such as solar and wind power units, the risk for a single unit to participate in the electricity market is very high. If the actual generated energy is smaller than the scheduled energy output, the renewable power producer has to buy expensive balancing energy to cover the shortage. On the contrary, if the actual generated energy is larger than the scheduled energy, the excess energy has to be sold at a less competitive price or even curtailed in some circumstances. The uncertainty is often too high to make the market participation impossible. Another problem with RDGs is that their capacities (e.g., in scale of kW) are often too small to participate in the electricity market,

where the minimum trading amount of hourly contract is in unit of MW for energy.

To deal with these issues, the framework of virtual power plant (VPP) [3] has been proposed to facilitate cost-efficient integration of distributed energy resources (DERs) into the existing power systems. DERs, including distributed generation, distributed storage, and controllable loads, have the benefits of reducing carbon emissions and improving the power quality and reliability due to the presence of generation close to demand. VPP can enhance the visibility and control of DERs to system operators and other market players by providing an appropriate interface between these system components. By adding many DER units in one cluster and connecting them with an information network, the stochastic variations can be balanced between lots of single DER units. Therefore, the cluster of DERs is comparable to a conventional power plant and can participate in the electricity market.

In this paper, we consider a VPP consisting of RDGs and end consumers, who have both inelastic and elastic energy demand. Specifically, we investigate the optimal renewable power curtailment and demand response strategy for the VPP in the balancing market after the day-ahead bid has been submitted (i.e., after the day-ahead market has been cleared) so that the imbalance cost incurred in the balancing market is minimized. Our work is complementary to previous research [4]–[6] on the optimal day-ahead bidding strategy for renewable energy producers or VPPs by focusing on the real-time operation strategy in the balancing market. Also, different from [7], which does not consider any electricity market operation when selling renewable energy to delay-tolerant consumers, our work focuses on the market participation of a virtual power plant with guaranteed level of renewable energy utilization.

The rest of this paper is organized as follows: In Section II, we describe the models we use in this paper and present the mathematical formulation of the problem. Section III introduces the proposed control algorithm based on the Lyapunov optimization technique. Performance properties of the proposed algorithm is analyzed in Section IV. We conduct a numerical study of our proposed algorithm on real-world data sets in Section V. Conclusions are made in Section VI.

II. PROBLEM FORMULATION

We consider a VPP consisting of non-dispatchable RDGs and end consumers (which have both inelastic and elastic energy demand). An example is the distribution company (DISCO), which both owns RDGs and has an obligation to

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meet the loads in its territory. The VPP would participate into a competitive electricity market, either as a consumer or a producer. In this paper, we consider a sufficiently long scheduling horizon.

A. Renewable Distributed Generator Model

We assume that one day ahead, the VPP can forecast the aggregated output of RDGs $\bar{W}(t)$ for each time period t of the coming day. However, the actual output may not be exactly the same as the forecasted value due to the forecast error. We model this effect by assuming that the actual aggregated renewable output of RDGs is as follows:

$$W(t) = \bar{W}(t) + N_w(t), \quad (1)$$

where $N_w(t)$ is a random variable with unknown distribution. Any forecasting method can be used. The real value of the renewable power generation $W(t)$ is assumed to be known only at the beginning of the time period t during the operation day. Denote the nameplate capacity of the aggregated RDGs as W_{max} . We know that $0 \leq W(t) \leq W_{max}$ for all t .

As analyzed in [5], it may be necessary to curtail some of the generated renewable power in order to reduce the imbalance cost when transmission congestion exists or there is overproduction in the whole system. Since we do not consider the storage in this work, the unused renewable power is spilled. Denote the actual utilized renewable power at time t as $Y(t)$. Obviously, we have

$$0 \leq Y(t) \leq W(t). \quad (2)$$

To comply with some renewable energy utilization regulations (e.g., collecting enough renewable energy credits to satisfy RPS), the VPP has to ensure that at least a certain fraction of actual generated renewable power from RDGs is utilized [8]. To model it, we assume that the time-average renewable utilization level must be no smaller than some threshold ρ_w (e.g., 90%). Therefore, the constraint for renewable energy utilization can be stated as

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} Y(t) \geq \rho_w \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} W(t). \quad (3)$$

B. Demand Model

We divide the energy demand of end consumers in the VPP into two categories: inelastic demand and elastic demand. Inelastic demand is the base loads that cannot be delayed and need to be served immediately. We model it as the following discrete-time stochastic process $D_b(t)$, representing the total amount of requested energy at time t :

$$D_b(t) = \bar{D}_b(t) + N_b(t), \quad (4)$$

where $\bar{D}_b(t)$ is the day-ahead forecast of inelastic demand and $N_b(t)$ is the forecast error for time t of the coming day. Once again, $N_b(t)$ is assumed to be a random variable with some unknown distribution, which can be observed only at the beginning of time period t during the operation day. Moreover, we assume that $D_b(t) \leq D_{b,max}$ for all t .

On the other hand, elastic demand is flexible and can be adjusted in real-time when needed. Typical examples of elastic demand include EV charging and air conditioning. This energy

demand is elastic in the sense that as long as the required amount of energy is satisfied within some periods, it would not impact the user comfort. Assume the elastic demand arrives randomly according to a process $D_e(t)$ (being the amount of energy that is requested in time period t) characterized as follows:

$$D_e(t) = \bar{D}_e(t) + N_e(t), \quad (5)$$

where $\bar{D}_e(t)$ can be forecasted one day ahead and $N_e(t)$ is the forecast error for time period t of the coming day. Since we assume that some consumers requesting energy are flexible and can tolerate their energy requests being satisfied with some delay, the unserved requests are buffered in a queue. Let $Q(t)$ denote the total energy requests (in unit of required energy) in the queue at time t , we have the following update equation:

$$Q(t+1) = \max \{Q(t) - X(t) + D_e(t), 0\}, \quad (6)$$

where $X(t)$ is a control variable, representing the amount of energy that is allocated to serve the buffered requests at time t . We need to ensure the queue $Q(t)$ is stabilized so that the buffered energy requests are not delayed infinitely long.

We also assume an upper bound X_{max} on the allocated energy amount during one time period for the elastic demand. The maximum amount of requested energy for elastic demand during one time slot is denoted as $D_{e,max}$, and we further assume that $X_{max} \geq D_{e,max}$ so that it is always possible to stabilize the queue $Q(t)$. Therefore, we have

$$0 \leq X(t) \leq X_{max}. \quad (7)$$

C. Market Model

As with [5], we consider a conventional two-settlement market system consisting of an ex-ante day-ahead forward market with an ex-post imbalance settlement mechanism to penalize uninstructed deviations from bids submitted ex-ante. In the market, the VPP needs to submit hourly bidding schedules $B(t)$ of production ($B(t) > 0$) or consumption ($B(t) < 0$) in the day-ahead market (e.g., by 10 AM of the previous day) for the next day based on its forecast. We assume that $B_{min} \leq B(t) \leq B_{max}, \forall t$. The schedules cleared in the day-ahead market are financially binding and are subject to deviation penalties. In the balancing market, the system operator would employ a settlement mechanism to compute the imbalance prices for positive and negative deviations from the day-ahead bid. Note that renewable power curtailment can help reduce the deviation in some circumstances. However, the VPP can not spill too much renewable power as it may be subject to certain regulations that require renewable energy accounting for a certain share of their total generation output. Moreover, the VPP can exploit the elastic demand in its portfolio to help reduce the imbalance cost.

For each time t , the related market prices are denoted as follows:

- $p(t)$: settlement price in the day-ahead market
- $q^+(t)$: positive imbalance price in the balancing market
- $q^-(t)$: negative imbalance price in the balancing market

As analyzed in [4], the imbalance prices and the day-ahead prices satisfy the following relationship in most electricity markets:

$$-q^+(t) \leq p(t) \leq q^-(t). \quad (8)$$

The above inequality illustrates the following two facts:

- the VPP cannot gain more profit by trading in the balancing market rather than trading in the day-ahead market.
- the negative imbalance cost is no less than the cost of purchasing electricity in the day-ahead market.

Moreover, we assume that $q^+(t) \leq q_{max}^+$ and $q^-(t) \leq q_{max}^-$.

With the notations defined above, the power balance equation in each time period t is as follows:

$$Y(t) = B(t) + \Delta(t) + D_b(t) + X(t), \quad (9)$$

where $\Delta(t)$ is the net imbalance between the day-ahead market and the balancing market. More specifically, we denote the positive imbalance $\Delta^+(t)$ and the negative imbalance $\Delta^-(t)$, respectively, as follows:

$$\Delta^+(t) = \max[Y(t) - B(t) - D_b(t) - X(t), 0], \quad (10)$$

$$\Delta^-(t) = \max[B(t) + D_b(t) + X(t) - Y(t), 0]. \quad (11)$$

Obviously,

$$\Delta(t) = \Delta^+(t) - \Delta^-(t). \quad (12)$$

D. VPP Operation Model

In this paper, we assume that VPP is a price taker since its capacity is small relative to the whole market. Our system works as follows: one day ahead, the VPP can forecast the output from RDGs and the inelastic and elastic energy demand for the next day. Since we focus primarily on the real-time operating of VPP, we assume in this paper that the VPP will bid the forecasted production minus the forecasted consumption into the day-ahead market for each time t of the coming day¹:

$$B(t) = \overline{W}(t) - \overline{D}_b(t) - \overline{D}_e(t). \quad (13)$$

In real-time at time t , after the VPP knows the accurate renewable power output, demand arrivals, as well as the imbalance prices, it would decide the amount of curtailed renewable power and energy allocated to serve the elastic demand. The profit of VPP acquired at time t can be stated as

$$f(t) = p(t)B(t) - q^+(t)\Delta^+(t) - q^-(t)\Delta^-(t). \quad (14)$$

Since $B(t)$ is given by (13), the profit gained from the day-ahead market is independent of the control decisions in our problem. Therefore, we focus on minimizing the following imbalance cost incurred in the balancing market at time t :

$$c(t) = q^+(t)\Delta^+(t) + q^-(t)\Delta^-(t) \quad (15)$$

Considering a sufficiently long scheduling horizon, the problem of minimizing imbalance cost can be formulated as follows:

$$\text{Minimize : } \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{c(t)\} \quad (16)$$

¹Other bidding strategies such as [5] are possible and our work primarily focuses on the real-time operation strategy rather than the day-ahead bidding strategy.

subject to constraints (2), (3), (7), (9), (10), (11), (12), and

$$\overline{Q} \triangleq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q(t)\} < \infty. \quad (17)$$

Note that the expectation here is w.r.t. the random renewable power output, imbalance prices, demand arrivals, as well as possibly randomized control decisions. The above problem is challenging to solve, especially considering that the future information is unknown and the probability distributions of renewable power output, market prices, and energy demand arrivals may be hard to obtain. To address this challenge, we will construct a robust algorithm with a parameter V that achieves the average cost within $O(1/V)$ of the optimal value while the queue length is no more than $O(V)$ in the following.

III. PROPOSED ALGORITHM

First, we introduce the following virtual queue to ensure the constraint for renewable energy utilization level (3):

$$Z_w(t+1) = \max\{Z_w(t) - Y(t), 0\} + \rho_w W(t). \quad (18)$$

Intuitively, from the results of queueing theory [9], we know that the queue $Z_w(t)$ is stabilized as long as the average arrival rate is less than the average service rate, which reduces to (3). The detailed proof is omitted here for brevity.

Following the framework of Lyapunov optimization, we define a Lyapunov function as follows:

$$L(t) \triangleq \frac{1}{2} (Q^2(t) + Z_w^2(t)). \quad (19)$$

Let $\mathbf{S}(t) \triangleq (Z_w(t), Q(t))$, then the one-slot conditional Lyapunov drift is as follows:

$$\Delta L(t) \triangleq \mathbb{E}\{L(t+1) - L(t) | \mathbf{S}(t)\}. \quad (20)$$

Here, the expectation is taken over the randomness of market prices, renewable power generation, demand arrivals, as well as the randomness in choosing the control actions. We add a function of the expected imbalance cost over one slot (i.e., the penalty function) to (20) to obtain the following *drift-plus-penalty* term:

$$\Delta L(t) + V \mathbb{E}\{q^+(t)\Delta^+(t) + q^-(t)\Delta^-(t) | \mathbf{S}(t)\}, \quad (21)$$

where V is a positive control parameter to be specified later. We obtain the following lemma regarding the *drift-plus-penalty* term:

Lemma 1: For any feasible action under constraints (2) and (7) that can be implemented at time t , we have

$$\begin{aligned} \Delta L(t) + V \mathbb{E}\{q^+(t)\Delta^+(t) + q^-(t)\Delta^-(t) | \mathbf{S}(t)\} &\leq A \\ &+ \mathbb{E}\{Q(t)D_e(t) + \rho_w Z_w(t)W(t) | \mathbf{S}(t)\} \\ &+ \mathbb{E}\{V(q^+(t)\Delta^+(t) + q^-(t)\Delta^-(t)) | \mathbf{S}(t)\} \\ &- \mathbb{E}\{Q(t)X(t) + Z_w(t)Y(t) | \mathbf{S}(t)\} \end{aligned} \quad (22)$$

where A is a constant given by

$$A \triangleq \frac{(1 + \rho_w^2)W_{max}^2 + X_{max}^2}{2}. \quad (23)$$

Proof: See our technical report [10]. ■

The design principle of our control algorithm is to minimize the R.H.S. of (22) in each time period t such that the cost is minimized while stabilizing the queues. The relative importance of the above two objectives are adjusted by the parameter V . Note that constraints (10) and (11) are nonlinear, which add difficulty into the optimization problem. However, with the price relationship (8), for a given total energy deviation $\Delta(t) = \Delta^+(t) - \Delta^-(t)$, the optimal solution is guaranteed to be achieved with one of the variables $\Delta^+(t)$ or $\Delta^-(t)$ equals zero. Our control algorithm is described in Algorithm 1.

Algorithm 1: Proposed Algorithm

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1 Initialization: Choose parameters  $V > 0$ ; Set  $Q(0)$  and  $Z_w(0)$  to be zero;
2 foreach time period  $t$  do
3   Observe the current system state  $\mathbf{S}(t)$ ,  $q^+(t)$ ,  $q^-(t)$ ,  $W(t)$ ,  $D_b(t)$ ,  $D_e(t)$ , and day-ahead bidding  $B(t)$  given by (13);
4   Choose  $X(t)$  and  $Y(t)$  to minimize the following:
      
$$Vq^+(t)\Delta^+(t) + Vq^-(t)\Delta^-(t) - Z_w(t)Y(t) - Q(t)X(t) \quad (24)$$

      subject to (2), (7), (9), and
      
$$0 \leq \Delta^+(t) \leq W(t) - B_{min} - D_b(t), \quad (25)$$

      
$$0 \leq \Delta^-(t) \leq B_{max} + D_b(t) + X_{max} \quad (26)$$

      ;
5   Update queues  $Q(t)$  and  $Z_w(t)$  accordingly;
6 end

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Note that the proposed algorithm only needs to solve a linear program at each time period t and requires no detailed statistics of related random processes except the value at current time. Therefore, it is much simpler to implement than the dynamic programming approach. Before analyzing the performance of our algorithm, we give some results regarding the optimal solution to the above optimization problem.

Lemma 2: The optimal solution to the optimization problem (24) satisfies the following:

- 1) If $Q(t) > Vq^-(t)$, $X^*(t) = X_{max}$.
- 2) If $Z_w(t) > Vq^+(t)$, $Y^*(t) = W(t)$.

Proof: See our technical report [10]. ■

Remark 1: The lemma above describes some insights of our control algorithm. Specifically, when the length of buffered elastic demand requests is large, the elastic demand requests are more likely to be served so as to reduce their waiting time. Also, when the negative imbalance price is low, that means, it incurs less penalty to produce less than the agreed power output in the day-ahead market. Therefore, it is more likely to serve the buffered elastic energy demand at the current time. The relativeness between the queue length and the price is adjusted by the parameter V .

Similarly, when the queue length $Z_w(t)$, which corresponds to more spilled renewable energy in the past, the actual renewable utilization is more likely to be large at the current time

so as to satisfy the renewable energy utilization level. Also, when the positive imbalance price is low, that means, it incurs less penalty (or gains more profit) by producing more than the agreed power output in the day-ahead market. Therefore, it is more likely to fully utilize the current renewable power production. Once again, the relativeness between the queue length and the price is adjusted by the parameter V .

IV. PERFORMANCE ANALYSIS

Theorem 1: (Performance Analysis) If $Q(0) = 0$ and $Z_w(0) = 0$, then, under our control algorithm with any fixed $V > 0$, we have

- 1) The queues $Q(t)$ and $Z_w(t)$ are deterministically upper bounded as follows:

$$0 \leq Q(t) \leq Vq_{max}^- + D_{e,max}, \forall t \quad (27)$$

$$0 \leq Z_w(t) \leq Vq_{max}^+ + \rho_w W_{max}, \forall t. \quad (28)$$

- 2) If the vector $(q^+(t), q^-(t), N_w(t), N_b(t), N_e(t))$ is *i.i.d.* over slots, then for all time slots $T > 0$, the time average imbalance cost satisfies:

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{c(t)\} \leq c^* + A/V, \quad (29)$$

where c^* is the minimum time average cost with the problem (16) and A is a constant given by (23).

Proof: See our technical report [10]. ■

Remark 2: Note that the proof for result 1) is based only on the sample paths without any assumption on the distribution of underlying random processes. Although the analytical performance result 2) requires the *i.i.d.* assumption, we want to emphasize that the result is quite general and can be extended to treat the non-*i.i.d.* case too with minor modifications by using the universal scheduling results in [11]. The detailed discussion is omitted here for brevity. Moreover, our simulations are based on the real-world traces without any assumption on the probability distributions of underlying random processes.

V. NUMERICAL RESULTS

We demonstrate the performance of our proposed algorithm through extensive numerical evaluations. We simulate a VPP consisting of a wind plant and elastic and inelastic energy demand. We use a wind power time series data set provided by the BPA [12], which contains both actual wind power output and forecasted wind power output in the BPA control area from Jan. 1, 2013 to Jan. 31, 2013. The data set is scaled down such that the installed wind plant nameplate capacity $W_{max} = 2$ MW. The inelastic demand $D_b(t)$ in each time slot t is set to be uniformly distributed in $[0, 200]$ kWh. The elastic demand $D_e(t)$ in each time slot t is uniformly distributed in $[0, 640]$ kWh to simulate the charging demand of 40 PHEVs. We set $X_{max} = 2$ MWh. To better illustrate the impact of random wind power output on the imbalance cost, we assume that both inelastic demand $D_b(t)$ and elastic demand $D_e(t)$ can be predicted accurately one day ahead, i.e., $N_b(t) = N_e(t) = 0, \forall t$. More detailed simulation results with other settings can be found in our technical report [10].

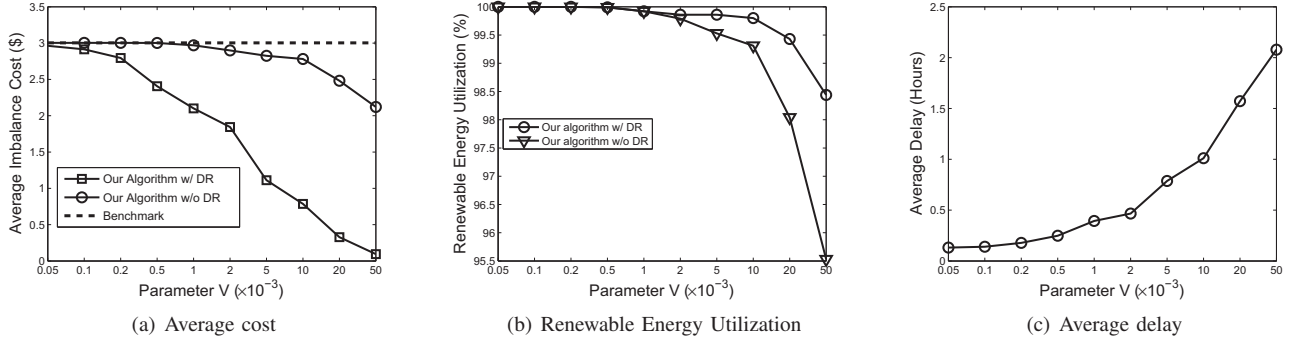


Fig. 1. Comparison between our algorithm and greedy algorithm with different V .

The day-ahead market price $p(t)$ is obtained from the New York ISO [13], also from Jan. 1, 2013 to Jan. 31, 2013. As with [14], we assume that deviations from the day-ahead bid, either positive or negative, in the balancing market are always penalized. We set the positive imbalance price $q^+(t)$ as follows:

$$q^+(t) = -(1 + \gamma^+(t))p(t), \quad (30)$$

where $\gamma^+(t)$ is uniformly distributed in $[-2, -1]$. Note that with this setting, the surplus renewable power in the balancing market cannot make any profit. Similarly, we set the negative imbalance price $q^-(t)$ as

$$q^-(t) = (1 + \gamma^-(t))p(t), \quad (31)$$

where $\gamma^-(t)$ is uniformly distributed in $[0, 1]$. The minimum renewable energy utilization level ρ_w is set to be 90%. The time slot duration is 1 hour. With these default settings, we evaluate the performance of our algorithm in the following with different values of the parameter V .

To show the effectiveness of our algorithm, we choose a greedy approach which fully utilizes all the renewable power generation and serves the elastic demand without delay as the benchmark. We also show the performance of our algorithm when there is no demand response (DR) resources, i.e., all energy loads are considered as inelastic. As we can see from the Figure 1(a), our proposed algorithm can largely reduce the imbalance cost with the increase of the parameter V . Specifically, by comparing our algorithm without DR with the greedy benchmark, we can observe that the real-time renewable energy curtailment can help reduce the imbalance cost. On the other hand, by comparing our algorithms with DR and without DR, we can observe the great benefits brought by aggregating the renewable generators and elastic demand. Meanwhile, from the Figure 1(b), our algorithms with DR and without DR can both achieve a high renewable energy utilization level while reducing the imbalance cost. Moreover, DR resources can improve the renewable energy utilization besides reducing the imbalance cost. However, our algorithm with DR does have a trade-off, as shown in Figure 1(c). The average delay increases as parameter V increases, which matches the results in Theorem 1. Since the average delay is relatively small (around 2 hours) even for the largest value of V we choose in the numerical evaluations, it further augments the effectiveness of our algorithm.

VI. CONCLUSION

In this paper, we have solved the problem of minimizing the imbalance cost of a VPP participating in a two-settlement electricity market. The stochastic nature of the imbalance prices, renewable power generation, and energy demand arrivals is considered. An efficient online algorithm, which does not require the statistical properties of the related random processes, has been proposed and evaluated both analytically and numerically.

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