Coordinated Energy Scheduling for Residential Households in the Smart Grid

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***Abstract*—In this paper, we investigate the minimization of the total energy cost of multiple residential households in a smart grid neighborhood sharing a load serving entity. Specifically, each household may have renewable generation, energy storage as well as inelastic and elastic energy loads, and the load serving entity attempts to schedule the energy consumption of these households. To minimize the total energy cost in this neighborhood, we propose an online algorithm, called Lyapunov-based cost minimization algorithm (LCMA), which jointly considers the energy management and demand response decisions. We prove that LCMA can achieve close-to-optimal performance and is robust to the uncertainty of system dynamics. Numerical results based on the real-world trace data show its cost saving effectiveness.**

I. Introduction

The grid modernization, which transforms the current power grids to the future "smart grid", is motivated by the growing demands of electricity and concerns over global climate change and carbon emission. The smart grid will enable deep penetration of renewable generation, customer driven demand response, widespread adoption of electric vehicles, and electric energy storage [1]. Sensing, communication, computation, and control technologies in conjunction with advances in renewable generation, energy storage, power electronics, etc. are critical to realizing the vision and promise of the smart grid.

Demand side management (DSM) is a key component in the smart grid, which can help reduce peak load, increase grid reliability, and lower generation cost [2]. There are mainly two types of demand side management techniques: direct load control (DLC) and demand response based on time-varying pricing [3]. In DLC, the load serving entity, usually a utility company, enters into a contract with the consumers beforehand, so that certain amount of energy load can be curtailed during the peak hours in order to release the congestion on the power grid or to avoid the operation of high cost peak generators. Currently, it is mainly adopted by large industrial and commercial customers. On the other hand, the demand response based on time-varying pricing encourages the customers to adjust their normal energy consumption, either reducing or shifting consumption, in return for some benefits, such as reduced electricity bill. Several popular schemes already exist in this regard, such as critical-peak pricing, time-of-use pricing, and real-time pricing. In the smart grid, it is expected that there will be a widespread deployment of such demand response programs for residential customers due to the existence of advanced metering infrastructure (AMI), which can provide two-way communication between utility companies and smart meters [4].

Meanwhile, nearly 7% of electricity is lost during transmission and distribution (T&D) from remote power plants to distant homes [5]. To decrease both T&D losses and carbon emissions, distributed generation (DG) from many small on-site energy sources deployed at individual homes and businesses can be used. Typical examples of these small on-site energy sources include rooftop solar panels, fuel cells, microturbines, and micro-wind generators. Distributed energy storage devices are usually used in combination with these renewable sources to better utilize them. We envision that residential households in the smart grid use on-site renewable generation, modest energy storage, and the electric grid to meet their energy demands, within which some are elastic and can be served in a flexible manner. Considering the dynamic nature of the system, it is quite challenging to simultaneously manage these components for households within a smart grid neighborhood in order to reduce the total energy consumption and cost, as well as the impact on the power distribution network.

In this paper, we develop an efficient online algorithm, called LCAM, to solve the problem above. In contrast to previous work [6], LCMA does not require the assumption of perfect future information of the underlying stochastic processes. We first provide theoretical results on the performance of LCMA under the i.i.d. case. Furthermore, the algorithm is provably robust to non-i.i.d. situations. Through numerical evaluations with real-world data, we show that our algorithm can efficiently reduce the total energy cost in the neighborhood.

This paper is organized as follows. In Section II, we describe our system model and formulate the problem as a stochastic programming problem. We describe the design principle behind our algorithm and present an online algorithm in Section III. We analyze our algorithm in Section IV. We present numerical results based on real-world data in Section V. Finally, some concluding remarks are presented

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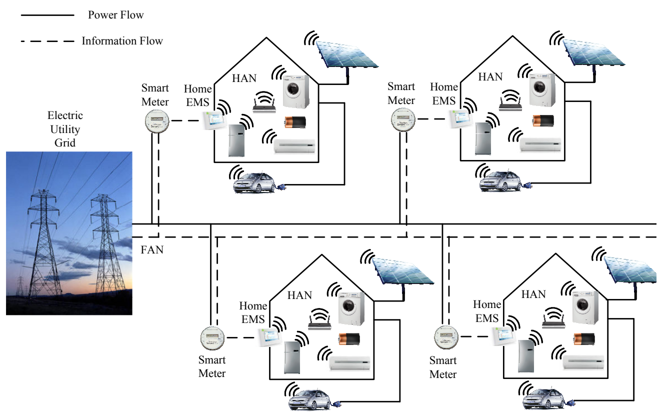


Fig. 1. Schematic of Household Energy Management in a Smart Grid Neighborhood

in Section VI.

II. System Model and Problem Formulation

Consider a set of N households/customers that are served by a single load serving entity (LSE) in a smart grid neighborhood setting as depicted in Fig. 1. The LSE may be a utility company and the smart grid neighborhood may cover all households connected to a step-down transformer in the distribution network connected to the electric grid. The LSE participates into wholesale electricity markets (day-ahead, hour-ahead, real-time balancing, ancillary service) to purchase electricity from power generators and then sell it to the N customers in the retail market. Currently, the electricity price in the retail market is usually flat because of the simplicity and predictability. However, it does not encourage efficient usage of electricity, causing high peak demand and low load factor. We consider a time-slotted system with dynamic pricing in this work. Each slot represents a suitable period for control decisions and is indexed by $t = \{0, 1, \dots\}$.[1]

A. Load Serving Entity

The LSE serves as an agent that is responsible for purchasing enough electricity from wholesale electricity markets to serve the energy demand of the households in its service area. The retail price is set in order to at least recover the running cost of the LSE. In the future smart grid, field area network (FAN) would be deployed, which can provide convenient communications between utility companies and smart meters of residential households. For simplicity, we make the assumption that the cost of the LSE can be represented by a time-varying cost function $C_t(D)$ that specifies the cost of providing amount D of electricity to the N customers at time slot t. We assume that the cost function $C_t(D)$ is increasing, continuously differentiable, and convex in D for any t with a bounded first derivative. We use α^{min} and α^{max} to denote the minimum and the maximum first derivatives of $C_t(D)$, respectively.

[1]In this paper, all power quantities such as $r_i(t)$, $s_i(t)$, $y_i(t)$, $d_{i,1}(t)$, $d_{i,2}(t)$ are in the unit of energy per slot, so the energy produced/consumed in time period t are $r_i(t)$, $s_i(t)$, $y_i(t)$, $d_{i,1}(t)$, $d_{i,2}(t)$, respectively.

B. Energy Load

In general, the energy loads in a household can be roughly divided into two categories: inelastic and elastic loads. Examples of inelastic energy loads include lights, TVs, microwaves, and computers. For this type of energy loads, the energy requests must be met exactly at the time t when needed. In contrast, there are some energy loads in households that are elastic in the sense that they can be controlled (using smart appliances, for example,) to adjust the times of their operations and the amount of their energy usage without impacting the satisfaction of customers. Examples include refrigerators, dehumidifiers, air conditioners, and electric vehicles. Actually, the vast majority of household loads are inelastic. However, as observed in [7], while the elastic energy loads comprise less than 7.5% of the total loads in a household, they account for 59% of the average energy consumption. Therefore, there is great hidden potential in exploiting the inherent flexibility of such elastic loads for various important individual and system level objectives.

Inside a household, electric loads can communicate with the smart meter via the home area network (HAN), which may be Wi-Fi or ZigBee. For each household $i \in N$, denote by $d_{i,1}(t)$ the inelastic energy loads (in unit of kWh) and by $d_{i,2}(t)$ the elastic energy loads (in unit of kWh) at time t. As in [8], we assume that the elastic energy loads are "buffered" (i.e., the energy requests are held or delayed) first in a queue $Q_i(t)$ before being served. Denote by $y_i(t)$ the amount of energy that is used for serving the queued energy loads at time t. Then the dynamics of $Q_i(t)$ is as follows:

$$Q_i(t+1) = \max\{Q_i(t) - y_i(t), 0\} + d_{i,2}(t), \quad \forall i. \tag{1}$$

For each i, we assume that

$$0 \le y_i(t) \le y_i^{max}, \tag{2}$$

where $y_i^{max} \ge d_{i,2}^{max}$ so that the queue Q_i can always be stabilized. For any feasible control decision, we need to ensure that the average delay of the elastic loads in the queue is finite. In other words, we cannot delay arbitrarily long time for the service of elastic energy loads. This can be stated as follows:

$$\overline{Q_i} \doteq \limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q_i(t)\} < \infty. \tag{3}$$

C. Energy Storage

In addition to energy loads, each household may have some kind of energy storage device, possibly in the form of the battery in PHEV. For each household i, we denote by E_i^{max} the battery capacity, by $E_i(t)$ the energy level of the battery at time t, and by $r_i(t)$, the power charged to (when $r_i(t) > 0$) or discharged from (when $r_i(t) < 0$) the battery during slot t. Assume that the battery energy leakage is negligible and batteries at households operate independently of each other. Then we model the dynamics of the battery energy level by

$$E_i(t+1) = E_i(t) + r_i(t). \tag{4}$$

For each household i, the battery usually has an upper bound on the charge rate, denoted by r_i^{max}, and an upper bound on the discharge rate, denoted by $-r_i^{min}$, where r_i^{max} and $-r_i^{min}$ are positive constants depending on the physical properties of the battery. Therefore, we have the following constraint on $r_i(t)$:

$$r_i^{min} \leq r_i(t) \leq r_i^{max}. \quad (5)$$

The battery energy level should be always nonnegative and cannot exceed the battery capacity. So in each time slot t, we need to ensure that for each household i,

$$0 \leq E_i(t) \leq E_i^{max}. \quad (6)$$

However, the cost of battery use cannot be ignored. In practice, there are limited times of charging/discharing cycles for each battery. Besides, conversion loss occurs both in charging and discharging processes. Stored energy is also subject to leakage with time. All these factors depend on how fast/much/often it is charged and discharged. Instead of modeling these factors exactly, we use an amortized time-invariant cost function $F_i(r_i)$ (in unit of dollars) to model the impact of charging or discharging operation r_i on the battery during one slot for household i. Each battery cost function $F_i(r_i)$ is assumed to be increasing, continuously differentiable, and convex in r_i with a bounded first derivative and $F_i(0) = 0$. We use β_i^{min} and β_i^{max} to denote the minimum and the maximum first derivatives of $F_i(r_i)$ for each household i, respectively.

D. Renewable Distributed Generation (DG)

Each household i may possess a distributed renewable generator installed on its site, such as rooftop PV panel. Denote $s_i(t)$ as the renewable energy generated in slot t by the renewable DG, which is usually intermittent, uncertain, and uncontrollable. We assume it is stochastic in different slots and has the maximum value given by its rated capacity s_i^{max}. Therefore, we have

$$0 \leq s_i(t) \leq s_i^{max} \quad \forall i, t. \quad (7)$$

Note that the energy generation from renewable generator is usually lower than the normal energy consumption density of households. Households need to connect to the utility electric grid for backup power and, therefore, are mostly grid-tied systems. In this paper, we assume that the renewable energy is free and should be utilized as much as possible.

E. Problem Formulation

With the above models for the battery and the distributed renewable generator, at each time t, the total power demand of household i needed from the utility electric grid is

$$g_i(t) \doteq \max\{d_{i,1}(t) + y_i(t) + r_i(t) - s_i(t), 0\}. \quad (8)$$

Note that in the formula above, we have assumed that power cannot be fed from the household into the utility electric grid through, for example, net metering. We plan to incorporate the option of two-way energy flow in our future investigation.

In this paper, we are interested in minimizing the LSE's total cost of providing the electricity to the whole smart grid neighborhood in a sufficiently long horizon. Note that reducing the supply cost of LSE is both beneficial to the LSE as well as individual customers since the cost will be finally transferred to the customers' electricity bill. Therefore, the control problem can be stated as follows: for the dynamic system defined by equations (1) and (4), design a control strategy which, given the past and the present random renewable supplies, the battery energy levels, the energy demands, and the energy cost function, chooses the battery charge/discharge vector $\mathbf{r}$ and the elastic load serving rate vector $\mathbf{y}$ such that the time-average total energy cost of the whole smart grid neighborhood is minimized. It can be formulated as the following stochastic programming problem, called **P1**:

$$\min_{\mathbf{y},\mathbf{r}} : \limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C_t(\sum_{i=1}^{N} g_i(t)) + \sum_{i=1}^{N} F_i(r_i(t))\}, \quad (9)$$

subject to constraints (2), (3), (4), (5), and (6).

Here the expectation in the objective is w.r.t. the random renewable generation $s_i(t)$, the random inelastic energy loads $d_{i,1}(t)$, the random elastic energy loads $d_{i,2}(t)$ for each household, and the random cost function C_t. Define **P1*** as the infimum time average cost associated with **P1**, considering all feasible control actions subject to the queue stability and the finite battery energy level. We will design a control algorithm, parameterized by a constant $V > 0$, that satisfies the constraints above and achieves the average cost within $O(1/V)$ of the optimal value **P1***. Moreover, it can guarantee that the worst-case delay is within $O(1/V)$.

III. Online Control Algorithm

In this section, we design an algorithm to solve **P1**. One challenge of solving the stochastic optimization problem above is the uncertainty of future renewable generation, time-varying cost function, inelastic or elastic energy loads. Moreover, the constraints on $E_i(t)$ bring the "time-coupling" property to the stochastic optimization problem above. That is to say, the current control action may impact the future control actions, making it more challenging to solve. Our solution is based on the technique of Lyapunov optimization [9] and requires minimum information on the random dynamics in the system.

A. Delay-Aware Virtual Queue

Since the constraint $\overline{Q_i} < \infty$ only ensures finite average delay for the elastic energy loads in household i, worst-case delay guarantee is usually desired in practice. For this purpose, we leverage the technique of "virtual queue" in the Lyapunov optimization framework. Specifically, the following virtual queues $Z_i(t), i = 1, 2, \ldots, N$ are defined to provide the worst-case delay guarantee on any buffered elastic energy loads in $Q_i(t)$:

$$Z_i(t+1) = \max\{Z_i(t) - y_i(t) + \epsilon_i 1_{\{Q_i(t)>0\}}, 0\}, \quad (10)$$

where $1_{\{Q_i(t)>0\}}$ is an indicator function that is 1 if $Q_i(t) > 0$ or 0 otherwise; ϵ_i is a fixed positive parameter to be specified later. The intuition behind this virtual queue is that since $Z_i(t)$ has the same service process as $Q_i(t)$, but has an arrival process that adds ϵ_i whenever the actual backlog is nonempty, this ensures that $Z_i(t)$ grows if there are energy loads in the queue $Q_i(t)$ that have not been serviced for a long time. The following lemma shows that if we can control the system to ensure that the queues $Q_i(t)$ and $Z_i(t)$ have finite upper bounds, then any buffered energy load is served within a worst-case delay as follows:

***Lemma** 1:* Suppose we can control the system to ensure that $Z_i(t) \leq Z_i^{max}$ and $Q_i(t) \leq Q_i^{max}$ for all slots t, where Z_i^{max} and Q_i^{max} are some positive constants. Then, the worst-case delay for all buffered energy loads in household i is upper bounded by δ_i^{max} slots where

$$\delta_i^{max} \triangleq \lceil \frac{(Q_i^{max} + Z_i^{max})}{\epsilon_i} \rceil. \tag{11}$$

Proof: The proof follows the framework of Lyapunov optimization [9] and is given in our technical report [10]. ■

We will show that there indeed exist such constants Z_i^{max} and Q_i^{max} for all households i later.

B. The Lyapunov-based Approach

The idea of our algorithm is to construct a Lyapunov-based scheduling algorithm with perturbed weights for determining the optimal energy usage. By carefully perturbing the weights, we can ensure that whenever we charge or discharge the battery, the energy level in the battery always lies in the feasible region.

First, we choose a perturbation vector $\theta = (\theta_i, \ \forall i)$ (to be specified later). We define a perturbed Lyapunov function as follows:

$$L(t) \doteq \frac{1}{2}\sum_{i=1}^{N}[(E_i(t) - \theta_i)^2 + Q_i^2(t) + Z_i^2(t)]. \tag{12}$$

Now define $\mathbf{K}(t) = (\mathbf{Q}(t), \mathbf{Z}(t), \mathbf{E}(t))$, and define a one-slot conditional Lyapunov drift as follows:

$$\triangle(t) = \mathbb{E}\{L(t+1) - L(t) \mid \mathbf{K}(t)\}. \tag{13}$$

Here the expectation is taken over the randomness of load arrivals, cost function, and renewable generation, as well as the randomness in choosing the control actions. Then, following the Lyapunov optimization framework, we add a function of the expected cost over one slot (i.e., the penalty function) to (13) to obtain the following *drift-plus-penalty* term:

$$\triangle_V(t) \doteq \triangle(t) + V\mathbb{E}\{C_t(\sum_{i=1}^{N} g_i(t)) + \sum_{i=1}^{N} F_i(r_i(t)) \mid \mathbf{K}(t)\}, \tag{14}$$

where V is a positive control parameter to be specified later. Then, we have the following lemma regarding the *drift-plus-penalty* term:

***Lemma** 2:* For any feasible action under constraints (2), (5), and (6) that can be implemented at slot t, we have

$$\begin{aligned}\triangle_V(t) \leq\ & B + \sum_{i=1}^{N}\mathbb{E}\{(E_i(t) - \theta_i)r_i(t) \mid \mathbf{K}(t)\} \\ & + \sum_{i=1}^{N}\mathbb{E}\{Q_i(t)(d_{i,2}(t) - y_i(t)) + Z_i(t)(\epsilon_i - y_i(t)) \mid \mathbf{K}(t)\} \\ & + V\mathbb{E}\{C_t(\sum_{i=1}^{N} g_i(t)) + \sum_{i=1}^{N} F_i(r_i(t)) \mid \mathbf{K}(t)\},\end{aligned} \tag{15}$$

where B is a constant given by

$$\begin{aligned}B \doteq \sum_{i=1}^{N}\Big\{ & \frac{\max\{(r_i^{min})^2, (r_i^{max})^2\}}{2} + \frac{\max\{(y_i^{max})^2, \epsilon_i^2\}}{2} \\ & + \frac{(y_i^{max})^2 + (d_{i,2}^{max})^2}{2}\Big\}.\end{aligned} \tag{16}$$

Proof: See our technical report [10]. ■

We now present the LCMA algorithm. The main design principle of the algorithm is to approximately minimize the R.H.S. of (15).

Lyapunov-based Cost Minimization Algorithm (LCMA): Initialize (θ_i, ϵ_i), $\forall i$ and V. At each slot t, observe $(d_{i,1}(t), d_{i,2}(t), s_i(t))$, $\forall i$, C_t, $\mathbf{K}(t)$, and do:

- Choose control decisions $\mathbf{y}^*$ and $\mathbf{r}^*$ as the optimal solution to the following optimization, called **P3**:

$$\begin{aligned}\min: \sum_{i=1}^{N}\Big\{ & (E_i(t) - \theta_i)r_i(t) + VF_i(r_i(t)) \\ & - (Q_i(t) + Z_i(t))y_i(t)\Big\} + VC_t(\sum_{i=1}^{N} g_i(t)),\end{aligned}$$

s.t.

$$r_i^{min} \leq r_i(t) \leq r_i^{max}, \quad \forall i,$$

$$0 \leq y_i(t) \leq y_i^{max}, \quad \forall i.$$

- Update $\mathbf{K}(t)$ according to the dynamics (1), (4), and (10), respectively.

As an intuitive explanation, our algorithm is trying to store excess renewable energy for later use, recharge the battery during the period of low electricity price while discharging it during the period of high electricity price, and delay elastic energy loads to later slots with lower electricity price. Note that we do not need to consider the time-coupling constraints (6) of the battery energy level in the algorithm, since they can be automatically satisfied during our operation of the queues, as proven in Theorem 1 below. Moreover, the algorithm only requires the knowledge of the instantaneous values of system dynamics and does not require any knowledge of the statistics of these stochastic processes.

IV. Performance Analysis

In this section, we analyze the performance of LCMA under the case that when the cost function $C_t(\cdot)$, renewable energy generation $s_i(t), \forall i$, energy load arrival processes $d_{i,1}(t), \forall i$ and $d_{i,2}(t), \forall i$ are all i.i.d.. We recognize that these assumptions are restrictive but are made only for the purposes of theoretical analysis; the LCMA algorithm (or some variant thereof) can be implemented even though these assumptions may not be satisfied. Additional results on the non i.i.d. case are presented in our technical report [10].

Theorem 1: If $Q_i(0) = Z_i(0) = 0$ and $\theta_i = V(\alpha^{max} + \beta_i^{max}) - r_i^{min}$ for all households i, then under the LCMA algorithm for any fixed parameters $0 \le \epsilon_i \le \mathbb{E}\{d_{i,2}(t)\}$, and $0 < V \le V^{max}$, where

$$V^{max} \doteq \min_i \frac{E_i^{max} - r_i^{max} + r_i^{min}}{\alpha^{max} + \beta_i^{max} - \alpha^{min} - \beta_i^{min}}, \tag{17}$$

we have the following properties:

1) The queues $Q_i(t)$ and $Z_i(t)$ are deterministically upper bounded by Q_i^{max} and Z_i^{max} at every slot, where

$$Q_i^{max} \doteq V\alpha^{max} + d_{i,2}^{max}, \tag{18}$$

$$Z_i^{max} \doteq V\alpha^{max} + \epsilon_i. \tag{19}$$

2) The worst-case delay of any buffered elastic energy load is given by:

$$\delta_i^{max} = \lceil \frac{2V\alpha^{max} + d_{i,2}^{max} + \epsilon_i}{\epsilon_i} \rceil. \tag{20}$$

3) The energy queue $E_i(t)$ satisfies the following for all time slots t:

$$0 \le E_i(t) \le E_i^{max}. \tag{21}$$

4) All control decisions are feasible.
5) If $C_t(\cdot)$, $s_i(t), \forall i$, $d_{i,1}(t), \forall i$, and $d_{i,2}(t), \forall i$ are i.i.d. over slots, then the time-average expected operating cost under our algorithm is within bound B/V of the optimal value, i.e.,

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C_t(\sum_{i=1}^{N} g_i(t)) + \sum_{i=1}^{N} F_i(r_i(t))\} \le \mathbf{P1}^* + B/V, \tag{22}$$

where B is the constant specified in (16).

Proof: We only provide the sketch of the proof here.

1) First, we prove $Q_i(t) \le Q_i^{max}$ for every time slot t. Once again, we will use induction method. Obviously, $Q_i(0) \le Q_i^{max}$. Suppose it holds at time slot t, we need to show that it also holds at time slot $t+1$. As $Q_i(t+1) = \max\{Q_i(t) - y_i(t), 0\} + d_{i,2}(t)$, if $Q_i(t) \le V\alpha^{max}$, and the maximum amount of inelastic energy load arrival is $d_{i,2}^{max}$, we have $Q_i(t+1) \le V\alpha^{max} + d_{i,2}^{max}$. If $V\alpha^{max} < Q_i(t) \le V\alpha^{max} + d_{i,2}^{max}$, LCMA will choose the maximum possible value for $y_i(t)$ since the partial derivative of the objective function in **P3** w.r.t. $y_i(t)$ is negative. If $Q_i(t) - y_i^*(t) > 0$, then, in time slot t the amount of energy demand being served is at least y_i^{max}, which is larger than the maximum amount of arrival during time slot t. Hence, the queue cannot increase, i.e., $Q_i(t+1) \le Q_i(t) \le V\alpha^{max} + d_{i,2}^{max}$. If $Q(t) - y_i^*(t) \le 0$, then $Q_i(t+1) \le d_{i,2}^{max} \le V\alpha^{max} + d_{i,2}^{max}$. Therefore, we have proved $Q_i(t) \le Q_i^{max}$. Similarly, we can prove $Z_i(t) \le Z_i^{max}$.
2) This directly follows Lemma 1.
3) Once again, we prove the result by induction. When $t = 0$, $E_i(0) = 0 \le E_i^{max}$. Now suppose that the bound above holds for time slot t. We need to show that it also holds for time slot $t+1$. First, assuming that $0 \le E_i(t) < \theta_i - V(\alpha^{max} + \beta_i^{max})$, then LCMA will choose the maximum value for $r_i(t)$ because the partial derivative of the objective function in **P3** w.r.t. $r_i(t)$ is always negative. Therefore, the battery would charge as much as possible, i.e., $0 \le E_i(t) \le E_i(t+1) < \theta_i - V(\alpha^{max} + \beta_i^{max}) + r_i^{max} \le E_i^{max}$. Second, assuming that $\theta_i - V(\alpha^{max} + \beta_i^{max}) \le E_i(t) \le \theta_i - V(\alpha^{min} + \beta_i^{min})$, then the maximum charge and discharge rates for the battery are r_i^{max} and $-r_i^{min}$, respectively. Hence, $0 = \theta_i - V(\alpha^{max} + \beta_i^{max}) + r_i^{min} \le E_i(t+1) \le \theta_i - V(\alpha^{min} + \beta_i^{min}) + r_i^{max} \le E_i^{max}$, where we have used the upper bound V_{max} of V. Third, suppose $\theta_i - V(\alpha^{min} + \beta_i^{min}) \le E_i(t) \le E_i^{max}$, then LCMA will choose the minimum value for $r_i(t)$ because the partial derivative of the objective function in **P3** w.r.t. $r_i(t)$ is always positive. Therefore, the battery would discharge as much as possible, i.e., $0 \le \theta_i - V(\alpha^{min} + \beta_i^{min}) + r_i^{min} \le E_i(t+1) \le E_i(t) \le E_i^{max}$. This completes the proof.
4) Since we choose our decisions to satisfy all constraints in **P3**, in combination with the results in 1) and 3), all constraints of **P1** are satisfied. Therefore, our control decisions are feasible to **P1**.
5) See our technical report [10].

■

V. Numerical Experiments

In this section, we provide numerical results based on real-world data sets to complement the analysis in the previous sections.

A. Experiment Setup

We consider a simple power system consisting of eight households in one neighborhood. The households are divided into two categories. For the first type of households (indexed by $i = 1, 2, 3, 4$), both the elastic and inelastic energy load arrivals during one slot are i.i.d. and take value from $[1, 5]$ kWh uniformly at random. For the second type of households (indexed by $i = 5, 6, 7, 8$), both the elastic and inelastic energy load arrivals during one slot are also i.i.d. and take value from $[1.5, 7.5]$ kWh uniformly at random. For the renewable generation, we use the hourly average solar irradiance data for Los Angels area from the Measurement and Instrumentation Data center [11] at National Renewable Energy Laboratory. The period we consider in this paper is half year from January

1, 2011 to June 30, 2011. In total, this duration includes 181 days or 4344 1-hour slots. The control interval is chosen to be 1-hour. We fix the maximum charge and discharge rates of batteries in households as follows: for $i \in \{1,2,3,4\}$, $r_i^{max} = 1$kWh, $r_i^{min} = -1$kWh, and for $i \in \{5,6,7,8\}$, $r_i^{max} = 1.5$kWh, $r_i^{min} = -1.5$kWh. Also, we choose $y_i^{max} = d_{i,2}^{max}$ for all i. The battery cost is assumed to be a simple quadratic function as $F_i(r_i) = b_1 r_i^2, \forall i$, where b_1 is a constant coefficient. For the LSE, we assume that the energy cost function is a smooth quadratic function as $C_t(D) = c_1(t)D^2 + c_2 D + c_3$, where $c_1(t)$ is a time-varying coefficient used to model different electricity marginal costs across time slots. In this evaluation, $c_1(t)$ takes value from $[0.1, 0.2]$ uniformly at random, $c_2 = 0.1$, and $c_3 = 0.2$.

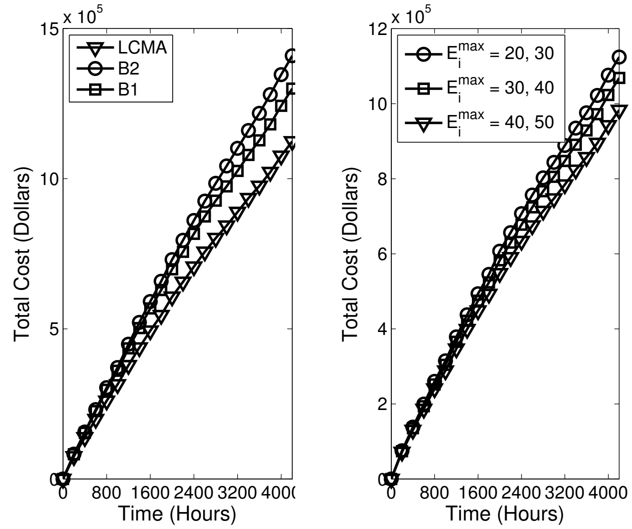
B. Results and Analysis

In order to analyze the performance improvement due to our LCMA, we compare it with the following two approaches: (i) No storage, no demand response (B1): The household tries to use the renewable energy as much as possible. When the renewable energy is not sufficient, the household draws energy from the utility grid. Unused renewable energy is wasted; (ii) Storage, no demand response (B2): The household uses renewable energy only as a supplement to the grid by consuming it whenever it is available. The household stores any extra renewable energy in its battery, but never charge the battery from the grid. The stored energy would be used to serve the future demands.

First, we compare our algorithm with the two approaches above using the real-world solar power data. Note that the performance of LCMA depends on the battery capacity, the battery cost, and the control parameters V and ϵ_i. We choose $b_1 = 0.5$, $E_i^{max} = 20$kWh$, i \in \{1,2,3,4\}$, and $E_i^{max} = 30$kWh$, i \in \{5,6,7,8\}$. The initial battery energy level at each household is chosen to be zero. Let $V = V^{max}$ and $\epsilon_i = \mathbb{E}\{d_{i,2}(t)\}, \forall i$. As can be seen in Figure 2(a), our proposed LCMA can reduce the total energy cost by approximately 20% compared with B1 and 13% compared with B2 in the six-month period. Also, the slopes of the lines are different, meaning that the savings are unbounded as the time increases. Secondly, we adjust the battery capacity and observe the energy cost while setting $V = V^{max}$. From Fig. 2(b), we can see that the larger the battery is, the more cost saving our algorithm can achieve, which matches the analytical results presented above. More detailed simulation results such as the delay performance of LCMA and the impacts of battery cost and ϵ_i on the performance of LCMA can be found in our technical report [10].

VI. Conclusions

In this paper, we present an algorithm (LCMA) for coordinated stochastic optimization of flexible energy resources in a smart grid setting. The total system cost can be reduced if more energy loads are elastic and can tolerate being served with some delay. Our algorithm is simple and was shown to be able to operate without knowing the statistical properties of



(a) (b)

Fig. 2. (a) Comparison of the total energy cost in three approaches; (b) The impact of battery capacity on the cost saving

the underlying dynamics in the system. With the increase of energy storage capacities, the performance of our algorithm is proved to be arbitrarily close to the optimal value. Moreover, our algorithm provides an explicit relationship between energy storage capacity, worst-case delay, and cost saving. Extensive numerical evaluations based on the real-world data show the effectiveness of our approach.

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