CTR-LORA: CURVATURE-AWARE AND TRUST-REGION GUIDED LOW-RANK ADAPTATION FOR LARGE LANGUAGE MODELS

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ABSTRACT

Parameter-efficient fine-tuning (PEFT) has become the standard approach for adapting large language models under limited compute and memory budgets. Although previous methods improve efficiency through low-rank updates, quantization, or heuristic budget reallocation, they often decouple the allocation of capacity from the way updates evolve during training. In this work, we introduce CTR-LoRA, a framework guided by curvature trust region that integrates rank scheduling with stability-aware optimization. CTR-LoRA allocates parameters based on marginal utility derived from lightweight second-order proxies and constrains updates using a Fisher/Hessian-metric trust region. Experiments on multiple open-source backbones (7B-13B), evaluated on both in-distribution and out-of-distribution benchmarks, show consistent improvements over strong PEFT baselines. In addition to increased accuracy, CTR-LoRA enhances training stability, reduces memory requirements, and achieves higher throughput, positioning it on the Pareto frontier of performance and efficiency. These results highlight a principled path toward more robust and deployable PEFT.

Index Terms— Parameter-efficient fine-tuning (PEFT), low-rank adaptation (LoRA), large language models (LLMs)

1 Introduction

Parameter-efficient fine-tuning (PEFT) has become the standard route to adapt billion-parameter language models under strict compute and memory budgets [1, 2, 3, 4, 5]. Among PEFT designs, low-rank adaptation (LoRA) injects a pair of thin adapters into selected projections and freezes the backbone, achieving strong transfer with *no* inference-time latency and a minimal trainable footprint [6]. Quantization-aware PEFT pushes this boundary further: QLoRA enables 4-bit training and makes even 65B models attainable on a single 48 GB GPU [7]. Despite its ubiquity, LoRA still faces two pain points: (1) rank schedules are typically uniform or hand-tuned, ignoring the heterogeneous importance of
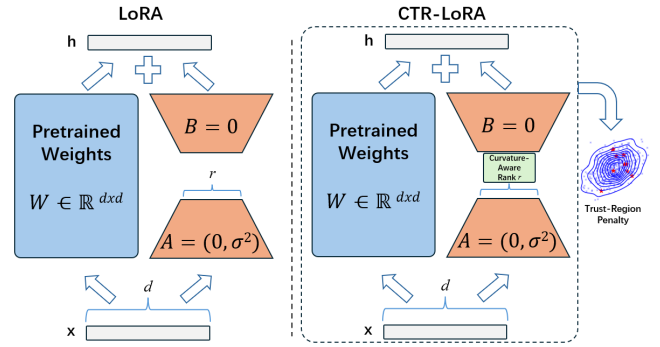

Fig. 1. Comparison between LoRA and CTR-LoRA. Left: standard LoRA decomposes the low-rank update $\Delta W = AB^\top$ with $A \sim \mathcal{N}(0, \sigma^2)$ and $B = 0$, merged into frozen pretrained weights $W \in \mathbb{R}^{d \times d}$. Right: CTR-LoRA introduces curvature-aware rank scheduling (guiding the choice of rank r) and a trust-region penalty that regularizes the update trajectory, improving stability and robustness during fine-tuning.

layers and matrices, and (2) training stability depends on update geometry and step size, leaving low-rank updates vulnerable in high-curvature regions. Recent studies also highlight pronounced hyperparameter sensitivity and a trade-off between acquiring new skills and retaining base knowledge [8]. A series of variants tackle these challenges from different angles. *Adaptive-budget* methods redistribute rank across matrices (e.g., AdaLoRA) under a global parameter cap [9]. *Optimization-aware* refinements adjust the update rule: DoRA decouples magnitude and direction [10], while LoRA+ applies asymmetric learning rates to correct imbalanced dynamics [11]. *Initialization-aware* schemes such as PiSSA seed adapters with principal components to accelerate convergence [12]. More recently, *sensitivity-driven* allocation leverages second-order signals for rank selection with low overhead (Sensitivity-LoRA), underscoring the value of curvature information [13]. Yet most methods still optimize rank *separately* from training dynamics: they decide *where* to place parameters but not *how* low-rank updates evolve across

the loss landscape.

This work proposes a purely adapter-based solution that unifies *where* to allocate capacity with *how* to move in the parameter space. We introduce a curvature-informed, trust-region-guided framework that (1) scores marginal utility of rank-1 directions under a global budget for rank scheduling, and (2) regularizes training with a curvature-metric penalty to keep updates stable in high-curvature regions. The approach is plug-and-play with existing LoRA stacks (compatible with LoRA+, DoRA, and 4-bit training), incurs *zero* inference-time cost, and preserves the simplicity practitioners expect. Our contributions are summarized as follows:

- We introduce a principled criterion based on marginal utility under curvature proxies, replacing uniform or heuristic schedules. This shifts PEFT from manual tuning toward theoretically guided allocation, setting a foundation for more systematic use of limited parameters.
- We establish a stability-oriented objective that constrains updates in curvature space, ensuring reliable optimization and quantization compatibility. This provides a unifying perspective that links PEFT to trust-region and natural-gradient methods, bridging practice with theory.
- Our framework is fully compatible with existing LoRA ecosystems, requires no architectural changes, and preserves zero inference overhead. This positions CTR-LoRA not merely as another variant, but as a deployable and extensible paradigm for robust, large-scale adaptation.

2 Method

Problem Definition We adapt a pretrained transformer with parameter matrices $\{W_\ell\}_{\ell=1}^L$ using low-rank adapters. For each target matrix $W_\ell \in \mathbb{R}^{d_{\text{out}} \times d_{\text{in}}}$, LoRA parameterizes an update $\Delta W_\ell = A_\ell B_\ell^\top$ with rank $r_\ell \ll \min(d_{\text{in}}, d_{\text{out}})$ and injects $W_\ell \leftarrow W_\ell + \Delta W_\ell$ while freezing the backbone [6]. We assume a global trainable-parameter budget $B = \sum_\ell r_\ell(d_{\text{in}} + d_{\text{out}})$, or equivalently a fixed ratio of base parameters. The central goal of CTR-LoRA is to allocate ranks such that each parameter provides the highest marginal reduction in loss, while simultaneously constraining the update geometry to remain stable in high-curvature regions, all without incurring inference-time latency.

Curvature Proxies Second-Order Signals Let $\mathcal{L}$ denote the training loss and $G_\ell = \nabla_{W_\ell} \mathcal{L}$ the gradient. To capture local geometry, we approximate curvature with a block-diagonal proxy $M_\ell \succeq 0$ for each matrix. In practice, we employ either Kronecker-factored approximations (K-FAC) [14], which decompose the Fisher block as $\mathbb{E}[gg^\top] \otimes \mathbb{E}[aa^\top]$ based on layer inputs a and backpropagated gradients g, or Hutchinson-style estimators [15, 16], which use stochastic Hessian-vector products to approximate diagonals and quadratic forms. These statistics are precomputed on a small calibration set and refreshed intermittently during

training. With M_ℓ in place, we define the whitened gradient $\tilde{G}_\ell = M_\ell^{-1/2} G_\ell M_\ell^{-1/2}$, which normalizes directions by curvature and aligns with natural-gradient intuition [17].

Curvature-Aware Rank Scheduling We evaluate candidate rank-one directions using a local quadratic model. The second-order expansion at W_ℓ is given by Eqn (1).

$$
\Delta \mathcal{L} \approx -\langle G_\ell, \Delta W_\ell \rangle + \tfrac{1}{2} \|\Delta W_\ell\|_{M_\ell}^2 \\
\|X\|_{M_\ell}^2 = \langle X, M_\ell X \rangle
\tag{1}
$$

Confining ΔW_ℓ to a form $\alpha U V^\top$ with $\|U\|_2 = \|V\|_2 = 1$, the optimal step size is $\alpha^\star = \langle G_\ell, U V^\top \rangle / \|U V^\top\|_{M_\ell}^2$, yielding a predicted decrease as shown in Eqn (2).

$$
\begin{aligned}
\mathcal{D}_\ell(U, V) &= \tfrac{1}{2} \frac{\langle G_\ell, U V^\top \rangle^2}{\|U V^\top\|_{M_\ell}^2} \\
&= \tfrac{1}{2} \sigma^2(M_\ell^{-1/2} G_\ell M_\ell^{-1/2})
\end{aligned}
\tag{2}
$$

Here, the equality holds when (U, V) corresponds to a singular pair of the whitened gradient. Thus the leading singular vectors of $\tilde{G}_\ell$ naturally provide descending marginal utilities. To respect the global budget B, we collect candidate directions across layers and select them greedily by their marginal utility scores. Each selection assigns one unit of rank, optionally followed by rank-one deflation to update the residual spectrum. In practice, randomized SVD [18] with small oversampling suffices to approximate these spectra efficiently. Compared with uniform or hand-tuned schedules, this strategy explicitly favors matrices where curvature-adjusted gradients carry the most useful information.

Trust-Region PEFT Objective Rank allocation alone does not constrain how updates traverse the loss landscape. To further stabilize training, we augment the objective with a curvature-metric regularizer (Eqn (3)).

$$
\min_{\{A_\ell, B_\ell\}} \mathcal{L}_{\text{task}} + \lambda \sum_{\ell=1}^L \|A_\ell B_\ell^\top\|_{M_\ell}^2
\tag{3}
$$

This term enforces a trust region in the ellipsoidal norm induced by M_ℓ, damping unstable directions and improving quantization-friendliness. The additional cost per batch is linear in adapter size, as it only requires quadratic forms. We anneal λ toward zero during training, ensuring strong regularization early for stability and relaxed constraints later for expressiveness.

Complexity and Memory For each matrix, K-FAC maintains two covariance factors whose inverses are computed efficiently by eigendecomposition or Cholesky. Randomized SVD with target rank k_ℓ adds cost $\mathcal{O}(d_{\text{out}} d_{\text{in}} k_\ell)$ with small constants, while curvature refreshes every few hundred steps amortize overhead. At inference, adapters are merged into the base model, preserving zero latency. When M_ℓ is the
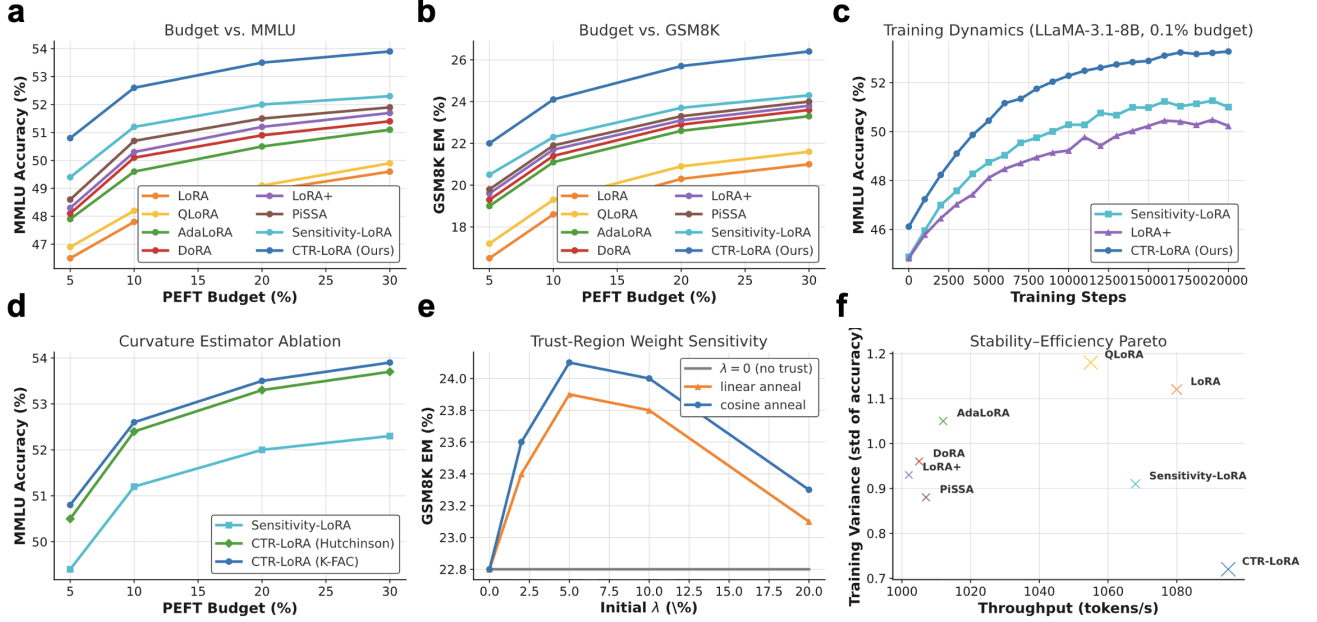
Fig. 2. Comprehensive analyses of CTR-LoRA. **(a-b)** Budget-performance curves show consistent improvements. **(c)** CTR-LoRA has smoother training dynamics and converges to higher accuracy. **(d)** K-FAC and Hutchinson proxies outperform sensitivity-only allocation, confirming robustness to curvature estimators. **(e)** Performance is stable within a moderate trust-region range. **(f)** CTR-LoRA occupies the Pareto frontier by reducing variance, lowering memory, and increasing throughput.

Fisher block, our scoring coincides with selecting dominant natural-gradient modes, and the trust-region regularizer reduces to Tikhonov regularization in this metric [17]. Compared with AdaLoRA [9] and Sensitivity-LoRA [13], CTR-LoRA explicitly couples rank placement with update geometry, achieving higher stability and memory efficiency under identical budgets.

3 Experiments

Experimental Setup We evaluate CTR-LoRA on several open-source LLMs to ensure reproducibility and scalability. Our main backbones are LLaMA-3.1-8B and Mistral-7B, with additional results on LLaMA-2-13B and Mixtral-8x7B to study scaling. All models are taken from HuggingFace Transformers, with frozen backbones and only adapter parameters updated. Experiments cover a broad set of benchmarks: `MMLU` [19] (5-shot), `GSM8K` [20] (8-shot), `ARC-C` [21] (25-shot), and `HellaSwag` [22] (10-shot), as well as `BoolQ` [23] (zero-shot) and `WinoGrande` [24] for out-of-distribution evaluation. We compare against representative PEFT baselines, including LoRA [6], QLoRA [7], AdaLoRA [9], DoRA [10], LoRA+ [11], PiSSA [12], and Sensitivity-LoRA [13]. All methods are trained under matched budgets of 0.1% and 0.3% of backbone parameters. Training uses AdamW with learning rate 2×10^{-4}, cosine decay, and 0.03 warm-up. Batch size is 128 with sequence length 2,048, gradient checkpointing, and FlashAttention.

For CTR-LoRA, curvature proxies are estimated on a 2k-sample calibration set, refreshed every 500 steps, and the trust-region weight λ is annealed from 0.1 to 0. Evaluation follows `lm-eval-harness` [25], averaging results over three seeds. Besides accuracy, we also report trainable parameter ratio, peak memory, and throughput. Ablations study different curvature estimators (K-FAC vs. Hutchinson), values of λ, the effect of LoRA+ and DoRA refinements, and budget–performance trade-offs, along with visualizations of rank allocation, curvature spectra, and training stability.

Main Results Table 1 reports results on four benchmarks across four open LLM backbones. LoRA and QLoRA offer only modest improvements, as they lack mechanisms to adaptively allocate capacity. AdaLoRA and DoRA provide incremental gains by redistributing rank or decoupling update magnitude and direction, while LoRA+ and PiSSA accelerate convergence through optimization refinements. Sensitivity-LoRA improves further by leveraging second-order information, but its benefits are uneven: `MMLU` and `HellaSwag` improve, whereas `GSM8K` and `ARC-C` see limited gains, revealing the limitation of sensitivity-only allocation under high-curvature updates. CTR-LoRA consistently outperforms all baselines. On LLaMA-2-13B, it surpasses Sensitivity-LoRA by +1.3 on `MMLU`, +1.7 on `GSM8K`, +1.4 on `ARC-C`, and +0.8 on `HellaSwag`; on Qwen2.5-7B, the margins remain +1.2-1.7. These results confirm that coupling curvature-aware rank scheduling with a trust-region constraint is essential: the for-

Table 1. Comparison of parameter-efficient fine-tuning methods across four open LLM backbones (LLaMA-3.1-8B, Mistral-7B, LLaMA-2-13B, Qwen2.5-7B). Results are reported as accuracy (%) on four representative benchmarks.

Method	LLaMA-3.1-8B				Mistral-7B				LLaMA-2-13B				Qwen2.5-7B			
	MMLU	GSM8K	ARC-C	HellaSwag	MMLU	GSM8K	ARC-C	HellaSwag	MMLU	GSM8K	ARC-C	HellaSwag	MMLU	GSM8K	ARC-C	HellaSwag
LoRA [6]	47.8	18.6	45.2	77.5	46.9	17.9	44.8	76.8	49.1	19.4	46.0	78.0	48.3	18.7	45.5	77.2
QLoRA [7]	48.2	19.3	45.9	78.0	47.5	18.8	45.4	77.4	49.6	20.0	46.5	78.5	48.9	19.4	46.1	77.8
AdaLoRA [9]	49.6	21.1	46.5	78.8	48.7	20.2	45.9	77.9	50.4	21.2	47.2	79.1	50.1	20.7	46.8	78.6
DoRA [10]	50.1	21.4	46.9	79.1	49.0	20.8	46.2	78.2	50.8	21.6	47.6	79.4	50.5	21.0	47.1	78.9
LoRA+ [11]	50.3	21.7	47.2	79.5	49.4	21.0	46.5	78.6	51.2	22.0	47.8	79.7	50.9	21.3	47.4	79.2
PiSSA [12]	50.7	21.9	47.3	79.6	49.8	21.2	46.7	78.8	51.5	22.2	48.1	79.9	51.3	21.6	47.6	79.4
Sensitivity-LoRA [13]	51.2	22.3	47.9	79.9	50.3	21.8	47.2	79.1	52.0	22.6	48.4	80.0	51.8	22.1	48.0	79.7
CTR-LoRA (ours)	**52.6**	**24.1**	**49.2**	**80.8**	**51.8**	**23.5**	**48.7**	**80.1**	**53.3**	**24.3**	**49.8**	**81.0**	**53.0**	**23.8**	**49.1**	**80.5**

Table 2. Accuracies (%) on out-of-distribution benchmarks (BoolQ / WinoGrande) under a 0.1% PEFT budget.

Method	LLaMA 3.1-8B		Mistral 7B		LLaMA 2-13B		Qwen 2.5-7B	
	BoolQ	WG	BoolQ	WG	BoolQ	WG	BoolQ	WG
LoRA [6]	70.2	65.8	69.7	65.2	71.5	66.9	70.8	66.1
QLoRA [7]	70.9	66.3	70.4	65.9	72.1	67.3	71.5	66.6
AdaLoRA [9]	72.3	67.1	71.5	66.8	73.0	68.0	72.5	67.2
DoRA [10]	72.8	67.6	71.9	67.2	73.4	68.5	72.9	67.7
LoRA+ [11]	73.0	67.9	72.2	67.5	73.8	68.7	73.2	68.0
PiSSA [12]	73.2	68.0	72.5	67.8	74.1	68.9	73.5	68.2
Sensitivity-LoRA [13]	73.8	68.4	73.0	68.2	74.6	69.2	74.0	68.6
CTR-LoRA (ours)	**75.1**	**69.7**	**74.2**	**69.4**	**76.0**	**70.3**	**75.5**	**69.8**

Table 3. Ablations on CTR-LoRA components with LLaMA-3.1-8B at 0.1% budget. Accuracies (%) are reported.

Variant	MMLU	GSM8K	ARC-C	HellaSwag	**Average**
CTR-LoRA (full)	52.6	24.1	49.2	80.8	51.7
w/o trust region ($\lambda=0$)	51.3	22.9	48.3	80.0	50.6
uniform rank (no sched.)	51.5	23.0	48.0	79.9	50.6
w/o both (no trust & uniform)	50.6	22.1	47.5	79.4	49.9
+ LoRA+ (asym. LR)	52.9	24.3	49.4	80.9	51.9
+ DoRA (mag/dir decouple)	53.0	24.4	49.5	81.0	52.0
+ PiSSA (SVD init)	**53.1**	**24.6**	**49.6**	**81.1**	**52.1**

mer determines *where* to allocate parameters, while the latter governs *how* updates evolve, producing both stronger accuracy and greater stability. Notably, CTR-LoRA's superiority holds across backbones with different scales and training corpora, highlighting its robustness and generalizability as a practical PEFT strategy.

Out-of-Distribution Generalization Table 2 reports results on BoolQ and WinoGrande, which probe out-of-distribution robustness. Sensitivity-LoRA remains the strongest baseline, confirming the utility of sensitivity-guided allocation, but its margins shrink particularly on WinoGrande, where it is only slightly better than PiSSA. In contrast, CTR-LoRA consistently achieves the highest scores across all models. On BoolQ, it improves over Sensitivity-LoRA by 1.2 to 1.5 points; on WinoGrande, by 1.2 to 1.5 as well. These results highlight that robust generalization in PEFT requires more than rank allocation alone: coupling curvature-aware scheduling with a trust-region constraint stabilizes updates under distributional shift, making CTR-LoRA more reliable for deployment in dynamic real-world settings.

Ablation and Diagnostic Analysis Table 3 evaluates the contribution of each component on LLaMA-3.1-8B under

a 0.1% budget. Removing either curvature-aware scheduling or the trust-region penalty reduces performance by more than one point, and discarding both leads to further degradation, confirming that allocation (*where*) and regulation (*how*) must be coupled. Adding LoRA+, DoRA, or PiSSA yields small but consistent gains, with PiSSA giving the best average, showing that CTR-LoRA is compatible with recent refinements. Fig. 2 further probes CTR-LoRA from multiple perspectives. Budget-performance curves on MMLU and GSM8K show consistent improvements, particularly under tight budgets and in multi-step reasoning tasks. Training dynamics reveal faster and more stable convergence. Ablations with different curvature proxies (K-FAC vs. Hutchinson) show both outperform sensitivity-only allocation, with minor differences, highlighting robustness. Varying λ confirms that a moderate range yields the best results, while overly large values slightly suppress learning. Finally, efficiency analysis places CTR-LoRA on the Pareto frontier, combining lower memory, higher throughput, and reduced variance. These results demonstrate that CTR-LoRA's advantage comes from the synergy of curvature-aware rank scheduling and trust-region regularization, which jointly ensure effective allocation, stable optimization, and broad robustness.

4 Conclusion

We introduced CTR-LoRA, a curvature-trust region guided framework for parameter-efficient fine-tuning of large language models. By combining curvature-aware rank scheduling with a stability-oriented trust-region objective, our method addresses both allocation and optimization challenges inherent in prior PEFT approaches. Experiments on four open backbones and multiple benchmarks, including out-of-distribution settings, show that CTR-LoRA consistently surpasses strong baselines while improving efficiency and stability. These results highlight CTR-LoRA as a principled and practical step toward robust and deployable PEFT, with future opportunities in multi-task and continual adaptation.

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