6 Commnuications

1 Karft Inequality

哈夫曼编码时，肯定希望尽可能地用短代码，但是短代码有限，所以出现Kraft 不等式，Li指的是第i个输入符号的代码长度。

$$\sum_i \frac{1}{2^{L_i}} \le 1$$

例如：4个输入符号 1.用00 01 10 11表示 则为1<=1
2.用0 10 110 111表示，则为15/16 <1

2 Source Entropy

n gained when the next symbol is ma

$$H = \sum_i p(A_i) \log_2 \left(\frac{1}{p(A_i)} \right)$$

2.1 Gibbs Inequality

$$\sum_i p(A_i)\log_2\left(\frac{1}{p(A_i)}\right) \leq \sum_i p(A_i)\log_2\left(\frac{1}{p'(A_i)}\right)$$

ility distribution (we will use it for source events a
lity distribution, or more generally any set of numbe

$$0 \leq p'(A_i) \leq 1$$

$$\sum_i p'(A_i) \leq 1.$$

y distributions,

$$\sum_i p(A_i) = 1.$$

proof :

$$\sum_i p(A_i) = 1. \tag{6.7}$$

Equation 6.4 can be proven by noting that the natural logarithm has the property that it is less than or equal to a straight line that is tangent to it at any point, (for example the point $x = 1$ is shown in Figure 6.2). This property is sometimes referred to as concavity or convexity. Thus

$$\ln x \leq (x - 1) \tag{6.8}$$

and therefore, by converting the logarithm base e to the logarithm base 2, we have

$$\log_2 x \leq (x - 1) \log_2 e \tag{6.9}$$

Then

$$\begin{aligned}
\sum_i p(A_i) \log_2 \left(\frac{1}{p(A_i)} \right) - \sum_i p(A_i) \log_2 \left(\frac{1}{p'(A_i)} \right) &= \sum_i p(A_i) \log_2 \left(\frac{p'(A_i)}{p(A_i)} \right) \\
&\leq \log_2 e \sum_i p(A_i) \left[\frac{p'(A_i)}{p(A_i)} - 1 \right] \\
&= \log_2 e \left(\sum_i p'(A_i) - \sum_i p(A_i) \right) \\
&= \log_2 e \left(\sum_i p'(A_i) - 1 \right) \\
&\leq 0 \qquad (6.10)
\end{aligned}$$

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3 Source Coding Theorem

$$L = \sum_i p(A_i) L_i$$

then : **H<=L**,H 是哈夫曼编码后的所有符号的总长度，L是等长编码后的所有符号的总长度

$$
\begin{aligned}
H &= \sum_i p(A_i)\log_2\left(\frac{1}{p(A_i)}\right) \\
&\le \sum_i p(A_i)\log_2\left(\frac{1}{p'(A_i)}\right) \\
&= \sum_i p(A_i)\log_2 2^{L_i} \\
&= \sum_i p(A_i)L_i \\
&= L
\end{aligned}
$$

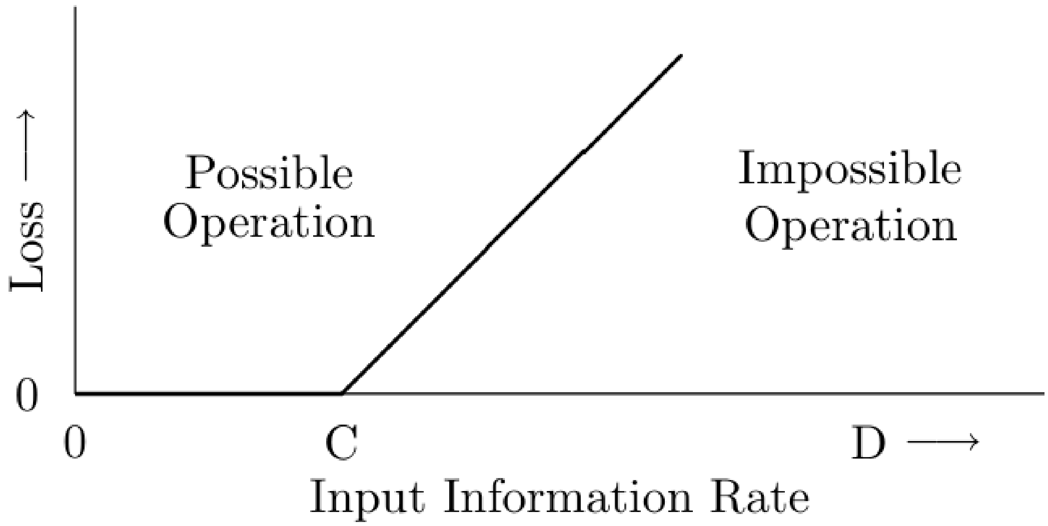
This inequal

其中: $p'(A_i) = 1/2^{L_i}$ (

e **channel capacity** $C = W\log_2 n$

C是通道容量，W是最大的传输速度，n代表每次输入时input的可能情况有多少，比如电脑每次输入时只有1 0 两种情况，所以n=2.

5 Noiseless Channel Theorem



6 Noisy Channel

transition probabilities cji

e input.

$$U_{\text{before}} = \sum_i p(A_i) \log_2 \left(\frac{1}{p(A_i)} \right)$$

icular output event B_j has been observed, what is the resid
ent? A similar formula applies, with $p(A_i)$ replaced by the co

$$U_{\text{after}}(B_j) = \sum_i p(A_i \mid B_j) \log_2 \left(\frac{1}{p(A_i \mid B_j)} \right)$$

learned in the case of this particular output event is the diff
tual information M is defined as the average, over all outputs

$$M = U_{\text{before}} - \sum_j p(B_j) U_{\text{after}}(B_j)$$

$$= U_{\text{before}} \tag{6.25}$$

We are now in a position to find M in terms of the input probability distribution and the properties of the channel. Substitution in Equation 6.23 and simplification leads to

$$M = \sum_j \left(\sum_i p(A_i) c_{ji} \right) \log_2 \left(\frac{1}{\sum_i p(A_i) c_{ji}} \right) - \sum_{ij} p(A_i) c_{ji} \log_2 \left(\frac{1}{c_{ji}} \right) \tag{6.26}$$

Note that Equation 6.26 was derived for the case where the input "causes" the output. At least, that was the way the description went. However, such a cause-and-effect relationship is not necessary. The term **mutual information** suggests (correctly) that it is just as valid to view the output as causing the input, or to ignore completely the question of what causes what. Two alternate formulas for M show that M can be interpreted in either direction:

$$\begin{aligned} M &= \sum_i p(A_i) \log_2 \left(\frac{1}{p(A_i)} \right) - \sum_j p(B_j) \sum_i p(A_i \mid B_j) \log_2 \left(\frac{1}{p(A_i \mid B_j)} \right) \\ &= \sum_j p(B_j) \log_2 \left(\frac{1}{p(B_j)} \right) - \sum_i p(A_i) \sum_j p(B_j \mid A_i) \log_2 \left(\frac{1}{p(B_j \mid A_i)} \right) \end{aligned} \tag{6.27}$$

Rather than give a general interpretation of these or similar formulas, let's simply look at the symmetric binary channel. In this case both $p(A_i)$ and $p(B_j)$ are equal to 0.5 and so the first term in the expression for M in Equation 6.26 is 1 and the second term is found in terms of ε:

$$M = 1 - \varepsilon \log_2 \left(\frac{1}{\varepsilon} \right) - (1 - \varepsilon) \log_2 \left(\frac{1}{(1 - \varepsilon)} \right) \tag{6.28}$$

剩下的内容没有什么有价值的