A Gravitational Twist to Quantum Entanglement

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- QFT usually assumes a field propagating in a background spacetime.

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- Bose et al. (2017) and Marletto & Vedral (2017) focus on using Quantum Entanglement to probe the nature of gravity.
- In this talk, focus is on how gravity could entangle two masses, provided gravity was quantum in nature.

BMV Experiment

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- Classical interactions $\neq$ Entanglement![1]
- The experiment is called the the Bose-Marletto-Vedral (BMV) experiment (nice review in Christodoulou & Rovelli (2020)).

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- Quantum interactions will induce entanglement!
- Erwin Schrödinger defined this as the characteristic trait of a quantum interaction Schrödinger (1935).
- Assume local interactions, not action at a distance.
- Bose et al. (2022) uses a principle of quantum information theory, where local classical interactions cannot entangle particles.

LOCC and LOQC

- LOCC stands for "Local Operations and Classical Communication".

[2] This is under the assumption that the "notion of classicity" itself is not extended. (Hall & Reginatto, 2005, 2018)

LOCC and LOQC

- "Classical Communication" means the subsystems "communicate" through classical means.

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LOCC and LOQC

- A central principle of quantum information theory is LOCC keeps any initially untangled state unentangled (Bose et al., 2017).[2]

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LOCC and LOQC

- If LOCC can't entangle particles, the entanglement must occur through local operations and quantum communication.

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LOCC and LOQC

- "Classicity" $=$ Anything that is not quantum.

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LOCC and LOQC

- LOQC therefore is "Local Operations and Quantum Communication".

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LOCC and LOQC

- If mutual gravitational interaction between two masses entangles them, then the mediating gravitational field is necessarily quantum mechanical in nature.

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Mechanism for Gravitationally Induced Entanglement

- The Bose-Marletto-Vedral (BMV) experiment (Christodoulou & Rovelli, 2020) relies on two assumptions:
  1. The interaction between two masses is mediated only by a gravitational field (Bose et al., 2017), and
  2. Entanglement between two systems cannot be created by LOCC (Bose et al., 2017; Marletto & Vedral, 2017).
- Bose et al. (2022) further described a mechanism for gravity to entangle masses.
- They calculate a shift in the energy of the graviton vacuum due to the interaction.
- And subsequently a measure of entanglement called concurrence. (Hill & Wootters, 1997; Rungta et al., 2001)
- We further calculate Entanglement Entropy as another measure of entanglement for the same cases.

Induced Entanglement

- Consider two matter systems in harmonic traps separated by a distance d, and initial state $|\psi_i\rangle = |0\rangle_A|0\rangle_B$.
- Let the harmonic traps be in a superposition.
- $$\hat{x}_A = -\frac{d}{2} + \delta\hat{x}_A, \qquad \hat{x}_B = \frac{d}{2} + \delta\hat{x}_B,$$
  $$\hat{H}_{\text{matter}} = \frac{\hat{p}_A^2}{2m} + \frac{\hat{p}_B^2}{2m} + \frac{m\omega_m^2}{2}\delta\hat{x}_A^2 + \frac{m\omega_m^2}{2}\delta\hat{x}_B^2,$$
  $$\delta\hat{x}_A = \sqrt{\frac{\hbar}{2m\omega_m}}(\hat{a} + \hat{a}^\dagger), \qquad \delta\hat{x}_B = \sqrt{\frac{\hbar}{2m\omega_m}}(\hat{b} + \hat{b}^\dagger),$$
  $$\hat{p}_A = i\sqrt{\frac{\hbar m\omega_m}{2}}(\hat{a} - \hat{a}^\dagger), \qquad \hat{p}_B = i\sqrt{\frac{\hbar m\omega_m}{2}}(\hat{b} - \hat{b}^\dagger),$$

Induced Entanglement

- Introduce a small interaction potential, λH_{AB}, and the perturbed final state is given by,
$$|\psi_{\mathrm{f}}\rangle \equiv \frac{1}{\sqrt{\mathcal{N}}} \sum_{n,N} C_{nN} |n\rangle |N\rangle,$$
where $\mathcal{N} = \sum_{n,N} |C_{nN}|^2$, $|n\rangle$ and $|N\rangle$ are numbered states.
- The coefficient of the unperturbed state is $C_{00} = 1$, and the remaining are given by (Bose et al., 2022),
$$C_{nN} = \lambda \frac{\langle n|\langle N|\hat{H}_{AB}|0\rangle|0\rangle}{2E_0 - E_n - E_N}$$
- If the interaction was classical, the remaining coefficients would be 0 due to orthogonality of $|n\rangle|N\rangle$ and $|0\rangle|0\rangle$.

Induced Entanglement

If we set $A_n \equiv C_{n0}$ and $B_N \equiv C_{0N}$, then we can write the perturbed state as (Balasubramanian et al., 2012; Bose et al., 2022),

$$|\psi_f\rangle = \frac{1}{\sqrt{\mathcal{N}}}\left(|0\rangle|0\rangle + \sum_{n>0} A_n|n\rangle|0\rangle + \sum_{N>0} B_N|0\rangle|N\rangle + \sum_{n,N>0} C_{nN}|n\rangle|N\rangle\right)$$

$$= \frac{1}{\sqrt{\mathcal{N}}}\left(\left(|0\rangle + \sum_{n>0} A_n|n\rangle\right)\left(|0\rangle + \sum_{N>0} B_N|N\rangle\right) + \sum_{n,N>0}(C_{nN} - A_nB_N)|n\rangle|N\rangle\right)$$

- The first term would yield a separable state, second an entangled state.
- It shows the stark difference between LOCC and LOQC.

Concurrence

- The density matrix for the system is given by

$$\hat{\rho}_A = \sum_{N_B} \langle N_B|\psi_\mathrm{f}\rangle\langle\psi_\mathrm{f}|N_B\rangle = \frac{1}{\mathcal{N}} \sum_{n,n',N} C_{nN} C^*_{n'N} |n\rangle\langle n'|$$

(Bose et al., 2022)
- Thus the concurrence can be calculated by

$$C \equiv \sqrt{2(1 - \mathrm{tr}[\hat{\rho}_A^2])}$$

(Hill & Wootters, 1997; Rungta et al., 2001)
-

$$C = \sqrt{2\left(1 - \frac{1}{\mathcal{N}^2} \sum_{n,n',N,N'} C_{nN} C^*_{n'N} C_{n'N'} C^*_{nN'}\right)}$$

- For an unentangled system, $C = 0$, and $\sqrt{2}$ for a maximally entangled system[3].

[3]with an infinite dimensional Hilbert Space

Entanglement Entropy

- Another measurement of the degree of entanglement is the von Neumann Entanglement Entropy (Balasubramanian et al., 2012; Marshman et al., 2020; Brahma et al., 2020).
- $$\mathcal{S}(\hat{\rho}_A) = -\operatorname{Tr}(\hat{\rho}_A \log \hat{\rho}_A)$$
- This can be calculated relatively easily by calculating the logarithm of the density operator, which is diagonalizable.
- We have seen that quantum interactions induce entanglement, and that Concurrence and Entanglement Entropy.

Linearization of Gravity

- Now that we know quantum interactions induce entanglement, we want to substitute the general quantum interaction with a quantum gravitational interaction.
- To do that, we must first understand how gravity is quantized.
- And to do that, we must understand how Einstein's Field Equations can be linearized.

Linearization of Gravity

- Follow the same procedure as Gupta (1952), with minor modifications.
- Consider the Einstein Field Equations, $R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = -\frac{1}{2}\kappa^2 T^{\mu\nu}$.
- Covariant divergence of the Stress-Energy tensor is 0, so we can write a Stress-Energy tensor *density* $\mathfrak{T}^{\mu\nu} = \sqrt{-g}\,T^{\mu\nu}$ (where g is the determinant of the metric).
- $$\frac{\partial \mathfrak{T}^{\nu}_{\mu}}{\partial x^{\nu}} - \frac{1}{2}\mathfrak{T}^{\alpha\beta}\frac{\partial g_{\alpha\beta}}{\partial x^{\mu}} = 0.$$
- Define Stress-Energy *pseudo-tensor density* (Gupta, 1952) for the gravitational field, satisfying the equation,
$$\frac{\partial t^{\nu}_{\mu}}{\partial x^{\nu}} = -\frac{1}{2}\mathfrak{T}^{\alpha\beta}\frac{\partial g_{\alpha\beta}}{\partial x^{\mu}}.$$
- Then the conservation of stress-energy tensor can be written as,
$$\frac{\partial}{\partial x^{\nu}}(\mathfrak{T}^{\nu}_{\mu} + t^{\nu}_{\mu}) = 0$$

Linearization of Gravity

- Following Gupta (1952) and Einstein (1918) we can obtain a linear approximation for the field by considering small perturbations $h_{\mu\nu}$.
- $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$
  where $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$.
- It is also convenient to decompose $h_{\mu\nu}$ as $h_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\gamma$, where $\gamma = \gamma_{\lambda\lambda}$.
- Choose coordinate/supplementary conditions given by (Gupta, 1952),

$$\frac{\partial h_{\mu\nu}}{\partial x_\nu} - \frac{1}{2}\frac{\partial h_{\lambda\lambda}}{\partial x_\mu} = 0$$

Linearization of Gravity

- Thus, simplifying the Einstein equations with these constraints, we get,

$$\frac{\partial t_{\mu\nu}}{\partial x_\nu} = \frac{\kappa^2}{2}\frac{\partial \gamma_{\nu\lambda}}{\partial x_\mu} T_{\nu\lambda} - \frac{\kappa^2}{4}\frac{\partial \gamma}{\partial x_\mu} T_{\nu\nu}$$

$$\frac{\partial \gamma_{\mu\nu}}{\partial x_\nu} = 0$$

$$\Box^2 \gamma_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

- These are only valid in a first order approximation.
- One can now find the Hamiltonian density by solving for $t_{\mu\nu}$ and calculating t_{00}.
-

$$H = t_{00} = \frac{1}{2}\left[\frac{\partial \gamma_{\lambda\rho}}{\partial t}\frac{\partial \gamma_{\lambda\rho}}{\partial t} - \frac{1}{2}\frac{\partial \gamma}{\partial t}\frac{\partial \gamma_{\rho\rho}}{\partial t} + \frac{1}{2}\left(\frac{\partial \gamma_{\lambda\rho}}{\partial x_\sigma}\frac{\partial \gamma_{\lambda\rho}}{\partial x_\sigma} - \frac{1}{2}\frac{\partial \gamma}{\partial x_\sigma}\frac{\partial \gamma_{\rho\rho}}{\partial x_\sigma}\right)\right]$$

Canonical Quantization

- Now that we have the Hamiltonian, we can proceed to do a canonical quantization, again following Gupta (1952).
- The Lagrangian Density is given by,

$$L = -\frac{1}{4}\left(\frac{\partial \gamma_{\mu\nu}}{\partial x_\lambda}\frac{\partial \gamma_{\mu\nu}}{\partial x_\lambda} - \frac{1}{2}\frac{\partial \gamma}{\partial x_\lambda}\frac{\partial \gamma}{\partial x_\lambda}\right)$$

- We can calculate the canonical conjugate of $\gamma_{\mu\nu}$ and γ using,

$$\pi_{\mu\nu} = \frac{\partial L}{\partial(\partial \gamma_{\mu\nu}/\partial t)} = \frac{1}{2}\frac{\partial \gamma_{\mu\nu}}{\partial t}$$

$$\pi = \frac{\partial L}{\partial(\partial \gamma/\partial t)} = -\frac{1}{4}\frac{\partial \gamma}{\partial t}$$

Canonical Quantization

- The ETCRs are found to be,

$$[\gamma_{\mu\nu}(x), \pi_{\lambda\rho}(x')] = i(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda})\delta(\boldsymbol{x} - \boldsymbol{x'}),$$

$$[\gamma(x), \pi(x')] = -4i\delta(\boldsymbol{x} - \boldsymbol{x'}).$$

- The perturbations can be written in an operator valued form (Bose et al., 2022),

$$\hat{h}_{\mu\nu} = \mathcal{A}\int d\boldsymbol{k}\sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}(2\pi)^3}}(\hat{P}^{\dagger}_{\mu\nu}(\boldsymbol{k})e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} + \text{h.c}),$$

  where $\boldsymbol{k}$ is the three-vector, $d\boldsymbol{k} \equiv d^3k$.
- $\hat{P}_{\mu\nu}$ & $\hat{P}^{\dagger}_{\mu\nu}$ are the graviton annihilation and the creation operators, and $\mathcal{A} = \sqrt{16\pi G/c^2}$.

Canonical Quantization

- Thus we get an on-shell spin-2 graviton through $\gamma_{\mu\nu}$ and an off-shell spin-0 graviton through γ given by,

$$\hat{\gamma}_{\mu\nu} = \mathcal{A} \int d\boldsymbol{k} \sqrt{\frac{\hbar}{2\omega_k (2\pi)^3}} (\hat{P}^{\dagger}_{\mu\nu}(\boldsymbol{k}) e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} + \hat{P}_{\mu\nu}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{r}}),$$

$$\hat{\gamma} = 2\mathcal{A} \int d\boldsymbol{k} \sqrt{\frac{\hbar}{2\omega_k (2\pi)^3}} (\hat{P}^{\dagger}(\boldsymbol{k}) e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} + \hat{P}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{r}}).$$

- The ETCRs for the mode operators are,

$$[\hat{P}_{\mu\nu}(\boldsymbol{k}), \hat{P}^{\dagger}_{\lambda\rho}(\boldsymbol{k}')] = (\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda})\delta(\boldsymbol{k} - \boldsymbol{k}'),$$

$$[\hat{P}(\boldsymbol{k}), \hat{P}^{\dagger}(\boldsymbol{k}')] = -\delta(\boldsymbol{k} - \boldsymbol{k}').$$

- Finally, the graviton vacuum energy is given by,

$$\hat{H}_g = \int d\boldsymbol{k}\, \hbar\omega_k \left(\frac{1}{2} \hat{P}^{\dagger}_{\mu\nu}(\boldsymbol{k}) \hat{P}^{\mu\nu}(\boldsymbol{k}) - \hat{P}^{\dagger}(\boldsymbol{k}) \hat{P}(\boldsymbol{k}) \right).$$

Quantum Gravitational Interaction

- $$\hat{H}_{\text{int}} = -\frac{1}{2}\int d\boldsymbol{r}\,\hat{h}^{\mu\nu}(\boldsymbol{r})\hat{T}_{\mu\nu}(\boldsymbol{r})$$
- The first order contribution to the shift in energy is zero, i.e. $\langle 0|\hat{H}_{\text{int}}|0\rangle = 0$.
- Thus the shift in the graviton vacuum energy is given by the second order perturbation theory,

$$\Delta\hat{H}_g \equiv \sum\int d\boldsymbol{k}\,\frac{\langle 0|\hat{H}_{\text{int}}|\boldsymbol{k}\rangle\langle\boldsymbol{k}|\hat{H}_{\text{int}}|0\rangle}{E_0 - E_{\boldsymbol{k}}}.$$

- The sum indicates summation over all one particle projectors $|\boldsymbol{k}\rangle\langle\boldsymbol{k}|$, and $E_{\boldsymbol{k}} = E_0 + \hbar\omega_k$.
- This defines the quantum gravitational interaction and now we can find the induced entanglement.

Entanglement in Static Limit

- In this case, stress energy tensor is

$$\hat{T}_{00}(\boldsymbol{r}) \equiv mc^2(\delta(\boldsymbol{r} - \hat{\boldsymbol{r}}_A) + \delta(\boldsymbol{r} - \hat{\boldsymbol{r}}_B))$$

  (Bose et al., 2022). This is the non-relativistic limit.

- $$\hat{H}_{\text{int}} = -\frac{1}{2}\int d\boldsymbol{r}\mathcal{A}\int d\boldsymbol{k}'\sqrt{\frac{\hbar}{2\omega_{k'}(2\pi)^3}}\Big(\hat{P}^{\dagger}_{00}(\boldsymbol{k}')e^{-i\boldsymbol{k}'\cdot\boldsymbol{r}} + \hat{P}^{\dagger}(\boldsymbol{k}')e^{-i\boldsymbol{k}'\cdot\boldsymbol{r}} + \text{H.c.}\Big)\hat{T}_{00}(\boldsymbol{r})$$

- The shift in the graviton vacuum energy in this case is then given by,

$$\Delta\hat{H}_g \equiv \int d\boldsymbol{k}\frac{\langle 0|\hat{H}_{\text{int}}|\boldsymbol{k}\rangle\langle\boldsymbol{k}|\hat{H}_{\text{int}}|0\rangle}{E_0 - E_{\boldsymbol{k}}}$$

- $|k\rangle = (\hat{P}^{\dagger}_{00}(\boldsymbol{k}) + \hat{P}^{\dagger}(\boldsymbol{k}))|0\rangle$ is the one particle state constructed in the unperturbed background, $E_{\boldsymbol{k}} = E_0 + \hbar\omega_k$

Entanglement in Static Limit

- Thus, we get,

$$\Delta\hat{H}_g = -\frac{Gm^2}{|\hat{x}_a - \hat{x}_b|}.$$

- This is the Newtonian potential!
- The lowest order interaction term is $\hat{H}_{AB} = \frac{2Gm^2}{d^3}\,\delta\hat{x}_A\delta\hat{x}_B$
- The final state is given by,

$$|\psi_f\rangle \equiv \frac{1}{\sqrt{1+(\mathfrak{g}/(2\omega_m))^2}}[|0\rangle|0\rangle - \frac{\mathfrak{g}}{2\omega_m}|1\rangle|1\rangle].$$

- Where we define the coupling, $\mathfrak{g}$, as, $\mathfrak{g} = \frac{Gm}{d^3\omega_m}$.
- Thus, the concurrence is given by,

$$C = \frac{Gm}{d^3\omega_m^2}$$

Entanglement in Non-Static Case

- In this case, stress energy tensor once again given by the interaction Hamiltonian, but is more complicated since the system is no longer static.
- Specifically, we consider two particles in QHOs moving along the x-axis such that the only non-zero components of $T_{\mu\nu}$ are given by T_{00}, $T_{01} = T_{10}$, and T_{11}.
- Following the exact same method as in the previous case, we get that the shift in the graviton vacuum energy in this case is then given by,

$$\Delta \hat{H}_g = -\frac{Gm^2}{|\hat{x}_A - \hat{x}_B|} - \frac{G(3\hat{p}_A^2 - 8\hat{p}_A\hat{p}_B + 3\hat{p}_B^2)}{2c^2|\hat{x}_A - \hat{x}_B|} - \frac{G(5\hat{p}_A^4 - 18\hat{p}_A^2\hat{p}_B^2 + 5\hat{p}_B^4)}{8c^4m^2|\hat{x}_A - \hat{x}_B|}$$

- There are now relativistic corrections to the Newtonian potential.
- Let us term them "Correction-1" and "Correction-2".

Correction-1

- The lowest order interaction term in this correction is given by,

$$\hat{H}_{AB} \approx \frac{4G\hat{p}_A\hat{p}_B}{c^2 d} + \dots$$

- The final state can now be calculated to be,

$$|\psi_f\rangle \equiv \frac{1}{\sqrt{1 + (\mathfrak{g}/(2\omega_m))^2}}[|0\rangle|0\rangle + \frac{\mathfrak{g}}{2\omega_m}|1\rangle|1\rangle].$$

- Where the coupling is, $\mathfrak{g} = \frac{2Gm\omega_m}{c^2 d}$.
- The concurrence for this correction then is

$$C = \frac{2Gm}{c^2 d}.$$

Correction-2

- The lowest order interction term in this correction is given by,

$$\hat{H}_{AB} \approx \frac{9G\hat{p}_A^2\hat{p}_B^2}{4c^4m^2d} + \dots$$

- The final state can now be calculated to be,

$$|\psi_f\rangle \equiv \frac{1}{\sqrt{1+(\mathfrak{g}/(2\omega_m))^2}}[|0\rangle|0\rangle - \frac{\mathfrak{g}}{2\omega_m}|2\rangle|2\rangle].$$

- Where the coupling is, $\mathfrak{g} = \frac{9G\hbar\omega_m^2}{16c^4d}$.
- The concurrence for this correction then is

$$C = \frac{9G\hbar\omega_m}{16c^4d}.$$

Calculation of Entanglement Entropy

- Each of the previous cases has similar final state and a common expression for the density matrix as a function of the coupling.
- If we were to define $\xi = \frac{\mathfrak{g}}{2\omega_m}$, then the density matrix for each case is given by,

$$\rho_A(\xi(\mathfrak{g})) = \frac{1}{1+\xi^2}\begin{bmatrix} 1 & 0 \\ 0 & \xi^2 \end{bmatrix},$$

- Common expression for the entanglement entropy,

$$\mathcal{S}(\rho_A(\mathfrak{g})) = \log\left(1 + \left(\frac{\mathfrak{g}}{2\omega_m}\right)^2\right) - \frac{2\left(\frac{\mathfrak{g}}{2\omega_m}\right)^2}{1+\left(\frac{\mathfrak{g}}{2\omega_m}\right)^2}\log\left(\frac{\mathfrak{g}}{2\omega_m}\right).$$

- We can further approximate this to be,

$$\mathcal{S}_A \approx \xi^2\Big((1-2\log(\xi)\Big) + \xi^4\left(2\log(\xi) - \frac{1}{2}\right) + \mathcal{O}(\xi^6),$$

Calculation of Entanglement Entropy

- Thus we can now simply plug in the coupling for each case, and get the entanglement entropy.
- Static case:

$$\mathcal{S}_A \approx \left(\frac{Gm}{2d^3\omega_m^2}\right)^2 \left(\left(1 - 2\log\left(\frac{Gm}{2d^3\omega_m^2}\right)\right) + \left(\frac{Gm}{2d^3\omega_m^2}\right)^4 \left(2\log\left(\frac{Gm}{2d^3\omega_m^2}\right) - \frac{1}{2}\right)\right.$$

- Non-static Correction-1:

$$\mathcal{S}_A \approx \left(\frac{Gm}{c^2 d}\right)^2 \left(\left(1 - 2\log\left(\frac{Gm}{c^2 d}\right)\right) + \left(\frac{Gm}{c^2 d}\right)^4 \left(2\log\left(\frac{Gm}{c^2 d}\right) - \frac{1}{2}\right)\right.$$

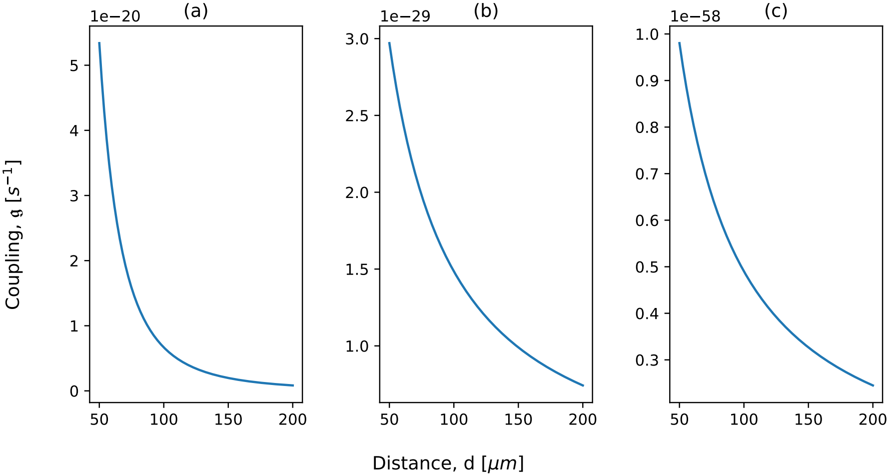
- Non-Static Correction-2:

$$\mathcal{S}_A \approx \left(\frac{9G\hbar\omega_m}{32c^4 d}\right)^2 \left(\left(1 - 2\log\left(\frac{9G\hbar\omega_m}{32c^4 d}\right)\right) + \left(\frac{9G\hbar\omega_m}{32c^4 d}\right)^4 \left(2\log\left(\frac{9G\hbar\omega_m}{32c^4 d}\right) - \frac{1}{2}\right)\right..$$

Comparisons

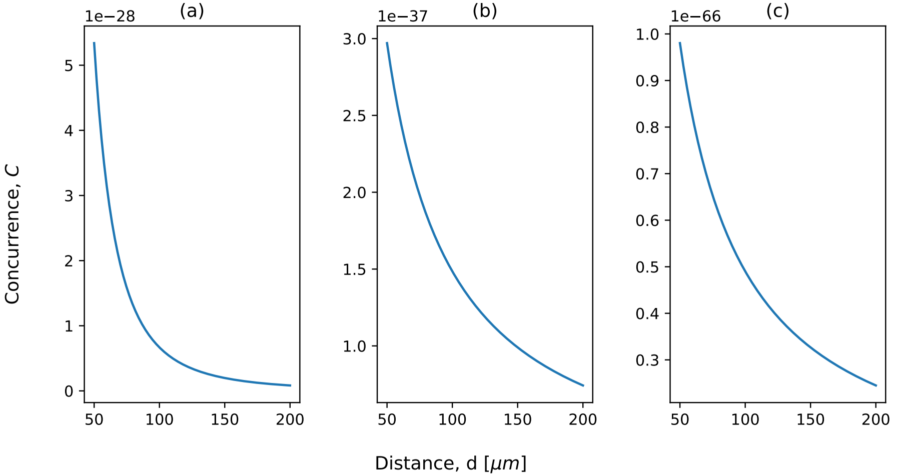
- Both Concurrence and Entanglement Entropy have non-zero positive values for each case.
- Expressions for Coupling, Concurrence and Entanglement Entropy easier to understand with a visualisation.
- Take mass as 10^{-14} Kg, and $\omega_m = 10^8$ Hz as fixed. Then we can plot the quantities against distance.

Comparisons



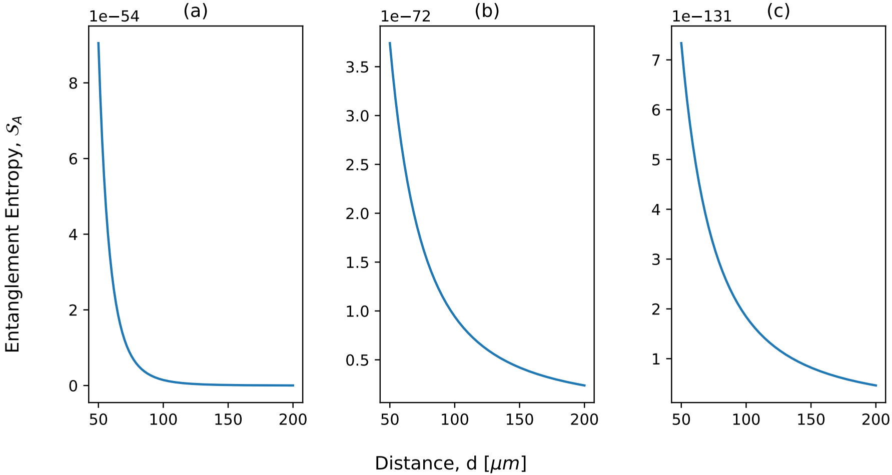
Comparison of Coupling for Different Cases: (a) Static Case; (b) Non-static Correction 1 (c) Non Static Correction 2

Comparisons



Comparison of Concurrence for Different Cases: (a) Static Case; (b) Non-static Correction 1 (c) Non Static Correction 2

Comparisons



Comparison of Entanglement Entropy for Different Cases: (a) Static Case; (b) Non-static Correction 1 (c) Non Static Correction 2

Summary

- We have talked about the recent table-top experiments to probe the quantum nature of gravity.
- We then talked about how Entanglement can be used to determine if gravity is a quantum entity.
- Next, we explored how entanglement is induced by quantum interactions.
- Then, in order to use a quantum gravitational interaction, we saw how gravity can be linearized and then quantized.
- We then calculated the shift in the graviton vacuum energy for an interaction between two particles in different cases, static and non-static.
- We finally calculated the two measures of entanglement, Concurrence and Entanglement Entropy.
- For each case, they yielded non-zero positive values. Thus, if gravity is a quantum entity, we can detect entanglement between two particles.

Conclusion

- Quantum entanglement can be used to confirm that gravity indeed is a quantum entity!
- This can be verified using table-top experiments.
- If classical interactions cannot entangle masses, and gravity does entangle masses, then gravity must be quantum natured. (Bose et al., 2017; Marletto & Vedral, 2017)
- This study was mainly done using a flat background spacetime. It will be interesting to see the results of these calculations done on a curved spacetime.
- Results on deSitter spacetime particularly interesting due to cosmological reasons.

References I

Balasubramanian, V., McDermott, M. B., & Van Raamsdonk, M. 2012, Phys. Rev. D, 86, 045014, doi: 10.1103/PhysRevD.86.045014

Belenchia, A., Wald, R. M., Giacomini, F., et al. 2018, Phys. Rev. D, 98, 126009, doi: 10.1103/PhysRevD.98.126009

—. 2019, International Journal of Modern Physics D, 28, 1943001, doi: 10.1142/S0218271819430016

Bose, S., Mazumdar, A., Schut, M., & Toroš, M. 2022, Phys. Rev. D, 105, 106028, doi: 10.1103/PhysRevD.105.106028

Bose, S., Mazumdar, A., Morley, G. W., et al. 2017, Phys. Rev. Lett., 119, 240401, doi: 10.1103/PhysRevLett.119.240401

Brahma, S., Alaryani, O., & Brandenberger, R. 2020, Phys. Rev. D, 102, 043529, doi: 10.1103/PhysRevD.102.043529

Christodoulou, M., & Rovelli, C. 2020, Frontiers in Physics, 8, doi: 10.3389/fphy.2020.00207

References II

Einstein, A. 1918, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften, (Berlin), 154. `https://ui.adsabs.harvard.edu/abs/1918SPAW.......154E`

Gupta, S. N. 1952, Proceedings of the Physical Society. Section A, 65, 161, doi: `10.1088/0370-1298/65/3/301`

Hall, M. J. W., & Reginatto, M. 2005, Phys. Rev. A, 72, 062109, doi: `10.1103/PhysRevA.72.062109`

—. 2018, Journal of Physics A: Mathematical and Theoretical, 51, 085303, doi: `10.1088/1751-8121/aaa734`

Hill, S. A., & Wootters, W. K. 1997, Phys. Rev. Lett., 78, 5022, doi: `10.1103/PhysRevLett.78.5022`

Marletto, C., & Vedral, V. 2017, Phys. Rev. Lett., 119, 240402, doi: `10.1103/PhysRevLett.119.240402`

Marshman, R. J., Mazumdar, A., & Bose, S. 2020, Phys. Rev. A, 101, 052110, doi: `10.1103/PhysRevA.101.052110`

References III

Rungta, P., Bužek, V., Caves, C. M., Hillery, M., & Milburn, G. J. 2001, Phys. Rev. A, 64, 042315, doi: 10.1103/PhysRevA.64.042315

Schrödinger, E. 1935, Mathematical Proceedings of the Cambridge Philosophical Society, 31, 555–563, doi: 10.1017/S0305004100013554