Practical 5 -- Problems

Monday, October 19, 2020 7:32 PM

Section 2.3 Finding Position from Velocity

5. | FIGURE EX2.5 shows the position graph of a particle.
   a. Draw the particle's velocity graph for the interval $0 \text{ s} \leq t \leq 4 \text{ s}$.
   b. Does this particle have a turning point or points? If so, at what time or times?

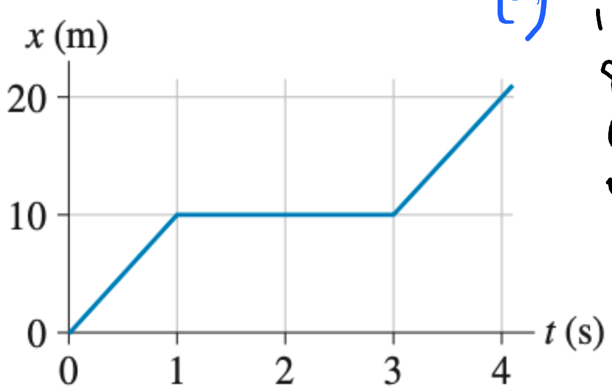
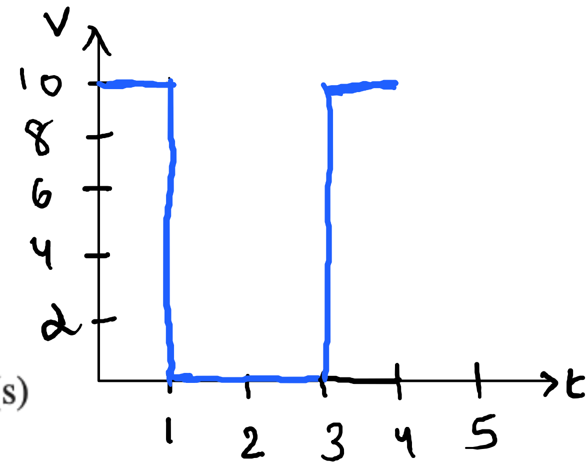


FIGURE EX2.5

(a)



(b) One stop point, at $t = 1$ s

75. ‖ The two masses in FIGURE P2.75 slide on frictionless wires. They are connected by a pivoting rigid rod of length L. Prove that $v_{2x} = -v_{1y} \tan\theta$.

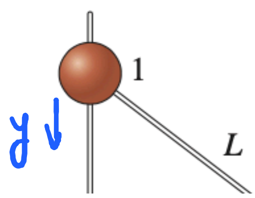
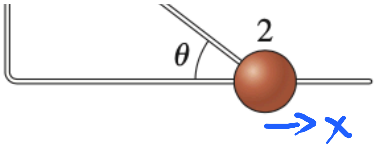


FIGURE P2.75



$$\frac{dy}{dt} = v_{1y} \qquad \frac{dx}{dt} = V_{2x} \qquad \tan\theta = \frac{y}{x}$$

$$x^2 + y^2 = l^2 \text{ (Pythagorean Theorem)}$$

Differentiating w.r.t t

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

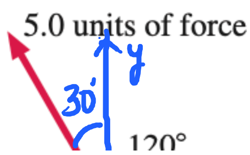
$$\Rightarrow \quad x V_{2x} + y v_{1y} = 0$$

$$\Rightarrow \quad V_{2x} = -\frac{y}{x} v_{1y}$$

$$\Rightarrow \quad \boxed{V_{2x} = -v_{1y}\tan\theta}$$

Q.E.D

43. ‖ **FIGURE P3.43** shows three ropes tied together in a knot. One of your friends pulls on a rope with 3.0 units of force and another pulls on a second rope with 5.0 units of force. How hard and in what direction must you pull on the third rope to keep the knot from moving?



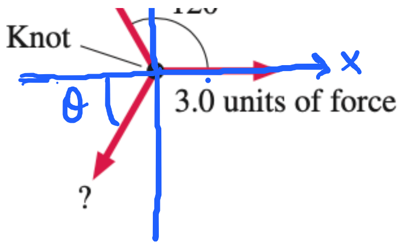


FIGURE P3.43

We Know that

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

where $\vec{F}_1 = 3\hat{i}$ N

$\vec{F}_2 = -5\sin 30^\circ \hat{i} + 5\cos 30 \hat{j}$ N

$\vec{F}_3$ = unknown force.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 \Rightarrow \vec{F}_3 = -\left(\vec{F}_1 + \vec{F}_2\right)$$

$$= -\left(3\hat{i} - 5\sin 30^\circ \hat{i} + 5\cos 30 \hat{j}\right) \text{ N}$$

$$= -\left(0.5\hat{i} + (2.5)\sqrt{3}\hat{j}\right) \text{ N}$$

$$\therefore F_3 = |\vec{F}_3| = \sqrt{0.6^2 + (2.5\sqrt{3})^2} = \underline{4.4\text{N}}$$

Angle, $\theta = \tan^{-1}\left(\frac{2.5\sqrt{3}}{0.3}\right) = 83.4^\circ$ below $-$ve $\underline{x\text{-axis}}$.

4. ‖ A velocity vector 40° below the positive x-axis has a y-component of -10 m/s. What is the value of its x-component?

$V_x = ?$ $V_y = -10$ m/s

$$V_y = -V\sin 40^\circ$$

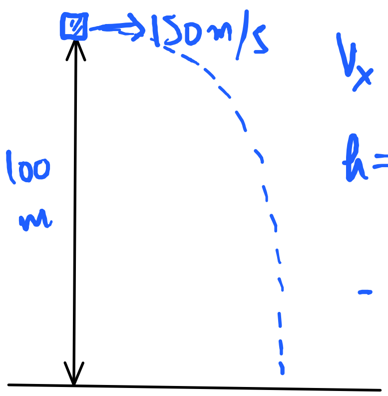
$$\Rightarrow \cancel{-}V\sin 40^\circ = \cancel{-}10$$

$$\Rightarrow V = \frac{10}{\sin 40^\circ} = 15.56 \text{ m/s}$$

$$V_x = V\cos 40^\circ$$

$$= \frac{10}{\sin 40^\circ}\cos 40^\circ = 10\cot 40^\circ = \underline{\underline{11.92 \text{ m/s}}}$$

13. ‖ A supply plane needs to drop a package of food to scientists working on a glacier in Greenland. The plane flies 100 m above the glacier at a speed of 150 m/s. How far short of the target should it drop the package?



(Not to scale)

$V_x = 50$ m/s | $V_y = 0$ m/s

$h = 100$ m $a_x = 0$ m/s^2 $a_y = -g$

$$-100 = -\frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{200}{g}} = 4.52 \text{ s}$$

$$R = V_x t$$

$$= 150t = \underline{\underline{677.2}} \text{ m}$$

∴ The package should be dropped $\underline{\underline{677.2}}$ m short of the target.

14. ‖ A rifle is aimed horizontally at a target 50 m away. The bullet hits the target 2.0 cm below the aim point.

hits the target 2.0 cm below the aim point.

a. What was the bullet's flight time?

b. What was the bullet's speed as it left the barrel?

v_x target $v_y = 0$ $a_y = -g$

$$\cancel{+}0.02 = \cancel{+}\frac{1}{2} g t^2$$

a) $t = \left(\frac{0.04}{9.8}\right)^{1/2} = \underline{\underline{0.064\cancel{8}}}$

b) $x = 50$

$$x = v_x t \Rightarrow v_x = \frac{x}{t} = 782.6 \text{ m/s} \approx \underline{\underline{783 \text{ m/s}}}$$

59. || A rubber ball is dropped onto a ramp that is tilted at 20°, as shown in **FIGURE P4.59**. A bouncing ball obeys the "law of reflection," which says that the ball leaves the surface at the same angle it approached the surface. The ball's next bounce is 3.0 m to the right of its first bounce. What is the ball's rebound speed on its first bounce?

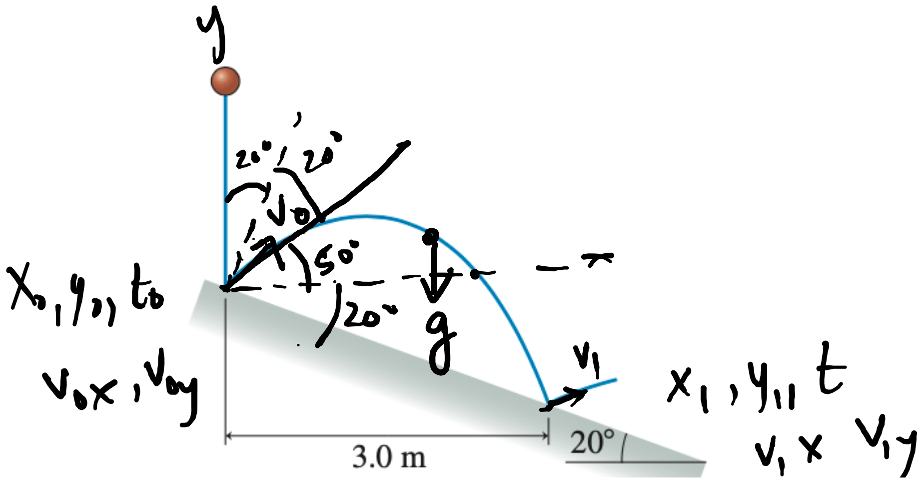


FIGURE P4.59

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \cancel{\frac{1}{2}a_x(t-t_0)^2}$$

$$3 = 0 + (v_0 \cos 50)(t_1 - \cancel{t_0})$$

$$\Rightarrow t_1 = \frac{3}{v_0 \cos 50^\circ} \quad \text{—} \; (1)$$

$$y_1 - y_0 = -(3)\tan 20^\circ$$
$$= -1.092\,\mathrm{m}$$

$$y_1 - y_0 = v_{0y}(t_1 - \cancel{t_0}) - \frac{1}{2}g(t_1 - \cancel{t_0})^2$$

$$-1.092 = v_0 \sin 50^\circ t_1 - \frac{1}{2}g t_1^2$$

$$-1.092 = \cancel{v_0} \sin 50 \cdot \frac{3}{\cancel{v_0}\cos 50} - \frac{1}{2}g\frac{3^2}{v_0^2\cos^2 50}$$

$$-1.092 - 3\tan 50^\circ = -\frac{1}{2}g\frac{9}{v_0^2\cos^2 50}$$

$$v_0 = \underline{\underline{4.8\,\mathrm{m/s}}}$$

64. ‖ A circular track has several concentric rings where people can run at their leisure. Phil runs on the outermost track with radius r_P while Annie runs on an inner track with radius $r_A = 0.80r_P$. The runners start side by side, along a radial line, and run at the same speed in a counterclockwise direction. How many revolutions has Annie made when Annie's and Phil's velocity vectors point in opposite directions for the first time?

They are running with the same speed.

$\therefore V_A = V_p \Rightarrow \omega_A r_A = \omega_p r_p$

$\Rightarrow 0.8\,\omega_A = \omega_p$.

Since they are in opposite directions,

$\theta_A - \theta_p = \pi$

$\omega_A t - \omega_p t = \pi$

$\Rightarrow 0.2\,\omega_A t = \pi$ ($\omega_p = 0.8\,\omega_A$; subsitute and simplify)

$\Rightarrow 0.2\,\theta_A = \pi$

$\Rightarrow \theta_A = 5\pi$

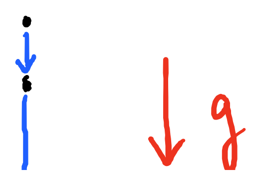
$1\text{ rev} = 2\pi$

$\Rightarrow 2.5\text{ rev} = 5\pi$

$\therefore$ Annie makes 2.5 revolutions.

52. ‖ A person on a bridge throws a rock straight down toward the water. The rock has just been released.

Draw the motion diagram.



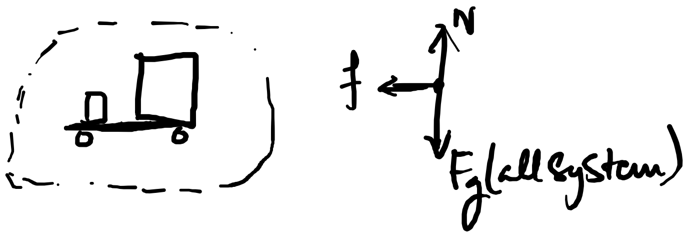
55. ‖ You're driving along at 25 m/s with your aunt's valuable antiques in the back of your pickup truck when suddenly you see a giant hole in the road 55 m ahead of you. Fortunately, your foot is right beside the brake and your reaction time is zero!

a. Can you stop the truck before it falls into the hole?

b. If your answer to part a is yes, can you stop without the antiques sliding and being damaged? Their coefficients of friction are $\mu_s = 0.60$ and $\mu_k = 0.30$.

Hint: You're not trying to stop in the shortest possible distance. What's your best strategy for avoiding damage to the antiques?

a) let us first consider the truck.



$$F_{net\,x} = \Sigma F_x = (m+M)a$$

$$\Rightarrow -f = (m+M)a.$$

$$\Sigma F_y = 0 \Rightarrow N - (m+M)g = 0$$

$$\Rightarrow N = (m+M)g.$$

$$f = \mu N$$

$$\Rightarrow f = \mu(m+M)g$$

$\Rightarrow -\mu(\cancel{m+M})g = (\cancel{m+M})a$

$\Rightarrow a = -\mu g.$

$\cancel{v_f^2}^{\,0} = v_i^2 + 2a\,\Delta x$

$\Rightarrow \cancel{+}v_i^2 = \cancel{+}2\mu g(55)$

$\Rightarrow \dfrac{25^2}{2(9.8)(55)} = \mu$

$\Rightarrow \underline{\underline{\mu = 0.58}} \leftarrow$ Smallest coefficient of friction needed.

Since the coefficient for rubber is 1.00 and 0.80 the truck can stop.

b) The coefficient also applies for the antiques system, and can be calculated in a similar fashion.

Since the static coefficient of friction for the box is given to be 0.60,

the antiques will not be

harmed!

56. ‖ The 2.0 kg wood box in **FIGURE P6.56** slides down a vertical wood wall while you push on it at a 45° angle. What magnitude of force should you apply to cause the box to slide down at a constant speed?

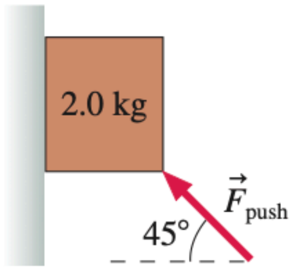
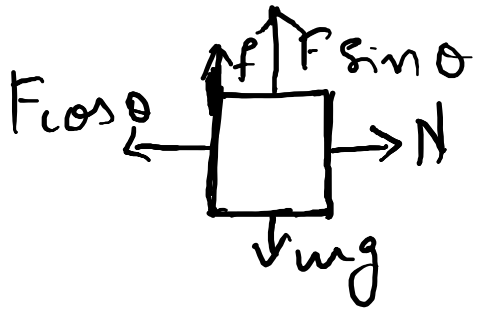


FIGURE P6.56



$\theta = 45^\circ$

$\Sigma F_x = 0$

$\Rightarrow F\cos\theta = N$

$\Sigma F_y = 0$

$\Rightarrow f + F\sin\theta - mg = 0$

μ_K for wood on wall = 0.2 (refer table of coefficients)

$\Rightarrow \mu_K F\cos\theta + F\sin\theta = mg$

$$\Rightarrow F = \frac{mg}{\mu\cos\theta + \sin\theta} = \frac{2.0 \times 9.8}{\frac{0.2}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \underline{\underline{23\,N}}$$

$\therefore$ Minimum magnitude is $\underline{\underline{23\,N}}$.

35. ‖ The coefficient of static friction is 0.60 between the two blocks in **FIGURE P7.35**. The coefficient of kinetic friction between the lower block and the floor is 0.20. Force $\vec{F}$ causes both blocks to cross a distance of 5.0 m, starting from rest. What is the least amount of time in which this motion can be completed without the top block sliding on the lower block?

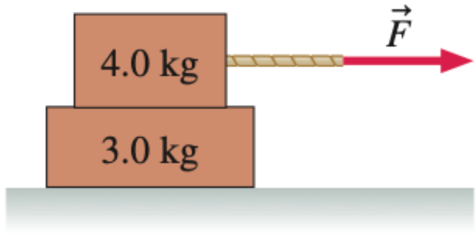
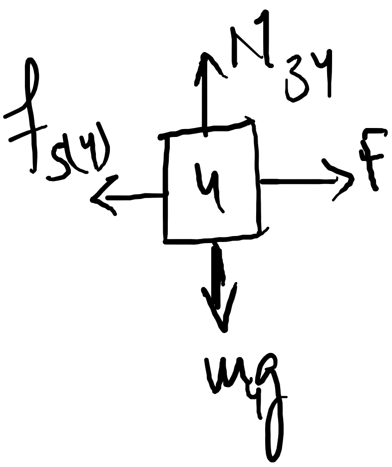


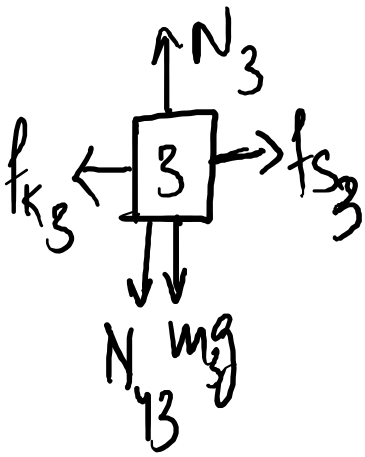
FIGURE P7.35

f_{s_3} → force of block static friction on 3

N_3 → Normal on 3

N_{34} → Normal on 4 due to 3.





Minimum time $\Leftrightarrow$ Maximum static friction

Consider 4 kg

$\Sigma F_y = N_{34} - m_4 g = 0 \Rightarrow m_4 g = N_{34} = (4.0)(9.8) = 39.2N$

$f_{s4} = \mu N_{34} = (0.6)(39.2) = 23.5 N.$

N_{34} & $_3$ $\rightarrow$ Action | Reaction pair
f_{s_3} & f_{s_4} — " " "

Consider 3 kg

$\Sigma F_y = N_3 - N_{43} - m_3 g = 0$

$\Rightarrow N_3 = N_{43} + m_3 g$

$= 39.2 + (3)(9.8)$

$= 68.6 N$

$f_{k_3} = \mu_k N_3 = (0.2)(68.6) = 13.72$

$\Sigma F_x = f_{s_3} - f_{k_3} = m_3 a_3$

$\Rightarrow a_3 = \dfrac{23.5 - 13.72}{3.0} = 3.27 m/s^2$

Since the blocks are connected (one on the other)

$$a_y = a_3 = 3.27 \, m/s^2$$

$$\Delta x = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2\Delta x}{a}}$$

$$= \sqrt{\frac{2(5)}{3.27}}$$

$$= 1.8 \, s$$

∴ Maximum time is 1.8 s.

30. || Three cars are driving at 25 m/s along the road shown in FIGURE EX8.30. Car B is at the bottom of a hill and car C is at the top. Both hills have a 200 m radius of curvature. Suppose each car suddenly brakes hard and starts to skid. What is the tangential acceleration (i.e., the acceleration parallel to the road) of each car? Assume $\mu_k = 1.0$.

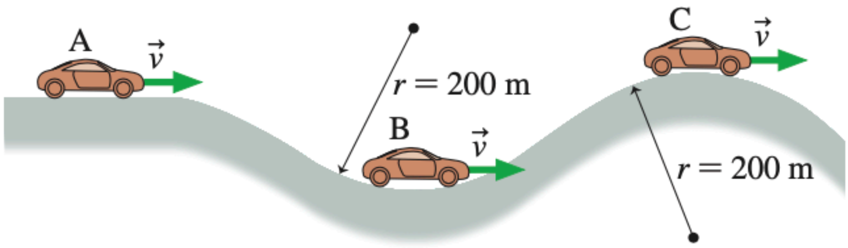
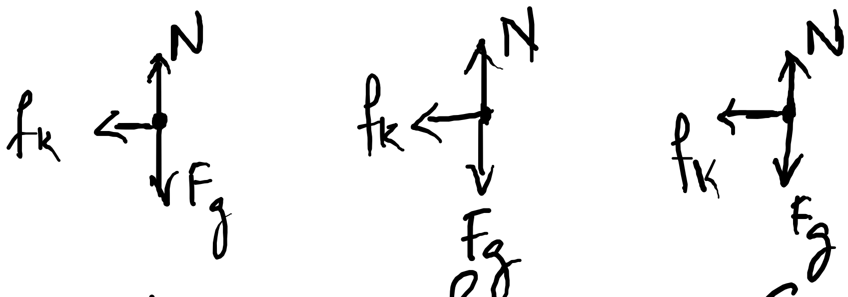


FIGURE EX8.30



A. B. C.

Car A

$$\sum F_x = -f_k = ma_x$$

$$\sum F_y = N - mg = 0 \Rightarrow N = mg$$

$$\Rightarrow f_K = \mu_K mg$$

$$\Rightarrow -\mu_K \cancel{m} g = \cancel{m} a_x$$

$$\Rightarrow \underline{\underline{a_x = -9.8\, m/s^2}}$$

(You can also think about the flat surface as a circle with ∞ radius!)

<u>Car B</u>

This car is in circular motion.

$$\sum F_r = N - mg = \frac{mv^2}{r}$$

$$\Rightarrow N = m\left(\frac{v^2}{r} + g\right)$$

$$\sum F_t = ma_t \text{ (tangential)}$$

$$\Rightarrow -f_K = ma_t$$

$$\Rightarrow -\mu_K\left(\cancel{m}\left(\frac{v^2}{r} + g\right)\right) = \cancel{m} a_t$$

$$\Rightarrow \quad a_t = -12.9\, m/s^2$$

Car C: Also under circular motion.

$$\Sigma F_r = -N + mg = \frac{mv^2}{r}$$

$$\Rightarrow \quad N = m\left(g - \frac{v^2}{r}\right)$$

$$\Rightarrow f_K = \mu_K m\left(g - \frac{v^2}{r}\right)$$

$$\Sigma F_t = -f_K = m a_t$$

$$\Rightarrow \quad -\mu_K \cancel{m}\left(g - \frac{v^2}{r}\right) = \cancel{m} a_t$$

$$\Rightarrow \quad a_t = -6.7\, m/s^2$$

$$a_{t_C} < a_{t_A} < a_{t_B}$$