

Problem Set 10, Math 191 Fall '15

This problem set is due Tuesday, November 3, 2015 at **the beginning of class**. All class guide rules apply. **Please remember to set aside “self-work time”** before consulting Piazza or working with others.

(Before doing the problems, look up and understand “Cauchy-Schwarz inequality” in Wikipedia/Google for 20 minutes.)

1. Four cars A, B, C, D travel at constant speeds on the same road. A passed B and C at 8 and 9 AM, respectively, and met D at 10 AM; D met B and C at 12 PM and 2 PM, respectively. When did B meet C ?
2. If a 7×7 matrix A has all diagonal entries equal to 0 and other entries ± 1 , can its determinant be 0? What about an 8×8 matrix?
3. Call a number *all-encompassing* if **every** positive integral multiple of the number contains all the digits from 0 to 9 at least once. Prove or disprove (no computers!): 526315789473684210 is an all-encompassing number.
4. Show that if $f : \mathbf{R} \rightarrow \mathbf{R}$ has a continuous derivative then

$$\int_{-\infty}^{\infty} |f(x)|^2 dx \leq 2 \left(\int_{-\infty}^{\infty} x^2 |f(x)|^2 dx \right)^{1/2} \left(\int_{-\infty}^{\infty} |f'(x)|^2 dx \right)^{1/2}.$$

5. Prove or disprove: it is possible to load three 6-sided dice (meaning give different probabilities of landing each side) so that when you roll them, the first will probably show a higher number than the second, the second will probably show a higher number than the third, and the third will probably show a higher number than the first.
6. Can every nonnegative integer be represented in the form $a^2 + b^2 - c^2$, where $a < b < c$ are positive integers?
7. Solve in positive integers the equation

$$2^x 3^y = 1 + 5^z.$$

8. Given a constant C , find all functions f such that

$$f(x) + Cf(2-x) = (x-1)^3$$

for all x .

9. **(required)** How much time (including self-work time) did you spend on this problem set? What comments do you have of the problems? (difficulty, type, enjoyment, etc.) Please confirm you have the basic idea of the boxed principle above (and would be able to show me on the board a basic example and some properties if I called on you =D). You can, of course, ask people to explain specific questions on Piazza.