Problem Set 1, Math 191 Fall '15

This problem set is due Tuesday, September 1, 2014 at **the beginning of class**. All class guide rules apply. **Please remember to set aside "self-work time"** before consulting Piazza or working with others.

Remember that the point of the math problems is to give honest effort; you do not have to solve all of them (though you must attempt all of them). **Non-math problems are always required and considered a part of the classroom participation grade.**

1. Solve the system of equations (in $\mathbf{R}$):

$$\begin{aligned} a+b &= 8 \\ ab+c+d &= 23 \\ ad+bc &= 28 \\ cd &= 12. \end{aligned}$$

2. Find a set (or show that it is impossible) of positive integers a, b, c, $n > 2$ such that $a^n - b^n = c^n$.

3. There is a tribe of people that dislike the digit 9 who call an integer *evil* if it contains the digit 9. Prove that $\sum_x (1/x) < 80$, where the sum is over all non-evil positive integers x.

4. Two players play a game: there is a positive integer n on the board. Each player can subtract a divisor of n from n, and this difference becomes thew new n (for example: I can replace 20 by $20 - 2 = 18$). The player who gets to 0 loses. For which numbers can you force a win?

5. On the surface of a ball are 5 points. Prove that you can find a *closed* hemisphere (i.e. including the boundary) containing at least 4 of the points.

6. A group of K mathematicians and K musicians sit around the table at a party. Some always tell the truth and some always lie. It is known that the number of liar mathematicans equal the number of liar musicians. when asked: "what is your right-hand neighbor?" Everyone replies: "musician!". Prove that K is even.

7. Let the "baby AM-GM inequality" be the AM-GM for $n = 2$; i.e. for nonnegative a_i, we have:

$$\frac{a_1 + a_2}{2} \geq \sqrt{a_1 a_2}.$$

First, prove it. Then, use only the baby AM-GM inequality and manipulation to prove the AM-GM,

$$\frac{a_1 + a_2 + \cdots + a_n}{n} \geq (a_1 a_2 \cdots a_n)^{1/n},$$

for arbitrary n.

8. Take a set of $n+1$ positive integers in $\{1, 2, \ldots, 2n\}$. Show that there are at least two elements of the set, a and b, such that $a|b$.

9. (**You will not receive a passing grade for this course without turning in this problem**) Acknowledge (**in your own words, through signature**) that you have:

   (a) read and understood the Class Guide and will hold yourself responsible to it;

   (b) understood that if you have any unresolved questions/complaints about the class policy, that you will tell me ASAP so we can resolve it, and if you are silent now then I assume we have agreed on class policy **(If you have an issue, DON'T put it on this homework, since it requires speedy grader turnaround. Instead, bring it up on Piazza or class so we can settle it.)**.

10. How much time did you spend on this problem set? What comments do you have of the problems? (difficulty, type, enjoyment, fairness, etc.)

11. Successfully make at least one question/statement about something **you care about** (for example: question about lecture material, asking someone to borrow notes because you missed class, homework question help, answering a question from someone else, introducing yourself and indicating something you would like to eventually learn in this class, class policy questions / suggestions, etc.) on Piazza by 11:59PM on **Monday, 8/31/15**. You do not have to copy this text to the physical problem set you turn in; it will be graded by us looking on Piazza. Acknowledge (**through signature**) that you have gotten your Piazza account to work, know that you are responsible for what goes on there (for example, errata for homework), and understand how to post / view / reply. (**For grading: we will disgard any Piazza action where it is obvious that the person did not care about the assignment, such as finding a random question from the internet at 11:59:59 PM or simply answering "no" to a question without any explanation**)