1)
[**Lemma 0**] The most number of steps you have to take to go from one square to another square is 7 steps.
Proof: Define the 1st ring of a square s as all squares touching s and define the ith ring of a square s as all squares touching the $i-1$th ring of s but that aren't in any of the previous rings. Clearly for any square s, it has at most 7 rings (7 if you choose an corner piece for s). Thus via at most 7 steps you can go from one square to any other square.

Getting back to our grand proof, suppose towards a contradiction that there doesn't exist two adjacent squares such the different of the numbers placed on them is at least 9.

We know by [**Lemma 0**] that it takes at most 7 steps to go from the square labeled 1 to the square labeled 64.

However since the different between two squares is less than 9, each square on the path can at most be 8 greater than the last number.

But that means in seven steps, we at most can use the number $1 + 8 * 7 = 57 < 64$.

Thus it's impossible to ever reach 64 from 1 in less than or equal to 7 steps but that's a contradiction.

So we know that there must exist two adjacent squares such the different of the numbers placed on them is at least 9 $\square$.

3) Partial Proof Suppose there exists a positive integer m for which $a_m = 0$.

First note since $a_1 = f(0)$, $\boxed{a_1 \text{ is the constant term in } f}$.

For every a_i for $i > 0$, a_0 appears in every term of the sum a_i.

Thus $\boxed{a_0 \text{ divides every } a_i \text{ for } i > 0}$.

Also since a_0 appears in every term of the sum a_i, if $a_0 = 0$, then every a_i is 0.

So suppose moving forward that $a_0 \neq 0$ and we'll work to show that $a_2 = 0$.

We are given that $0 = a_m = f(a_{m-1})$.

Since a_{m-1} is an integer, we know using the rational roots theorem that a_{m-1} must divides the constant term of f.

But the constant term is a_1 by the first box, so a_{m-1} divides a_1.

We also know that a_0 divides all a_i where $i > 0$ by the second box, so a_1 divides a_{m-1}.

Since a_{m-1} divides a_1 and a_1 divides a_{m-1}, we know either:
(1) $a_{m-1} = a_1$
(2) $a_{m-1} = -a_1$

In (1), we know that $a_2 = f(a_1) = f(a_{m-1}) = 0$ so we get as desired $a_2 = 0$.

I wasn't able to figure out (2).

5) We can apply Hall's Theorem.

One set consists of the polygons on the back sheet and the other set is the polygons on the front sheet.

We want to show that every subset of the polygons on the backsheet, call any subset B, cover a set of polygons on the front sheet, F, that is at least as big as B.

Since the total area covered by B is B, there must exist at least B polygons on the front sheet in that area since the polygons on both sheets are the same size.

Thus by Hall's Theorem, a matching exists between both sets. $\square$

9) Total of 10 hours spent on this. I thought this problem set was a lot harder. I found boxing insights I have in my scratch work particularly helpful.