	Grading
Your PRINTED name is:	1
	2
	3
	4
	5
	6

Each problem is worth 10 points. Each problem has roughly 2 pages of space for your solution. If you want more space, CLEARLY mark a piece of scratch paper with the problem number and your name and attach it to the exam.

In such a competition, you are graded on the PROOF, not just the answer. You will not receive any partial credit without showing the REASONING behind significant nontrivial insight. (It is strongly recommended that you try to write up strategically complete solutions instead of generating text to receive partial credit.)

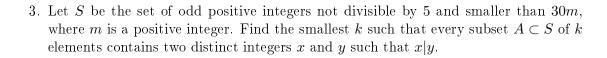
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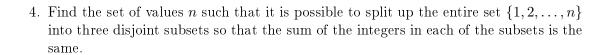
3

1. Prove there is no equilateral triangle in the plane with all vertices at integral lattice

points.

2. Is there a function f, differentiable for all real x, such that |f(x)| < 2 and $f(x)f'(x) \ge \sin(x)$ for all x?





5. Let $n \geq 3$ be an integer. Let f(x) and g(x) be polynomials with real coefficients such that the points $(f(1), g(1)), (f(2), g(2)), \ldots, (f(n), g(n))$ in \mathbb{R}^2 are the vertices of a regular n-gon in counterclockwise order. Prove that at least one of f(x) and g(x) has degree greater than or equal to n-1.

6. The diameter of a set is the supremum of the possible distances between any two points in the set. Prove that any set in the plane of diameter 1 can be partitioned into three parts, each with diameter no more than $\frac{\sqrt{3}}{2}$.