3) If n is odd, $P(n+1) = 0$. Otherwise (ie when n is even), $P(n+1) = -1$.

Proof: Using Lagrange Interpolation, we get that:

$$P(x) = \sum_{k=0}^{n} \binom{n+1}{k}^{-1} \prod_{j=0 \text{ st } j \neq k}^{n} \frac{(x-j)}{(k-j)}$$

Thus putting as input $(n+1)$, we get:

$$\begin{aligned}
P(n+1) &= \sum_{k=0}^{n} \binom{n+1}{k}^{-1} \prod_{j=0 \text{ st } j \neq k}^{n} \frac{(n+1-j)}{(k-j)} \\
&= \sum_{k=0}^{n} \frac{(k)!(n+1-k)!}{(n+1)!} \frac{\prod_{i=n}^{k+1}(n+1-i) \prod_{j=k-1}^{1}(n+1-j)}{\prod_{i=n}^{k+1}(k-i) \prod_{j=k-1}^{1}(k-j)} \\
&= \sum_{k=0}^{n} \frac{(k)!(n+1-k)!}{(n+1)!} \frac{(n-k)! \prod_{j=k-1}^{1}(n+1-j)}{\prod_{i=n}^{k+1}(k-i) \prod_{j=k-1}^{1}(k-j)} \\
&= \sum_{k=0}^{n} \frac{(k)!(n+1-k)!}{(n+1)!} \frac{(n-k)! \frac{(n+1)!}{(n+1-k)!}}{\prod_{i=n}^{k+1}(k-i) \prod_{j=k-1}^{1}(k-j)} \\
&= \sum_{k=0}^{n} \frac{(k)!(n+1-k)!}{(n+1)!} \frac{(n-k)! \frac{(n+1)!}{(n+1-k)!}}{(n-k)!(-1)^{n-k} \prod_{j=k-1}^{1}(k-j)} \\
&= \sum_{k=0}^{n} \frac{(k)!(n+1-k)!}{(n+1)!} \frac{(n-k)! \frac{(n+1)!}{(n+1-k)!}}{(n-k)!(-1)^{n-k}(k)!} \\
&= \sum_{k=0}^{n} (-1)^{n-k} \\
&= \begin{cases} 0 & \text{if n is odd} \\ 1 & \text{o.w.} \end{cases} \quad \square
\end{aligned}$$

9)