Problem Set 8, Math 191 Fall '15

This problem set is due Tuesday, Oct 20, 2015 at **the beginning of class**. All class guide rules apply. **Please remember to set aside "self-work time"** before consulting Piazza or working with others.

(Before doing the problems, look up and understand "Linearity of Expectation" in Wikipedia/Google for 20 minutes. Note the (continued) change in Problem 9. From now on, problem 9 will stay the way it is currently and these last two sentences will self-destruct.)

1. If A and B are square matrices of the same size such that $ABAB = 0$, prove or disprove: $BABA = 0$.

2. Design the rules of a 2-player game, where players use just a single fair coin, such that the first player has a probability of $1/\pi$ of winning. (one of the player must win with probability 1; the game also doesn't have to be fun)

3. Call an integer *good* if each of its prime factors p appears in its factorization as p^2 or higher.

   (a) Prove or disprove: there are infinitely many pairs of consecutive good numbers.

   (b) Then do the same for triples.

4. The team members of Squadron 191 fly off in n identical mini-battleships into interstellar battle. The home base sent barely enough battleships, so if there were only $n-1$ battleships, then the team would not fit. After the battle, the team members fly back in the same n battleships, but not necessarily in the same ships as they left. Prove that there are n team members such that for any pair of them, they did not sit with each other on either ride.

5. Let $F(x) = \sum_{n=1}^{\infty}(x^2 + n^4)^{-1}$. Compute $\int_0^{\infty} F(t)dt$.

6. There are mn potato chips, each one labeled by a different divisor of $2^{m-1}3^{n-1}$. Each turn, a player must take a potato chip... and eat it! But if he/she eats the potato chip labeled k, he/she must also eat all potato chips with labels $d|k$. The potato chip labeled $2^{m-1}3^{n-1}$ is a Death Chip and the first player to eat it loses. Prove that the first player can win, and find a winning strategy for him/her for all m and n.

7. Starting with an ordered quadruple of integers, repeatedly perform

$$(a, b, c, d) \to (|a-b|, |b-c|, |c-d|, |d-a|).$$

   Prove that after finitely many steps, we get to $(0, 0, 0, 0)$.

8. Show that for an n-gon inscribed in a circle, the regular n-gon has the greatest area.

9. **(required)** How much time (including self-work time) did you spend on this problem set? What comments do you have of the problems? (difficulty, type, enjoyment, etc.) Please confirm you have the basic idea of the boxed principle above (and would be able to show me on the board a basic example and some properties if I called on you =D). You can, of course, ask people to explain specific questions on Piazza.