1)
Thoughts: It seems like the answer is only $1, 2, \cdots$ but I'm having trouble proving that. I tried letting n_i be the first number such that $n_i \neq i$ and showing that leads to a contradiction but that seems difficult. I also was going to try to show it's increasing, by 1, and show it must start at 1 but that also seems hard. It's very possible that my proposition is wrong but ya those were my thoughts.

3)
Thoughts: If you compare this to the taylor series expansion of e, then you know this sequence goes to e if you can keep all n positive where $1/n = 1/k!$ for all k and then show you can make the rest the terms sum to 0. I wasn't sure though how to make the terms sum to 0.

4) $\boxed{\text{The determinant is 0}}$. Let the ith row equal ith row $-(i+1)$th row and let last equal the last row minus the first row. Now for each row, add all other rows to it. You now have the 0 matrix which has determinant 0 and since multiples of rows and columns can be added together without changing the determinant's value, the value of the original matrix is also 0. I can write this more rigorously using an arbitrary index and showing it's true for that, but that seemed really tedious so I was wondering if my explanation was enough.

5)

$\boxed{\text{B wins and I'm really sorry I accidently threw out my scratch work for this problem!}}$

[**Lemma 0**] If both A and B play optimally, A wins by playing first iff by subtracting 2, 5, or 6, A can get to a number of coins where if A and B both played optimally and the second player to place his tiles would would win.

Proof of Lemma 0: Suppose by subtracting 2, 5, or 6 A can gets to a position where the second player to place his tiles would win. If A chooses this spot, then B will lose so playing optimally A will always choose to recurse to this spot. Suppose by subtracting 2, 5, or 6 A can only get to a position where the first player to place his tiles would win. A must choose one of these spots thus B playing optimally will win. $\square$

[**Lemma 1**] Let x be the number of coins on the table when A starts their turn. If x mod $11 \equiv 0, 1, 4, 8$, then B wins if both A and B play optimally.

Proof of Lemma 1: We'll prove this statement using induction.

For $n \in \mathbf{Z}_{\geq 0}$, let $P(n)$ be the proposition that for n coins are on the table when the first player starts their turn and Player A and B play optimally, $n \mod 11 \equiv 0, 1, 4, 8$ iff the second player wins.

For $n < 11$, it's easy to see using [**Lemma 0**] that the second player only wins if $n = 0, 1, 4,$ or 8 thus our base cases are covered.

We now want to show that $P(k)$ is true for all $k > m$ for some $m \geq 11$ implies $P(k+1)$ is true.

First note we can rewrite $k + 1$ as $11a + b$ for $b \in [0, 10]$.

For $b = 0$, the first player can go back to $11a-2, 11a-5, 11a-6$ which are all less than $k+1$ and equivalent to $9, 6, 5$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the second player will win.

For $b = 1$, the first player can go back to $11a-1, 11a-4, 11a-5$ which are all less than $k+1$ and equivalent to $10, 7, 6$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the second player will win.

For $b = 2$, the first player can go back to $11a, 11a-3, 11a-4$ which are all less than $k+1$ and equivalent to $0, 8, 7$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the first player will win.

For $b = 3$, the first player can go back to $11a+1, 11a-2, 11a-3$ which are all less than $k+1$ and equivalent to $1, 9, 8$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the first player will win.

For $b = 4$, the first player can go back to $11a+2, 11a-1, 11a-2$ which are all less than $k+1$ and equivalent to $2, 10, 9$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the second player will win.

For $b = 5$, the first player can go back to $11a + 3, 11a + 0, 11a - -1$ which are all less than $k+1$ and equivalent to $3, 0, 10$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the first player will win.

For $b = 6$, the first player can go back to $11a+4, 11a+1, 11a-0$ which are all less than $k+1$ and equivalent to $4, 1, 0$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the first player will win.

For $b = 7$, the first player can go back to $11a+5, 11a+2, 11a-1$ which are all less than $k+1$ and equivalent to $5, 2, 1$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the firstplayer will win.

For $b = 8$, the first player can go back to $11a+6, 11a+3, 11a-2$ which are all less than $k+1$ and equivalent to $6, 3, 2$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the second player will win.

For $b = 9$, the first player can go back to $11a+7, 11a+4, 11a-3$ which are all less than $k+1$ and equivalent to $7, 4, 3$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the first player will win.

For $b = 10$, the first player can go back to $11a+8, 11a+5, 11a-4$ which are all less than $k+1$ and equivalent to $8, 5, 4$ (respectively) in $\mod 11$ land. Thus by our inductive hypothesis and using [**Lemma 0**], the first player will win.

This completes our inductive proof. Sorry I'm sure there was a cleaner way to do this then just going through all the cases but this works!

By [**Lemma 1**], since $100 \mod 11$ is 1, B wins. $\square$

8)
We'll prove this statement using induction.

Let $P(n)$ be the proposition that on a 2^n by 2^n chessboard if you remove one square, the remaining $2^{2n} - 1$ squares can be tiled using L-tiles for $n \in \mathbf{Z}_{>0}$.

Base Case: For n = 1, if you remove one square from a 2 by 2 board, you have a L-tile so clearly it can be covered by a L-tile.

We now want to show that $P(k)$ is true for all $k > m$ for some $m \geq 1$ implies $P(k+1)$ is true.

Suppose we have a 2^{k+1} by 2^{k+1} chessboard.

Firstly, note that a 2^{k+1} by 2^{k+1} chessboard is the same as 4 2^k by 2^k chessboards so let's split the 2^{k+1} by 2^{k+1} chessboard into 4 equal size chessboards that are 2^k by 2^k.

We know one of the 4 2^k by 2^k chessboards has a square cut out, and thus that 2^k by 2^k chessboard by our inductive hypothesis can be covered by L-tiles.

Now in the remaining 3 2^k by 2^k chessboards, cut out their square piece most close the center of the giant 2^{k+1} by 2^{k+1} chessboard.

Clearly each of the 3 2^k by 2^k maimed chessboards (maimed because each has 1 square removed) are by our inductive hypothesis able to be covered by L-tiles.

Now note, that the way we chose the 3 squares to cut out, they are in the shape of an L and thus we can cover those 3 squares using a L-tile.

Thus we have shown $P(k+1)$ is true.

This completes our inductive proof and our statement. $\square$

9) Total of 4.5 hours. I feel like I'm getting better at induction and just not putting enough effort into the analysis and algebra questions :(so probably something I should/can work on.

A good geometric trueism about determinants is that the determinant of a linear transformation is the signed volume of the region gotten by applying the linear transfromation to the unit cube.

I think though the first thing I learned about determinants is that if the determinant of a matrix is non-0, then the system of equations described by the matrix has a unique solution.

The most straight forward way to calculate the determinant of a matrix is to do Laplace expansion on the matrix.

An important fact when calculating the determinant of a matrix is that multiples of rows and columns can be added together without changing the determinant's value.

I also was thinking that instead of being called super models, they should be called super-duper models. That's it from me.