UC Berkeley Mock Putnam 2015

Grading
1
2
3
4
5
6

Your PRINTED name is:________________________

Each problem is worth 10 points. Each problem has roughly 2 pages of space for your solution. If you want more space, CLEARLY mark a piece of scratch paper with the problem number and your name and attach it to the exam.

In such a competition, you are graded on the PROOF, not just the answer. You will not receive any partial credit without showing the REASONING behind significant nontrivial insight. (It is strongly recommended that you try to write up strategically complete solutions instead of generating text to receive partial credit.)

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1. Prove there is no equilateral triangle in the plane with all vertices at integral lattice points.

(continue your solution here)

2. Is there a function f, differentiable for all real x, such that $|f(x)| < 2$ and $f(x)f'(x) \geq \sin(x)$ for all x?

(continue your solution here)

3. Let S be the set of odd positive integers not divisible by 5 and smaller than $30m$, where m is a positive integer. Find the smallest k such that every subset $A \subset S$ of k elements contains two distinct integers x and y such that $x|y$.

(continue your solution here)

4. Find the set of values n such that it is possible to split up the entire set $\{1, 2, \ldots, n\}$ into three disjoint subsets so that the sum of the integers in each of the subsets is the same.

(continue your solution here)

5. Let $n \geq 3$ be an integer. Let $f(x)$ and $g(x)$ be polynomials with real coefficients such that the points $(f(1), g(1)), (f(2), g(2)), \ldots, (f(n), g(n))$ in $\mathbb{R}^2$ are the vertices of a regular n-gon in counterclockwise order. Prove that at least one of $f(x)$ and $g(x)$ has degree greater than or equal to $n-1$.

(continue your solution here)

6. The diameter of a set is the supremum of the possible distances between any two points in the set. Prove that any set in the plane of diameter 1 can be partitioned into three parts, each with diameter no more than $\frac{\sqrt{3}}{2}$.

(continue your solution here)