Problem Set 11, Math 191 Fall '15

This problem set is due Tuesday, November 10, 2015 at **the beginning of class**. All class guide rules apply. **Please remember to set aside "self-work time"** before consulting Piazza or working with others.

(Before doing the problems, look up and understand "Lagrange Interpolation" in Wikipedia/Google for 20 minutes.)

1. Suppose we have a set of real numbers a_n. Let the k-th "elementary symmetric function" e_k be the sum of all $a_{i_1}a_{i_2}\cdots a_{i_k}$, where $i_1 < i_2 < \cdots < i_k \leq n$. For example, e_2 of $a_1, \ldots, a_4$ is equal to

$$a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4.$$

   If all the e_i's are positive for $1 \leq i \leq n$, must all the a_i's be positive?

2. The entry in the i-th row and j-th column of an $n \times n$ matrix equals $a_i + b_j$, where the a_i and b_i form a set of distinct real numbers. The products of numbers in each row of the matrix are equal. Prove that the products of the numbers in each column are also equal.

3. Let P be a polynomial of degree n satisfying for $k = 0, 1, \ldots, n$, $P(k) = \binom{n+1}{k}^{-1}$. Determine $P(n+1)$.

4. On a 10×10 chessboard, in each move, we pick 2 rows and 2 columns, and look at the 4 squares where they intersect. if at least one of them has no pawn on it, we put a pawn on all 4 squares. What is the maximum number of moves we can make before getting stuck?

5. Let $\alpha = \pi/(n+1)$. Is the $n \times n$ matrix A, where $A_{ij} = \sin(ij\alpha)$, invertible?

6. Is it possible to write every natural number as a sum of numbers of the form 2^i3^j, where i, j are nonnegative integers, such that no summand divides any other summand? For example, $19 = 4 + 6 + 9$, and no pair of those 3 numbers divide evenly.

7. Color the points of the plane red or yellow. Prove that for one of the two colors C, it is true that for any distance d, there are 2 points of color C which are d apart.

8. Let $f(n)$ be the number of ways of writing n as a sum of an odd number of positive integers, and $g(n)$ be the number of ways of writing n as a sum of even number of positive integers. Find $f(n) - g(n)$ for all n.

9. **(required)** How much time (including self-work time) did you spend on this problem set? What comments do you have of the problems? (difficulty, type, enjoyment, etc.) Please confirm you have the basic idea of the boxed principle above (and would be able to show me on the board a basic example and some properties if I called on you =D). You can, of course, ask people to explain specific questions on Piazza.