# Problem Set 5, Math 191 Fall '15

This problem set is due Tuesday, September 29, 2014 at **the beginning of class**. All class guide rules apply. **Please remember to set aside "self-work time"** before consulting Piazza or working with others.

**(Before doing the problems, look up and understand "infinite descent" in Wikipedia.)**

1. Place the positive integers from 1 to 64 on the squares of a chessboard. Prove that there exist two adjacent **(diagonal counts!)** squares such that the difference of the numbers placed on them is at least 9.

2. Show that positive $a_1, a_2, a_3, a_4$ satisfy the bound
$$2 \leq \frac{a_1}{a_2 + a_3} + \frac{a_2}{a_3 + a_4} + \frac{a_3}{a_4 + a_1} + \frac{a_4}{a_1 + a_2}.$$

3. Let $f(x)$ be a polynomial with integral coefficients. Define a sequence $a_0, a_1, \ldots$ of integers such that $a_0 = 0$ and $a_{n+1} = f(a_n)$ for all $n \geq 0$. Prove that if there exists a positive integer $m$ for which $a_m = 0$, then either $a_1 = 0$ or $a_2 = 0$.

4. Suppose you have $2n + 1$ football players, such that if you remove any player the remaining $2n$ players can always be split up into two teams of equal size $n$ but also equal total weight. Must all the players have equal weight, assuming they have integral weight? What about real weight?

5. Two square sheets have area equal to 2014. Each sheet is divided into 2014 non-overlapping polygons with area 1 each. Then, overlay the two sheets. Prove that the double layer sheet you get can be punctured 2014 times, so each of the 4028 polygons are punctured exactly once.

6. Solve
$$\sin^7(x) + 1/\sin^3(x) = \cos^7(x) + 1/\cos^3(x).$$

7. Evaluate
$$\int_2^4 \frac{\sqrt{\log(9-x)}dx}{\sqrt{\log(9-x)} + \sqrt{\log(x+3)}}.$$

8. Given a finite collection of squares of total area 1/2, show that they can be arranged to fit inside a unit square with no overlaps.

9. **(required)** How much time (including self-work time) did you spend on this problem set? What comments do you have of the problems? (difficulty, type, enjoyment, etc.)