Problem Set 4, Math 191 Fall '14

This problem set is due Tuesday, September 22, 2014 at **the beginning of class**. All class guide rules apply. **Please remember to set aside "self-work time"** before consulting Piazza or working with others.

(Before doing the problems, look up and understand Hall's Marriage Theorem/Lemma.)

1. Let $P(t)$ be a polynomial of degree 2014 with real coefficients. Prove that $f(X) = P(X)$ defined on $n \times n$ matrices is not surjective.
2. A $m \times n$ array is filled with $1, 2, \ldots, n$, each used exactly m times. Show that one can always permute the numbers **within columns** so that each row contains every number $1, 2, \ldots, n$ at least once.
3. Find all triples of consecutive positive integers such that their product is a perfect square.
4. Show that a convex polygon with $2n$ sides has at least n diagonals not parallel with any of its sides.
5. Let $(x_1, y_1) = (0.8, 0.6)$, and then $x_{n+1} = x_n \cos y_n - y_n \sin y_n$ and $y_{n+1} = x_n \sin y_n + y_n \cos y_n$ for higher n. Find (or prove the limit doesn't exist) $\lim_{n\to\infty} x_n$ and $\lim_{n\to\infty} y_n$.
6. Compute the first 3 decimal places of $\int_0^1 \cos \sqrt{x} dx$.
7. Can one find two biased dice (with sides 1 to 6 still, just potentially weighted differently) in such a way that the probability of getting a sum j for all $2 \leq j \leq 12$ when rolling two dice is in $(2/33, 4/33)$?
8. Given a finite collection of squares of total area 3, prove that you can move them to cover the unit square (overlaps allowed).
9. **(required)** How much time (including self-work time) did you spend on this problem set? What comments do you have of the problems? (difficulty, type, enjoyment, etc.)