3) If n is odd, $P(n+1)=0$. Otherwise (ie when n is even), $P(n+1)=-1$.

Proof: Using Lagrange Interpolation, we get that:

$$P(x)=\sum_{k=0}^{n}\binom{n+1}{k}^{-1}\prod_{j=0 \text{ st } j\neq k}^{n}\frac{(x-j)}{(k-j)}$$

Thus putting as input $(n+1)$, we get:

$$\begin{aligned}
P(n+1) &= \sum_{k=0}^{n}\binom{n+1}{k}^{-1}\prod_{j=0 \text{ st } j\neq k}^{n}\frac{(n+1-j)}{(k-j)}\\
&= \sum_{k=0}^{n}\frac{(k)!(n+1-k)!}{(n+1)!}\frac{\prod_{i=n}^{k+1}(n+1-i)\prod_{j=k-1}^{1}(n+1-j)}{\prod_{i=n}^{k+1}(k-i)\prod_{j=k-1}^{1}(k-j)}\\
&= \sum_{k=0}^{n}\frac{(k)!(n+1-k)!}{(n+1)!}\frac{(n-k)!\prod_{j=k-1}^{1}(n+1-j)}{\prod_{i=n}^{k+1}(k-i)\prod_{j=k-1}^{1}(k-j)}\\
&= \sum_{k=0}^{n}\frac{(k)!(n+1-k)!}{(n+1)!}\frac{(n-k)!\frac{(n+1)!}{(n+1-k)!}}{\prod_{i=n}^{k+1}(k-i)\prod_{j=k-1}^{1}(k-j)}\\
&= \sum_{k=0}^{n}\frac{(k)!(n+1-k)!}{(n+1)!}\frac{(n-k)!\frac{(n+1)!}{(n+1-k)!}}{(n-k)!(-1)^{n-k}\prod_{j=k-1}^{1}(k-j)}\\
&= \sum_{k=0}^{n}\frac{(k)!(n+1-k)!}{(n+1)!}\frac{(n-k)!\frac{(n+1)!}{(n+1-k)!}}{(n-k)!(-1)^{n-k}(k)!}\\
&= \sum_{k=0}^{n}(-1)^{n-k}\\
&= \begin{cases}0 & \text{if n is odd}\\ 1 & \text{o.w.}\end{cases} \qquad \square
\end{aligned}$$

7) Suppose towards a contradiction, that for any distance d, there are no 2 points of color red or yellow which are d apart.

Color a point yellow and draw a circle, C_y, whose center is the yellow point and with radius d.

Clearly in order for this condition to hold all points on the circumference of the circle must be red.

Now, draw another circle, C_r, with radius d and whose center point is on the circumference of C_y.

All the points on the circumference of C_r must be yellow but the circle intersects the C_y so clearly at least 2 points are red.

Thus we have a contradiction and it must be true that for one of the two colors C, it is strue that for any distance d, there are 2 points of color C which are d apart. $\square$

8) The answer is 0 but I wasn't able to rigoriously prove why that's true.

9) I spent probably 12 or so hours on it. I liked the variety in problems this week! I thought the matrix problems were hard.

Lagrange Interpolation can be used to find the least degree polynomial such that at each point x_j assumes the corresponding value y_i. The form the polynomial actually takes is exemplified in my solution to number 3.