

1. (a) See txt file  
 (b) See txt file  
 (c) See txt file  
 (d) See txt file  
 (e)  $\rho(temp, (Sells \bowtie_{Sells.iid=Item.iid} Item) \bowtie_{storeid} (Sells \bowtie_{Sells.iid=Item.iid} Item))$   
 $\rho(temp1, temp \bowtie_{storeid} temp)$   
 $\rho(tempf(2 \rightarrow iid1, 4 \rightarrow color1, 5 \rightarrow iid2, 7 \rightarrow color2, 8 \rightarrow iid3, 10 \rightarrow color3, 11 \rightarrow iid4, 13 \rightarrow color4), temp1)$   
 $\rho(check1, \sigma_{color1 \neq color2 \neq color3 \neq color4} tempf)$   
 $\rho(check2, \sigma_{color1 \neq color2 \neq color3} (\pi_{storeid, color1, color2, color3} tempf))$   
 $\rho(ans, \pi_{storeid}(check2) - \pi_{storeid}(check1))$   
 Table ans is our final result  
 (f) Sells S1 and Sells S2  
 $\rho(temp1, \pi_{iid}(\sigma_{storeid=S2.storeid} Sells))$   
 $\rho(temp2, \pi_{storeid}(\sigma_{iid=temp1.storeid} Sells))$   
 $\rho(temp3, \pi_{S1.storeid, S2.storeid}(\sigma_{S1.storeid \neq temp2.storeid} (S1 \times S2)))$   
 $\rho(temp4(1 \rightarrow storeid1, 2 \rightarrow storeid2), temp3)$   
 $\rho(ans, \pi_{storeid1, storeid2}(\sigma_{storeid1 > storeid2} temp4))$   
 Table ans is our final result  
 (g)  $\rho(sells2, sells)$   
 $\rho(cross, sells2 \times sells)$   
 $\rho(cross2(1 \rightarrow iid1, 2 \rightarrow storeid1, 3 \rightarrow price1, 4 \rightarrow iid2, 5 \rightarrow storeid2, 6 \rightarrow price2), cross)$   
 $\rho(notmin1, \sigma_{price2 > price1 \wedge iid2 = iid1 \wedge storeid2 \neq storeid1} cross2)$   
 $\rho(notmin2, \pi_{iid2, storeid2, price2} notmin1)$   
 $\rho(ans, Sells - notmin2)$   
 Table ans is our final result
2. see txt file
3. (a) True  
 $\delta(\sigma_C(R)) \equiv \sigma_C(\delta(R))$   
 LHS  
 $\delta(\sigma_C(R)) \equiv \{t \in R \mid c \text{ is true for } t\}$   
 $\sigma_C(R) \equiv \{kt \in R \mid k \text{ is the multiplicity of } t \text{ and } c \text{ is true for } t\}$   
 $R \equiv \{kt \in R \mid k \text{ is the multiplicity for } t\}$   
 RHS:  
 $\sigma_C(\delta(R)) \equiv \{t \in R \mid \text{where } c \text{ is true for } t\}$   
 $\delta(R) \equiv \{kt \in R \mid k \text{ is the multiplicity for } t\}$   
 $R \equiv \{kt \in R \mid k \text{ is the multiplicity for } t\}$   
 $LHS \equiv RHS$
- (b) True  
 $\delta(\pi_A(R)) \equiv \pi_A(\delta(R))$   
 LHS:  
 $\delta(\pi_A(R)) \equiv \{t[A] \mid t \in R\}$   
 $\pi_A(R) \equiv \{kt[A] \mid kt \in R \mid k \text{ is the multiplicity of } t\}$   
 $R \equiv \{kt \in R \mid k \text{ is the multiplicity of } t\}$   
 RHS:

$$\pi_A(\delta(R)) \equiv \{t[A] \mid t \in R\}$$

$$\text{delta}(R) \equiv \{t \in R\}$$

$$R \equiv \{kt \in R \mid k \text{ is the multiplicity of } t\}$$

$$LHS \equiv RHS$$

(c) True

$$\delta(R \times S) \equiv \delta(R) \times \delta(S)$$

LHS:

$$\delta(R \times S) \equiv \{r \cdot s \mid r \in R \wedge s \in S\}$$

$$R \times S \equiv \{k(r \cdot s) \mid r \in R \wedge s \in S \wedge k \text{ is the multiplicity of } (r \cdot s)\}$$

RHS:

$$\delta(R) \times \delta(S) \equiv \{r \cdot s \mid r \in R \wedge s \in S\}$$

$$R \times \delta(S) \equiv \{m(r) \cdot s \mid r \in R \wedge s \in S \wedge m \text{ is the multiplicity of } r\}$$

$$R \times S \equiv \{m(r) \cdot n(s) \mid r \in R \wedge s \in S \wedge m \text{ is the multiplicity of } r \text{ and } n \text{ is the multiplicity of } s\}$$

$$R \times S \equiv \{k(r \cdot s) \mid r \in R \wedge s \in S \wedge k=mn \text{ is the multiplicity of } (r \cdot s)\}$$

$$LHS \equiv RHS$$

(d) True

$$\delta(R \bowtie_C S) \equiv \delta(R) \bowtie_C \delta(S)$$

LHS:

$$\delta(R \bowtie_C S) \equiv \delta(\sigma_C(R \times S))$$

$$\delta(\sigma_C(R \times S)) \equiv \{r \cdot s \mid r \in R \wedge s \in S \wedge c \text{ is true for } r \cdot s\}$$

$$\sigma_C(R \times S) \equiv \{k(r \cdot s) \mid r \in R \wedge s \in S \wedge c \text{ is true for } r \cdot s \wedge k \text{ is the multiplicity of } r \cdot s\}$$

$$R \times S \equiv \{k(r \cdot s) \mid r \in R \wedge s \in S \wedge k=mn \text{ is the multiplicity of } (r \cdot s)\}$$

RHS:

$$\delta(R) \bowtie_C \delta(S) \equiv \{r \cdot s \mid r \in R \wedge s \in S \wedge c \text{ is true for } r \cdot s\}$$

$$R \bowtie_C \delta(S) \equiv \{m(r) \cdot s \mid r \in R \wedge s \in S \wedge c \text{ is true for } m(r) \cdot s \wedge m \text{ is the multiplicity of } r\}$$

$$R \bowtie_C S \equiv \{m(r) \cdot n(s) \mid r \in R \wedge s \in S \wedge c \text{ is true for } m(r) \cdot s \wedge m \text{ is the multiplicity of } r \text{ and } n \text{ is the multiplicity of } s\}$$

$$R \bowtie_C S \equiv \{k(r \cdot s) \mid r \in R \wedge s \in S \wedge c \text{ is true for } k(r \cdot s) \wedge k=mn \text{ is the multiplicity of } (r \cdot s)\}$$

$$R \bowtie_C S \equiv \sigma_C(R \times S)$$

$$\sigma_C(R \times S) \equiv \{k(r \cdot s) \mid r \in R \wedge s \in S \wedge c \text{ is true for } k(r \cdot s) \wedge k=mn \text{ is the multiplicity of } (r \cdot s)\}$$

$$R \times S \equiv \{k(r \cdot s) \mid r \in R \wedge s \in S \wedge k=mn \text{ is the multiplicity of } (r \cdot s)\}$$

$$LHS \equiv RHS$$

(e) False

$$\delta(R \cup_B S) \neq \delta(R) \cup_B \delta(S)$$

$$R = \{(1, 2), (1, 2), (3, 4)\} \text{ and } S = \{(1, 2), (3, 4), (3, 4)\}$$

LHS:

$$R \cup_B S = \{(1, 2), (1, 2), (1, 2), (3, 4), (3, 4), (3, 4)\}$$

$$\delta(R \cup_B S) = \{(1, 2), (3, 4)\}$$

RHS:

$$\delta(R) = \{(1, 2), (3, 4)\} \text{ and } \delta(S) = \{(1, 2), (3, 4)\}$$

$$\delta(R) \cup_B \delta(S) = \{(1, 2), (1, 2), (3, 4), (3, 4)\}$$

$$LHS \neq RHS \text{ so this is false}$$

(f) True

LHS:

$$\delta(R \cap_B S) \equiv \delta(R) \cap_B \delta(S)$$

$$\delta(R \cap_B S) \equiv \{t \in R \vee t \in S \mid t \text{ occurs in both } R \text{ and } S\}$$

$$R \cap_B S \equiv \{kt \in R \vee t \in S \mid t \text{ occurs in both } R \text{ and } S \text{ and } k \text{ is the minimum number of occurrences of } t \text{ in either } R \text{ or } S\}$$

RHS:

$$\delta(R) \cap_B \delta(S) \equiv \{t \in R \vee t \in S \mid t \text{ occurs in both } R \text{ and } S\}$$

$$R \cap_B \delta(S) \equiv \{mt \in R \vee t \in S \mid t \text{ occurs in both } R \text{ and } S \text{ and } m \text{ is the amount of occurrence of } t \text{ in } R\}$$

$R \cap_B S \equiv \{mt \in R \vee nt \in S \mid t \text{ occurs in both } R \text{ and } S \text{ and } m \text{ is the amount of occurrence of } t \text{ in } R \text{ and } n \text{ is the } n \text{ amount of occurrences of } t \text{ in } S\}$

$R \cap_B S \equiv \{kt \in R \vee t \in S \mid t \text{ occurs in both } R \text{ and } S \text{ and } k \text{ is the minimum number of occurrences of } t \text{ in either } R \text{ or } S\}$

$LHS \equiv RHS$

(g) False

$\delta(R -_B S) \equiv \delta(R) -_B \delta(S)$

$R = \{(1, 2), (1, 2), (3, 4)\}$  and  $S = \{(1, 2), (3, 4), (3, 4)\}$

LHS:

$R -_B S = \{(1, 2)\}$

$\delta(R -_B S) = \{(1, 2)\}$

RHS:

$\delta(R) = \{(1, 2), (3, 4)\}$  and  $\delta(S) = \{(1, 2), (3, 4)\}$

$\delta(R) -_B \delta(S) = \{\}$  empty set

$LHS \neq RHS$