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HW3

Question 1

a) If $F \subseteq G$ then $F^+ \subseteq G^+$.
   1) Assume an arbitrary functional dependency $X \rightarrow Y \in F^+$
   2) By completeness, $X \rightarrow Y \in F$
   3) Since $F \subseteq G$, $X \rightarrow Y \in G$ by Reflexivity
   4) By soundness, $X \rightarrow Y \in G^+$

b) $F^+ = (F^+)^+$.
   1) $F^+ \subseteq (F^+)^+$ by the definition of a closure
   2) Let there be an arbitrary f.d. $X \rightarrow Y \in (F^+)^+$
   3) By completeness, $X \rightarrow Y \in F^+$

c) If for all $f \in F$, $G \models f$ then $F^+ \subseteq G^+$.
   1) If for all $f \in F$, $G \models f$, $F \subseteq G$ by Reflexivity
   2) By a), $F^+ \subseteq G^+$

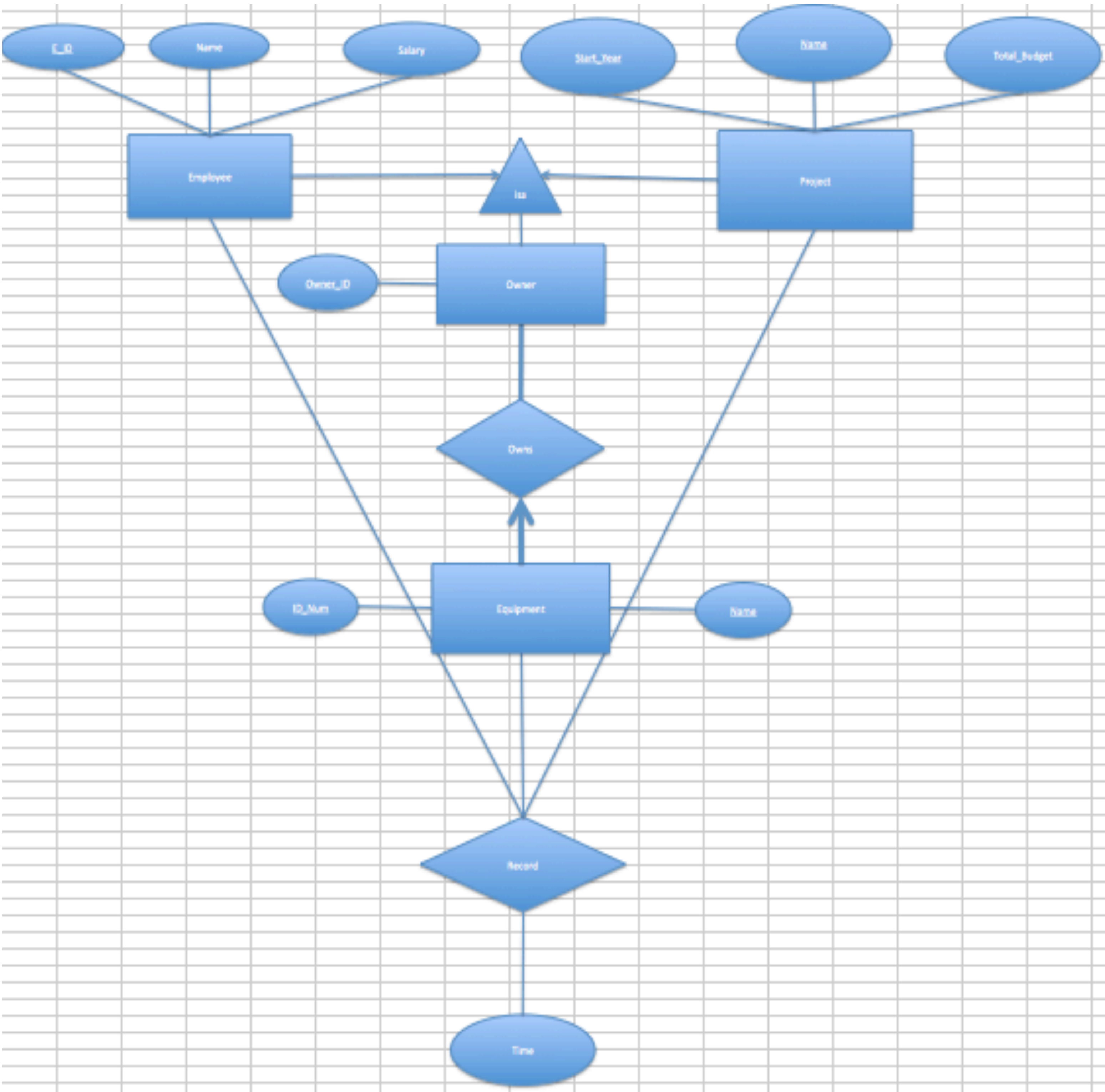
d) The reflexivity and augmentation Armstrong axioms are sound (we proved soundess of the transitivity axiom in class).
   Reflexivity: Prove that if $Y \subseteq X$, then $X \rightarrow Y$ for arbitrary attribute sets X and Y
   1) Take arbitrary relation R
   2) For arbitrary tuples s & t $\in R$, $s(X) = t(X)$
   3) Since s & t agree on all attributes in X and $Y \subseteq X$, then they agree on all attributes in Y
   4) $s(Y) = t(Y)$, QED

   Augmentation: Prove that if $X \rightarrow Y$, then $XZ \rightarrow YZ$ for arbitrary attribute sets X, Y, and Z
   1) Assume an arbitrary relation R with arbitrary tuples s & t
   2) Assume $s(XZ) = t(XZ)$
   3) Thus, $s(X) = t(X)$ and $s(Z) = t(Z)$
   4) Since R satisfies $X \rightarrow Y$, $s(X) = t(X)$ implies $s(Y) = t(Y)$ by reflexivity
   5) Thus, $s(YZ) = t(YZ)$, QED

e) Consider the criterion for testing whether a decomposition of a relation is dependency-preserving, on page 621 of your textbook. Let X, Y, F, FX, FY be as in your textbook. The criterion given in the textbook is $(FX \cup FY)^+ = F^+$. Show that the forward direction always holds, i.e. it is always true that $(FX \cup FY)^+ \subseteq F^+$.
   1) FX is the projection of F on X, which is the set of FD's in the closure F^+ that involve only attributes in X therefore by definition $FX \subseteq F^+$.
   2) FY is the projection of F on Y, which is the set of FD's in the closure F^+ that involve only attributes in Y therefore by definition $FY \subseteq F^+$.

3) $FX \cup FY$ is by the definition of union the set composed of everything in the two sets, therefore $FX \cup FY \subseteq F^+$.
4) By 1b we know $F^+ = (F^+)^+$ so all using these we can prove that $FX \cup FY=(FX \cup FY)^+$ and $(FX \cup FY)^+ \subseteq F^+$.

Question 2



The above graph shows our ER diagram. Employee is an entity with primary key composed of E_ID and attributes name and salary. Project is an entity with primary key composed of Start Year and Name and attribute Total_Budget. Both Employee and Project are subclasses of the superclass Owner that has a key of Owner_ID. The relationship of the subclasses to the super class is many to one because each employee and project can only have one Owner_ID. There is also an entity Equipment whose key is composed of ID_Num and Name. Equipment and Owner form a relationship Owns where Equipment has a strong many to one relationship because each piece of equipment needs exactly one owner. Then the Record relationship keeps track of the record every time a piece of equipment is used. A Record takes in a piece of equipment, an employee, and a project, and also has a time attribute. This keeps track of the equipment used, the employees responsible, the active project, and the time.

Question 3

The first thing to accomplish is to find the keys for each set of functional dependencies. The following is the algorithm we used to do so:

1) Find all attributes not in one of the functional dependencies (f.d.)
2) Find all attributes present only on the right side of a f.d.
3) Find all attributes present only on the left side of a f.d.
4) Combine the attributes from 1) and 3)
5) Look for closures amongst the attributes from 4) and those present on both sides of some f.d.(s)

a) ABCDE, with $A \rightarrow B$, $BC \rightarrow E$, $ED \rightarrow A$
   I. Keys are ACD, BCD, CDE
   II. Since $A \rightarrow B$ has A on the left, which is not a key, this relation is not in BCNF
   III. Since all of the right hand sides are part of some key ($B \in BCD$, $E \in CDE$, $A \in ACD$) this relation is in 3NF
   IV. The relation is in 3NF (but not in BCNF)

b) ABCDE, with $A \rightarrow BC$, $C \rightarrow DE$, $CE \rightarrow A$
   I. Keys are A and C
   II. Since $CE \rightarrow A$ has CE on the left, which is not a key, this relation is not in BCNF
   III. Since $C \rightarrow DE$ has DE on the right, which is not part of some key, this relation is not in 3NF
   IV. The relation is in neither 3NF nor BCNF