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Ss2654 and jdr289

CS 4320

HW3

1. If F ⊆ G then F+ ⊆ G+.
2. Assume an arbitrary functional dependency X → Y∈F+
3. By completeness, X → Y∈F
4. Since F ⊆ G, X → Y∈G by Reflexivity
5. By soundness, X → Y∈G+
6. F+ = (F+)+.
7. F+ ⊆ (F+)+ by the definition of a closure
8. Let there be an arbitrary f.d. X → Y∈ (F+)+
9. By completeness, X → Y∈F+
10. If for all f ∈ F, G |= f then F+ ⊆ G+.
11. If for all f ∈ F, G |= f, F ⊆ G by Reflexivity
12. By a), F+ ⊆ G+
13. The reflexivity and augmentation Armstrong axioms are sound (we proved soundess of the transitivity axiom in class).

Reflexivity: Prove that if Y ⊆ X, then X → Y for arbitrary attribute sets X and Y

1. Take arbitrary relation R
2. For arbitrary tuples s & t ∈R, s(X) = t(X)
3. Since s & t agree on all attributes in X and Y ⊆ X, then they agree on all attributes in Y
4. s(Y) = t(Y), QED

Augmentation: Prove that if X → Y, then XZ → YZ for arbitrary attribute sets X, Y, and Z

1. Assume an arbitrary relation R with arbitrary tuples s & t
2. Assume s(XZ) = t(XZ)
3. Thus, s(X) = t(X) and s(Z) = t(Z)
4. Since R satisfies X → Y, s(X) = t(X) implies s(Y) = t(Y) by reflexivity
5. Thus, s(YZ) = t(YZ), QED
6. Consider the criterion for testing whether a decomposition of a relation is dependency-preserving, on page 621 of your textbook. Let X, Y, F, FX, FY be as in your textbook. The criterion given in the textbook is (FX ∪ FY )+ =F+ . Show that the forward direction always holds, i.e. it is always true that (FX ∪ FY )+ ⊆ F+ .
7. FX is the projection of F on X, which is the set of FD’s in the closure F+ that involve only attributes in X therefore FX ⊆ F+.
8. FY is the projection of F on Y, which is the set of FD’s in the closure F+ that involve only attributes in Y therefore FY ⊆ F+.
9. FX ∪ FY is by the definition of union is the set composed of everything in the two sets, therefore FX ∪ FY⊆ F+.
10. By 1b we know F+ = (F+)+ so all using these we can prove that FX ∪ FY=(FX ∪ FY )+ and (FX ∪ FY )+ ⊆ F+ .