On the well-posedness of the two-fluid equations for cosmic rays and thermal plasma

Siddhartha Gupta[1,2], Prateek Sharma[1] & Andrea Mignone[3]

1. Joint Astronomy Programme and Department of Physics, Indian Institute of Science, Bangalore 560012, India
2. Raman Research Institute, Sadashiva Nagar, Bangalore 560080, India
3. Dipartimento di Fisica Generale, Universita degli Studi di Torino, Via Pietro Giuria 1, I-10125 Torino, Italy

Introduction

The role of non-thermal pressure from the relativistic particles (e.g. cosmic rays) can be as important in the stellar and/or AGN feedback studies as they suffer less losses compared to the thermal plasma, especially in a dense medium.

To study the effects of CRs, we consider the two fluid approach where the non-thermal fluid (e.g., cosmic ray fluid) is coupled with the thermal fluid/plasma. We investigate various numerical methods and find that, in general, the solution of the two-fluid equations depends on the choice of the numerical method. We have implemented the two-fluid equations in the **PLUTO** code. Our conclusions are important for the completeness of CR-HD/MHD systems.

Two-fluid HD system

Fluid consideration

- The relativistic particles are coupled to the background plasma due to magnetic fluctuations generated by the streaming instability.
- CR Gyroradius is much smaller than length scale of interest.

Macroscopic system:

1. Mass: $$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho\,\vec{v}) = 0$$
2. Momentum: $$\frac{\partial}{\partial t}(\rho\,\vec{v}) + \vec{\nabla}.(\rho\,\vec{v}\otimes\vec{v}) + \vec{\nabla}\,(p_{\rm tot}) = 0$$
3. Total energy: $$\frac{\partial e_{\rm tot}}{\partial t} + \vec{\nabla}.\left[(e_{\rm tot}+p_{\rm tot})\,\vec{v} + \vec{F}_{\rm crd}\right] = 0$$
4. CR energy: $$\frac{\partial e_{\rm cr}}{\partial t} + \vec{\nabla}.\left[(e_{\rm cr}+p_{\rm cr})\,\vec{v} + \vec{F}_{\rm crd}\right] = \underbrace{\vec{v}.\vec{\nabla}p_{\rm cr}}_{\text{Source term}}$$

Total energy $e_{\rm tot} = \rho v^2/2 + p_{\rm th}/(\gamma_{\rm th}-1) + p_{\rm cr}/(\gamma_{\rm cr}-1)$

Total pressure $p_{\rm tot} = p_{\rm th} + p_{\rm cr}$

- Three conservation laws + CR energy evolution.
- The source term represents the interaction of relativistic fluid with thermal fluid.
- The shock interaction can cause diffusive acceleration.

Characteristics & Numerical Methods

Eigen system without CR diffusion:

- Real eigenvalues - Hyperbolic system
- Effective sound speed $a_{\rm eff} = \sqrt{\dfrac{\gamma_{\rm th}\,p_{\rm th} + \gamma_{\rm cr}\,p_{\rm cr}}{\rho}}$

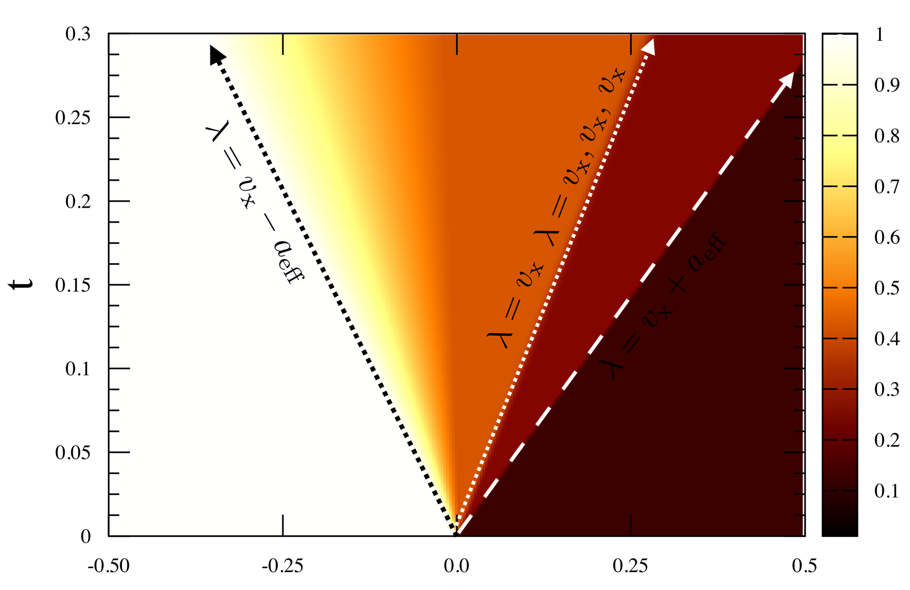

Figure 1: Space time diagram of CR-HD system

Methods to deal with the source term of Eq. (4)

1. Direct Euler update: vdp
2. Operator splitting: pdv
3. Passive scaler: crden $\rho_{\rm cr} \equiv p_{\rm cr}^{1/\gamma_{\rm cr}}$

Result: cartesian geometry

Initial Conditions	Left state	Right state
Density	1.0	0.2
Velocity	0.0	0.0
Thrm. Pressure	2.0	0.02
CR Pressure	1.0	0.1

Table 1: Initial condition. Left and right states are defined w.r.t. X= 0.0

- Results depend on the choice of method!
- Solution

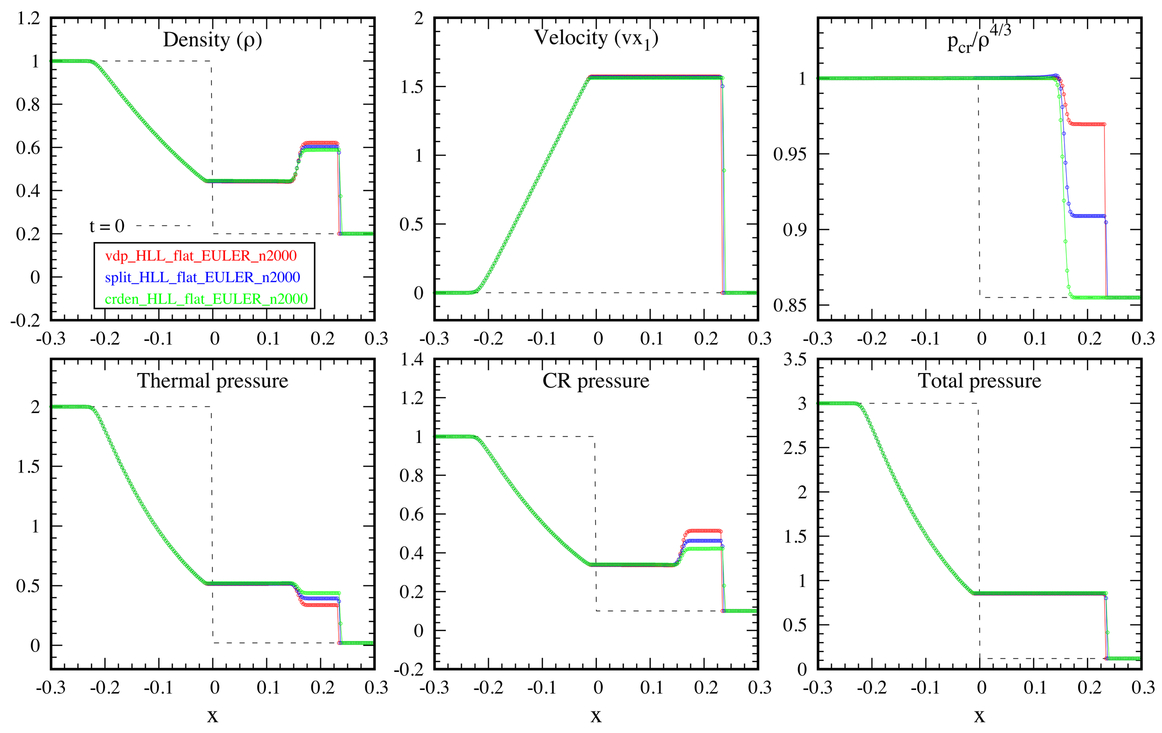

Figure 2 : 1D Shock tube with a high upstream CR pressure. Nomenclature of the labels is `Solver_Reconstruction_TimeStepping_GridPoints'.

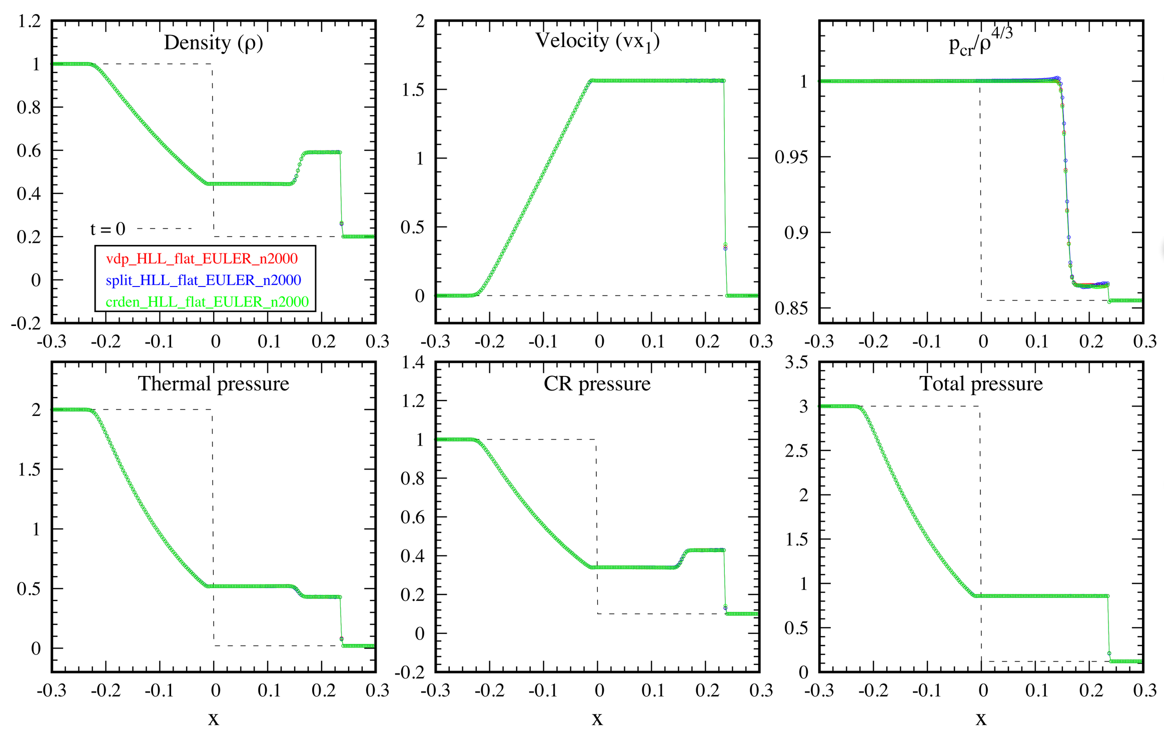

Figure 3 : Solution of the post shock region is fixed *by hand.*

- Shock detection — numerically challenging.
- Fixes the energy transfer from thermal fluid to CR fluid at the shock.
- vdp method shows much better results in other test problems.

Result: spherical geometry

- Supernova energy is injected in the central zone at t = 0.
- A shock detection program is used to inject CRs at the shocked zone (denoted by green symbols).
- The results do not depend on reconstruction. However, the split and crden methods show error at the center.

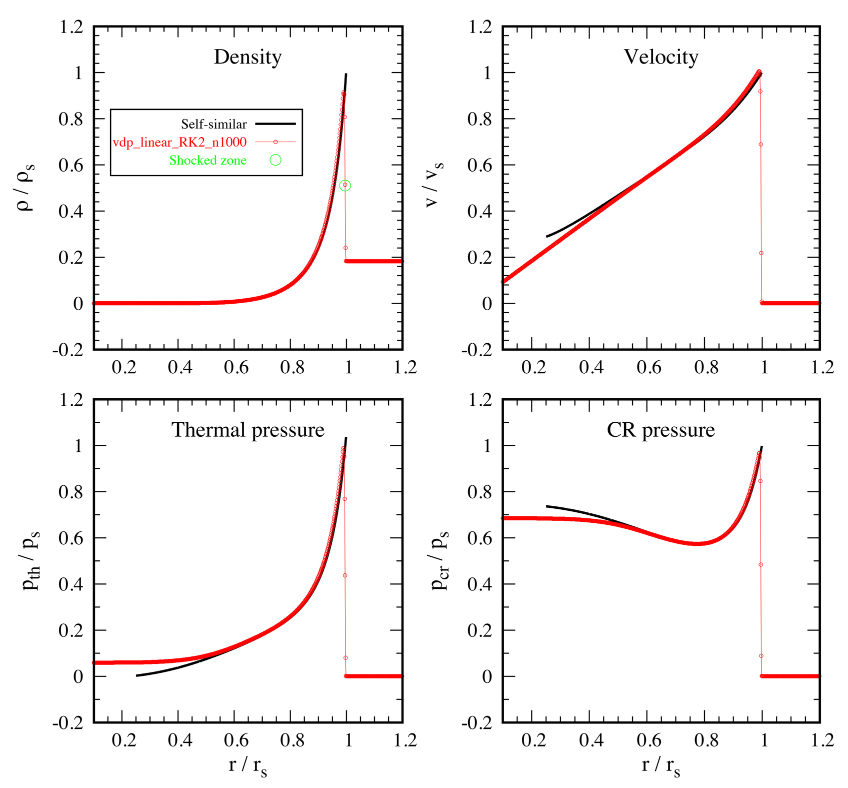

Figure 4 : The comparison of self-similar solution with simulation result.

Summary

1. Failure: (Fig 2)

For a high upstream CR pressure, the solution at the post shock region depends on the choice of numerical method.

2. Good news: (Figs 3, 4)

a. Two-fluid equations are implemented in the **PLUTO** code.
b. A shock detection module is also supplied to inject CRs.
c. Supports multi-dimensions.

3. Future direction:

The fraction of upstream energy which gets converted to CRs is still an open problem, needs input from PIC simulations. A technique to detect weak shocks and to estimate their Mach number in multi-D is needed.

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Contact:
E-mail: gsiddhartha@iisc.ac.in