Fibonacci sequence

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Parameter	Value	Description
$y(n)$	$y(n) = y(n-1) + y(n-2)$	$(n)^{th}$ term
$x(0)$	1	1^{st} term
$x(1)$	1	2^{nd} term

Input Table

$$y(0) = y(-1) + y(-2) = 1 \tag{1}$$
$$y(1) = y(0) + y(-1) = 1 \tag{2}$$

Hence,

$$y(-2) = 1 \tag{3}$$
$$y(-1) = 0 \tag{4}$$

Applying one sided z transform and considering the initial conditions,

$$Y^+(z) = z^{-1}Y^+(z) + y(-1) + z^{-2}Y^+(z) + y(-2) + z^{-1}y(-1) \tag{5}$$
$$= \frac{1}{1 - z^{-1} - z^{-2}} \tag{6}$$
$$= \frac{1}{(1 - \alpha z^{-1})(1 - \beta z^{-1})} \tag{7}$$

Where,

$$\alpha = \frac{1 + \sqrt{5}}{2} \tag{8}$$
$$\beta = \frac{1 - \sqrt{5}}{2} \tag{9}$$

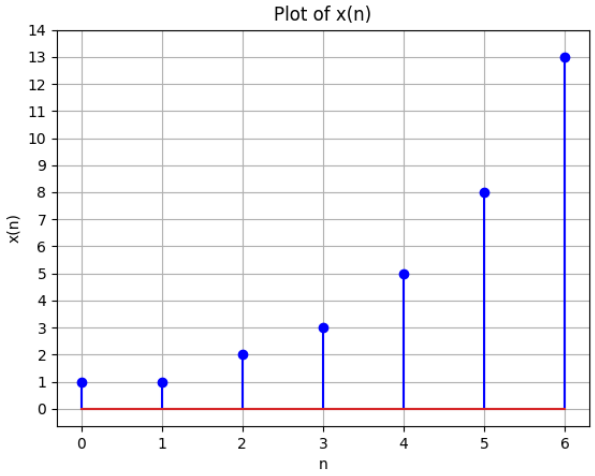
Using partial fractions,

$$X(z) = \frac{\alpha}{(\alpha - \beta)} \frac{1}{(1 - \alpha z^{-1})} - \frac{\beta}{(\alpha - \beta)} \frac{1}{(1 - \beta z^{-1})} \tag{10}$$

$$a^n u(n) \xleftrightarrow{z} \frac{1}{1 - az^{-1}} |z| > |a| \tag{11}$$

Substituting this result,

$$x(n) = \frac{\alpha}{(\alpha - \beta)}(\alpha^n u(n)) - \frac{\beta}{(\alpha - \beta)}(\beta^n u(n)) \tag{12}$$
$$= \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} u(n) \tag{13}$$
$$= \frac{(1 + \sqrt{5})^{n+1} - (1 - \sqrt{5})^{n+1}}{2^{n+1}\sqrt{5}} u(n) \tag{14}$$



(a) Plot of $x(n)$ *vs* n