Solution of Q12.13.3.21

SUJAL GUPTA - EE22BTECH11052

Ten coins are tossed. What is the probability of getting atleast 8 heads?

Solution: Let the event of getting a head on one coin toss be H. Then

$$\Pr(H) = \frac{1}{2} \tag{1}$$

Variable	Description	Value
n	Number of tosses	10
X_i	Result of ith coin	$X_i, i = \{1, 2, ..., 10\}$
X	No of heads	$\sum_{i=1}^{10} X_i$

$$X = \sum_{i=1}^{10} X_i \tag{2}$$

$$X \leq 10 \tag{3}$$

The probability of getting a head is:

$$p_X(k) = {}^{10}C_k (0.5)^k (0.5)^{10-k} \tag{4}$$

$$= {}^{10}C_k (0.5)^{10} \qquad \forall k = 0, 1, 2, ..., 10 \tag{5}$$

The above equation gives the PMF of getting k heads on 10 coint tosses. Let $F_X(k)$ denote the cumulative distribution function of X:

$$F_X(k) = p(X \leq k) \tag{6}$$

$$= \sum_{i=0}^{k} {}^{10}C_i \left(\frac{1}{2}\right)^{10} \tag{7}$$

Let $F_X(k)$ denote the cumulative distribution function of X:

$$F_X(k) = p(X \leq k) \tag{8}$$

$$F_X(10) - F_X(7) = \sum_{i=8}^{10} {}^{10}C_i \left(\frac{1}{2}\right)^{10} \tag{9}$$

$$= \frac{7}{128} \tag{10}$$

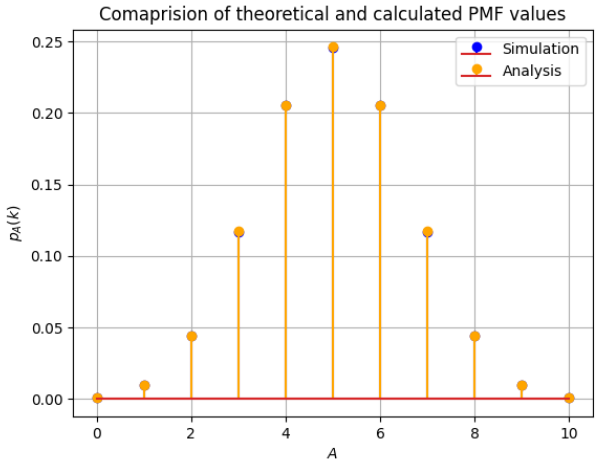

Fig. 0. PMF of X

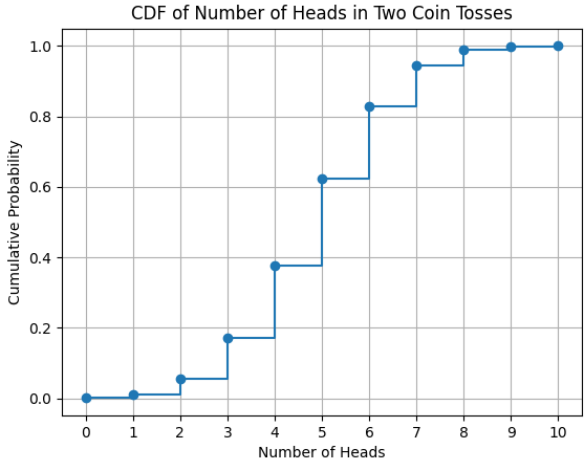

Fig. 0. CDF of X