

# Probability Assignment 1

EE22BTECH11052 - SUJAL GUPTA

Given in the question:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \quad (2)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (3)$$

The equation of the incircle is given by

$$\|\mathbf{x} - \mathbf{I}\|^2 = r^2 \quad (4)$$

Find the parametric equation of  $BC$  and use it to verify that  $BC$  intersects the incircle at exactly one point  $\mathbf{D}_3$ .  $BC$  is defined to be a tangent to the incircle.  $\mathbf{D}_3$  is defined to be point of contact.

**Solution:** Let us define

$$\mathbf{m} = \mathbf{C} - \mathbf{B} \quad (5)$$

and  $\mathbf{I}$  is the incentre of the  $\triangle ABC$

$$\mathbf{I} = \frac{1}{\sqrt{74} + \sqrt{32} + \sqrt{122}} \begin{pmatrix} \sqrt{122} - 4\sqrt{32} - 3\sqrt{74} \\ -\sqrt{122} + 6\sqrt{32} - 5\sqrt{74} \end{pmatrix} \quad (6)$$

$$= \begin{pmatrix} -1.47756217 \\ -0.79495069 \end{pmatrix} \quad (7)$$

The general position vector on the line  $BC$  (in parametric form) and the equation of incircle are:

$$\Rightarrow \mathbf{x} = \mathbf{B} + k\mathbf{m} \quad (8)$$

$$\Rightarrow \|\mathbf{x} - \mathbf{I}\|^2 = r^2 \quad (9)$$

Substituting the value of  $\mathbf{x}$  from (8) in (9)

$$\Rightarrow \|\mathbf{B} + k\mathbf{m} - \mathbf{I}\|^2 = r^2 \quad (10)$$

$$\Rightarrow [\mathbf{B} + k\mathbf{m} - \mathbf{I}]^T [\mathbf{B} + k\mathbf{m} - \mathbf{I}] = r^2 \quad (11)$$

On simplifying the above equation:

$$k^2\|\mathbf{m}\|^2 + 2k\mathbf{m}^T(\mathbf{B} - \mathbf{I}) + \|\mathbf{I}\|^2 + \|\mathbf{B}\|^2 - 2(\mathbf{B}^T\mathbf{I}) - r^2 = 0 \quad (12)$$

The above is a quadratic equation in  $k$ . The Discriminant of the quadratic equation is:

$$D = \left\{ 2[\mathbf{m}^T(\mathbf{B} - \mathbf{I})] \right\}^2 - 4(\|\mathbf{m}\|^2)(\|\mathbf{I}\|^2 + \|\mathbf{B}\|^2 - 2(\mathbf{B}^T\mathbf{I}) - r^2) \quad (13)$$

$$D = \left[ 2 \begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} -4 + 1.47756217 \\ 6 + 0.79495069 \end{pmatrix} \right]^2 - 4[122][2.8151 + 52 - 2(1.140544) - 3.59820] \quad (14)$$

$$= 4[(-2.522 - 74.744)^2 - 5970.173] \quad (15)$$

$$= 4[5970.173121 - 5970.173121] \quad (16)$$

$$= 0 \quad (17)$$

We substituted the values of  $\mathbf{B}, \mathbf{I}, \mathbf{m}$ . On solving this Discriminant turns out to be zero. Hence, the quadratic equation has only one solution. To find the unique solution of the equation (i.e. the unique value of  $k$ ), Set discriminant value to 0. And the solution is

$$k = \frac{-2\mathbf{m}^T(\mathbf{B} - \mathbf{I})}{2\|\mathbf{m}\|^2} \quad (18)$$

On substituting the values, the value of  $k$  is

$$k = - \begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} -4 - \frac{\sqrt{122} - 4\sqrt{32} - 3\sqrt{74}}{\sqrt{74} + \sqrt{32} + \sqrt{122}} \\ 6 - \frac{-\sqrt{122} + 6\sqrt{32} - 5\sqrt{74}}{\sqrt{74} + \sqrt{32} + \sqrt{122}} \end{pmatrix} \quad (19)$$

$$= \begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} \frac{5\sqrt{122} + \sqrt{74}}{\sqrt{74} + \sqrt{32} + \sqrt{122}} \\ \frac{-7\sqrt{122} - 11\sqrt{74}}{\sqrt{74} + \sqrt{32} + \sqrt{122}} \end{pmatrix} \quad (20)$$

$$k = \frac{122\sqrt{74} + 82\sqrt{122}}{122(\sqrt{74} + \sqrt{122} + \sqrt{32})} \quad (21)$$

$$= 0.6333352080102638 \quad (22)$$

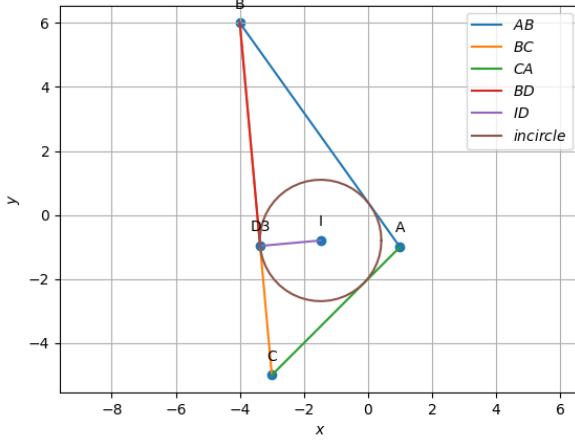


Fig. 0. Incircle generated using python

So on substituting this value in (8), we get the point  $\mathbf{D}_3$

$$\mathbf{D}_3 = \begin{pmatrix} -4 \\ 6 \end{pmatrix} + \frac{122\sqrt{74} + 82\sqrt{122}}{122(\sqrt{74} + \sqrt{122} + \sqrt{32})} \begin{pmatrix} 1 \\ -11 \end{pmatrix} \quad (23)$$

$$= \begin{pmatrix} -4 + \frac{122\sqrt{74} + 82\sqrt{122}}{122(\sqrt{74} + \sqrt{122} + \sqrt{32})} \\ 6 - 11 \frac{122\sqrt{74} + 82\sqrt{122}}{122(\sqrt{74} + \sqrt{122} + \sqrt{32})} \end{pmatrix} \quad (24)$$

$$\mathbf{D}_3 = \begin{pmatrix} \frac{-366\sqrt{74} - 406\sqrt{122} - 488\sqrt{32}}{122(\sqrt{74} + \sqrt{32} + \sqrt{122})} \\ \frac{-610\sqrt{74} + 732\sqrt{32} - 170\sqrt{122}}{122(\sqrt{74} + \sqrt{32} + \sqrt{122})} \end{pmatrix} \quad (25)$$

$$= \begin{pmatrix} -3.36666479 \\ -0.96668729 \end{pmatrix} \quad (26)$$