Solution of Q9.3.21

SUJAL GUPTA - EE22BTECH11052

It is known that 10% of certain articles manufactured are defective. What is probability that a random sample space of 12 such articles,9 are defective?

Solution:

Parameter	Values	Description
n	12	Number of articles
k	9	Number of defective articles
p	0.1	Probability of being defective
X	$1 \le X \le 12$	X defective elements out of 12

TABLE 0
TABLE 1

1) *Binomial Distribution :*
The X is the random variable, the pmf of X is given by

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \tag{1}$$

We require $\Pr(X = 9)$. Since $n = 12$,

$$\Pr(X = 9) = p_X(9) \tag{2}$$
$$= {}^nC_k p^k (1-p)^{n-k} \tag{3}$$
$$= {}^{12}C_9 \left(\frac{1}{10}\right)^9 \left(1 - \frac{1}{10}\right)^{12-9} \tag{4}$$
$$= 22\left(\frac{9^3}{10^{11}}\right) \tag{5}$$
$$= 1.60379(10^{-7}) \tag{6}$$

2) *Gaussian Distribution*
Let Y be gaussian variable

$$\mu = np \tag{7}$$
$$= 1.2 \tag{8}$$

$$\sigma^2 = np(1-p) \tag{9}$$
$$= 1.08 \tag{10}$$

Using Normal distribution at X=9.

$$Z = \frac{X-\mu}{\sigma} \tag{11}$$
$$= \frac{9-1.2}{\sqrt{1.08}} \tag{12}$$
$$= 7.5055534 \tag{13}$$

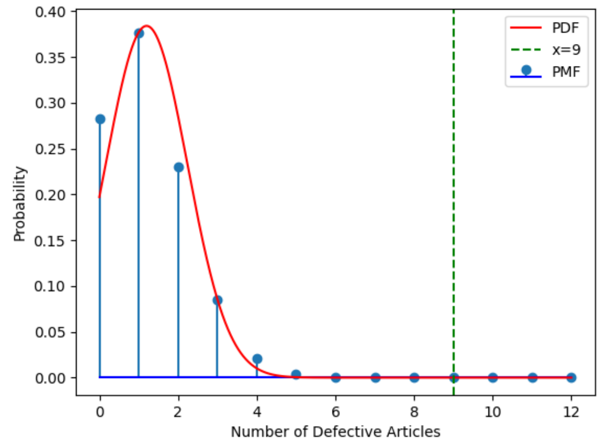

Fig. 2. Binomial-PMF and Gaussian-PDFof X

For pdf(probability density function) calculation

$$f_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{14}$$

$$p_Y(9) = p_Z(7.5055534) \tag{15}$$
$$= 3.89010(10^{-9}) \tag{16}$$

Hence we observe that the gaussian and binomial distribution have very less absolute error.