

Solution of Q10.13.3.24

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A coin is tossed two times. Find the probability of getting at most one head.

Solution: Let the event of getting a head on one coin toss be H. Then

$$\Pr(H) = \frac{1}{2} \quad (1)$$

| Variable | Description | Value |
|----------|-----------------------|-------------|
| n | Number of tosses | 2 |
| X_1 | Result of first coin | X_1 |
| X_2 | Result of second coin | X_2 |
| X | No of heads | $X_1 + X_2$ |

$$X = \sum_{i=1}^2 X_i \quad (2)$$

$$X = X_1 + X_2 \quad (3)$$

$$X \leq 2 \quad (4)$$

The probability of getting a head is:

$$p_X(k) = {}^2C_k(0.5)^k(0.5)^{2-k} \quad (5)$$

$$= {}^2C_k(0.5)^2 \quad (6)$$

The above equation gives the PMF of getting k heads on 2 coin tosses. Let $F_X(k)$ denote the cumulative distribution function of X:

$$F_X(k) = p(X \leq k) \quad (7)$$

$$= \sum_{i=0}^k {}^2C_i \left(\frac{1}{2}\right)^2 \quad (8)$$

Let $F_X(k)$ denote the cumulative distribution function of X:

$$F_X(k) = p(X \leq k) \quad (9)$$

$$F_X(1) = p(X \leq 1) \quad (10)$$

$$= F_X(1) \quad (11)$$

$$= \sum_{i=0}^1 {}^2C_i \left(\frac{1}{2}\right)^2 \quad (12)$$

$$= \frac{3}{4} \quad (13)$$

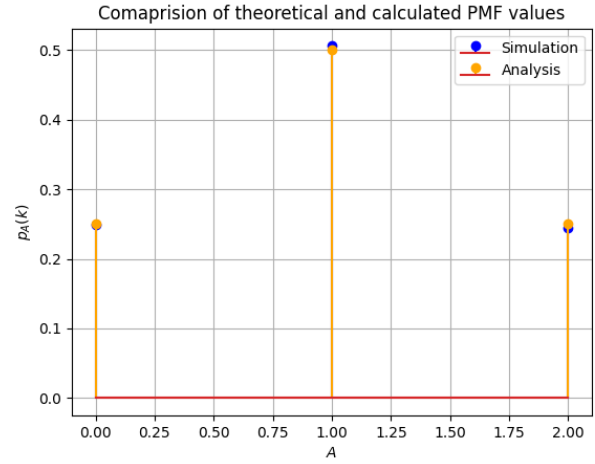


Fig. 0. PMF of X

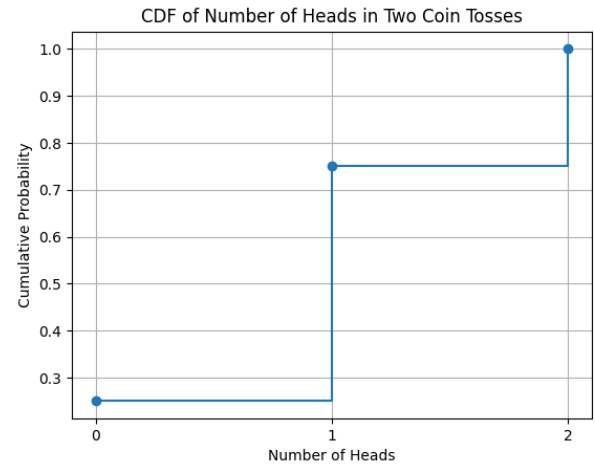


Fig. 0. CDF of X