Probability Assignment 1

EE22BTECH11052 - SUJAL GUPTA

Given in the question:

$$A = \begin{pmatrix} 1 \\ -1 \end{pmatrix} B = \begin{pmatrix} -4 \\ 6 \end{pmatrix} C = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

Let us define $\mathbf{n} = \mathbf{C} - \mathbf{B}$ and $\mathbf{I}$ is the incentre of the tri.ABC

$$\mathbf{I} = \frac{1}{\sqrt{74}+\sqrt{32}+\sqrt{122}} \begin{pmatrix} \sqrt{122} - 4\sqrt{32} - 3\sqrt{74} \\ -\sqrt{122} + 6\sqrt{32} - 5\sqrt{74} \end{pmatrix}$$

The general position vector on the line BC (in parametric form) and the equation of incircle are:

$$\mathbf{x} = \mathbf{B} + k(\mathbf{C} - \mathbf{B}) \tag{1}$$

$$\| \mathbf{x} - \mathbf{I} \|^2 = r^2 \tag{2}$$

Substituting the value of $\mathbf{x}$ from (1) in (2)

$$\left\|\mathbf{B} + k(\mathbf{C} - \mathbf{B}) - \mathbf{I}\right\|^2 = r^2$$

$$(\mathbf{B} + k(\mathbf{C} - \mathbf{B}) - \mathbf{I}) \cdot (\mathbf{B} + k(\mathbf{C} - \mathbf{B}) - \mathbf{I}) = r^2$$

On simplifying the above equation:

$$k^2\|n\|^2 + 2k\mathbf{n} \cdot (\mathbf{B} - \mathbf{I}) + \|I\|^2 + \|B\|^2 - 2(\mathbf{B} \cdot \mathbf{I}) - r^2 = 0$$

The above is a quadratic equation in k

The Discriminant of the quadratic equation is:

$$(2k\mathbf{n} \cdot (\mathbf{B} - \mathbf{I}))^2 - 4(\|n\|^2)(\|I\|^2 + \|B\|^2 - 2\mathbf{B} \cdot \mathbf{I} - r^2)$$

On solving this Discriminant turns out to be zero.

Hence the quadratic equation has only one solution.
To find the unique solution of the equation (i.e. the unique value of k), Put D=0
And the solution is

$$k = \frac{-(2\mathbf{n} \cdot (\mathbf{B} - \mathbf{I}))}{2\,\|n\|^2}$$

On substituting the values, the value of k is

$$k = \frac{122\sqrt{74} + 82\sqrt{122}}{122(\sqrt{74} + \sqrt{122} + \sqrt{32})}$$

So on substituting this value in (1), we get the point D3

$$D3 = \frac{1}{\sqrt{74}+\sqrt{32}+\sqrt{122}} \begin{pmatrix} -366\sqrt{74} - 406\sqrt{122} - 488\sqrt{32} \\ -610\sqrt{74} + 732\sqrt{32} - 170\sqrt{122} \end{pmatrix}$$