Solution of Q9.3.21

SUJAL GUPTA - EE22BTECH11052

It is known that 10% of certain articles manufactured are defective. What is probability that a random sample space of 12 such articles,9 are defective?

Solution:

Parameter	Values	Description
n	12	Number of articles
k	9	Number of defective articles
p	0.1	Probability of being defective
X	$1 \leq X \leq 12$	X defective elements out of 12
$\mu = np$	1.2	mean
$\sigma = \sqrt{np(1-p)}$	1.08	standard deviation

TABLE 0

TABLE 1

1) *Binomial Distribution :*
   The X is the random variable, the pmf of X is given by

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \tag{1}$$

   We require $\Pr(X = 9)$. Since $n = 12$,

$$p_X(9) = 1.60379(10^{-7}) \tag{2}$$

2) *Gaussian Distribution*
   Let Y be gaussian variable. The central limit theorem states that we can take a random variable Z such that,

$$Z \approx \frac{Y-\mu}{\sigma} \tag{3}$$

   Now, Z is a random variable with $\mathcal{N}(0,1)$. Hence, the gaussian distribution function changes to:

$$p_Z(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \qquad (x \in Z) \tag{4}$$

   Using Normal distribution at X=9.

$$= \frac{9-1.2}{\sqrt{1.08}} \tag{5}$$

$$= 7.5055534 \tag{6}$$

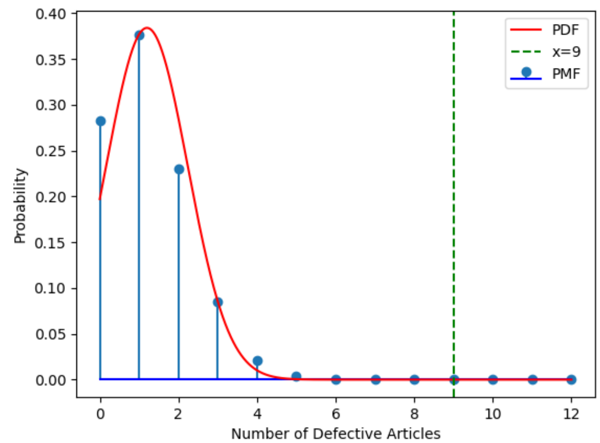

Fig. 2. Binomial-PMF and Gaussian-PDFof X

For pdf(probability density function) calculation

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{7}$$

$$p_Y(9) = p_Z(7.5055534) \tag{8}$$

$$= 3.89010(10^{-9}) \tag{9}$$

Hence we observe that the gaussian and binomial distribution have very less absolute error.