Solution of Q9.3.21

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It is known that 10% of certain articles man-
ufactured are defective. What is probability that
a random sample space of 12 such articles,9 are
defective?

Solution:

Parameter	Values	Description
n	12	Number of articles
k	9	Number of defective articles
p	0.1	Probability of being defective
X	$1 \leq X \leq 12$	X defective elements out of 12

TABLE 0
TABLE 1

1) *Binomial Distribution :*
   The X is the random variable, the pmf of X is
   given by

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \qquad (1)$$

   We require $\Pr(X = 9)$. Since $n = 12$,

$$p_X(9) = 1.60379(10^{-7}) \qquad (2)$$

2) *Gaussian Distribution*
   Let Y be gaussian variable

$$\mu = np \qquad (3)$$

$$\sigma^2 = np(1-p) \qquad (4)$$

   Using Normal distribution at X=9.

$$Z \approx \frac{X - \mu}{\sigma} \qquad (5)$$

$$= \frac{9 - 1.2}{\sqrt{1.08}} \qquad (6)$$

$$= 7.5055534 \qquad (7)$$

   For pdf(probability density function) calcula-
   tion

$$f_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (8)$$

$$p_Y(9) = p_Z(7.5055534) \qquad (9)$$
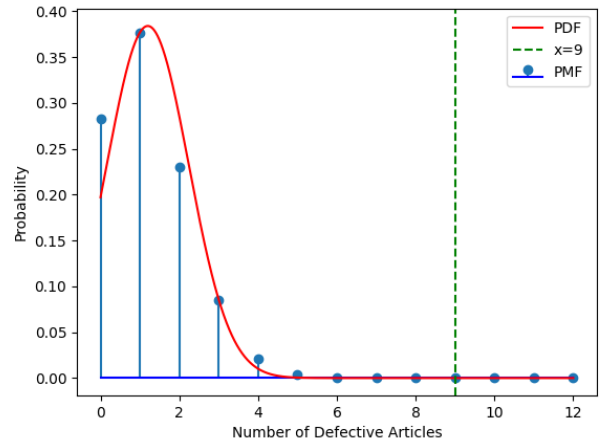
$$= 3.89010(10^{-9}) \qquad (10)$$

Hence we observe that the gaussian and binomial
distribution have very less absolute error.

Fig. 2.   Binomial-PMF and Gaussian-PDFof X