Solution of Q10.13.3.24

SUJAL GUPTA - EE22BTECH11052

A coin is tossed two times. Find the probability of getting at most one head.

Solution: Let the event of getting a head on one coin toss be H. Then

$$\Pr(H) = \frac{1}{2} \tag{1}$$

Variable	Description	Value
n	Number of tosses	2
X_i	Result of ith coin	$X_i, i = 1, 2$
X	No of heads	$X_1 + X_2$

$$X = \sum_{i=1}^{2} X_i \tag{2}$$

$$X = X_1 + X_2 \tag{3}$$

$$X \leq 2 \tag{4}$$

The probability of getting a head is:

$$p_X(k) = {}^2C_k (0.5)^k (0.5)^{2-k} \tag{5}$$

$$= {}^2C_k (0.5)^2 \qquad \forall k = 0, 1, 2 \tag{6}$$

The above equation gives the PMF of getting k heads on 2 coint tosses. Let $F_X(k)$ denote the cumulative distribution function of X:

$$F_X(k) = p(X \leq k) \tag{7}$$

$$= \sum_{i=0}^{k} {}^2C_i \left(\frac{1}{2}\right)^2 \tag{8}$$

Let $F_X(k)$ denote the cumulative distribution function of X:

$$F_X(k) = p(X \leq k) \tag{9}$$

$$F_X(1) = \sum_{i=0}^{1} {}^2C_i \left(\frac{1}{2}\right)^2 \tag{10}$$

$$= \frac{3}{4} \tag{11}$$

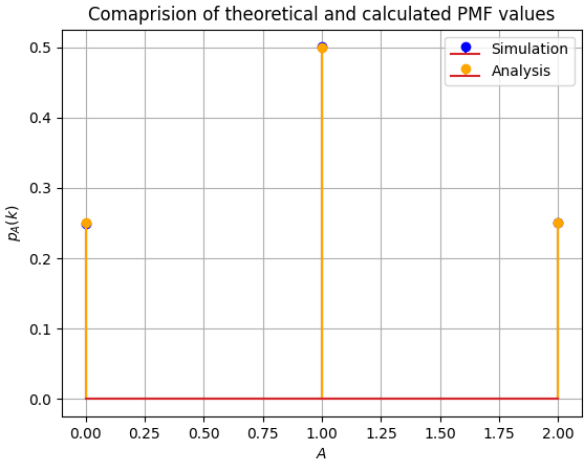

Fig. 0. PMF of X

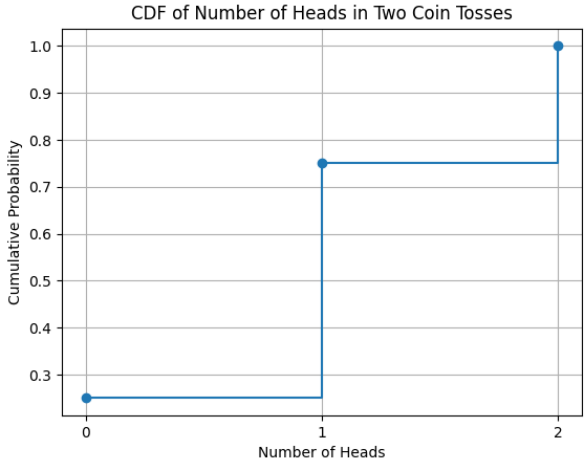

Fig. 0. CDF of X