Solution of Q9.3.21

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It is known that 10% of certain articles manufactured are defective. What is probability that a random sample space of 12 such articles,9 are defective?

Solution:

Parameter	Values	Description
n	12	Number of articles
k	9	Number of defective articles
p	0.1	Probability of being defective
X	$1 \leq X \leq 12$	X defective elements out of 12
Y	$1 \leq Y \leq 12$	gaussian variable
$\mu = np$	1.2	mean
$\sigma = \sqrt{np(1-p)}$	1.039	standard deviation

TABLE 0
TABLE 1

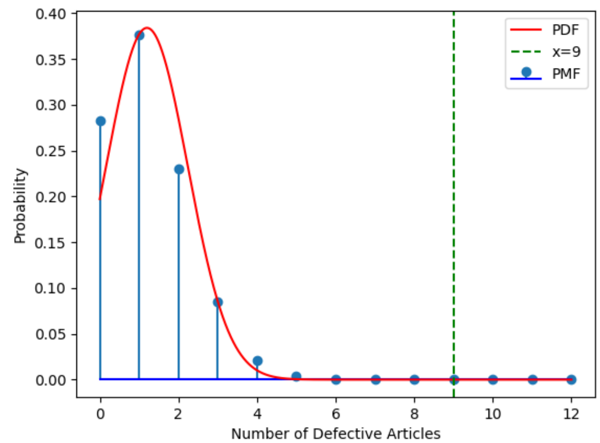

Fig. 3. Binomial-PMF and Gaussian-PDFof X

1) *Binomial Distribution :*
The X is the random variable, the pmf of X is given by

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \tag{1}$$

We require $\Pr(X = 9)$. Since $n = 12$,

$$p_X(9) = 1.60379(10^{-7}) \tag{2}$$

2) *Gaussian Distribution*
Let Y be gaussian variable. Using central limit theorem, we can use the gaussian distribution function:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (x \in Y) \tag{3}$$

Using Normal distribution at X=9.

$$p_Y(9) = \frac{1}{\sqrt{2\pi\left(\frac{27}{25}\right)}} e^{-\frac{\left(x-\frac{6}{5}\right)^2}{2\left(\frac{27}{25}\right)}} \tag{4}$$

$$= \frac{1}{\sqrt{2\pi\left(\frac{27}{25}\right)}} e^{-\frac{169}{3}} \tag{5}$$

$$= 3.89010(10^{-9}) \tag{6}$$

3) *using Q function:*

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{7}$$

The CDF of Y:

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y-\mu}{\sigma}\right), & y > \mu \\ Q\left(\frac{\mu-y}{\sigma}\right), & y < \mu \end{cases} \tag{8}$$

But,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{9}$$

$$\implies F_Y(y) = 1 - Q\left(\frac{y-\mu}{\sigma}\right) \tag{10}$$

to include correction of 0.5,

$$p_Y(8.5 < Y < 9.5) = F_Y(9.5) - F_Y(8.5) \tag{11}$$

$$= Q\left(\frac{8.5-\mu}{\sigma}\right) - Q\left(\frac{9.5-\mu}{\sigma}\right) \tag{12}$$

$$= Q(7.02) - Q(7.98) \tag{13}$$

$$= 1.2798(10^{-12}) \tag{14}$$