

Solution of Q12.13.3.21

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A shopkeeper sells three types of flower seeds A_1 , A_2 and A_3 . They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 0.45, 0.60 and 0.35. Calculate the probability:

- 1) of a randomly chosen seed to germinate
- 2) that it will not germinate given that the seed is of type A_3 ,
- 3) that it is of the type A_2 given that a randomly chosen seed does not germinate.

Solution: Let the random variables be X and Y .

Variable	Description	Value
X	Type of seed	0,1,2
Y	seed germinates or not	0,1

$$p_X(X) = \begin{cases} \frac{4}{10}, & \text{if } X = 1 \\ \frac{4}{10}, & \text{if } X = 2 \\ \frac{2}{10}, & \text{if } X = 3 \end{cases} \quad (1)$$

$$\Pr(Y = 0|X = 1) = 0.45 \quad (2)$$

$$\Pr(Y = 0|X = 2) = 0.60 \quad (3)$$

$$\Pr(Y = 0|X = 3) = 0.35 \quad (4)$$

$$(5)$$

1)

$$p_Y(0) = \Pr(Y = 0|X = 1) \Pr(X = 1) + \Pr(Y = 0|X = 2) \Pr(X = 2) + \Pr(Y = 0|X = 3) \Pr(X = 3) \quad (6)$$

$$= \frac{49}{100} \quad (7)$$

$$= 0.49 \quad (8)$$

2)

$$\Pr(Y = 1|X = 2) = 1 - \Pr(Y = 0|X = 2) \quad (9)$$

$$= 1 - 0.35 \quad (10)$$

$$= 0.65 \quad (11)$$

3)

$$\Pr(X = 2|Y = 1) = \frac{\Pr(Y = 1|X = 2) \Pr(X = 2)}{\Pr(Y = 1)} \quad (12)$$

$$= \frac{(1 - \Pr(Y = 0|X = 2)) \Pr(X = 2)}{1 - \Pr(Y = 0)} \quad (13)$$

$$= \frac{16}{51} \quad (14)$$