

Solution of Q9.3.21

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It is known that 10% of certain articles manufactured are defective. What is probability that a random sample space of 12 such articles, 9 are defective?

Solution:

Parameter	Values	Description
n	12	Number of articles
k	9	Number of defective articles
p	0.1	Probability of being defective
X	$1 \leq X \leq 12$	k defective elements out of 12

TABLE 0

TABLE 1

The X is the random variable, the pmf of X is given by

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \quad (1)$$

We require $\Pr(X = 9)$. Since $n = 12$,

$$\Pr(X = 9) = p_X(9) \quad (2)$$

$$= {}^nC_k p^k (1-p)^{n-k} \quad (3)$$

$$= {}^{12}C_9 \left(\frac{1}{10}\right)^9 \left(1 - \frac{1}{10}\right)^{12-9} \quad (4)$$

$$= 22 \left(\frac{9^3}{10^{11}}\right) \quad (5)$$

$$= 1.60379(10^{-7}) \quad (6)$$

Let Y be gaussian variable

$$\mu = np \quad (7)$$

$$= 12 \times 0.1 \quad (8)$$

$$= 1.2 \quad (9)$$

$$\sigma^2 = np(1-p) \quad (10)$$

$$= 12 \times 0.1 \times 0.9 \quad (11)$$

$$= 1.08 \quad (12)$$

Using Normal distribution at $X=9$.

$$Z = \frac{X - \mu}{\sigma} \quad (13)$$

$$= \frac{9 - 1.2}{\sqrt{1.08}} \quad (14)$$

$$= 7.5055534 \quad (15)$$

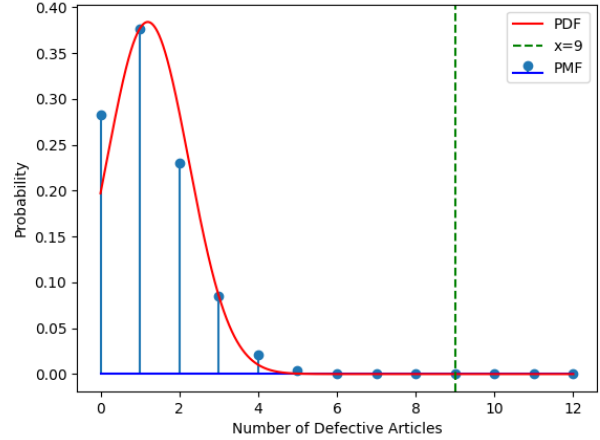


Fig. 0. Binomial-PMF and Gaussian-PDF of X

For pdf(probability density function) calculation

$$f_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (16)$$

$$p_Y(9) = p_Z(7.5055534) \quad (17)$$

$$= 3.89010(10^{-9}) \quad (18)$$

Hence we observe that the gaussian and binomial distribution have very less absolute error.