

Solution of Q12.13.3.21

SUJAL GUPTA - EE22BTECH11052

Ten coins are tossed. What is the probability of getting atleast 8 heads?

Solution: Let the event of getting a head on one coin toss be H. Then

$$\Pr(H) = \frac{1}{2} \quad (1)$$

Variable	Description	Value
n	Number of tosses	10
X_i	Result of ith coin	$X_i, i = \{1, 2, \dots, 10\}$
X	No of heads	$\sum_{i=1}^{10} X_i$

$$X = \sum_{i=1}^{10} X_i \quad (2)$$

$$X \leq 10 \quad (3)$$

The probability of getting a head is:

$$p_X(k) = {}^{10}C_k(0.5)^k(0.5)^{10-k} \quad (4)$$

$$= {}^{10}C_k(0.5)^{10} \quad \forall k = 0, 1, 2, \dots, 10 \quad (5)$$

The above equation gives the PMF of getting k heads on 10 coin tosses. Let $F_X(k)$ denote the cumulative distribution function of X:

$$F_X(k) = p(X \leq k) \quad (6)$$

$$= \sum_{i=0}^k {}^{10}C_i \left(\frac{1}{2}\right)^{10} \quad (7)$$

Let $F_X(k)$ denote the cumulative distribution function of X:

$$F_X(k) = p(X \leq k) \quad (8)$$

$$\Rightarrow F_X(10) - F_X(7) = \sum_{i=8}^{10} {}^{10}C_i \left(\frac{1}{2}\right)^{10} \quad (9)$$

$$= \frac{7}{128} \quad (10)$$

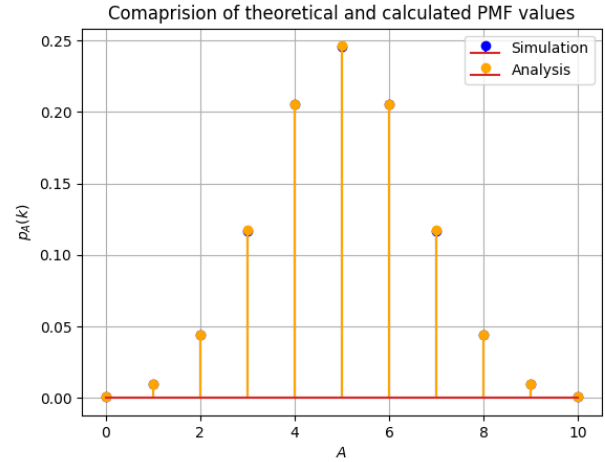


Fig. 0. PMF of X

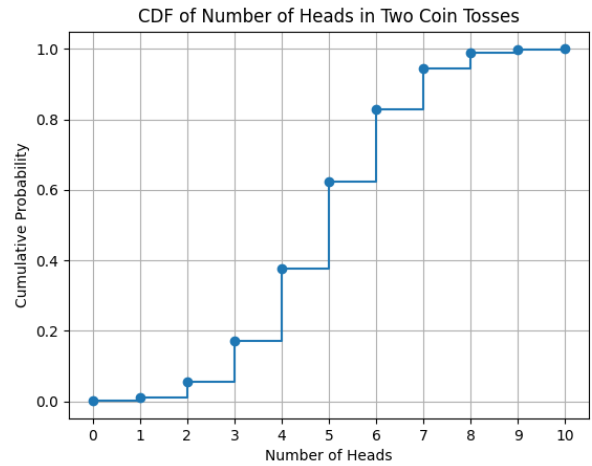


Fig. 0. CDF of X