Solution of Q12.13.3.43

SUJAL GUPTA - EE22BTECH11052

A shopkeeper sells three types of flower seeds A1, A2 and A3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 0.45, 0.60 and 0.35. Calculate the probability:

1) of a randomly chosen seed to germinate
2) that it will not germinate given that the seed is of type A_3,
3) that it is of the type A_2 given that a randomly chosen seed does not germinate.

Solution: Let the two random variables be X and Y for denoting type of seed and germination status.

Variable	Description	Value
X	Type of seed 1:A_1, 2:A_2, 3:A_3	$\{1, 2, 3\}$
Y	0:germinates, 1:not germinate	$\{0, 1\}$

$$p_X(X) = \begin{cases} \frac{4}{10}, & if X = 1 \\ \frac{4}{10}, & if X = 2 \\ \frac{2}{10}, & if X = 3 \end{cases} \tag{1}$$

$$\Pr(Y = 0|X = 1) = 0.45 \tag{2}$$

$$\Pr(Y = 0|X = 2) = 0.60 \tag{3}$$

$$\Pr(Y = 0|X = 3) = 0.35 \tag{4}$$

$$\tag{5}$$

1)

$$p_Y(0) = \Pr(Y = 0|X = 1)\Pr(X = 1) + \Pr(Y = 0|X = 2)\Pr(X = 2) + \Pr(Y = 0|X = 3)\Pr(X = 3) \tag{6}$$

$$= \frac{49}{100} \tag{7}$$

$$= 0.49 \tag{8}$$

2)

$$\Pr(Y = 1|X = 2) = 1 - \Pr(Y = 0|X = 2) \tag{9}$$

$$= 1 - 0.35 \tag{10}$$

$$= 0.65 \tag{11}$$

3)

$$\Pr(X = 2|Y = 1) = \frac{\Pr(Y = 1|X = 2)\Pr(X = 2)}{\Pr(Y = 1)} \tag{12}$$

$$= \frac{(1 - \Pr(Y = 0|X = 2))\Pr(X = 2)}{1 - \Pr(Y = 0)} \tag{13}$$

$$= \frac{16}{51} \tag{14}$$