


Prime Sieve

Prior knowledge → Some conceptual knowledge on prime no.

→ basic arrays, vectors & loops

Agenda → Basic of Prime sieve
Optimizations
Problem Solving
Segmented Sieve

Q

You have a number N . You have to print all the prime no. less than N .

$N=10$ \rightarrow 2, 3, 5, 7

Basic Solⁿ \rightarrow TC \rightarrow $O(n^2)$ \rightarrow Optimize??

$$2 \times 18$$

$$3 \times 12$$

$$4 \times 9$$

$$6 \times 6$$

$$9 \times 4$$

$$12 \times 3$$

$$18 \times 2$$

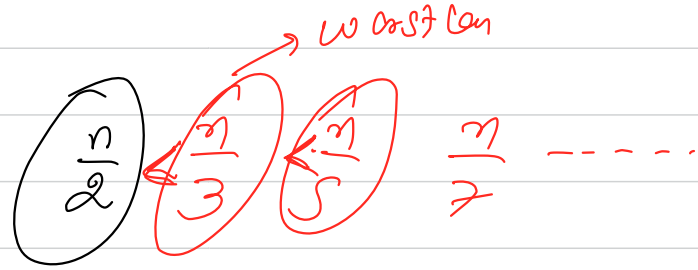
36

$$\Downarrow$$
$$\underline{\underline{O(n\sqrt{n})}}$$

\leftarrow Sqaure root of $n=36$

$$\underline{O(n^2)} \rightarrow \underline{O(n\sqrt{n})}$$

↳ Prime Sieve



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	...	<u>n</u>
\times	\times	\times	\times	\times	\times	\times	\times	\times	\times	\times	\times	\times	\times	\times	\times	...	<u>n</u>

2 is a prime

for any $x \rightarrow p_1^{a_1} \times p_2^{a_2} \times p_3^{a_3} \dots$ product of
 $x = 6 \Rightarrow 2^1 \times 3^1$ power of
prime

implant

Storage

result

vector<bool> → 12 byte
bool vector

F	A	T	T	F	T	F	T	F	F	F	T	F	T
0	1	2	3	4	5	6	7	8	9	10	11	12	13

{ 2, 3, 5, 7, 11, 13 } →

prime list

Bitset

in terms of space
8 times efficient

1 byte
arr[boolean]

STL

1 bit

8×2
 8×4
 8×6

3×2
 3×4
 3×8

3×6
 3×8

array of boolean ? ? \rightarrow for every index we need
1 byte space
(8 bit)

\hookrightarrow BitSet \rightarrow for every index it takes 1 bit space



$$L < \sqrt{N} \quad \underline{\underline{N}}$$

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots$$

$$= \sum_{p=2}^{p \leq N} \frac{1}{p}$$

Time Complexity

prime number theorem

→ No. of prime numbers less than N → $\frac{1}{\ln n}$

↳ K^{th} prime → approx $K \ln K$

↳ natural log

$$\left[\sum_{p=2}^{\infty} \frac{\eta}{p} \right]$$

$$\rightarrow \sum_{k=2}^{\infty} \frac{\eta}{k \ln k}$$

$$\approx \sum_{k=1}^{\infty} \frac{\eta}{k \ln k}$$

$$\Rightarrow \sum_{k=2}^{\infty} \frac{1}{k \ln k} \xrightarrow{\text{approx}} \int_2^{\infty} \frac{1}{k \ln k} dk$$

integral \approx

$$\Rightarrow \eta \left[\ln \ln k \right]_2^{\infty}$$

$$n \left(\ln \ln k \right)^{1/\ln n} \rightarrow n \left(\ln \ln \left(\frac{n}{\ln n} \right) - \underbrace{\ln \ln 2}_{\text{const}} \right)$$

$$\rightarrow n \left(\ln \ln n - \ln \ln \ln n - \ln \ln 2 \right)$$

$$\sum_{p \leq n} \frac{1}{p} \approx \text{approach} \quad \underline{\underline{n \ln \ln n}}$$

$$O(\underline{\underline{n \ln \ln(n)}})$$

$$\underline{\underline{n \ln \ln n}} < \underline{\underline{n \sqrt{n}}}$$

$$O(n^2) \rightarrow O(n\sqrt{n}) \rightarrow O(n \log \log n)$$

↳ natural log

Divisor of factorial ←

→ N=3

3! = 6

N → 5 × 10⁴

→ N! → that what's the highest power of any prime p that divides N!

↳ Every number can be represented as product of power of primes

↳ for any ith prime

n log log n

Sieve

5 × 10⁴

prime

5
10
20

$\textcircled{3} \hookrightarrow \textcircled{\frac{5!}{4!}} \rightarrow \text{highest power of } 2 \text{ that divides } \underline{5!}$
 $\textcircled{5!}$

$\textcircled{\frac{5!}{2!}}$

$\textcircled{2 \times 2 \times 2 \times 3}$

$$\left[\frac{5}{2} \right] + \left[\frac{5}{2^2} \right]$$

$$\underline{2} + 1 \Rightarrow \textcircled{3}$$

$$\left[\frac{5}{3} \right] \Rightarrow \underline{1}$$

$$\left[\frac{5}{3^2} \right] = 0$$

$$\underline{\underline{\textcircled{(3+1) \times (1+1) \times (1+1)}}}$$

$$\begin{aligned}
 &\textcircled{2, 2, 2} \rightarrow \\
 &\hookrightarrow (3) \rightarrow \\
 &\hookrightarrow \underline{(5)} \rightarrow \\
 &\text{Combinator}
 \end{aligned}$$

for any N , we want to calc the highest power of p that divides $\underline{\underline{N!}}$

$$\left\lfloor \frac{N}{p} \right\rfloor + \left\lfloor \frac{N}{p^2} \right\rfloor + \left\lfloor \frac{N}{p^3} \right\rfloor + \dots \rightarrow \underline{\underline{\text{zero}}}$$

$$\left\lfloor \frac{5}{2} \right\rfloor + \left\lfloor \frac{5}{2^2} \right\rfloor$$

$$\left\lfloor \frac{5}{3} \right\rfloor \Rightarrow \underline{\underline{1}}$$

$$\left(n \log \log n + \frac{n}{\log n} \times (\text{hyperpower}) \right)$$

→ $O(\underline{n \log \log n})$

N, m

↳ We can't generate primes less than max(N, m)

$(N - m) \leq 10^9$

$N \rightarrow m$

Sieve $\rightarrow \sqrt{N}$
prime $\leq \sqrt{N}$

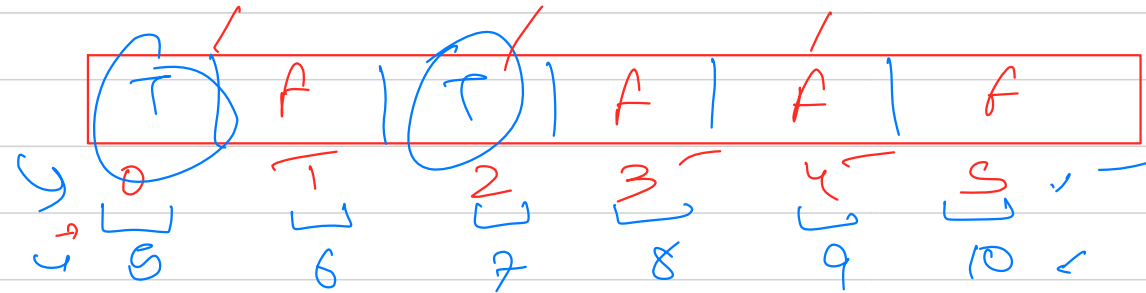
$\sqrt{\max(N, m)}$

$S = 10$

$\ll \sqrt{N}$

primes $\leq N$

$\begin{bmatrix} 2 & 3 \end{bmatrix}$
primes



$N - m$

max \rightarrow actual no

for any value $n \rightarrow \underline{\underline{\frac{n}{\log n}}}$

\sqrt{n}

3 + first rule

7



$\left(\frac{7}{3}\right) \times 3$

$\Rightarrow 2 \times 3$

$\Rightarrow \underline{\underline{6 + 3}}$

$\rightarrow \underline{\underline{9}}$

$n = 12$

$\left(\frac{12}{3}\right) \times 3$

4×3
 $\Rightarrow \underline{\underline{12}}$

for any prime ~~p~~ \rightarrow to get the first
multiple $\geq \min(n, m)$

$$\rightarrow x = \left(\frac{\min(n, m)}{p} \right) \times p$$

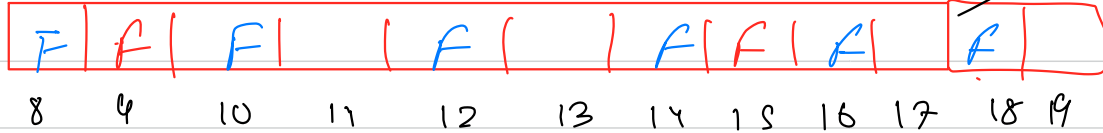
if $(x < \min(n, m))$ &

\exists ~~$x += p$~~

$$N = 8$$

$$m = 19$$

\sqrt{m}



$$N \rightarrow \sqrt{\log \log m}$$

$$\sqrt{\log \log m}$$

$$\left(\frac{8}{2} \right) \rightarrow 4$$

$$\left(\frac{8}{3} \right) \times 3$$

$$\Rightarrow 6$$

$$\boxed{2 \mid 3}$$

$$6 < 8$$

$$6 + 3 \Rightarrow 9$$

↳ Eliminate multiples of primes

Eliminate multiples of cubes ??

→ sieve cubes of
prime numbers

2 → 8 → 64

1 27 225 $\sqrt[3]{n}$

22
6x76x10

$i \cdot j \cdot k \leq n$ $i \neq j$
multiple → eliminate
cube

```

for (int i = 2; i * i ≤ n; i++) {
    if (arr[i] == true) {
        for (int j = 1; i * i * j ≤ n; j++) {
            arr[i * i * j] = false
        }
    }
}

```

3 → 4 → prime? 3, 4
 3
 3
 cube for
 Sieve