Prime Sieve

Prior knowledge → Some conceptual knowledge on prime no.

→ basic arrays , vectors & loops

Agenda → Basic of Prime sieve
Optimizations
Problem Solving
Segmented Sieve

Q => You have a number N. You have to print all the prime no. less than N.

$N = 10 \rightarrow 2, 3, 5, 7$

Basic Sol^n → TC → $O(n^2)$ → optimise??

$O(n\sqrt{n})$

2 x 18

3 x 12

4 x 9

6 x 6 ← squareroot of $n = 36$

9 x 4

12 x 3

18 x 2

36

$O(n^2) \longrightarrow O(n\sqrt{n})$

worst case

$\frac{n}{2} \leftarrow \frac{n}{3} \leftarrow \frac{n}{5} \quad \frac{n}{7} \quad - - - - -$

↳ Prime Sieve

2 2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	..	N
✗	✗	=	=	✗		✗	=	✗	✗	✗		✗		✗	✗		

2 is a prime

for any $x \rightarrow \left[p_i^{a_1} \times p_j^{a_2} \times p_k^{a_3} - - - \right] \ldots \rightarrow$ product of power of prime

$x = 6 \Rightarrow 2^1 \times 3^1$

implement

storage

result

vector<bool> → 1 byte

bool vector

0	1	2	3	4	5	6	7	8	9	10	11	12	13
F	F	T	T	F	T	F	T	F	F	F	T	F	T

{2, 3, 5, 7, 11, 13 - - - -} → prime list

Bitset → In terms of space

8 times effecent

STL

1 bit

1 byte

arr[boolean]

5x2

5x4

5x6

3x2

3x4

x*y

3x3

x*x

array of boolean ?? → for every index we need 1 byte space (8 bit)

↳ Bitset → for every index it takes 1 bit space

$i < \sqrt{N}$ $\qquad$ N

$$\frac{n}{2} + \frac{n}{3} + \frac{n}{5} + \frac{n}{7} \cdots\cdots$$

$$= \sum_{p=2}^{p\le n} \frac{n}{p}$$ } Time Complexity

Prime number theorem

→ No. of prime numbers less than N → $\frac{1}{\ln n}$

↳ K^{th} prime → approx $K \ln K$

↳ natural log

$$\left[\sum_{p=2}^{p \le n} \frac{n}{p} \right]$$

$$\boxed{\sum_{k=2}^{n/\ln n} \frac{n}{k \ln k}} \approx \sum_{k=1}^{n/\ln n} \frac{n}{k \ln k}$$

$$\Rightarrow n \sum_{k=2}^{n/\ln n} \frac{1}{k \ln k} \underset{\text{approx}}{\longrightarrow} \int_2^{n/\ln n} \frac{1}{k \ln k} \, dk = \text{integral??}$$

$$\Rightarrow n \left[\ln \ln k \right]_2^{n/\ln n} \longrightarrow$$

$$n\left[\ln \ln k\right]_{2}^{n/\ln n} \rightarrow n\left(\ln \ln \left(\frac{n}{\ln n}\right) - \underbrace{\ln \ln 2}_{\text{const}}\right)$$

$$\rightarrow n\left(\ln \ln n - \ln \ln \ln n - \ln \ln 2\right)$$

$$\sum_{p \le n} \frac{n}{p} = \text{approx} \Rightarrow n \ln \ln n$$

$$O\left(n \ln \ln (n)\right)$$

$$n \ln \ln n < n\sqrt{n}$$

$O(n^2) \longrightarrow O(n\sqrt{n}) \longrightarrow O(n \log \log n)$

natural log

Divisor of factorial

→ N = 3   3! = 6   N → 5×10^4

→ N! → that what's the highest power of any prime p that divides N!

↳ Every number can be represented as product of power of primes

↳ for any i^{th} prime

nloglogn

5×10^4

Sieve

prime

3 5! → highest power of 2 that does 5!

4!

5!

$\left\lfloor \frac{5}{2} \right\rfloor + \left\lfloor \frac{5}{2^2} \right\rfloor$

$2 + 1 \Rightarrow 3$

$(3+1) \times (1+1) \times (1+1)$

3

$2 \times 2 \times 2 \times 3$

$\left\lfloor \frac{5}{3} \right\rfloor \Rightarrow 1$

$\left\lfloor \frac{5}{5} \right\rfloor \Rightarrow 1$

(2, 2, 2) →

(3) →

(5) →

Combination

For any N, we want to calc the highest power of p that divides $\underline{\underline{N!}}$

$$\left\lfloor \frac{N}{p} \right\rfloor + \left\lfloor \frac{N}{p^2} \right\rfloor + \left\lfloor \frac{N}{p^3} \right\rfloor \ldots\ldots$$

$\rightarrow$ $\underline{\underline{\text{zero}}}$

$$\left\lfloor \frac{5}{2} \right\rfloor + \left\lfloor \frac{5}{2^2} \right\rfloor$$

$$\left\lfloor \frac{5}{3} \right\rfloor \Rightarrow \underline{\underline{1}}$$

$$\left(n \log \log n + \frac{n}{\log n} \times (\text{height of down}) \right)$$

$$O(n \log \log n)$$

(N, M)

↳ We can't generate primes less than max(N, M)

$N - M \le 10^5$

Siev → $\sqrt{N}$

prime $\le \sqrt{N}$

$N \to M$

$\sqrt{\max(N,M)}$

primes $< N$

$5 - 10$ $\quad \le \sqrt{N}$

2	3

prime

T	F	T	F	F	F
0	1	2	3	4	5
5	6	7	8	9	10

index

$N - M$

↳ actual no.

for any value $n \rightarrow \dfrac{n}{\log n}$

$\sqrt{n}$

3 → first rule

$7 \rightarrow \left(\dfrac{7}{3}\right) \times 3$

$\Rightarrow 2 \times 3$

$\Rightarrow 6 + 3 \rightarrow 9$

$n = 12$

$\left(\dfrac{12}{3}\right) \times 3$

4×3

$\Rightarrow 12$

For any prime $\underline{\underline{p}}$ $\rightarrow$ to get the first multiple $\geq \min(N, m)$

$$\rightarrow \quad x = \left(\frac{\min(N, m)}{p} \right) \times p$$

if $(x < \min(N, m))$ {

$\underline{\underline{x \mathrel{+}= p}}$

}

$N = \cancel{7}\ 8$ $m = 19$

$\sqrt{max}$

F	F	F		F		F	F	F		F	

8 9 10 11 12 13 14 15 16 17 18 19

$N ? \text{sqrt}(\text{Max})$

$n \log \log n$

$\frac{8}{\cancel{2}} ? \cancel{8} \Rightarrow 8$

$\boxed{2 \mid 3}$

$\frac{8}{3} \times 3 \Rightarrow 6$

$6 < 8$

$6 += 3 \Rightarrow 9$

↳ Eliminate multiples of primes

Eliminate multiples of cubes ???

→ given cubes of prime numbers

2 8 → 64 ×

8 27 125 $\sqrt[3]{n}$

22

$6 \times 16 \times 16$

$i = 8$

$i * i * i \le n$

i++

→ multiple → eliminate

cube

3 → 4 → prime ??.?

```
for (int i=2; i*i*i <= n ; i++) {
    if( arr[i] == true) {
        for (int j=1; i*i*i*j <= n ; j++) {
            arr[j*i*i*i] = false)
        }
    }
}
```

multiples

cube free

Sieve