Q => Fibonacci series

0th 1st 2nd 3rd 4th 5th

$\underbrace{0, 1}_{\text{[seed terms / starting values]}}, \underbrace{1, 2, 3, 5, 8, 13, 21, 34 \ldots\ldots}$

any n^{th} term is the sum of previous 2 terms

20^{th} Fibonacci

Binet's formula → Derive

$a = 1 \quad b = -1 \quad c = -1$ ← $x^2 - x - 1 = 0$] solution of this eqⁿ

$ax^2 + bx + c = 0 \longrightarrow \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\longrightarrow \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$

$\Rightarrow \dfrac{1 \pm \sqrt{1+4}}{2} \Rightarrow \dfrac{1 \pm \sqrt{5}}{2}$ → roots / solⁿ for abv equation

I am trying to calculate

x^n

$x^1 = 1x + 0$

a) $5x + 3$

b) $3x + 2$

c) $2x + 2$

d) None

$x^2 - x - 1 = 0$

$x^2 = 1x + 1$

0th 1st 2nd 3rd 4th 5th

$0, 1, 1, 2, 3, 5, 8, 13, 21, 34$

$n \geq 1$

$x^3 = x(x^2)$

$= x(x+1)$

$= x^2 + x$

$= x + 1 + x = 2x + 1$

$x^3 = 2x + 1$

$x^4 = x(x^3) = x(2x+1) = 2x^2 + x$

$x^4 = 2(x+1) + x = 3x + 2$

$x^5 = x(x^4) = x(3x+2) \Rightarrow 3x^2 + 2x$

$= 3(x+1) + 2x = 3x + 3 + 2x$

$x^5 = 5x + 3$

$x^6 = 8x + 5$

$x^7 = 13x + 8$

By above equation, for any value n:

$$x^n = f_n x + f_{n-1}$$

$$\alpha = \frac{1+\sqrt{5}}{2} \qquad \beta = \frac{1-\sqrt{5}}{2}$$ (sol^n of quad eq^n)

$$\alpha^n = f_n \alpha + f_{n-1} \quad (1) \qquad \beta^n = f_n \beta + f_{n-1} \quad (2)$$

(1) − (2)

$$\alpha^n - \beta^n = f_n \alpha + \cancel{f_{n-1}} - f_n \beta - \cancel{f_{n-1}}$$

$$(\alpha-\beta) f_n = \alpha^n - \beta^n$$

$$f_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$$

$$f_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$$

$$f_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\frac{\cancel{1}+\sqrt{5} - \cancel{1}+\sqrt{5}}{2}}$$

$$\boxed{f_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}}$$

$\hookrightarrow$ Benet's Formula

$$\boxed{f_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}}$$

$n=2$

$$f_2 = \left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^2 \Big/ \sqrt{5}$$

$$= \left(\frac{1+5+2\sqrt{5}}{4} - \left(\frac{1+5-2\sqrt{5}}{4}\right)\right) \Big/ \sqrt{5}$$

$$\Rightarrow \left(\frac{\cancel{1}+\cancel{5}+2\sqrt{5}-\cancel{1}-\cancel{5}+2\sqrt{5}}{4}\right) \Big/ \sqrt{5}$$

$$= \left(\frac{\cancel{4}\cancel{\sqrt{5}}}{\cancel{4}}\right) \Big/ \cancel{\sqrt{5}}$$

$$\Rightarrow \underline{\underline{1}}$$

$$f_3 = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^3 - \left(\frac{1-\sqrt{5}}{2}\right)^3}{\sqrt{5}}$$

$$= \left(\frac{1+5\sqrt{5}+3\sqrt{5}(1+\sqrt{5})}{8} - \frac{\left(1-5\sqrt{5}-3\sqrt{5}(1-\sqrt{5})\right)}{8} \right) \Big/ \sqrt{5}$$

$$\left(\frac{\cancel{1}+5\sqrt{5}+3\sqrt{5}\cancel{+15}\cancel{-1}+5\sqrt{5}+3\sqrt{5}\cancel{-15}}{8} \right) \Big/ \sqrt{5}$$

$$\left(\frac{\cancel{16}^{\,2}\cancel{\sqrt{5}}}{\cancel{8}} \right) \Big/ \cancel{\sqrt{5}}$$

$$\boxed{f_3 = 2}$$

Q ⇒ if you will implement fib using binets formula where you will implement exponent operation yourself what is the best case runtime for your algo

a) O(n)
b) O(log n) ←
c) O(1)
d) None

a^n

a^n

$2^4 = 2^2$

$$a^n = a^{n/2} \times a^{n/2}$$

$$n \rightarrow n/2 \rightarrow n/4 \rightarrow n/8 \text{ - - - - - - } \frac{n}{2^k}$$

last term

$$a^{n/2} = a^{n/4} \times a^{n/4}$$

$$a^{n/4} = a^{n/8} \times a^{n/8}$$

$$2^2 = 2^1 \times 2^1$$

$$\boxed{\frac{n}{2^k} = 1}$$

$$n = 2^k$$

$$\boxed{k = \log_2 n}$$

a) 10^{10}

b) 10^{100}

c) 10^{8} → 1 sec

d) None

a^n

$n \approx 10^9$

1 sec $\rightarrow$ 10^8 operation

1 operation $\rightarrow$ $\frac{1}{10^8}$ sec

$10^9 \rightarrow \frac{1}{\cancel{10^8}} \times \cancel{10^9}$ sec

10 sec

$\log_2 10^9$

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DSA → Data Structure & Algo

→ ACM ICPC
→ facebook hackercup
→ google kickstart
→ google hashcode
⋮

→ d → divisible by 4

→ total animals in band → c + d

exactly how many animals touch ground

a) $\boxed{c + d} - \boxed{d/4}$

b) c + d + d/4

c) c + d − d % 4

d) None

which of denotes no. of animals whose feet don't touch the ground?

total animal → c+d

animal lovely ground → ⌊d/4⌋ → floor

animal not lovely ground → c+d − ⌊d/4⌋ → x

for the situation to be true

1 dog → 2 cats

x ≥ 0

atmax how may animals can be there whose feet don't touch → 2*d

c ≥ 2*d

c < d → x will never be greater than c

$\boxed{x \geq 0 \text{ and } x \leq c \text{ and } x \leq 2*d}$

True

Yes

else $\longrightarrow$ NO

$x \geq 0$ and $x \leq \min(c, 2d)$

$10^5 \rightarrow$ divisibility $\underline{O(1)}$ $\underline{\text{for}}$

$\rightarrow d/4$

$\rightarrow x = c + d - d/4$

$\rightarrow x >= 0$ and $x \le \min(c, 2d)$

$\hookrightarrow$ Yes

else $\rightarrow$ NO

$\rightarrow$ Const

3×10^5 x $\underline{10^5}$

$\underline{10^5}$