

Analysis Of algorithms — Part - 1

↳ Computation



we want the systems to
be as fast as possible

↳ Processor
Ram
SSD / HDD
CPU
⋮

↳ Hardware config

No

You want your software to be as
efficient as possible.

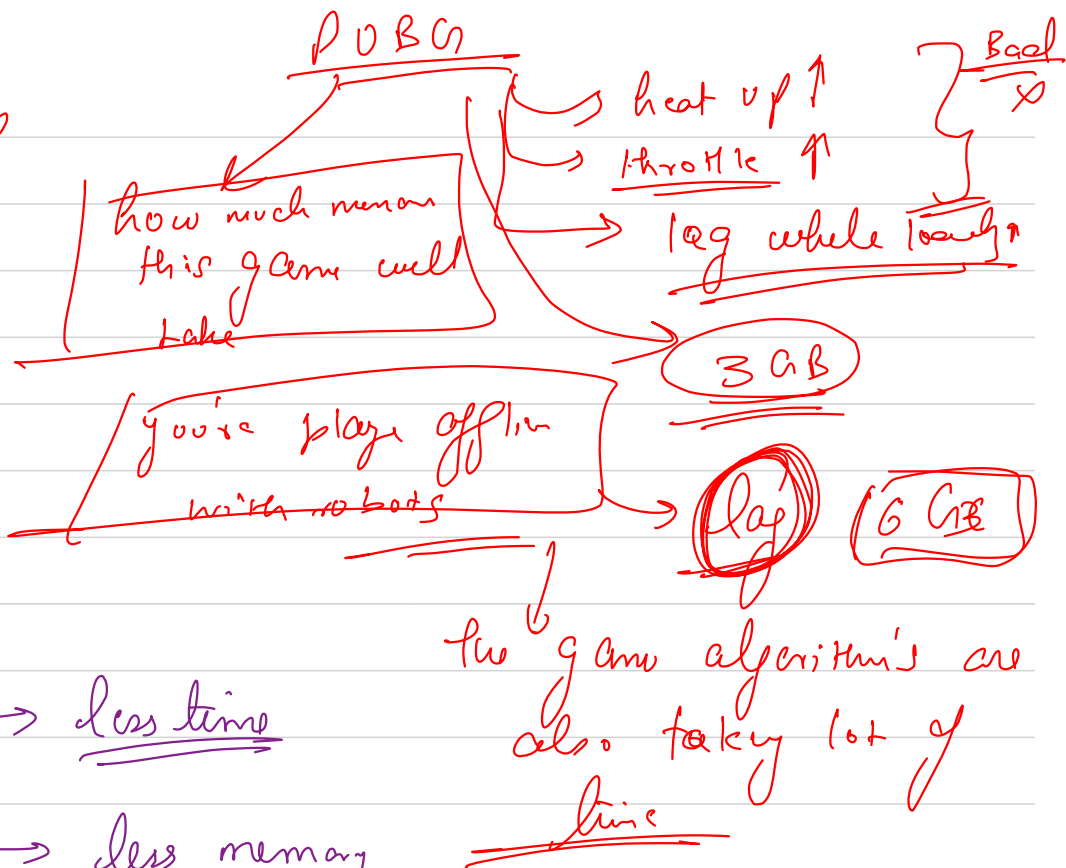
Course
efficient program

In what sense



✓ in terms of time → less time

✓ in terms of space → less memory space



How to analyse an algorithm??

↳ Can we compare algorithms based on no. of line of code??

No

Fib (4-5 line)
2-3
10 line

↳ Can we check the execution time of the algorithm??

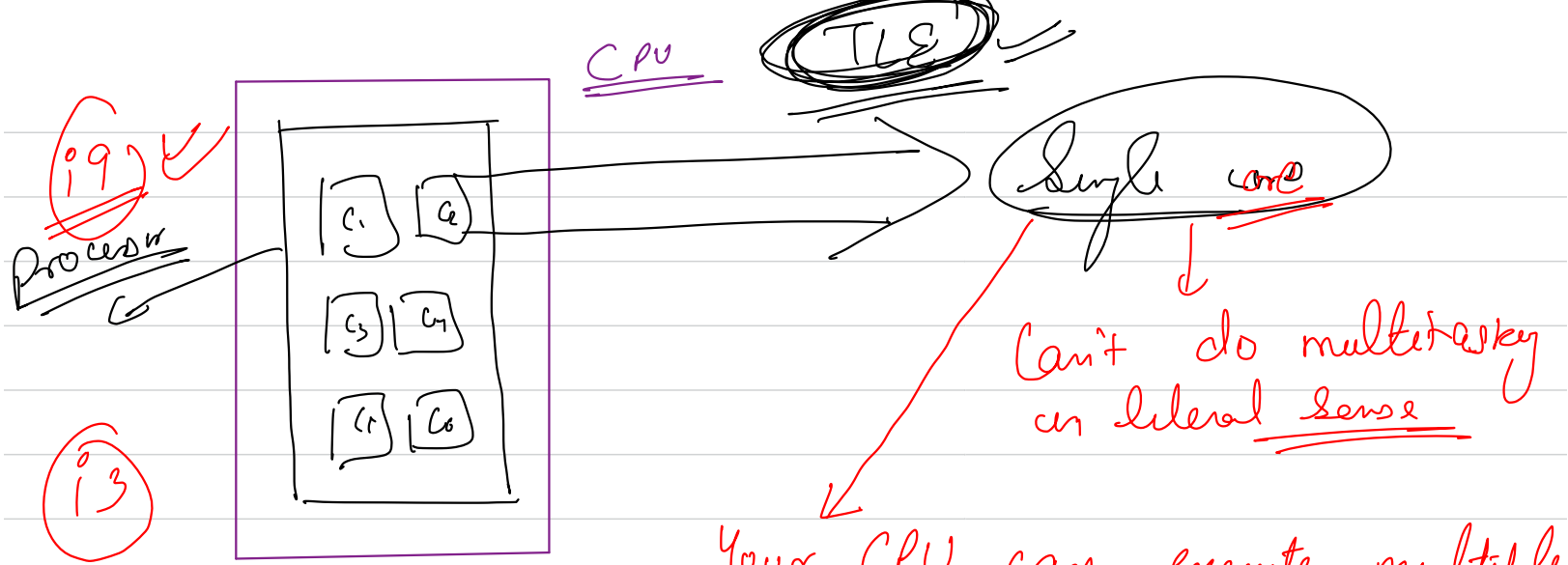
↳ No

$$\begin{array}{c} t = t_2 \\ \text{end time} - \text{start time} \end{array} \Rightarrow \text{time executed} \quad \underline{\underline{(t_2 - t_1)}}$$

→ Experimental Analysis

→ why was the time of execution change without even changing the input or code ??

Multi tasking ??



Your CPU can execute multiple process by the concept of scheduling.

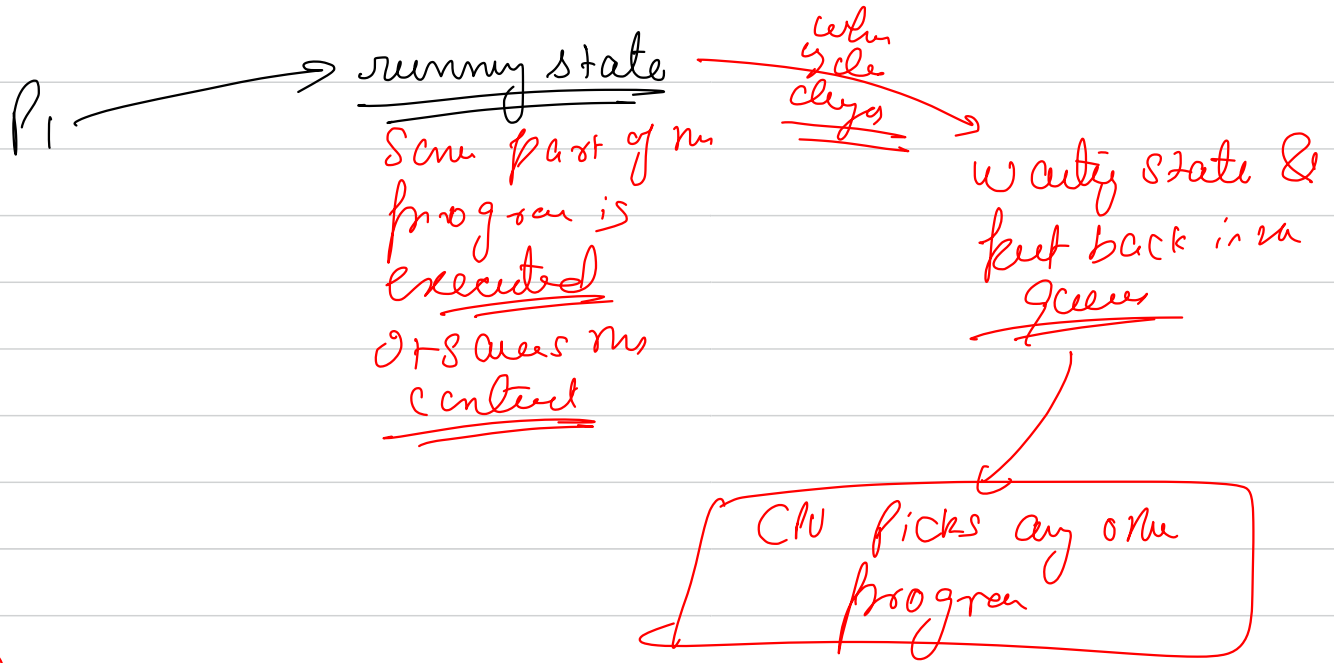
↓
queue of process

[FIFO]

P_1 | P_2 | P_3 ...

1 sec → 10^8 cycles

Process can have priority



Starve

That's why we can't do experimental analysis.

$$1\text{sec} \rightarrow 10^8 \text{ cycle}$$

$$\underline{\underline{10\text{sec}}} \rightarrow 10^9 \text{ cycle}$$

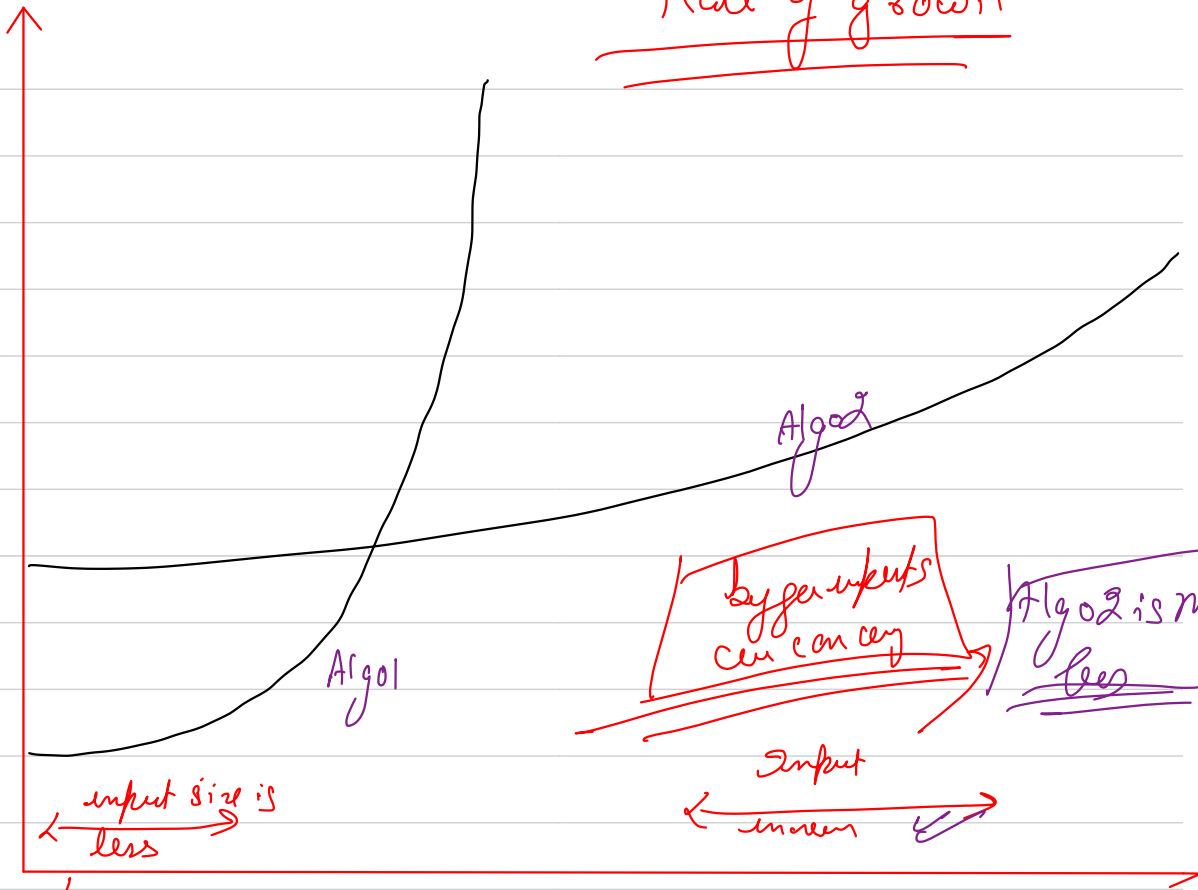
$$2\text{sec} \rightarrow 2 \times 10^8 \text{ cycle}$$

⋮



less
less

Rate of growth



Algol

Algol

By per inputs
can can say

Algol is more
less

input size is
less

input
more

Algol is better

input

What is Rate of growth??

The rate at which running time of algo increases as a function of input.

Assume,

$$f(n) = n^2$$
$$g(n) = \underline{\underline{10n}}$$
$$=$$

$$n = 30$$
$$f(n) = \underline{\underline{900}}$$
$$g(n) = 300$$

$$n = 10$$
$$\boxed{f(n) = 100}$$
$$g(n) = 100$$

$$\underline{\underline{n = 20}}$$
$$f(n) \Rightarrow 400$$
$$g(n) = 200$$

Rate of growth of a curve depends just on the highest degree term. All the lower degree terms for bigger input will be insignificant.

↳ So if we want to analyse algo, for given input, we need to extract the highest degree term from the polynomial curve representing time vs input of algo.

of algo-x $\rightarrow f(n) = 2n^2 + 3n \rightarrow n^2$
 $n \rightarrow \text{input size}$

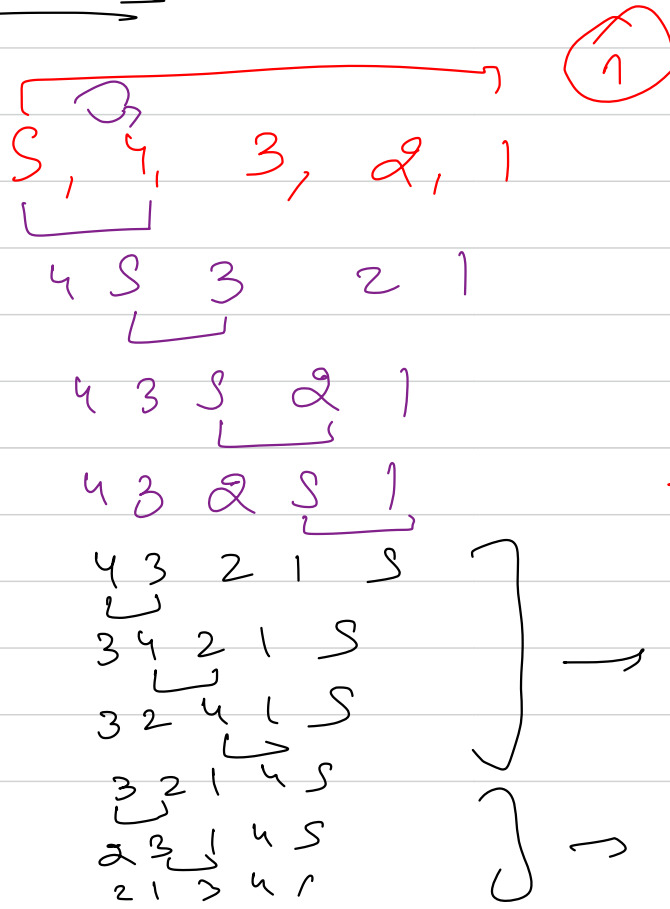
algo-y $\rightarrow f(n) = 3n^2 - 2n + 5 \rightarrow n^2$

runtime will be same for algo-x & algo-y

This type of analysis where we compare rate of growth w.r.t input is called

ASYMPTOTIC ANALYSIS

Bubble Sort



n swaps
n comparisons

n-1 swaps
n-1 comparisons

n-2 swaps
n-2 comparisons

$$n+n + (n-1)+(n-1) + (n-2)+(n-2) - - - - - 1)$$

$$2(n + n-1 + n-2 - - - - - 1)$$

$$\frac{n(n+1)}{2} \rightarrow n^2 + n$$

$n \rightarrow$ sing
let

$$f(n) = n^2 + n$$

Rate of growth depends
just on n^2

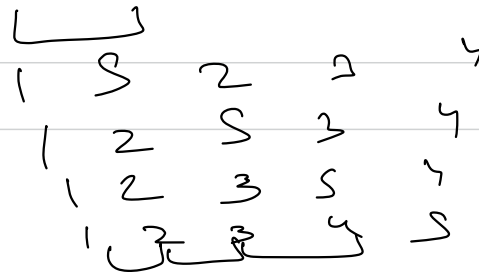
1 2 3 4 5



n comparisons
0 swap

$f(n) = n$ → Rate of growth is n

5 1 2 3 4



$n + n$

Based on type of input the rate of growth
can change.

3 types of analysis \rightarrow

Worst Case

It defines i/p for
which algo takes
long time
Here algo is the
slowest

Avg Case

Predicts an avg
approximation
any random
input

Best Case

Here the algo is
fastest & takes
least time

3 notations \rightarrow general notations

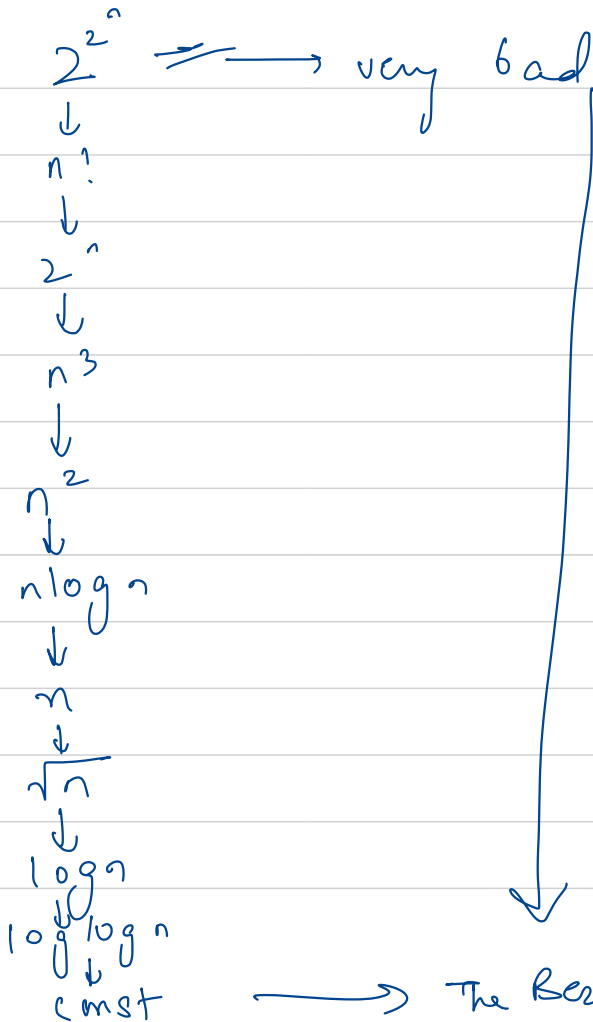
Worst \rightarrow Big O $\rightarrow O(n^2)$

Avg \rightarrow Big Theta $\rightarrow \underline{\underline{\Theta(n^2)}}$

Best \rightarrow Big omega $\rightarrow \Omega(n)$

Rate of growth

$$f(n) = \textcircled{0}$$

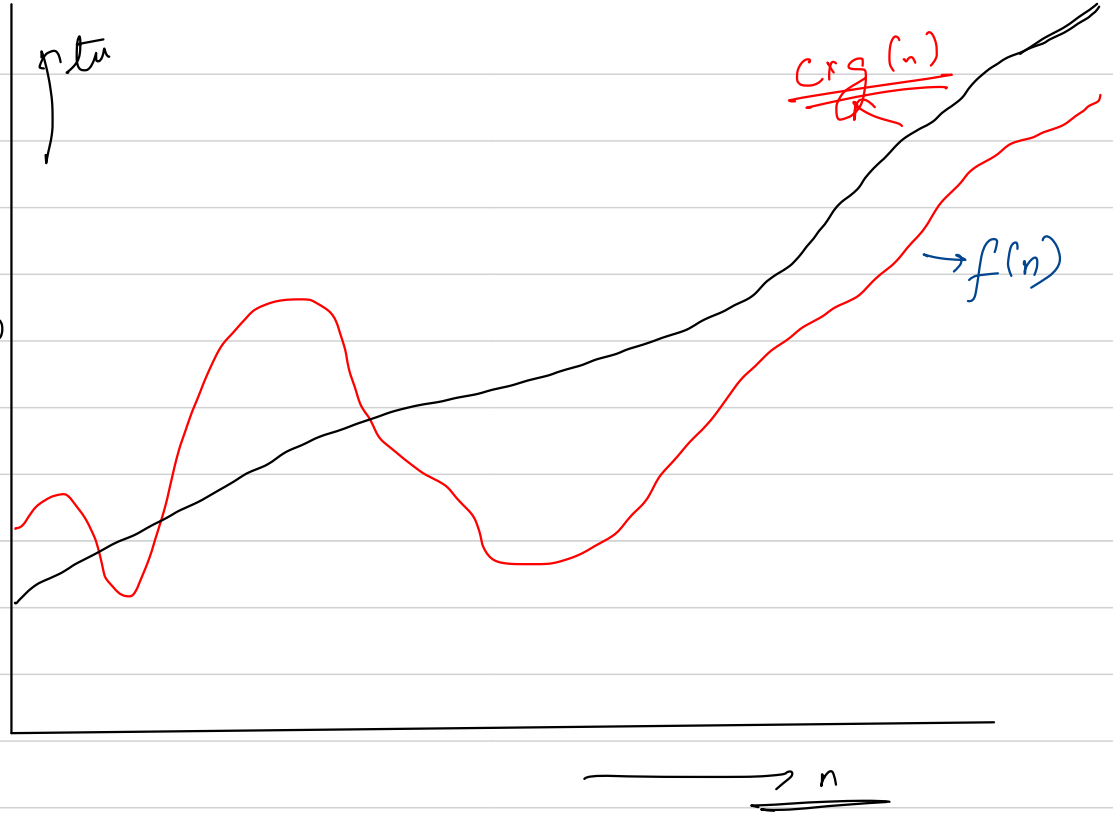


Big O → highest
upper
bound

$$f(n) = n^4 + 100n^2 + 10n + 50$$

$$g(n) = \underline{n^4} \quad \underline{\text{larger}}$$

$$f(n) = \underline{\underline{O(g(n))}}$$



Big Omega

highest lower
bound.

$$f(n) = n^4$$

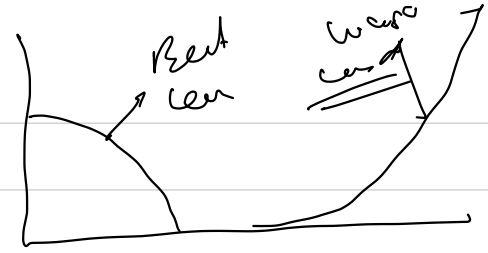
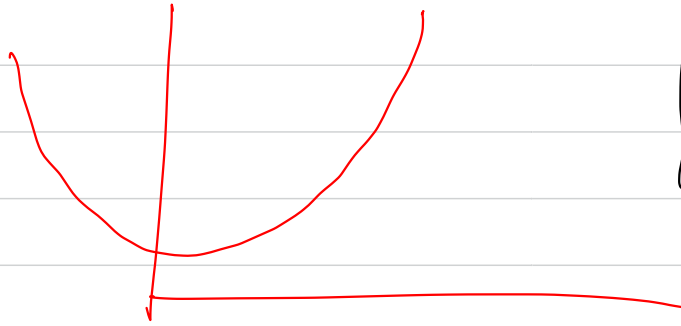
$$g(n) = n^4$$

$$\underline{\underline{\Omega(n^4)}}$$



$$f(n) = \begin{cases} n^2 & \text{faster} \\ \underline{\underline{\Omega(n)}} & \text{lower} \end{cases}$$

Bubble
sort



$f(n) = \underline{\underline{x^2}}$
 we need $x \rightarrow \underline{\underline{x^2}}$
 Big $x \rightarrow \underline{\underline{x^2}}$

