Agenda → Introduction to Bitmasking

→ Intuitive problems to club DP with Bitmask

Pre-requisite → Recursion
1D DP
2D DP

Bitmask → Bit → {0, 1} + mask → hidden information

101 → 5

Bit Manipulation →

included

exclude

included

→ Set → {1, 2, 3, 4}

↳ 1001 → Bit mask

↳ a subset of the given set

1001 is represents some different crucial information

{1, 4}

Bitwise And Or Xor Not ....

not 0 → 1

0 or 1 → 1

8 7 6 5 4 3 2 1

n = 0 1 0 0 1 0 1 1

K = 4

Set the K^{th} bit

0 1 0 0 0 0 1 1 or 0 0 0 1 0 0 0

1 << (K-1)

0 1 0 0 1 0 1 1

0 0 0 0 0 0 1

3 & 1

Clear the K^{th} bit

4 1

n = 0 1 0 0 1 0 1 1 K = 4

Clear

$\sim(1 << K-1)$

clearing kth bit

$n \,\&\, \sim(1<<K-1)$

→ power of 2 2^3 2^n

$1<<3$ → 1000 → 2^3

→ $1<<n$

Count the no. of set bits

10111 ≈ 23 ⇐ n

10110 ← 22

23 & 22 → 10110 → 22

21 → 10101

1111

19 → 10011

22 & 21 → 10100 → 20

20 & 19 → 10000 → 16

16 & 15 → 0

count = ~~0~~ ~~1~~ ~~2~~ ~~3~~ 4

O(no. of set bit)

(x) , $(x-1)$

The right most bit in x is flipped in $x-1$

& everything to it's right is also flipped in $x-1$

$10110 \leftarrow 22 \leftarrow x$

$10101 \leftarrow 21 \leftarrow x-1$

flipped

$x=6$ $x-1=5$

110 101

16 15

10000 01111

DP with Bitmasking

Q: N workers, and we have N Jobs. Each job can be allocated to one person, & one person can only do one job. We have a $N \times N$ cost matrix → cost when cost[i][j] represents the cost reqd for i^{th} worker to complete j^{th} job.

Allocate the jobs such that cost is minimum.

$$
\begin{array}{c|cccc}
 & J_1 & J_2 & J_3 & J_4 \\
\hline
W_1 & 3, & 2, & 7, & 4 \\
W_2 & 5, & 6, & 1, & 2 \\
W_3 & 5, & 3, & 1, & 5 \\
W_4 & 4, & 6, & 11, & 3
\end{array}
$$

$N \times N$

$N = 20$

Constraint

$O(N!) > 2^n$

J_1	J_2	J_3	J_4

$O(n!)$

w_1	w_2	w_3	w_4
w_1	w_2	w_4	w_3
w_1	w_3	w_2	w_4
⋮		⋮	

all permutation

$$
\begin{array}{c|cccc}
 & J_1 & J_2 & J_3 & J_4 \\
\hline
W_1 & 3, & 2, & 7, & 4 \\
W_2 & 5, & 6, & 1, & 2 \\
W_3 & 5, & 3, & 1, & 5 \\
W_4 & 4, & 6, & 11, & 3
\end{array}
$$

← 0^{th} row

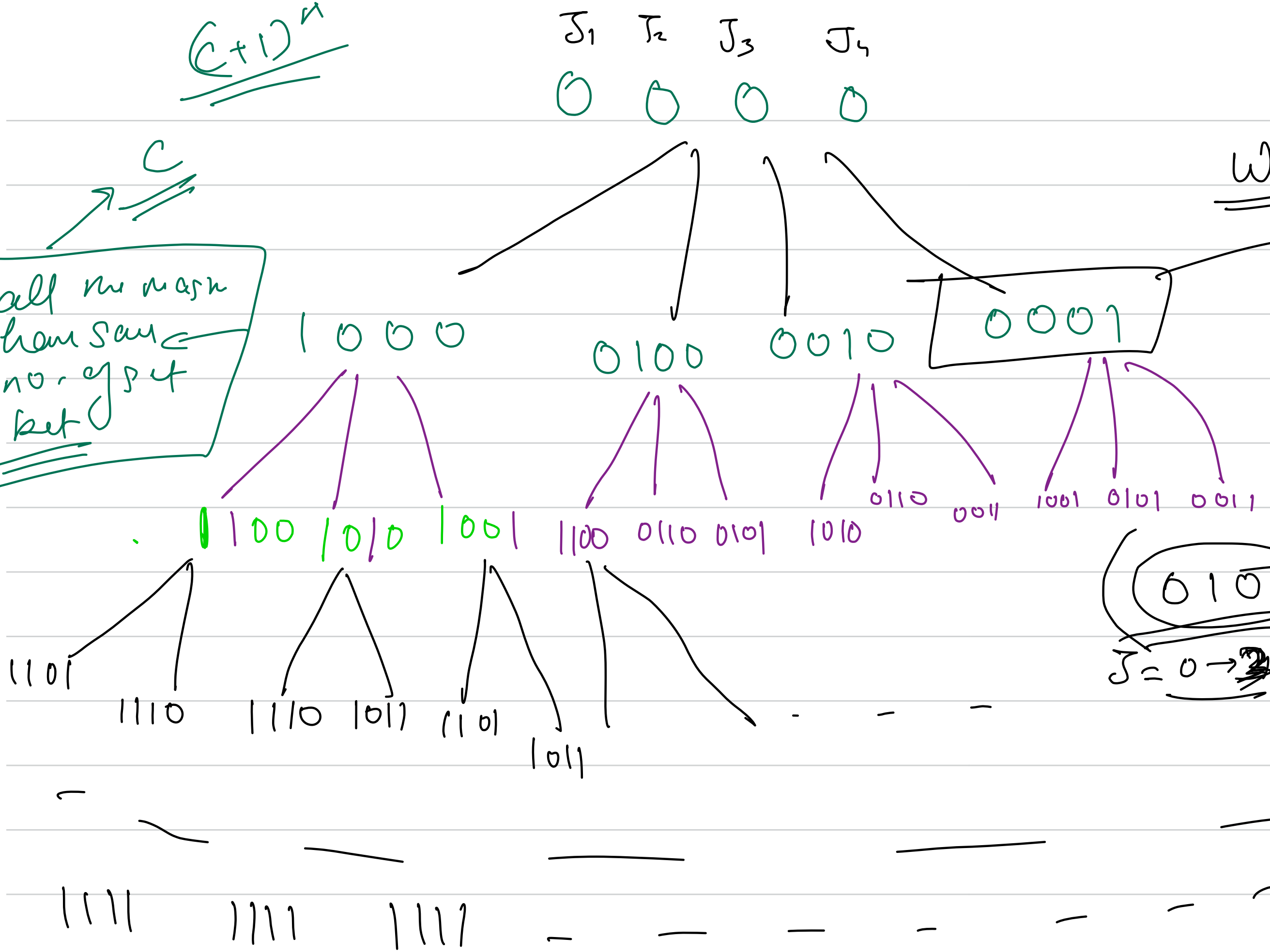
→ What to store in a mask → $\overset{J_1}{1}\ \overset{J_2}{0}\ \overset{J_3}{1}\ \overset{J_4}{0}$

The Job J_1 & J_3 are allocated already, the next worker will get a job either J_2 or J_4

$(C+1)^n$

C

all the mask have same no. of set bit



W_1

this is represent the min cost at this subproblem

W_2

$(0101 >> j) \& 1$

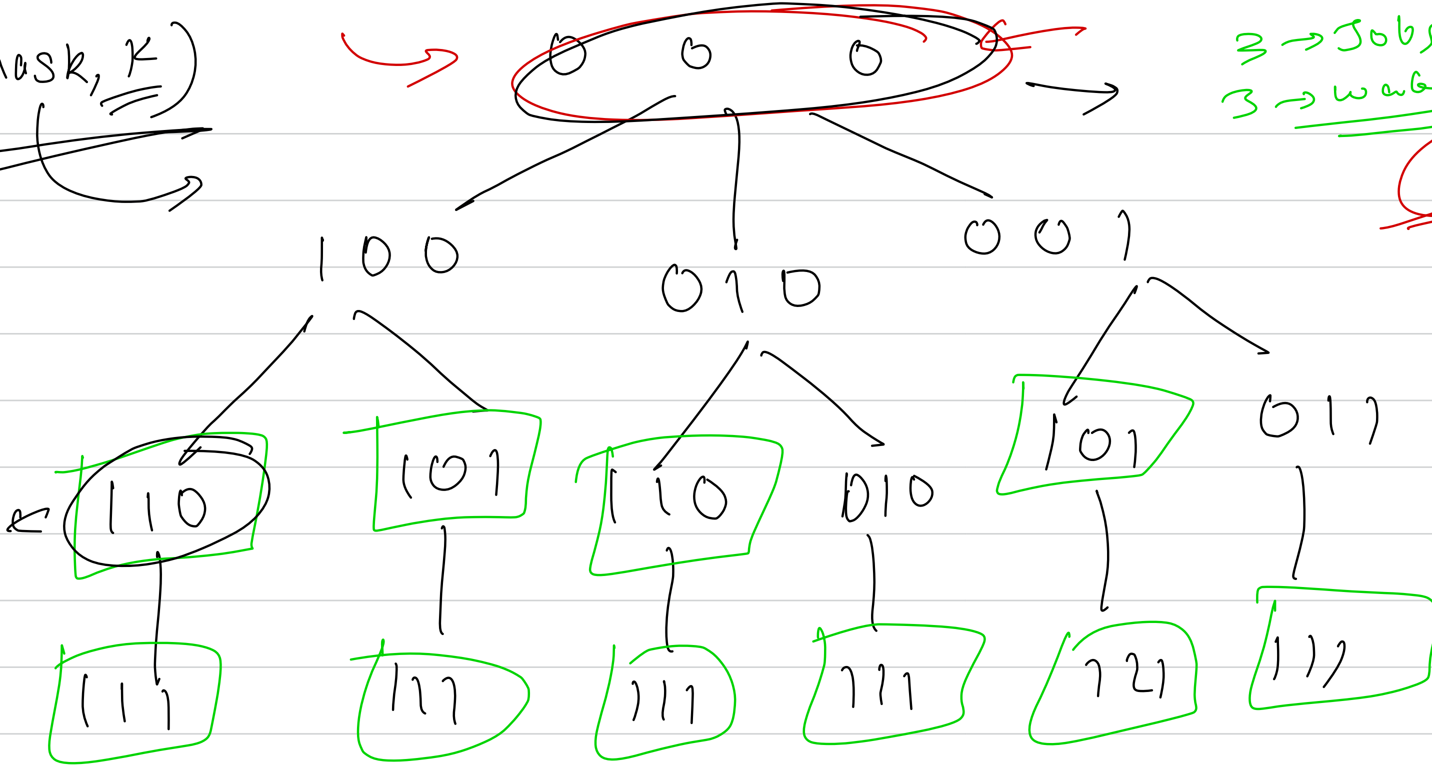
$j = 0 \rightarrow 3$

W_3

(mask, k)

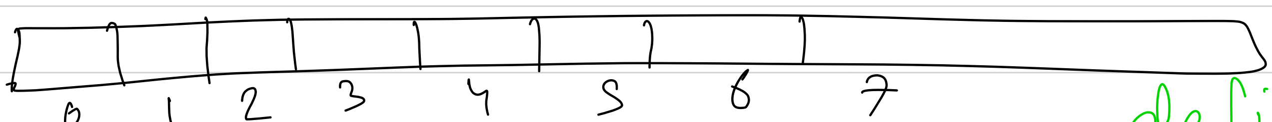
3 → Jobs
3 → work

1st



2^3

dp



101

dp[i]

i=5

unset bit of present unallocated job

for all the unset bits j

x = count_set_bit

$$dp[mask \mid (1<<j)] = \min(dp[mask \mid (1<<j)], cost[x][j] + dp[mask])$$

$$\begin{array}{c|c} & \begin{array}{ccc} J_1 & J_2 & J_3 \end{array} \\ \hline \begin{array}{c} w_0 \\ w_1 \\ w_2 \end{array} & \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \end{array}$$

$J_1 \rightarrow w_0$, $J_2 \rightarrow w_1$ → 2

$J_1 \rightarrow w_0$
$J_3 \rightarrow w_1$

$w_0 \rightarrow$ J_1 2

$w_1 =$
$w_1 \rightarrow J_2$

000

100 010 001

$J_3 \rightarrow w_0$

110 101 101 011

$J_1 \rightarrow w_0$, $J_3 \rightarrow w_1$

$J_3 \rightarrow w_0$, $J_1 \rightarrow w_1$

0	1	2	3	4	5	6	7
0	3	2	∞	1	3	2	∞

$dp[0] = 0$

$dp[mask] + cost[x][y]$

5

2^n

$2^{20} \approx 10^6$

mask → Subproblem uniquely

101 → J_1, J_3 are allocated with min cost, the next worker will get options from the unallocated job

5 4 3 2 1

$(11011) >> (2-1)$

0 1 1 0 1

& 1

---

1 → set

$1 << 2$

5 4 3 2 1

1 1 0 1 1 → set

& 1 0 0

---

0 0 0 0 0

Q →

kids

$$
\begin{array}{c|cccc}
 & \multicolumn{4}{c}{\text{chocolates}} \\
 & C_0 & C_1 & C_2 & C_3 \\
\hline
k_0 & & & & \\
k_1 & & & & \\
k_2 & & & & \\
k_3 & & & &
\end{array}
\quad
\begin{bmatrix}
1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0
\end{bmatrix}
$$

\# ways in which you can distribute chocolates to kids

Binary grid

→ like

$N \times N$

$N \approx 22$

$C \rightarrow K$

like[i][j] = 1 → i^{th} kid likes j^{th} chocolate

0 → i^{th} kid doesn't like j^{th} chocolate

C_1	C_2	C_3	C_4
4	3	2	1

$N!$

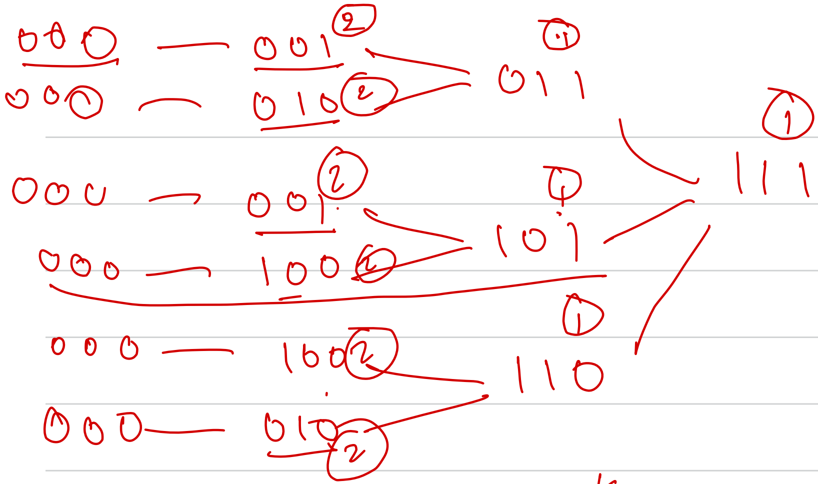
$C_1 \quad C_2$

$1 \quad 1 \quad \boxed{0} \quad \boxed{0}$

$C_1 \quad C_2 \quad C_3 \quad C_4$

$\boxed{1 \quad 1 \quad 1 \quad 1}$ — ~~1way~~

$\boxed{0 \quad 0 \quad 0 \quad 0}$



$C_2 \to K_2$

$3 \to$

$3 \to 6$

$C_0 \to K_0$

000 → 100 → ~~→~~ 101 $C_2 \to K_2$ ~~→~~

ways 5 nel nt

dp[i] → no. of candy assigned

001

4<<0

0 0 1 → count_set_bit

1

K,1

111 → 1

dp[1<<N -1] = 0

111 → 1

011 101 110

10110

& 100 ← 1<<2

00100

if (like[k][i] == true && !(mask & (1<<i))

101

C_0 C_1 C_2 → 1 way

mask $1<<N-2$ → 000

mask → count set bit → k

which kid will be given chocolate now

i kid → 0 — N-1

i

like[k][i] &&

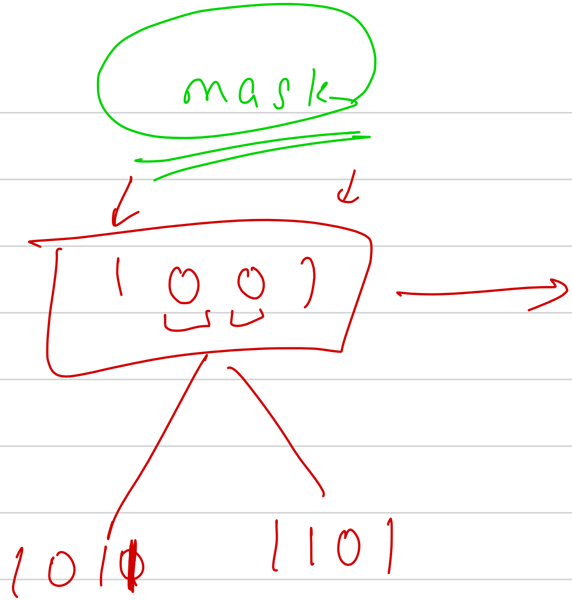
101 <<1

$dp[mask] += dp[mask | (1<<j)]$

OOD $\leftarrow$ 111

x_0 x_1 C_0 C_1 C_2 max2 x_2

110



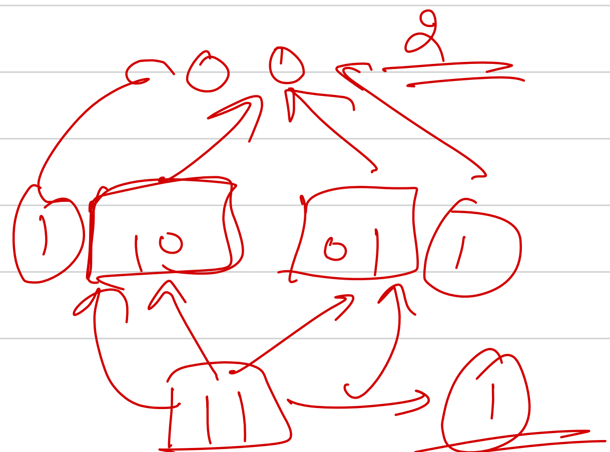
$C_0, C_3 \longleftrightarrow K_0, K_1$

K_2

C_1

K_2

C_2



$\longrightarrow 2$

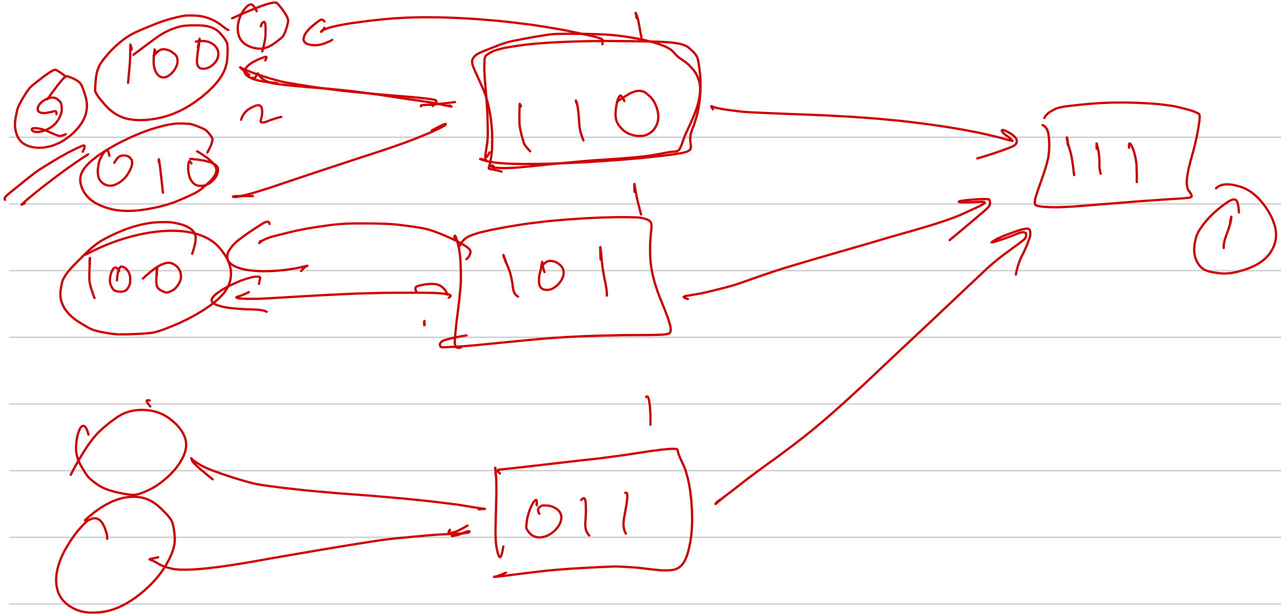
$K_0 \rightarrow C_0$

$K_1 \rightarrow C_1$

$K_0 \rightarrow C_1$

$K_1 \rightarrow C_0$

00
01
10
11


100
1
110
2
111
010
100
101
011
1

C_0 C_1 C_2

$1c_0$ $1c_1$ $1c_2$ → $N!$

5 4 3 2 1

1 0 1 1 0

0 0 0 0 1 0

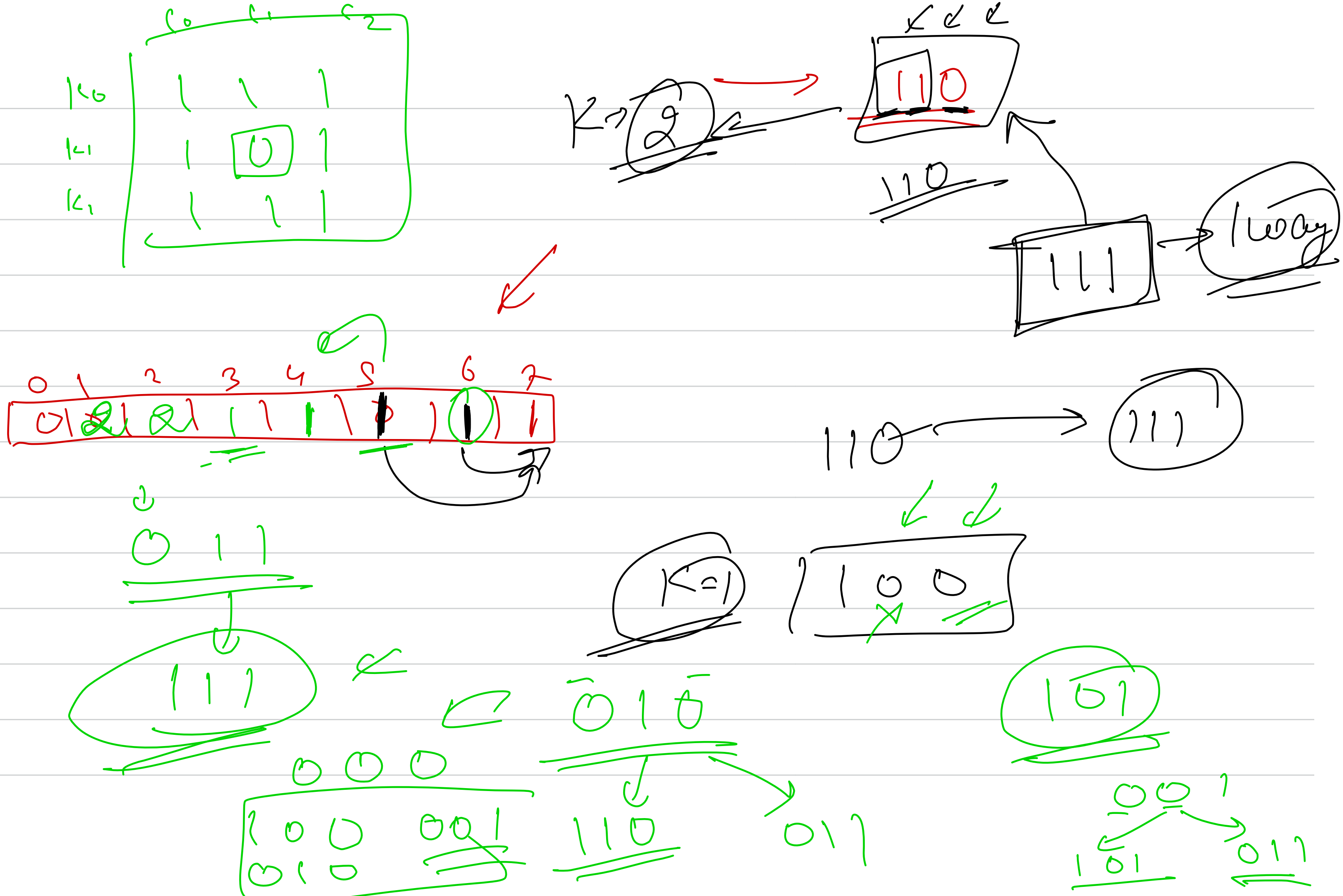
0 0 0 1 0 ← 0x

0

1<<(2-1)

1 0 1 1 0 → 0 1 0 1 1

0 0 0 0 1


C0 C1 C2
K0 1 1 1
K1 1 0 1
K1 1 1 1
K=2
110
110
111
0 1 2 3 4 5 6 7
011
111
100
K=1
010
101
000
110
011
00
101
011