### CSC258 Winter 2016 Lecture 2

#### Lab 1 prep

- You should be registered in a lab section by now; if not, get in now, there is a new section PRAo105 added.
- Lab 1 is about familiarize with everything
  - Create a circuit using Quartus
  - Simulate the circuit using Quartus
  - Burn the circuit into the FPGA board, and test the hardware for real.
- Work in groups of 2 students.

#### Lab tips

 Read the "Summary of TODOs" part of the handout to quickly know what to do.

#### 6 Summary of TODOs

Below is the summary of the steps to be completed for this lab:

- 1. Before the lab, read through the Quartus tutorial and/or install the tools on your own computer.
- 2. Find a partner in the lab to work together.
- 3. Predict the behaviour of the mystery circuit before implementing it.
- 4. Implement the mystery circuit using Quartus.
- 5. Simulate the circuit, explain the waveforms and show them to your TA.
- 6. Load your the circuit to the DE-2 board, make sure it's working as expected, and show it to your TA.

Evaluation (4 marks in total): 2 marks for attending and making an honest effort; 1 mark for showing and explaining the simulation waveforms; 1 mark for having the circuit working on the DE-2 board.

#### Lab tips

- Make sure to finish the work that needs to be done before the lab.
- Be ready to explain your work to the TA in order to get full mark.
- If your partner can answer but you can't, your partner gets more marks than you.
- Teach your partner! That makes you learn better, too!

#### Lab 1 tips

- Read the Quartus Tutorial (posted on course website) patiently.
- Follow the tutorial and you'll mostly be mostly fine.
- The DE2 pin assignment posted on course website is useful for loading to DE2 boards, you'll use in future labs over and over again.
- Sometime Quartus crashes unreasonably, just restart it.

#### More lab tips

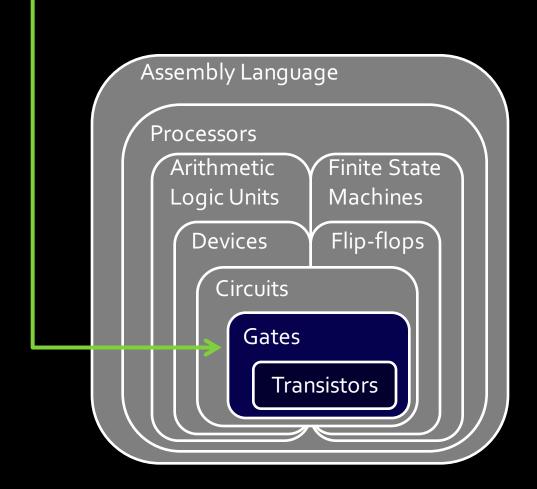
 Back up your work (copy to USB, upload to Dropbox, etc), you may need it for future labs.

- Send feedback using the Weekly Feedback Form after the lab.
  - http://goo.gl/forms/0248ETWgnS
  - Or just drop by my office to chat about how it went, while eating candies.

#### More lab tips

 Consider switching to the new lab section PRA0105, where you can get more help from the TA.

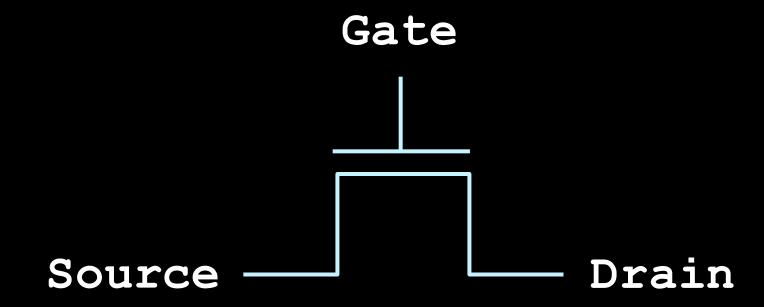
#### We are here



#### Recap: Transistors

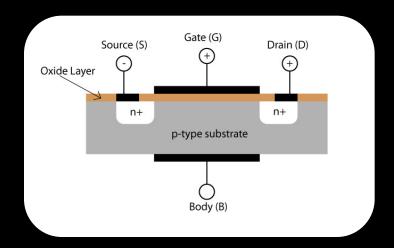
- Transistors, made of doped semiconductors put together (PN-Junctions), is like a resistor but can change its resistance.
- It has two state: connected (switched ON) or disconnected (switched OFF)
  - This is the origin of the 1's and o's that all current computers are built with. (Quantum computer may do it differently)
- The ON/OFF state of a transistor is control by an electrical signal, like in the MOSFET.

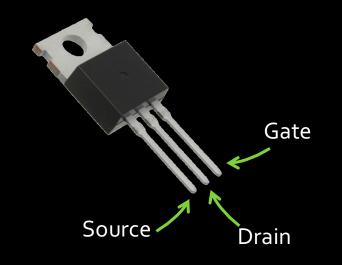
#### **MOSFET**



#### From transistors to gates

- Transistors are semiconductor circuits that can connect the source and the drain together, depending on the voltage value at the gate.
  - For NPN MOSFETs (nMOS), they are connected when the gate value is high.
  - For PNP MOSFETs (pMOS), they are connected when the gate value is low.
- These are then used to make digital logic gates.





#### Transistor notation

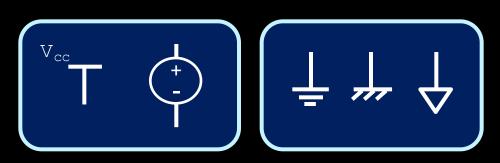
NPN transistor:



PNP transistor:

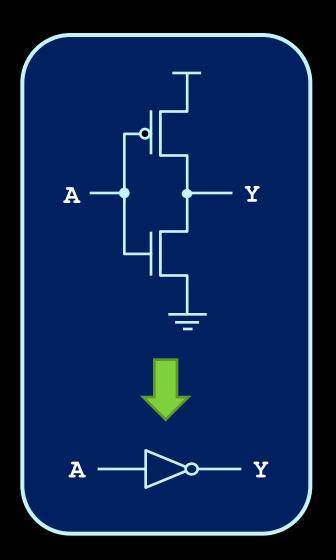


Voltage values:

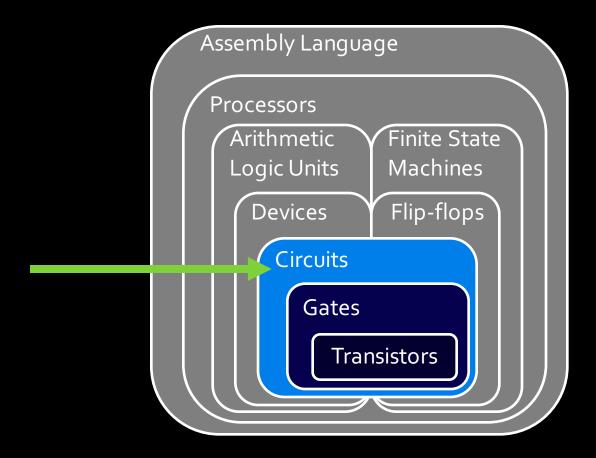


#### How gates are made

- To create logic gates:
  - Remember that transistors act like faucets for electricity.
  - The inputs to the logic gates determine if the outputs will be connected to high or low voltage.
  - <u>Example:</u> NOT gates:



### From gates to circuits



#### Making logic with gates

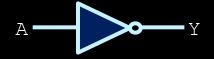
- Logic gates like the following allow us to create an output value, based on one or more input values.
  - Each corresponds to Boolean logic that we've seen before in CSC108 and CSC148:



A	В	Y
0	0	0
0	1	0
1	0	0
1	1	1



A	В	Y
0	0	0
0	1	1
1	0	1
1	1	1



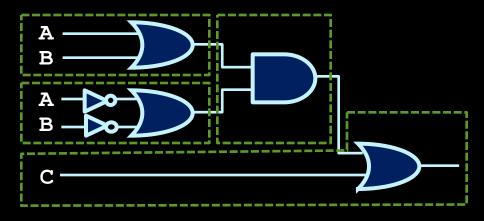
A	Y
0	1
1	0

#### Making boolean expressions

So how would you represent Boolean expressions using logic gates?

```
Y = (A or B) and (not A or not B) or C
```

Like so:



#### Creating complex circuits

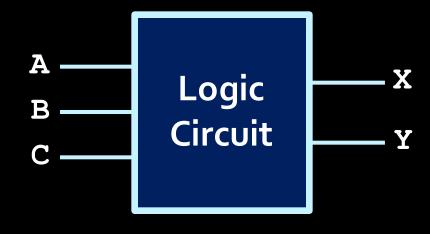
• What do we do in the case of more complex circuits, with several inputs and more than one output?

- If you're lucky, a truth table is provided to express the circuit.
- Usually the behaviour of the circuit is expressed in words, and the first step involves creating a truth table that represents the described behaviour.



#### Circuit example

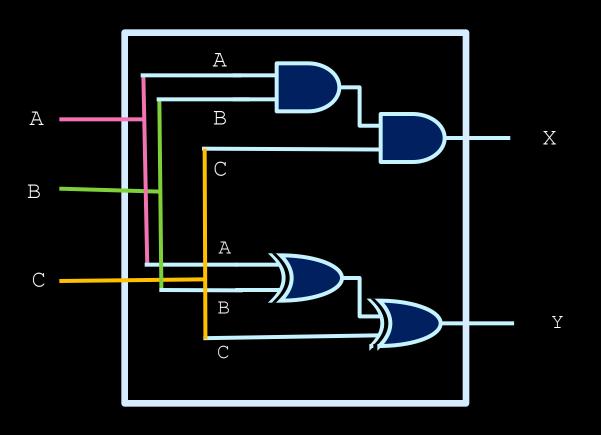
The circuit on the right has three inputs (A, B and C) and two outputs (X and Y).



- What logic is needed to set X high when all three inputs are high?
- What logic is needed to set Y high when the number of high inputs is odd?

#### Combinational circuits

Small problems can be solved easily.



X high when all three inputs are high

Y high when number of high is odd

# For more complicated circuits, we need a systematical approach

#### Creating complex logic

- The general approach
- Basic steps:
  - 1. Create truth tables based on the desired behaviour of the circuit.
  - 2. Come up with a "good" Boolean expression that has exactly that truth table.
  - 3. Convert Boolean expression to gates.
- The key to an efficient design?
  - Spending extra time on Step #2.

First,
a better way to represent
truth tables

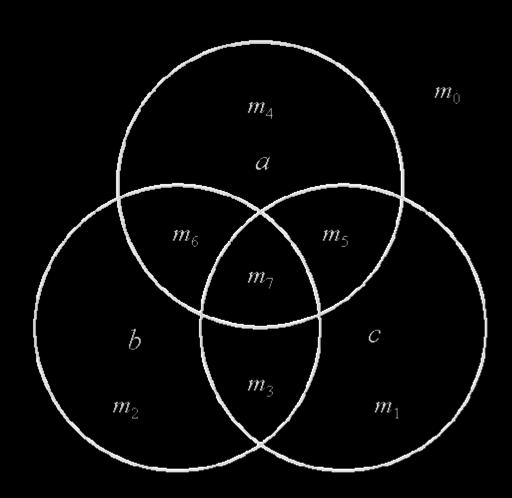
#### Example truth table

- Consider the following example:
  - "Y is high only when B and C are both high"
- This leads to the truth table on the right.
  - Do we always have to draw the whole table?
  - Is there a better way to describe the truth table?

A	В	С	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

This is all we needed to express!

#### Yes, use "Minterms" and "Maxterms"



#### Minterms, informally

- First, sort the rows according to the value of the number "ABC" represents
- Then for each row, we name the inputs as m<sub>{row number}</sub>
- m<sub>o</sub>, m<sub>1</sub>, m<sub>2</sub>, ..., are called minterms

A	В	С	Y	Minterm	Y
0	0	0	0	$\mathbf{m}_{0}$	0
0	0	1	1	$\mathtt{m_1}$	1
0	1	0	1	$\mathbf{m}_2$	1
0	1	1	1	$\mathbf{m}_3$	1
1	0	0	1	$m_4$	1
1	0	1	0	<b>m</b> <sub>5</sub>	0
1	1	0	1	$\mathbf{m}_6$	1
1	1	1	0	$m_7$	0

#### Minterm, a more formal description

Minterm: an AND expression with every input present in true or complemented form.

$$m_3$$
: A ·B ·C

$$m_7$$
: A ·B ·C

A	В	С	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Minterm	Y
$\mathbf{m}_0$	0
$\mathbf{m_1}$	1
$m_2$	1
$\mathbf{m}_3$	1
$m_4$	1
<b>m</b> <sub>5</sub>	0
$m_6$	1
m <sub>7</sub>	0

#### Minterm (m) and Maxterm (M)

Minterm: an AND expression with every input present in true or complemented form.

Maxterm: an OR expression with every input present in true or complemented form.

$$M_o$$
: A+B+C  $M_1$ : A+B+ $\overline{C}$ 

$$M_6$$
: A+B+C  $M_7$ : A+B+C

Feel something fishy?

#### Naming!

$$m_o$$
 is  $\overline{A} \cdot \overline{B} \cdot \overline{C}$ 

Minterm is about when the output is 1

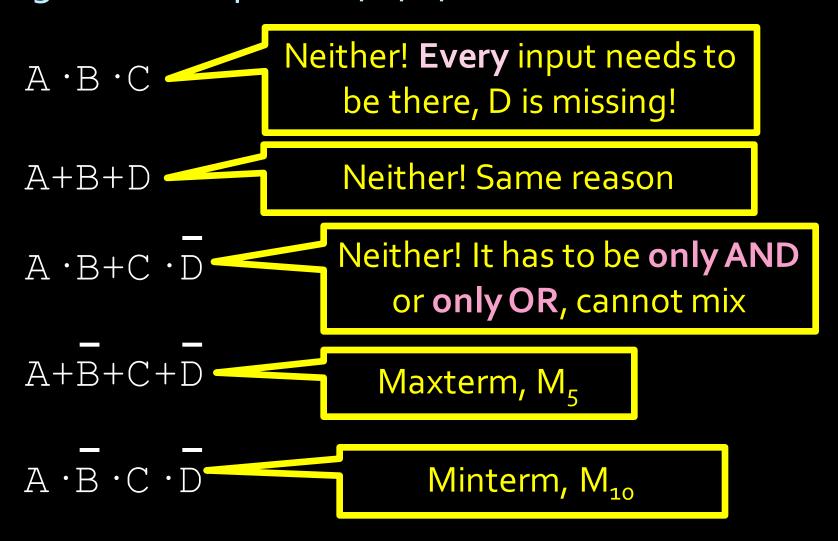
 $A \cdot B \cdot C$  is **1** only when A, B, C are o, o, o

$$M_o$$
 is  $A+B+C$ 

A+B+C is  $\bigcirc$  only when A, B, C are  $\bigcirc$ ,  $\bigcirc$ ,  $\bigcirc$ 

Maxterm is about when the output is o

## Exercise: Minterm or Maxterm? given four inputs: (A, B, C, D)



#### Quick fact

- Given n inputs, how many possible minterms and maxterms are there?
  - 2<sup>n</sup> minterms and 2<sup>n</sup> maxterms
     possible (same as the number of rows in a truth table).

#### Quick note about notations

- AND operations are denoted in these expressions by the multiplication symbol.
  - e.g.  $A \cdot B \cdot C$  or  $A*B*C \approx A \wedge B \wedge C$
- OR operations are denoted by the addition symbol.
  - e.g. A+B+C ≈ A∨B∨C
- NOT is denoted by multiple symbols.
  - e.g.  $\neg A$  or A' or  $\overline{A}$
- XOR occurs rarely in circuit expressions.
  - e.g. A ⊕ B

## Use minterms and maxterms to go from truth table to logic expression

#### Using minterms

- What are minterms used for?
  - A single minterm indicates a set of inputs that will make the output go high.
  - Example: Describe the truth table on the right using minterm:

M<sub>2</sub> A'B'CD'

A	В	С	D	$m_2$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

#### Using minterms

- What happens when you OR two minterms?
  - Result is output that goes high in both minterm cases.
  - Describe the truth table with the right-most column of outputs

m	_	m	
			0

A	В	С	D	$\mathbf{m}_2$	m <sub>8</sub>	m <sub>2</sub> +m <sub>8</sub>
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	1	0	1
0	0	1	1	0	0	0
0	1	0	0	0	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	1	1	0	0	0
1	0	0	0	0	1	1
1	0	0	1	0	0	0
1	0	1	0	0	0	0
1	0	1	1	0	0	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	0	0	0

We came up with a logic expression that has the desired truth table, easily: A'B'CD' + AB'C'D'

#### Creating boolean expressions

- Two canonical forms of boolean expressions:
  - Sum-of-Minterms (SOM): AB + A'B + AB'
    - Each minterm corresponds to a single high output in the truth table.
    - Also known as: Sum-of-Products.
  - Product-of-Maxterms (POM): (A+B)(A'+B)(A+B')
    - Each maxterm corresponds to a single low output in the truth table.
    - Also known as Product-of-Sums.

Every logic expression can be converted to a SOM, also to a POM.

#### $\overline{Y} = m_2 + m_6 + m_7 + m_{10}$ (SOM)

A	В	С	D	$m_2$	m <sub>6</sub>	m <sub>7</sub>	m <sub>10</sub>	Y
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	1	0	0	0	1
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	1	1	0	0	1	0	0	1
0	1	1	1	0	0	1	0	1
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0

## $Y = M_3 \cdot M_5 \cdot M_7 \cdot M_{10} \cdot M_{14}$ (POM)

A	В	С	D	<b>M</b> <sub>3</sub>	<b>M</b> <sub>5</sub>	<b>M</b> <sub>7</sub>	<b>M</b> <sub>10</sub>	M <sub>14</sub>	Y
0	0	0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1
0	0	1	0	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	0
0	1	0	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	0
0	1	1	0	1	1	1	1	1	1
0	1	1	1	1	1	0	1	1	0
1	0	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	1
1	0	1	0	1	1	1	0	1	0
1	0	1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	0	0
1	1	1	1	1	1	1	1	1	1

#### Sum-of-Minterms vs Product-of-Maxterm

- SOM expresses which inputs cause the output to go high.
- POM expresses which inputs cause the output to go low
- SOMs are useful in cases with very few input combinations that produce high output.
- POMs are useful when expressing truth tables that have very few low output cases...

#### What if we do this using POM?

$$Y = m_2 + m_6 + m_7 + m_{10}$$
 (SOM)

A	В	С	D	$m_2$	m <sub>6</sub>	m <sub>7</sub>	<b>m</b> <sub>10</sub>	Y
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	1	0	0	0	1
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	1	1	0	0	1	0	0	1
0	1	1	1	0	0	1	0	1
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0

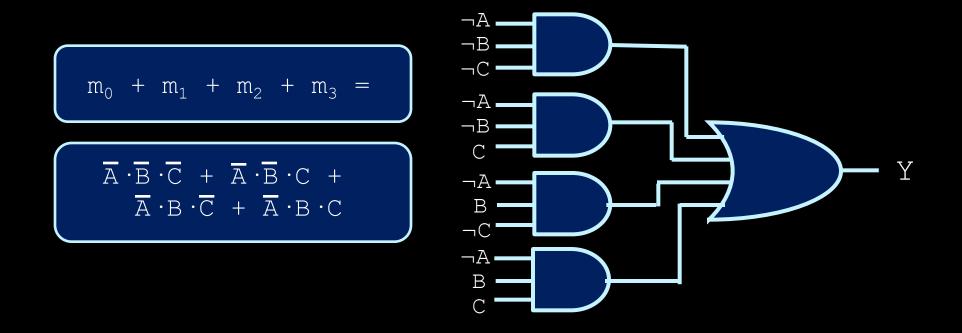
#### What if we do this using SOM?

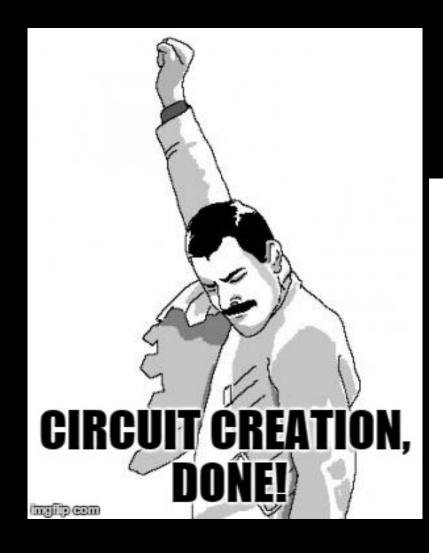
$$Y = M_3 \cdot M_5 \cdot M_7 \cdot M_{10} \cdot M_{14}$$
 (POM)

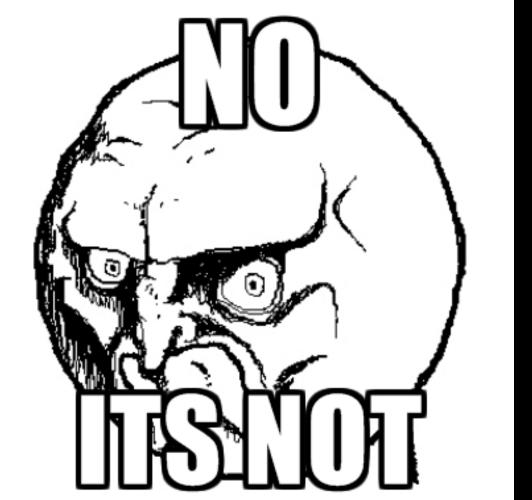
A	В	С	D	<b>M</b> <sub>3</sub>	<b>M</b> <sub>5</sub>	<b>M</b> <sub>7</sub>	<b>M</b> <sub>10</sub>	M <sub>14</sub>	Y
0	0	0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1
0	0	1	0	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	0
0	1	0	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	0
0	1	1	0	1	1	1	1	1	1
0	1	1	1	1	1	0	1	1	0
1	0	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	1
1	0	1	0	1	1	1	0	1	0
1	0	1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	0	0
1	1	1	1	1	1	1	1	1	1

### Converting SOM to gates

 Once you have a Sum-of-Minterms expression, it is easy to convert this to the equivalent combination of gates:



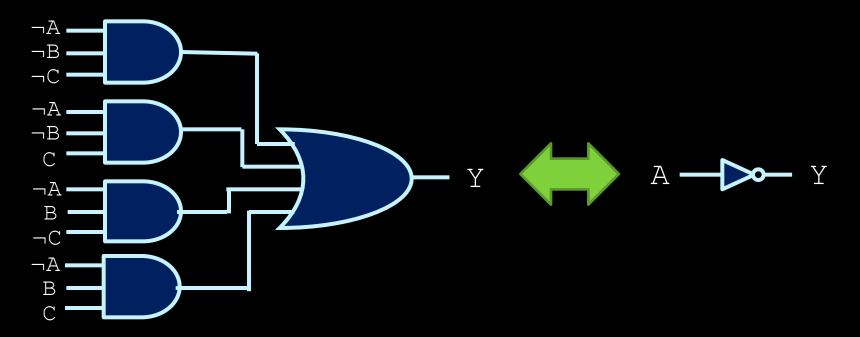




# Reducing circuits



## Reasons for reducing circuits



- To minimize the number of gates, we want to reduce the Boolean expression as much as possible from a collection of minterms to something smaller.
- This is where math skills come in handy ©

#### Boolean algebra review

Axioms:

$$0 \cdot 0 = 0$$
  $0 \cdot 1 = 1 \cdot 0 = 0$   
 $1 \cdot 1 = 1$  if  $x = 1$ ,  $\overline{x} = 0$ 

From this, we can extrapolate:

$$x \cdot 0 = 0$$
  $x+1 = 1$   
 $x \cdot 1 = x$   $x+0 = x$   
 $x \cdot x = x$   $x+x = x$   
 $x \cdot \overline{x} = 0$   $x+\overline{x} = 1$   
 $\overline{x} = x$ 

#### Other boolean identities

Commutative Law:

$$x \cdot y = y \cdot x$$
  $x+y = y+x$ 

Associative Law:

$$x \cdot (\lambda + z) = (x \cdot \lambda) \cdot z$$
  
 $x \cdot (\lambda \cdot z) = (x \cdot \lambda) \cdot z$ 

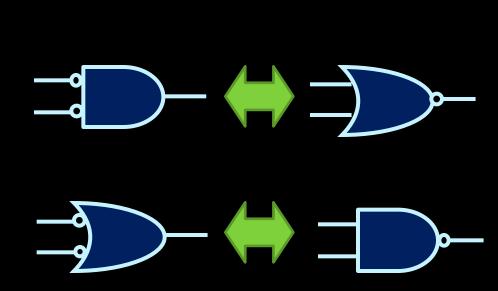
Distributive Law:

$$x \cdot (\lambda + z) = (x + \lambda) \cdot (x + z)$$
  
 $x \cdot (\lambda + z) = x \cdot \lambda + x \cdot z$ 

#### Other boolean identities

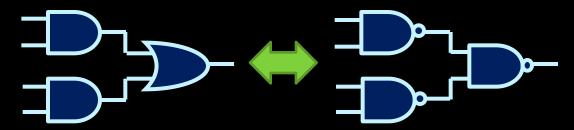
De Morgan's Laws:

$$\frac{X}{X} + \frac{\lambda}{\lambda} = \frac{X \cdot \lambda}{X + \lambda}$$

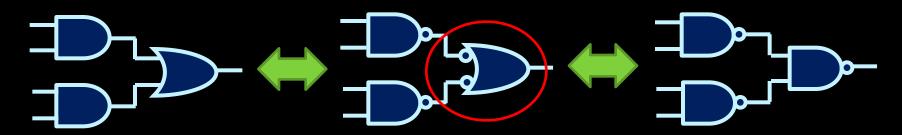


## De Morgan and NAND gates

- De Morgan's Law is important because out of all the gates, NANDs are the cheapest to fabricate.
  - a Sum-of-Products circuit could be converted into an equivalent circuit of NAND gates:



This is all based on de Morgan's Law:



#### Reducing boolean expressions

A	В	С	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

 Assuming logic specs at left, we get the following:

$$m_3 + m_4 + m_6 + m_7$$

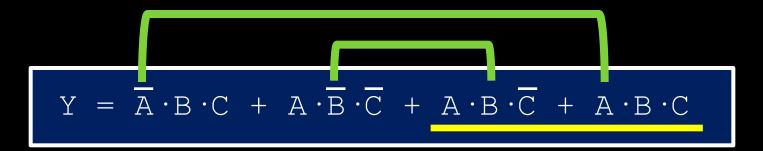
$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

## Warming up...

$$A \cdot B + A \cdot \overline{B} = A$$

Reduce by combing two terms that differ by a single literal.

#### Let's reduce this



Combine the last two terms...

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B$$

Combine the middle two and the end two ...

$$Y = B \cdot C + A \cdot \overline{C}$$

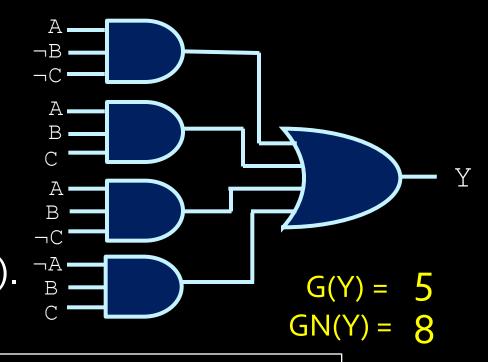
There could be different ways of combining, some are **simpler** than others.

How to get to the simplest expression?

Wait ... What does "simplest" mean?

### What is "simplest"?

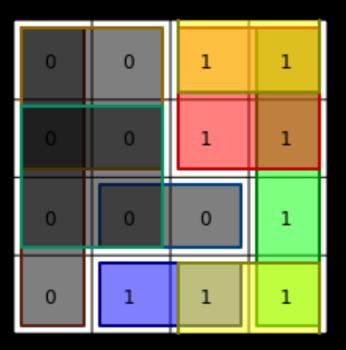
- In this case, "simple" denotes the lowest gate cost (G) or the lowest gate cost with NOTs (GN).
- To calculate the gate cost, simply add all the gates together (as well as the cost of the NOT gates, in the case of the GN cost).



Don't count ¬C twice!

### Karnaugh maps

Find the simplest expression, systematically.



#### Reducing boolean expressions

- How do we find the "simplest" expression for a circuit?
  - Technique called Karnaugh maps (or K-maps).
  - Karnaugh maps are a 2D grid of minterms, where adjacent minterm locations in the grid differ by a single literal.
  - Values of the grid are the output for that minterm.

	B·C	B·C	B·C	B⋅C
Ā	0	0	1	0
A	1	0	1	1

## Compare these...

A	В	С	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

	B·C	B·C	B·C	B⋅C
Ā	0	0	1	0
A	1	0	1	1

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

#### Karnaugh maps

- Karnaugh maps can be of any size, and have any number of inputs.
- Since adjacent minterms only differ by a single literal, they can be combined into a single term that omits that value.

	C·D	C·D	C ·D	C . D
A ·B	$m_{o}$	$m_1$	$m_3$	$m_2$
A·B	$m_4$	$m_5$	m <sub>7</sub>	$m_6$
A·B	m <sub>12</sub>	m <sub>13</sub>	m <sub>15</sub>	m <sub>14</sub>
A·B	m <sub>8</sub>	m <sub>9</sub>	m <sub>11</sub>	$m_{10}$

#### Using Karnaugh maps

- Once Karnaugh maps are created, draw boxes over groups of high output values.
  - Boxes must be rectangular, and aligned with map.
  - Number of values contained within each box must be a power of 2.
  - Boxes may overlap with each other.
  - Boxes may wrap across edges of map.

	B·C	B ⋅C	B·C	B ⋅C
Ā	0	0	1	0
A	1	0	1	1

	B·C	B·C	в∙с	В·С
Ā	0	1	1	0
A	0	0	1	0



Must be rectangle!

	B·C	B ⋅C	в.с	B⋅C
Ā	0	1	1	0
A	0	0	1	0



Two boxes overlapping each other is fine.

	B·C	<del>B</del> ⋅C	в∙с	B⋅C
Ā	0	1	1	1
A	0	0	0	0



Number of value contained must be power of 2.

	$\overline{B} \cdot \overline{C}$	B·C	В∙С	B⋅C
Ā	0	1	1	1
A	0	0	0	0



1 is a power of 2

1 = 2°

	B·C	<del>B</del> ⋅C	В∙С	B ⋅C̄
Ā	0	1	1	0
A	0	1	1	0



# Rectangle, with power of 2 entries

	B·C	B ⋅C	В∙С	B⋅C
Ā	0	1	0	0
A	0	0	1	0



Must be aligned with map.

	B·C	B ⋅C	в∙с	B ⋅C
Ā	0	0	0	0
A	1	0	0	1



Wrapping across edge is fine.

## So... how to find smallest expression

Minterms in one box can be combined into one term

	B·C	B·C	в·С	B⋅C
Ā	0	0	1	0
A	0	0	1	0

$$\overline{A} \cdot B \cdot C + A \cdot B \cdot C = B \cdot C$$

## So... how to find smallest expression

The simplest expression corresponds to the smallest number of boxes that cover all the high values (1's).

	B·C	<del>B</del> ⋅C	в∙с	B⋅C
Ā	0	0	1	0
A	1	0	1	1

	B·C	B·C	B·C	B⋅C
Ā	0	0	1	0
A	1	0	1	1



$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

	B·C	B·C	B·C	B⋅C
Ā	0	0	1	0
A	1	0	1	1

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B$$

	B·C	B·C	B·C	B⋅C
Ā	0	0	1	0
A	1	0	1	1

$$Y = B \cdot C + A \cdot \overline{C}$$

#### K-map: the steps

Given a complicated expression

- 1. Convert it to Sum-Of-Minterms
- 2. Draw the 2D grid
- 3. Mark all the high values (1's), according to which minterms are in the SOM.
- 4. Draw boxes that cover 1's.
- 5. Find the smallest set of boxes that cover all 1's.
- 6. Write out the simpfied result according to the boxes found.

#### Everything can be done using Maxterms, too

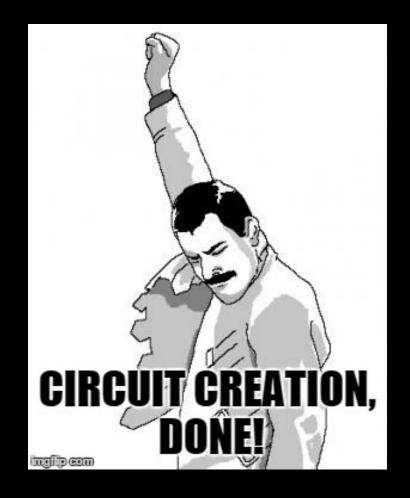
- Can also use this technique to group maxterms together as well.
- Karnaugh maps with maxterms involves grouping

	C+D	C+D	C+D	<del>C</del> +D
A+B	${ m M}_{\odot}$	$M_1$	$M_3$	$M_2$
A+B	$M_4$	$M_5$	$M_7$	$M_6$
Ā+B	M <sub>12</sub>	M <sub>13</sub>	M <sub>15</sub>	M <sub>14</sub>
Ā+B	$M_8$	$M_9$	$M_{11}$	$M_{10}$

the zero entries together, instead of grouping the entries with one values.

#### Circuit creation – the whole flow

- 1. Understand desired behaviour
- 2. Write the truth table based on the behaviour
- 3. Write the SOM (or POM) of that truth table
- 4. Simplify the SOM using K-map
- 5. Translate the simplified logic expression into circuit with gates.



#### Today we learned

- How to create a logic circuit from scratch, given a desired digital behaviour.
- Minterm & Maxterm
- K-Map use to reduce the circuit

#### Next Week:

Logical Devices

# Quiz Time!

#### Question 1

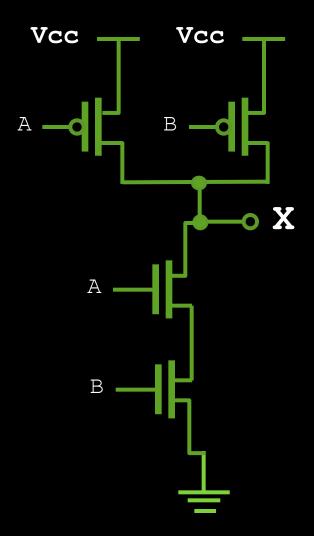
After doping a piece of silicon with n-type impurity (phosphorus), there overall electric charge of the material is

- A. Positive
- B. Negative
- C. Neutral
- D. None of above

## Question 2

- What gate is this?
- A and B are inputs, X is output.

#### **NAND**



#### Question 3

Write the following truth table as Sum-of-Minterms.

A	В	С	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Write in this form
A'B'C + A'BC'

Flip your quiz face-down and pass it to your right.